Systematic Uncertainties in Direct Dark Matter Searches

by Knut Dundas Morå

A (possible) response

PHYSTAT – Systematics | November 1, 2021

Alessandra R. Brazzale

alessandra.brazzale@unipd.it

Department of Statistical Sciences | University of Padova

First of all...

Baxter et al. (2021)

Eur. Phys. J. C (2021) 81:907
https://doi.org/10.1140/epjc/s10052-021-09655-y

THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Special Article - Tools for Experiment and Theory

Recommended conventions for reporting results from direct dark matter searches

D. Baxter[1], I. M. Bloch[2], E. Bodnia[3], X. Chen[4,5], J. Conrad[6], P. Di Gangi[7], J. E. Y. Dobson[8], D. Durnford[9], S. J. Haselschwardt[10], A. Kaboth[11,12], R. F. Lang[13], Q. Lin[14], W. H. Lippincott[3,a], J. Liu[4,5,15], A. Manalaysay[10], C. McCabe[16], K. D. Morå[17], D. Naim[18], R. Neilson[19], I. Olcina[10,20], M. -C. Piro[9], M. Selvi[7], B. von Krosigk[21], S. Westerdale[22], Y. Yang[4], N. Zhou[4]

Profile likelihood ratio analysis

2.1 Discovery (claims and look elsewhere effect)

2.2 Limit settings (power)

2.3 Asymptotic approximation

2.4 Contours

2.5 Modeling background and detector response

2.6 Experimenter bias mitigation

Profile likelihood ratio analysis

$$n \longrightarrow +\infty$$

LARGE-SAMPLE THEORY

(asymptotic approximation)

Algeri et al. (2020)

EXPERT
RECOMMENDATION

Check for updates

Searching for new phenomena with profile likelihood ratio tests

Sara Algeri[1], Jelle Aalbers[2,3], Knut Dundas Morå[2,3,4] and Jan Conrad[4]

Abstract | Likelihood ratio tests are standard statistical tools used in particle physics to perform tests of hypotheses. The null distribution of the likelihood ratio test statistic is often assumed to be χ^2, following Wilks' theorem. However, in many circumstances relevant to modern experiments this theorem is not applicable. In this Expert Recommendation, we overview practical ways to identify these situations and provide guidelines on how to construct valid inference. We use examples from particle physics, but the statistical constructs discussed here can be used in any scientific discipline that relies on data analysis.

“Regularity” conditions (Algeri et al., 2020)

Box 2 | **Necessary conditions for Wilks' theorem**

Asymptotic
Sufficient data are collected.

Interior
Only values of the parameters of interest μ and nuisance parameters θ that are not on the boundaries of their parameter space are admitted.

Identifiable
Different values of the parameters specify distinct models.

Nested
The null hypothesis H_0 is a limiting case of the general case hypothesis H_1, for example, with some parameter constrained to a subrange of the entire parameter space.

Correct
The true model is specified either under H_0 or under H_1.

Recommendations (Algeri et al., 2020)

Table 1 | **Statistical techniques and assumptions necessary for their applicability**

Technique	Asymptotic	Interior	Identifiable	Nested	Correct
Wilks' theorem	•	•	•	•	•
Higher-order asymptotics	–	•	•	•	•
Boundary corrections	•	–	•	•	•
Look-elsewhere corrections	•	○	–	•	•
Test for non-nested models	•	•	•	–	•
Bootstrap/Monte Carlo	–	○	–	–	•
Nuisance parameters	○	○	○	○	○
Non-parametric methods	–	–	–	–	–

The closed circles represent strict assumptions and open circles represent assumptions that can be relaxed with simple extensions. A dash indicates that the technique does not rely on that assumption.

Nuisance parameters (Algeri et al., 2020, p.251)

- If the true values of the parameters are not in the interior of their parameter space, one can implement boundary corrections for the *P* values. Conversely, when aiming to address the problem by means of simulations, there are situations where classical estimators of the nuisance parameters should be replaced by more efficient estimators to guarantee the consistency of the solution.
- If the nuisance parameters are not identifiable and a fully simulated solution is too computationally expensive, corrections for the look-elsewhere effect allow one to perform inference while drastically reducing the number of simulations required.
- If the models under comparison are non-nested, a simple solution is to specify a model that includes the models under study as special cases.
- If the likelihood specified is not correct, one may attempt to recover the structure of the true underlying model by adding nuisance parameters and applying any of the above-mentioned inferential methods. As for any other modelling strategy, however, the bias–variance trade-off must be taken into account.

Finally, if none of the assumptions above hold, or the correct models cannot be recovered by simply adding nuisance parameters, one may refer to non-parametric methods or other specialized procedures to correct mis-modelling.

LARGE-SAMPLE THEORY

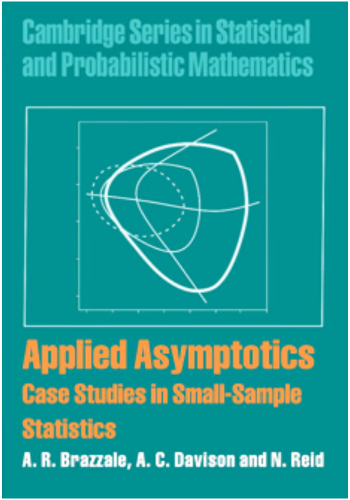
(asymptotic approximation)

$$\Downarrow$$

SMALL-SAMPLE THEORY

(higher order asymptotics)

Want to know more?



Cowan (2019)

Eur. Phys. J. C (2019) 79:133
https://doi.org/10.1140/epjc/s10052-019-6644-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Experimental Physics

Statistical models with uncertain error parameters

Glen Cowan[a]

Physics Department, Royal Holloway, University of London, Egham TW20 0EX, UK

Received: 31 October 2018 / Accepted: 3 February 2019 / Published online: 12 February 2019
© The Author(s) 2019

Error on errors (Cowan, 2019)

5 Single-measurement model

To investigate the asymptotic properties of the profile likelihood ratio it is useful to examine a simple model with a single measured value y following a Gaussian with mean μ and standard deviation σ. The parameter of interest is μ and we treat the variance σ^2 as a nuisance parameter, which is constrained by an independent gamma-distributed estimate v. Thus the likelihood is given by

$$L(\mu, \sigma^2) = f(y, v|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\mu)^2/2\sigma^2} \frac{\beta^\alpha}{\Gamma(\alpha)} v^{\alpha-1} e^{-\beta v}. \quad (29)$$

As before we set the parameters α and β of the gamma distribution so that $E[v] = \sigma^2$ and so that from Eq. (9) the standard deviation of v is $\sigma_v = 2r\sigma^2$, where r characterizes the relative error on the error. This gives

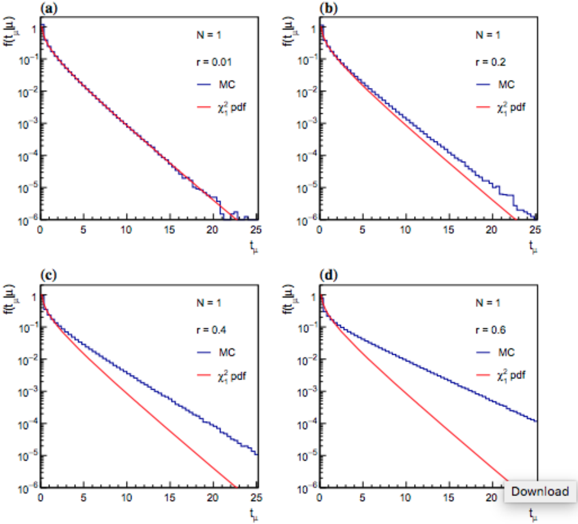
$$\alpha = \frac{1}{4r^2}, \quad (30)$$

$$\beta = \frac{1}{4r^2\sigma^2}. \quad (31)$$

The goal is to construct a confidence interval for μ by using the profile likelihood ratio

Likelihood ratio (Cowan, 2019)

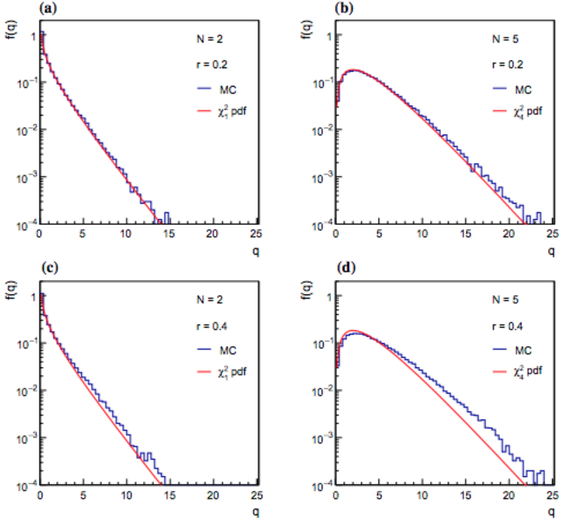
Fig. 2 Distributions of the test variable t_μ for a single Gaussian distributed measurement with relative error-on-error r



Bartlett correction (Cowan, 2019)

Eur. Phys. J. C (2019) 79:133

Fig. 5 Distributions of the test variable q for averages of $N = 2$ and 5 values using $r = 0.2$ and $r = 0.4$



Our proposal (joint work with Glen Cowan)

Modified likelihood root

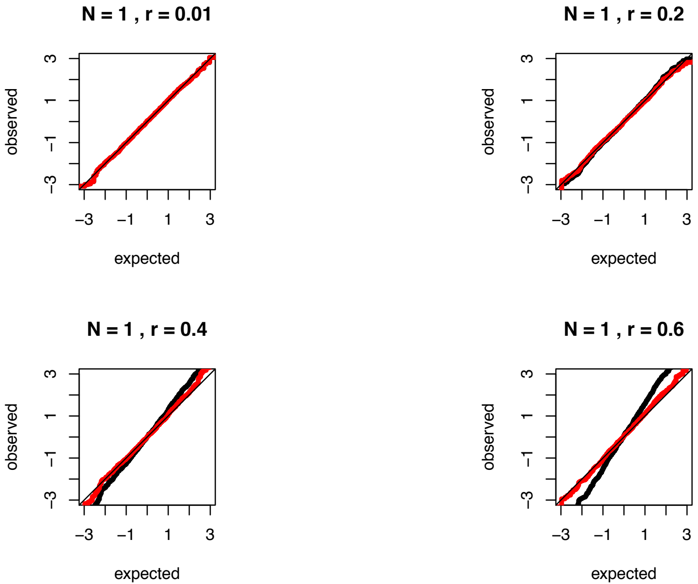
$$t^*(\mu) = t(\mu) + \frac{1}{t(\mu)} \log\left\{ \frac{q(\mu)}{t(\mu)} \right\}$$

$$\sim N(0,1) + O(n^{-3/2})$$

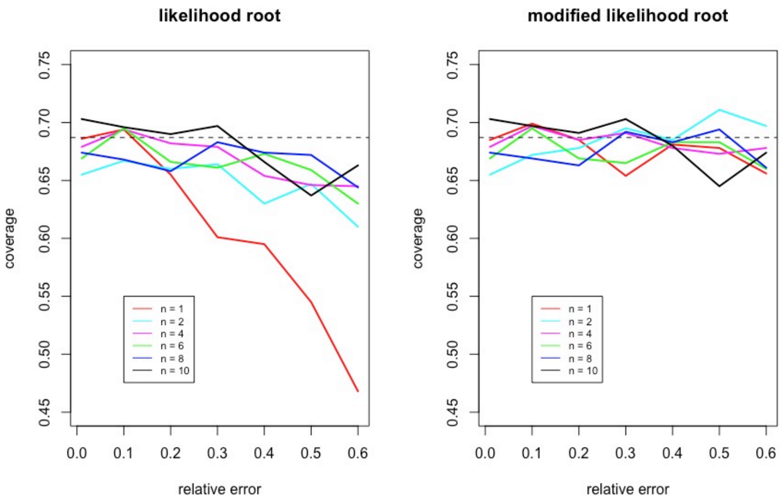
Lugannani-Rice tail approximation

$$\Phi^*(t_\mu) = \Phi(t_\mu) + \phi(t_\mu)\left(\frac{1}{t_\mu} - \frac{1}{q}\right)$$

Finite sample distribution



True coverage



Two-for-one

- **nuisance parameters**: $\theta = (\mu, \lambda)$

INCIDENTAL PARAMETERS !

$$\frac{p}{n} \longrightarrow 0$$

$$t^*(\mu) = t(\mu) + t_{inf}(\mu) + t_{np}(\mu)$$

But also...

$$n \longrightarrow +\infty$$

(small-sample asymptotics)

AND

$$r \longrightarrow 0$$

(small-dispersion asymptotics)

By-the-way

- Feasible for complex models
- Can be combined with simulation (empirical moments, pre-pivoting)
- Bayesian counterpart available
- Directional *p*-values

Questions?