Discussion: Model Misspecification

Larry Wasserman

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Confidence interval for θ

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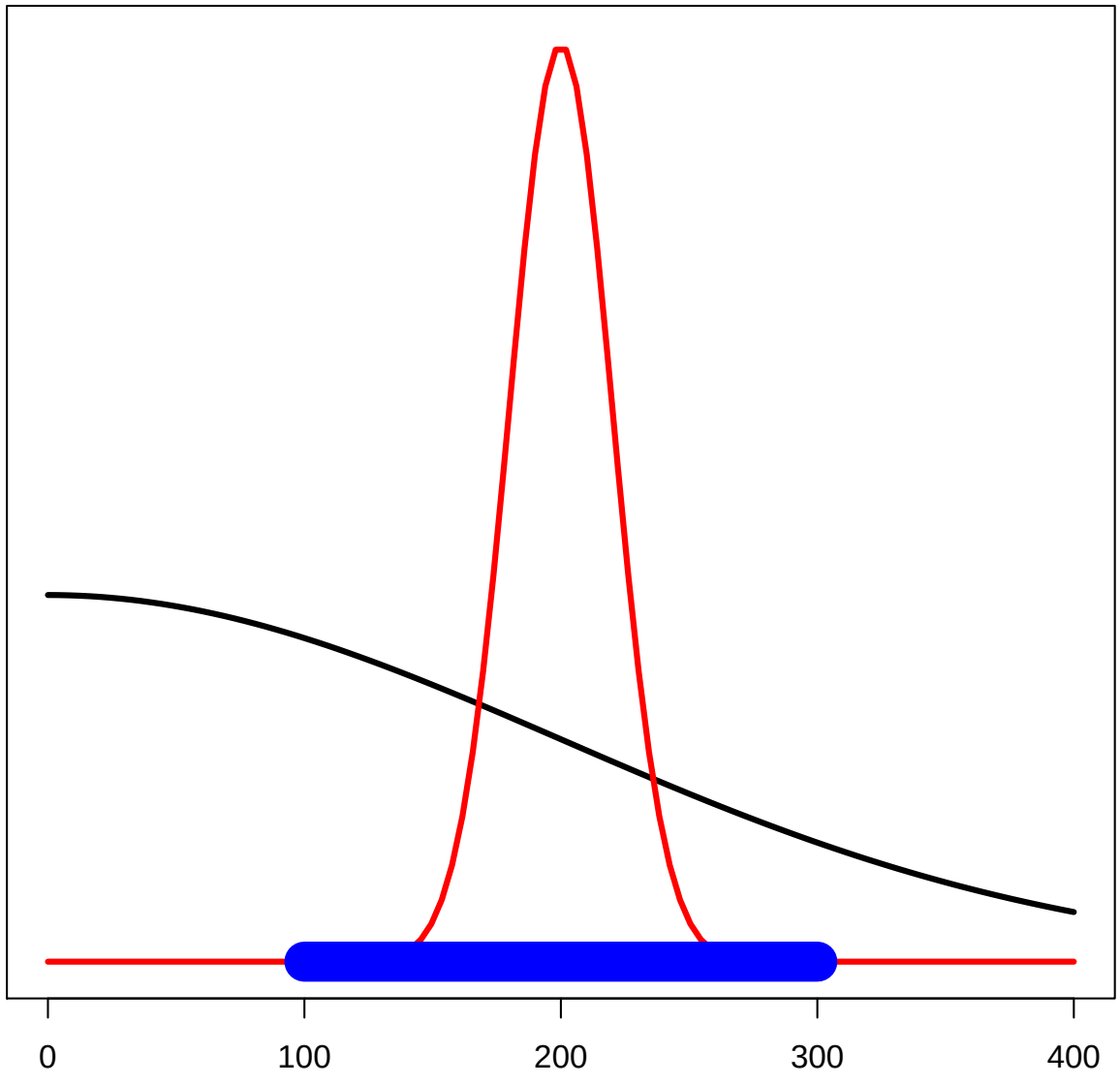
How to include this as part of the uncertainty? Very difficult.

Example



Estimating the Background

$$p_0(x;\gamma) = \sum_{j=1}^{k} \gamma_j \phi_j(x)$$

or

$$p_0(x;\gamma) = \frac{e^{\sum_{j=1}^{k} \gamma_j \phi_j(x)}}{\int e^{\sum_{j=1}^{k} \gamma_j \phi_j(x)}}$$

where $\phi_1, \phi_2, \ldots$ are basis functions.

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The mixture model is not identified if k is unrestricted.

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or use optimal transport (morphing). See Tudor Manole's talk on Wednesday for more on this.

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One solution: use power divergence (Basu, Harris, Hjort and Jones 1998):

Power Divergence

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$$S(\gamma) = \int p_0(x;\gamma)^{1+\alpha} - \left(1 + \frac{1}{\alpha}\right) \frac{1}{n} \sum_i p_0(x;\gamma)^{\alpha}$$

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$\alpha \uparrow$: robustness $\uparrow$ and efficiency $\downarrow$

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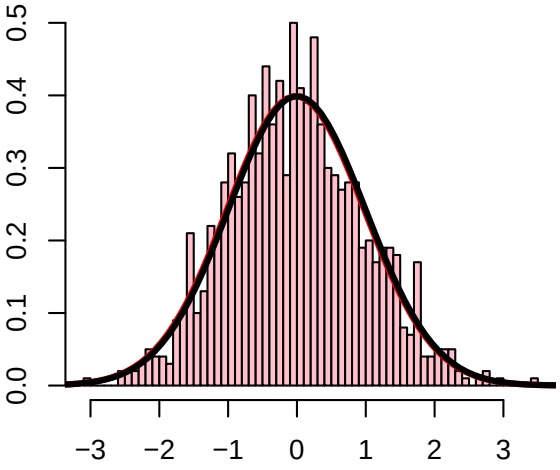
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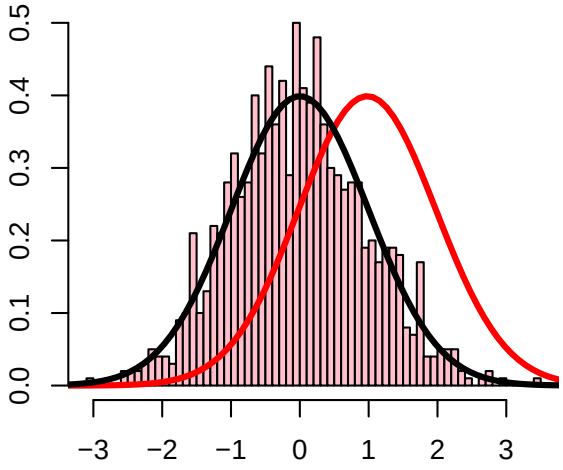
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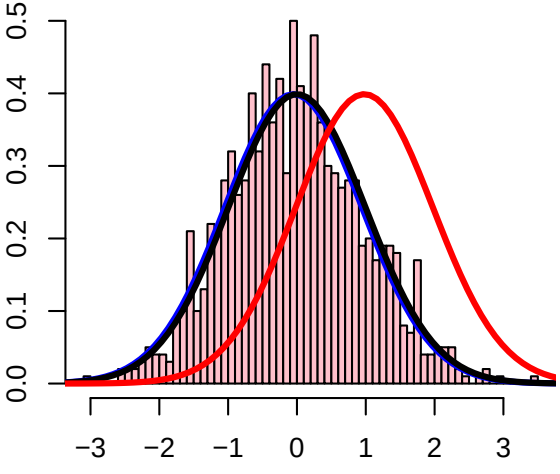
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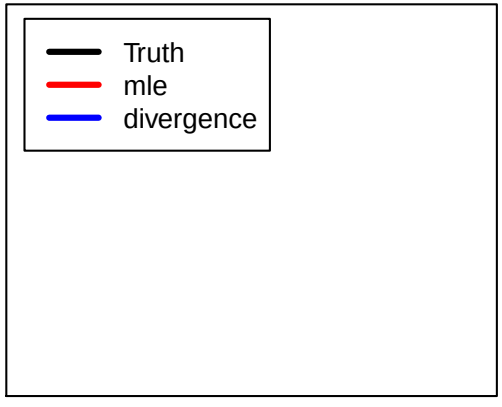
Truth
mle
divergence

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- THE END