Transfer Learning for Data-Driven Background Modelling: Optimal Transport v. Classifier Extrapolation

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Carnegie Mellon University

Joint work with: Patrick Bryant, John Alison, Mikael Kuusela and Larry Wasserman

PhyStat--Systematics, November 2021

Data-Driven Background Modelling

Setting: Observe collider events $X_1, X_2, \ldots$ arising from a Poisson point process with intensity function:

$$\lambda(x) = B(x) + \mu S(x).$$

- $S(\cdot)$ is the **known** intensity function of signal events;
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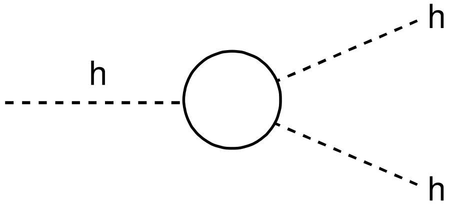
- Significant source of systematic uncertainties.
- Our focus will be on providing cross-checks for estimating B.

Motivating Example: di-Higgs Background Modelling



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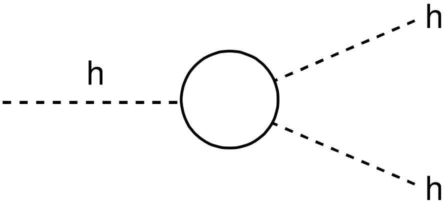

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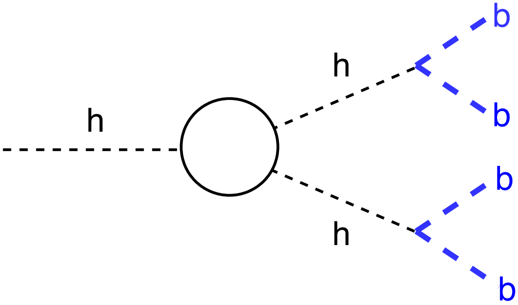

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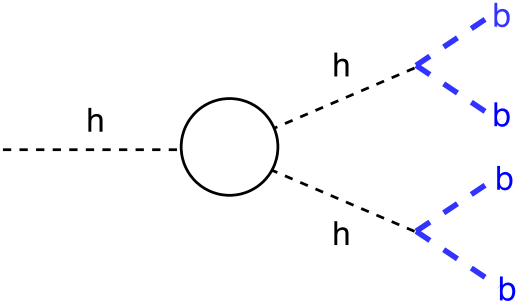

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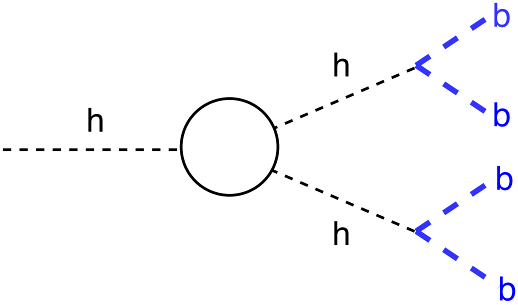

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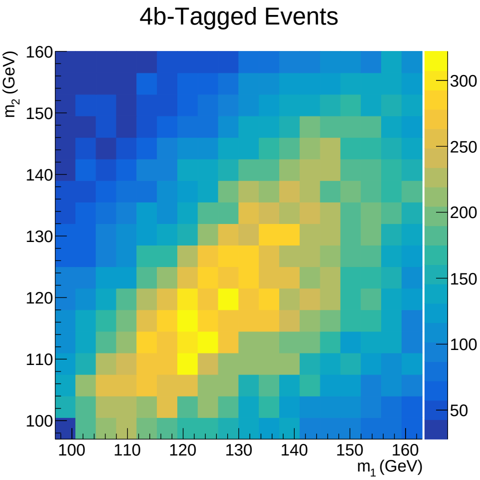
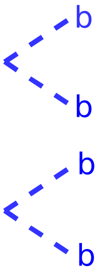
- 4b background is very large and cannot be simulated with high fidelity $\Longrightarrow$ needs to be estimated from data.

Control and Signal Regions





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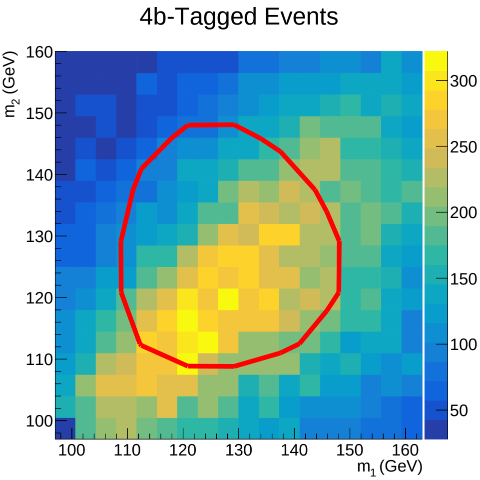

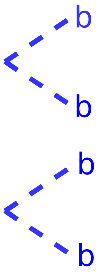

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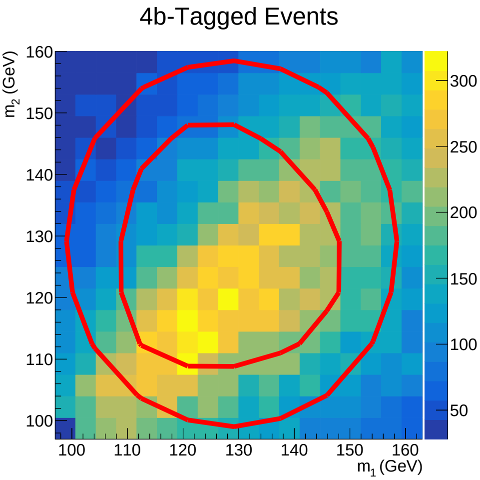

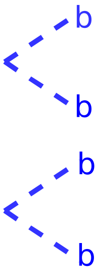

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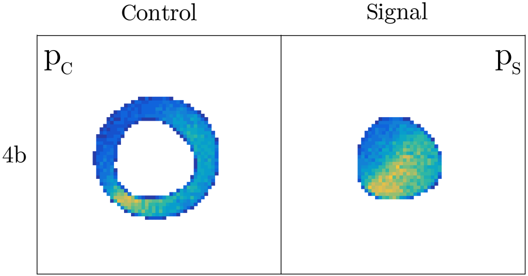
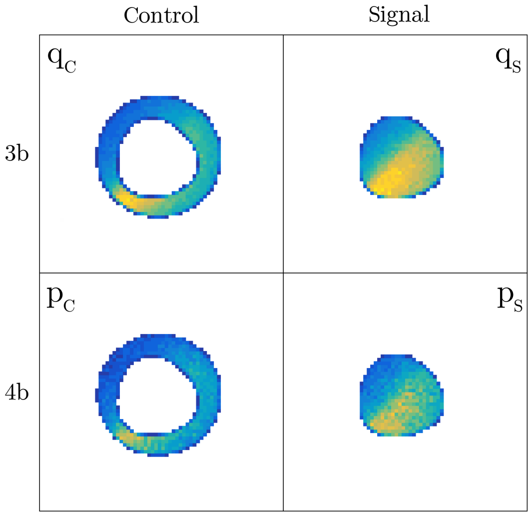


Introduce a Sample of Three-Tagged Collider Events



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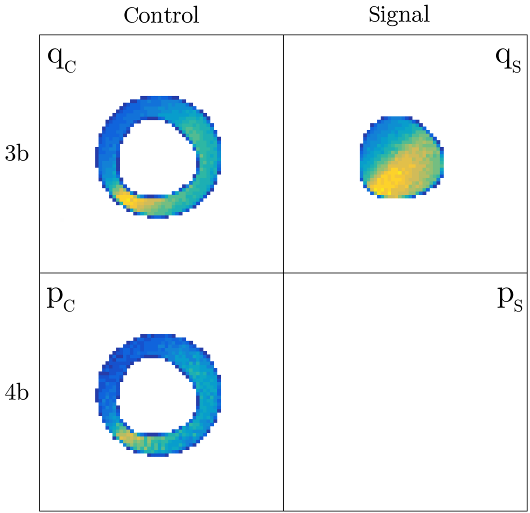
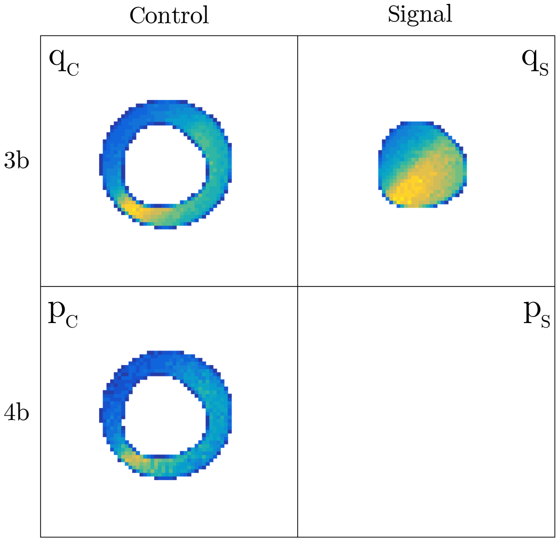


Existing Approach for Background Estimation: Density Ratio Extrapolation



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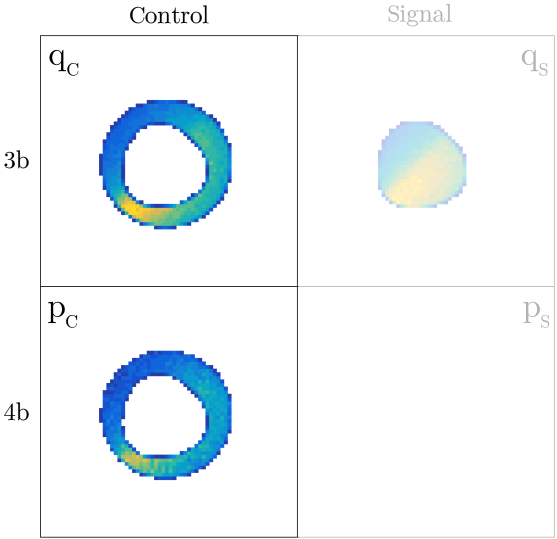

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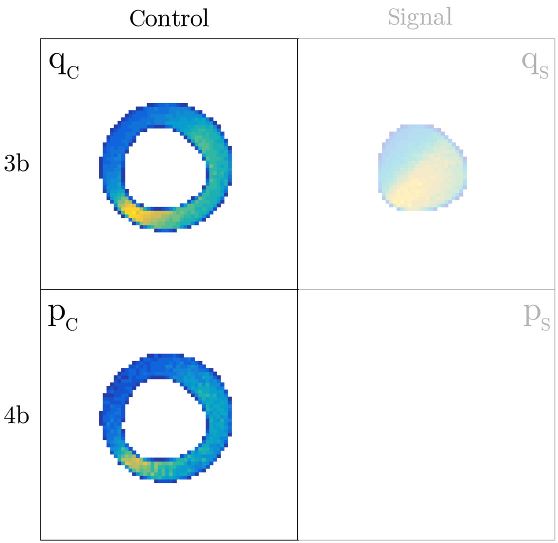

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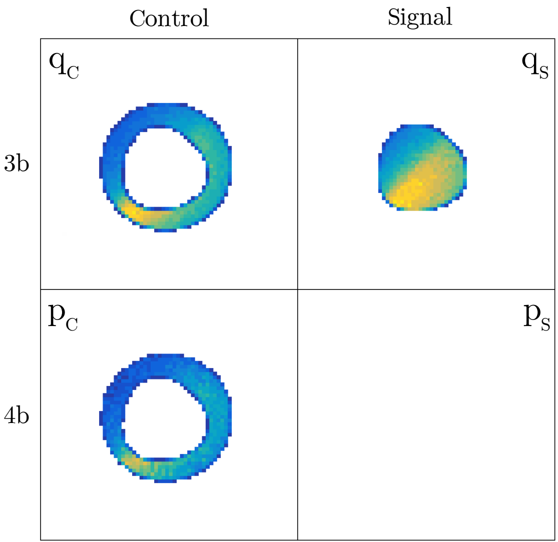

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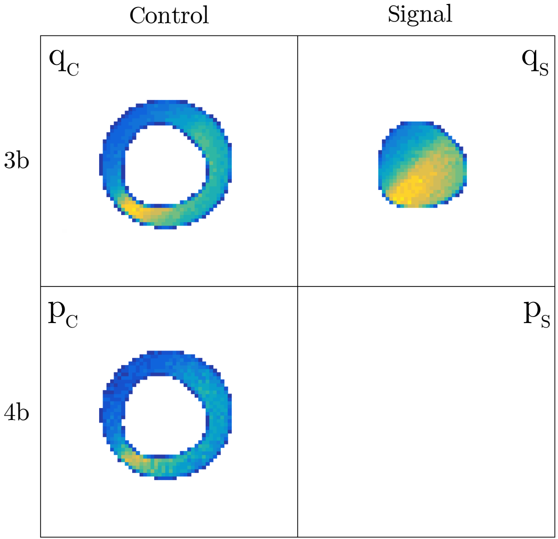
- $\hat{\psi}$ is often estimated using a classifier for discriminating 3b and 4b.



Implicit Modelling Assumptions

Assumption 1 (Informal): The following density ratio is "sufficiently" smooth over $\mathrm{CR} \cup \mathrm{SR}$

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- We develop a method which makes the following **orthogonal** assumption.

Assumption 2: There exists a map $T_0 : \mathrm{CR} \to \mathrm{SR}$ such that

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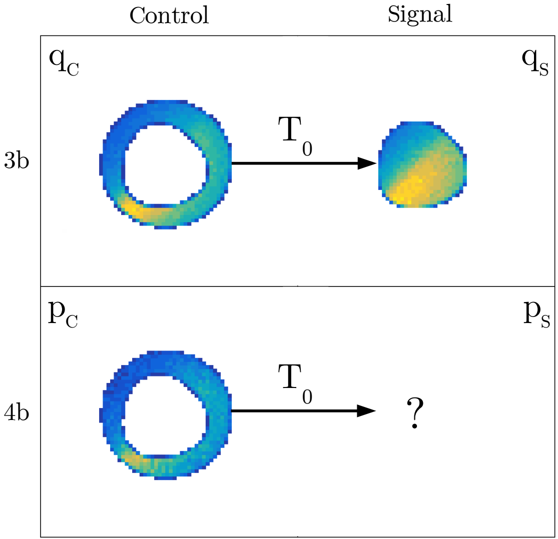

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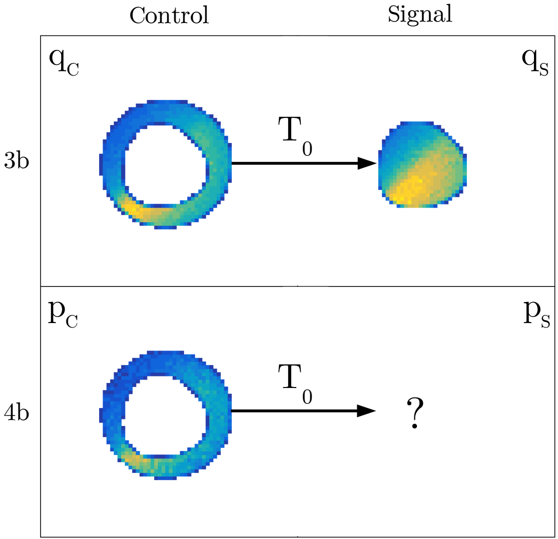

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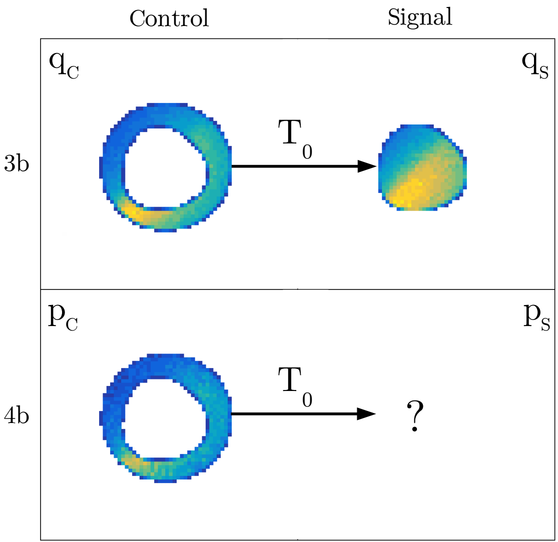
Here, D is a metric over collider events[4].



[4] Komiske, P. T., Metodiev, E. M., and Thaler, J. (2019). Metric Space of Collider Events. Physical Review Letters, 123(4).

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Assumption 2': The OT map from q_C to q_S also pushes p_C onto p_S, i.e.:

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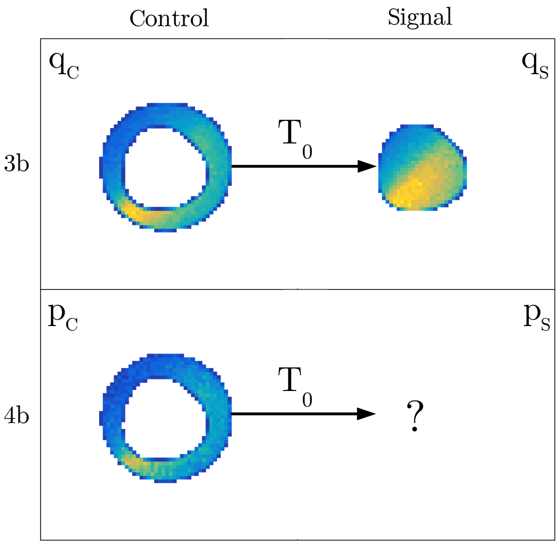

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Estimating p_S using Optimal Transport

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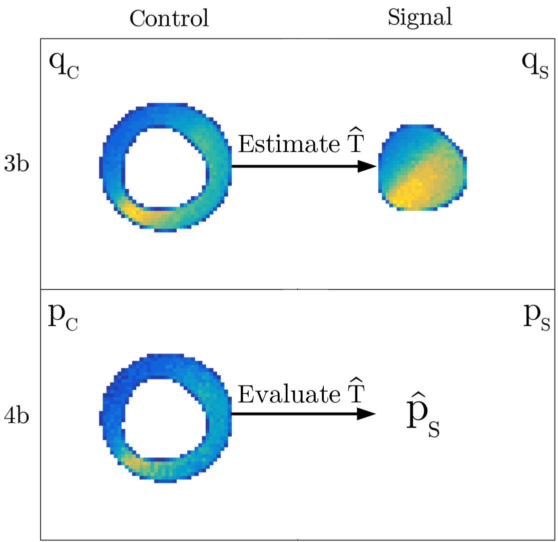

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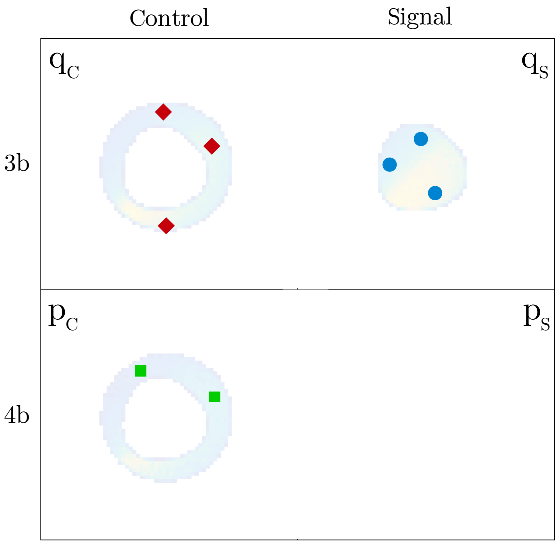

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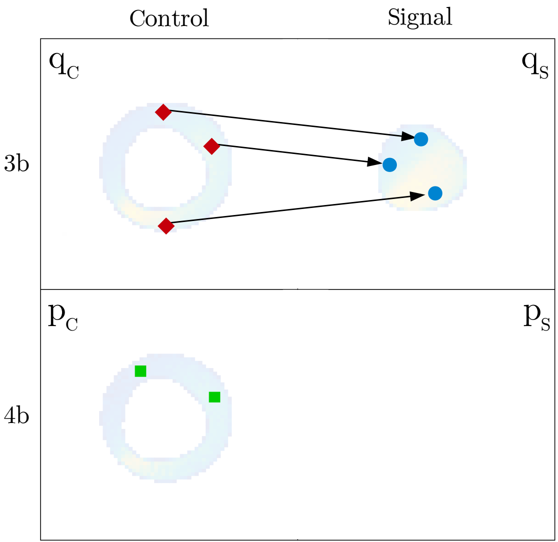

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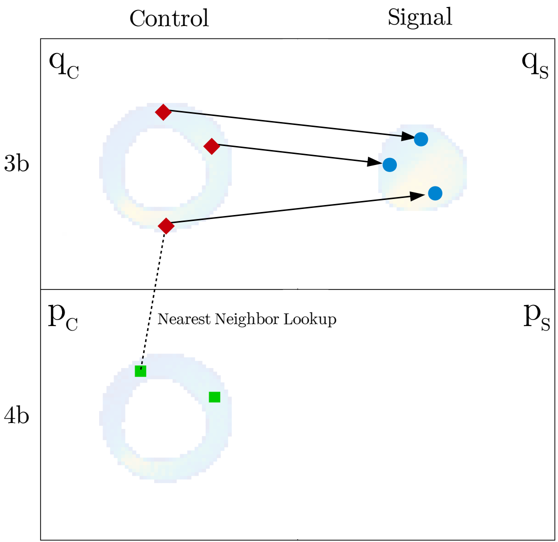

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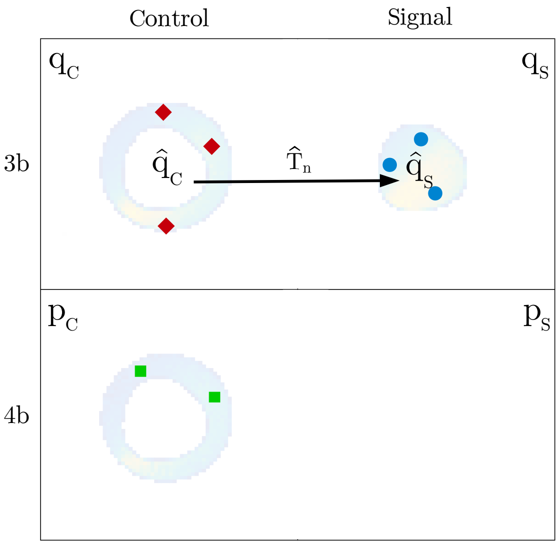
- **"Smooth-then-Map"**: Compute the unique OT map between density estimators $\hat{q}_C$ and $\hat{q}_S$.
  - We again avoid density estimation by using 3b-vs-4b classification.



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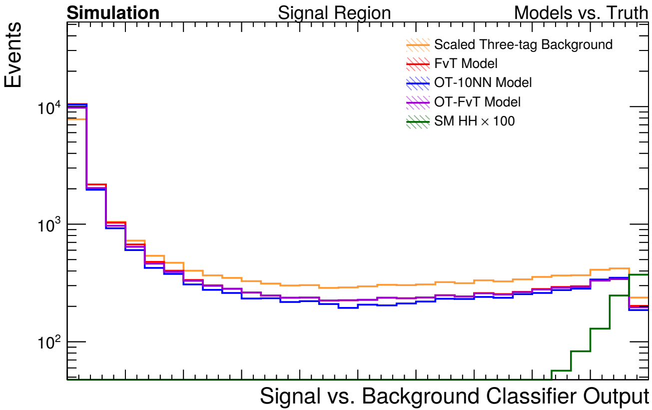

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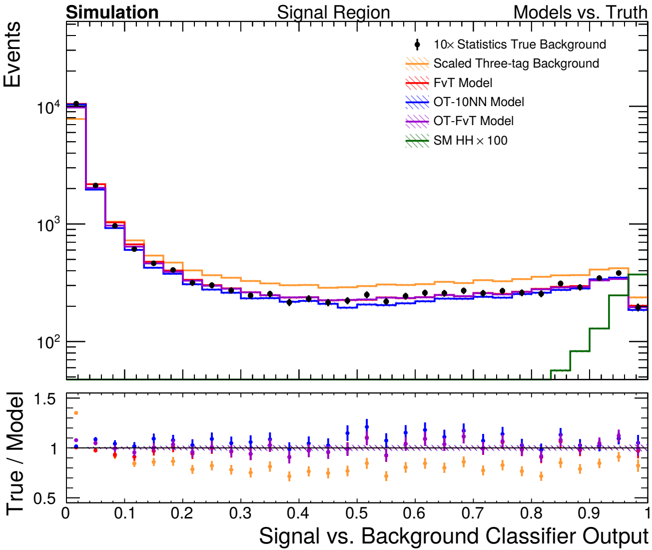

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- Quantifying the uncertainty in estimating optimal transport maps also remains a challenging open problem.
- This is only one of many potential applications of the optimal transport toolbox in particle physics...

Thank You

Backup

Some Theoretical Properties

Theorem (M. et al., 2021; Informal). Under some technical conditions, if T_0 is Lipschitz, then the "map-then-smooth" estimator $\hat{T}_n$ based on one-nearest neighbor extrapolation satisfies (for $d \geq 5$),

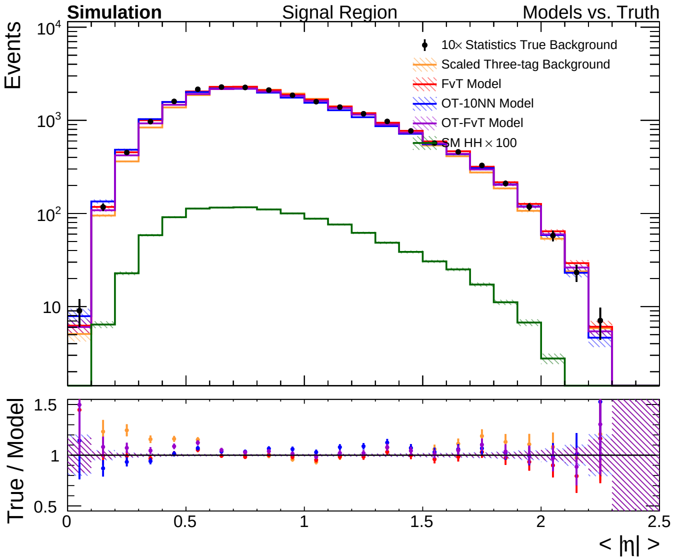
$$\mathbb{E}\|\hat{T}_n - T_0\|_{L^2}^2 \lesssim \left(\frac{\log n}{n}\right)^{\frac{2}{d}}.$$

- Convergence rates as fast as $1/n$ can be obtained for "smooth-then-map" estimators under stronger conditions.
- **Caveat**: This assumes $D = \|\cdot\|^2$, whereas we take D to be the metric of Komiske et al. (2019), itself arising from an optimal transport problem between event jets.

[6] Manole, T., Balakrishnan, S., Niles-Weed, J. and Wasserman, L. (2021). Plugin Estimation of Smooth Optimal Transport Maps. Preprint, arXiv:2107.12364.

[7] Komiske, P. T., Metodiev, E. M., and Thaler, J. (2019). Metric Space of Collider Events. Physical Review Letters, 123(4).

Further Simulation Results



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