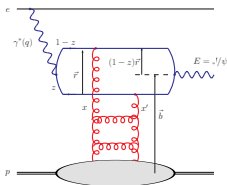


# Exclusive vector meson production in $ep$ collisions at the LHeC

Graeme Watt

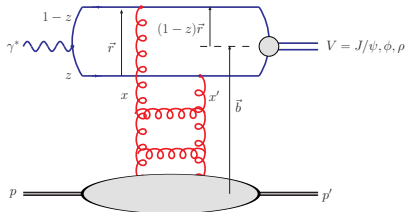
CERN PH-TH

*3rd CERN-ECFA-NuPECC Workshop on the LHeC*  
Chavannes-de-Bogis, Switzerland  
12th November 2010



# Introduction

- **Motivation:** exclusive vector meson (VM) production at the LHeC as a probe of unitarity effects at a perturbative scale.
- Increase unitarity corrections by (i) **increasing energy in  $ep$**  (this talk) or (ii) moving to  $eA$  collisions ( $\rightarrow$  H. Kowalski).



Aim to extract dipole–proton scattering amplitude:  $\mathcal{N}(x, r, b)$

[cf. Munier, Staśto, Mueller, hep-ph/0102291]

## “Experimental” variables

$$W^2 = (q + p)^2, \quad Q^2 = -q^2$$

$$t = (p - p')^2 = -|\vec{\Delta}|^2$$

## “Theoretical” variables

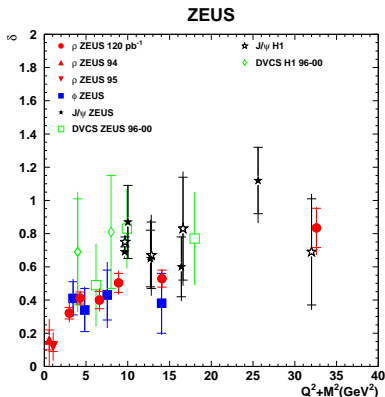
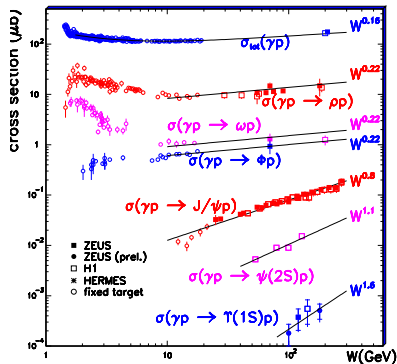
$$\text{Mom. frac., } x \simeq \frac{Q^2 + M_V^2}{Q^2 + W^2} \ll 1$$

$$\text{Dipole size, } r = |\vec{r}| \quad (\vec{r} \overset{F,T}{\leftrightarrow} \vec{k})$$

$$\text{Impact parameter, } \vec{b} \overset{F,T}{\leftrightarrow} \vec{\Delta}$$

# W dependence of exclusive VM cross sections at HERA

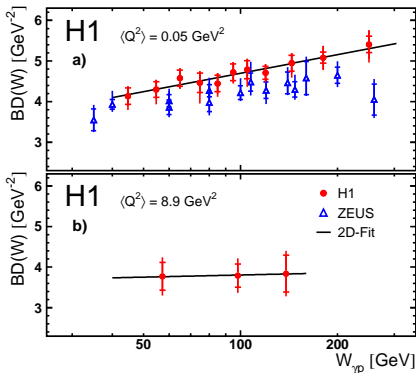
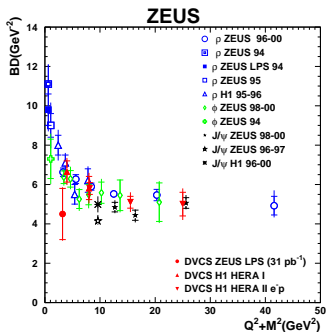
- Transition between “soft” and “hard” energy dependence.



Regge phenomenology for “effective” Pomeron trajectories

$$\sigma(\gamma p \rightarrow V + p) \propto W^\delta, \quad \delta = 4[\alpha_{\mathbb{P}}(\langle t \rangle) - 1], \quad \alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t$$

# $t$ dependence of exclusive VM cross sections at HERA

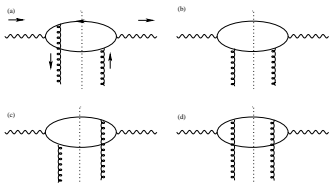


- Parameterise as exponential:  
 $d\sigma/dt \propto \exp(-B_D |t|)$ .
- $t$ -slope parameter  $B_D$  gives transverse proton size  $\langle b^2 \rangle$ .

- Regge phenomenology:  
 $B_D = B_0 + 4 \alpha'_P \ln(W/W_0)$ .
- Non-zero  $\alpha'_P$  for  $\gamma p \rightarrow J/\psi + p$ ,  
 $\sim$  half value from elastic  $p-p$ .

# Dipole picture in the non-forward direction

[Bartels, Golec-Biernat, Peters, hep-ph/0301192]



- Non-forward photon impact factor calculated in the high-energy limit.
- Fourier transform from momentum space to coordinate space ( $\vec{k} \rightarrow \vec{r}$ ), then to impact parameter space ( $\vec{\Delta} \rightarrow \vec{b}$ ), with  $t = -|\vec{\Delta}|^2$ .
- Results obtained in colour dipole picture: amplitude factorises into (wave function) · (dipole cross section) · (wave function).
- Non-forward wave functions can be written as forward wave functions multiplied by  $\exp[\pm i(1-z)\vec{r} \cdot \vec{\Delta}/2]$ .

$$\mathcal{A}_{T,L}^{\gamma^* P \rightarrow V+P}(x, Q, \Delta) = i \int d^2\vec{r} \int_0^1 \frac{dz}{4\pi} \int d^2\vec{b} (\Psi_V^* \Psi)_{T,L} e^{-i[\vec{b} - (1-z)\vec{r}] \cdot \vec{\Delta}} \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$$

# Unified description of exclusive and inclusive processes

## Exclusive diffractive processes

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow V+p}}{dt} = \frac{1}{16\pi} \left| \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V+p} \right|^2$$

with corrections from the *real part* and from *skewedness* ( $x \neq x'$ ).

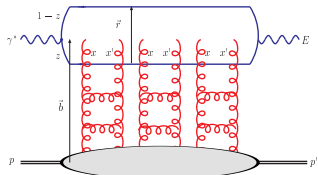
## Inclusive deep-inelastic scattering (DIS) at small $x$

$$\begin{aligned} \sigma_{T,L}^{\gamma^* p}(x, Q) &= \text{Im} \mathcal{A}_{T,L}^{\gamma^* p \rightarrow \gamma^* p}(x, Q, \Delta = 0) \\ &= \sum_f \int d^2\vec{r} \int_0^1 \frac{dz}{4\pi} (\Psi^* \Psi)_{T,L}^f \int d^2\vec{b} \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} \end{aligned}$$

$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2 \mathcal{N}(x, r, b)$ , where  $\mathcal{N} \in [0, 1]$  and  $\mathcal{N} = 1$  is the unitarity limit.

- $\mathcal{N}(x, r, b)$  should be determined from a **simultaneous** description of *inclusive* DIS and *exclusive* diffractive processes.

## Impact-parameter-dependent saturation (“b-Sat”) model



Golec-Biernat, Wüsthoff [[hep-ph/9807513](https://arxiv.org/abs/hep-ph/9807513)]

→ Bartels, Golec-Biernat, Kowalski [[hep-ph/0203258](https://arxiv.org/abs/hep-ph/0203258)]

→ Kowalski, Teaney [[hep-ph/0304189](https://arxiv.org/abs/hep-ph/0304189)]

→ Kowalski, Motyka, G.W. [[hep-ph/0606272](https://arxiv.org/abs/hep-ph/0606272)]

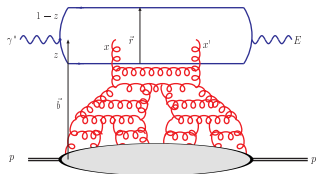
- Eikonalised DGLAP-evolved gluon density with Gaussian  $b$  dependence:

$$\mathcal{N}(x, r, b) = 1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) xg(x, \mu^2) T(b)\right)$$

$$xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}, \quad T(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}, \quad \mu^2 = \frac{4}{r^2} + \mu_0^2$$

- $B_G = 4 \text{ GeV}^{-2}$  from  $t$ -slope of exclusive  $J/\psi$  photoproduction.
- Fit to 163 ZEUS  $F_2$  points with  $x_{Bj} \leq 0.01$  and  $Q^2 \in [0.25, 650] \text{ GeV}^2$  gives a  $\chi^2/\text{d.o.f.} = 1.21$  with parameters:  
 $\mu_0^2 = 1.17 \text{ GeV}^2$ ,  $A_g = 2.55$ ,  $\lambda_g = 0.020$ .

# Impact-parameter-dependent CGC (“b-CGC”) model



- lancu, Itakura, Munier (IIM) [[hep-ph/0310338](#)]  
 → Kowalski, Motyka, G.W. [[hep-ph/0606272](#)]  
 → Soyez [[arXiv:0705.3672](#)]  
 → G.W., Kowalski [[arXiv:0712.2670](#)]  
 (N.B. Charm quarks not included by IIM.)

- Original CGC model of IIM assumed  $b$  dependence:  $\Theta(b_{\max} - b)$ .
- Introduce Gaussian  $b$  dependence into the saturation scale  $Q_s$ :

$$\mathcal{N}(x, r, b) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{\ln(2/rQ_s)}{9.9\lambda \ln(1/x)}\right)} & : rQ_s \leq 2 \\ 1 - e^{-A \ln^2(BrQ_s)} & : rQ_s > 2 \end{cases}$$

$$Q_s \equiv Q_s(x, b) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[ \exp\left(-\frac{b^2}{2B_{\text{CGC}}}\right) \right]^{\frac{1}{2\gamma_s}}$$

- $B_{\text{CGC}} = 7.5 \text{ GeV}^{-2}$  from  $t$ -slope of exclusive  $J/\psi$  photoproduction.
- Fit to 133 ZEUS  $F_2$  points with  $x_{\text{Bj}} \leq 0.01$  and  $Q^2 \in [0.25, 45] \text{ GeV}^2$  gives a  $\chi^2/\text{d.o.f.} = 0.92$  with parameters:

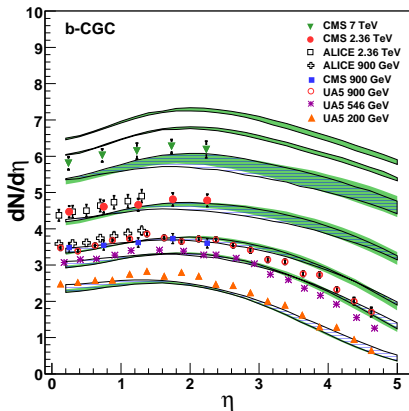
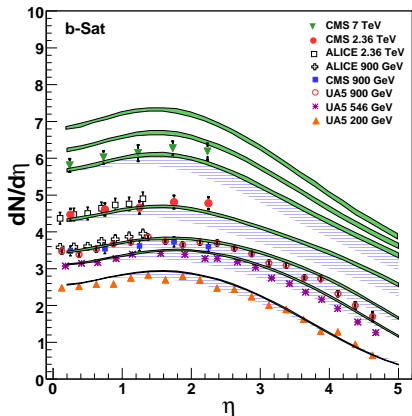
$$\mathcal{N}_0 = 0.558, \gamma_s = 0.46, x_0 = 1.84 \times 10^{-6}, \lambda = 0.119.$$



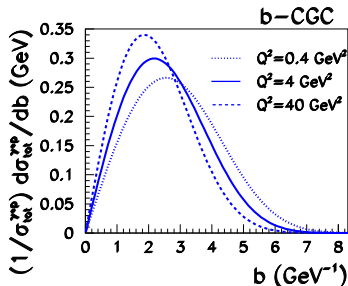
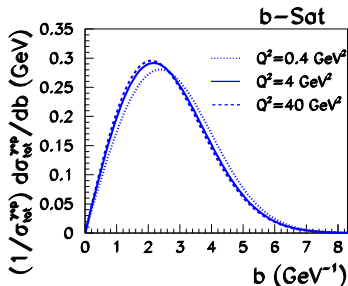
# Aside: b-Sat and b-CGC models in LHC $pp$ minimum-bias

[Tribedy, Venugopalan, arXiv:1011.1895 (see also Levin, Rezaeian, arXiv:1005.0631)]

- Charged-hadron pseudorapidity distributions,  $dN_{\text{ch}}/d\eta$ :

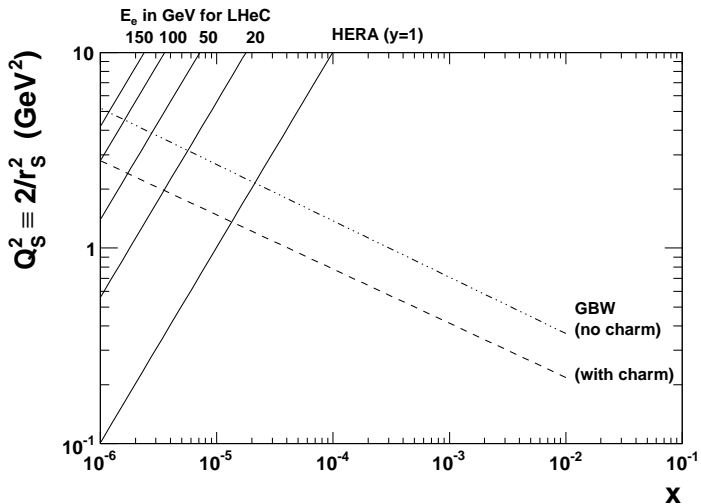


- b-dependent dipole cross sections* crucial for successful description.

Impact-parameter dependence of total  $\gamma^* p$  cross section

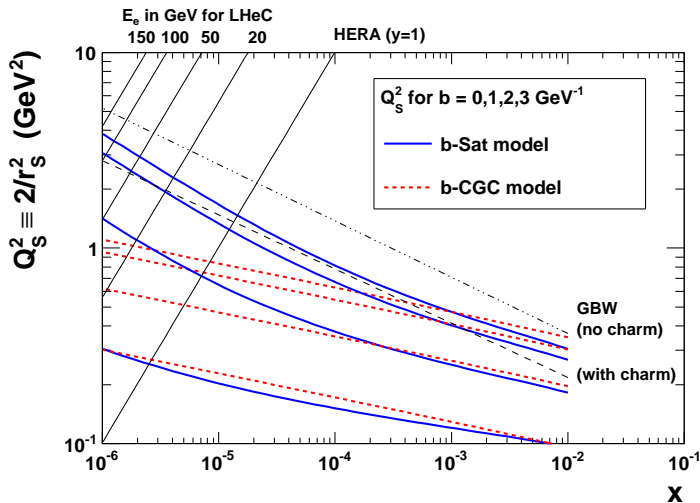
- $Q^2 = 0.4, 4, 40 \text{ GeV}^2$  with  $x = 10^{-4}, 10^{-3}, 10^{-2}$  respectively.
- Median impact parameters  $b$  are all between 2 and 3  $\text{GeV}^{-1}$ .

# Saturation scale $Q_S^2 \equiv 2/r_S^2$ from GBW dipole model



- Define  $r_S$  as the dipole size where  $\mathcal{N}(x, r_S, [b]) = 1 - e^{-1/2} \simeq 0.4$ .

# Saturation scale $Q_S^2 \equiv 2/r_S^2$ from b-Sat and b-CGC models



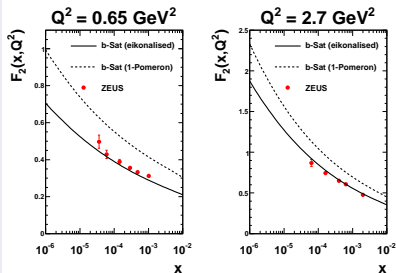
- $Q_S^2 \lesssim 0.5 \text{ GeV}^2$  in HERA regime for relevant  $b \sim 2-3 \text{ GeV}^{-1}$ .

# Size of unitarity corrections for $F_2$ and $F_L$ in b-Sat model

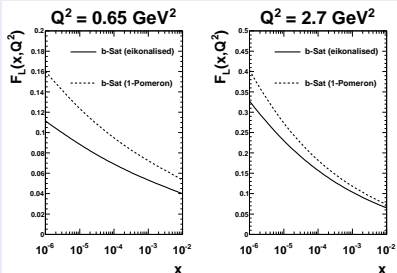
$$\mathcal{N}(x, r, b) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \left[ \frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) xg(x, \mu^2) T(b) \right]^n$$

Compare “**eikonalised**” ( $n = 1, \dots, \infty$ ) with “**1-Pomeron**” ( $n = 1$ ):

## Proton structure function $F_2$



## Longitudinal structure function



- Large unitarity corrections *only* at low  $Q^2 \lesssim 1 \text{ GeV}^2$ .

# Exclusive vector meson production at the LHeC

- Focus on **exclusive  $J/\psi$  photoproduction** ( $Q^2 = 0$ ).
- **Hard scale** provided by charm-quark mass,  $m_c = 1.4$  GeV.
- Overlap between photon and  $V (= J/\psi)$  wave functions:

$$(\Psi_V^* \Psi)_T = \frac{2}{3} e \frac{N_c}{\pi z(1-z)} \left\{ m_c^2 K_0(m_c r) \phi_T(r, z) - \left[ z^2 + (1-z)^2 \right] m_c K_1(m_c r) \partial_r \phi_T(r, z) \right\}$$

- Consider two alternatives for the scalar part of  $\Psi_V$ :

“Gaus-LC” [Dosch *et al.*, hep-ph/9608203; Kowalski *et al.*, hep-ph/0304189]

$$\phi_T(r, z) = N_T [z(1-z)]^2 \exp(-r^2/2R_T^2)$$

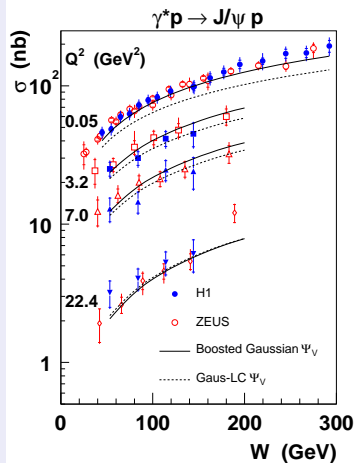
“Boosted Gaussian” [Nemchik *et al.*, hep-ph/9405355, hep-ph/9605231; Forshaw, Sandapen, Shaw, hep-ph/0312172 (set Coulomb part to zero)]

$$\phi_T(r, z) = N_T z(1-z) \exp\left(-\frac{m_c^2 \mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2} + \frac{m_c^2 \mathcal{R}^2}{2}\right)$$

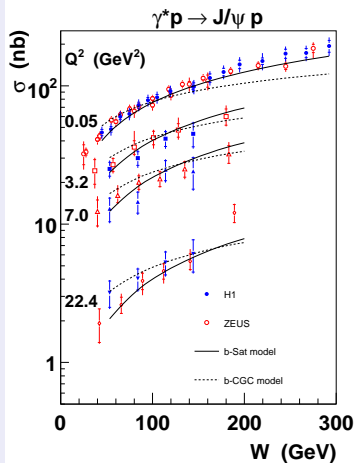
- **Parameters** determined by normalisation and measured  $\Gamma_{V \rightarrow e^+e^-}$ .

# Exclusive $J/\psi$ production at HERA

b-Sat model with different  $\Psi_V$

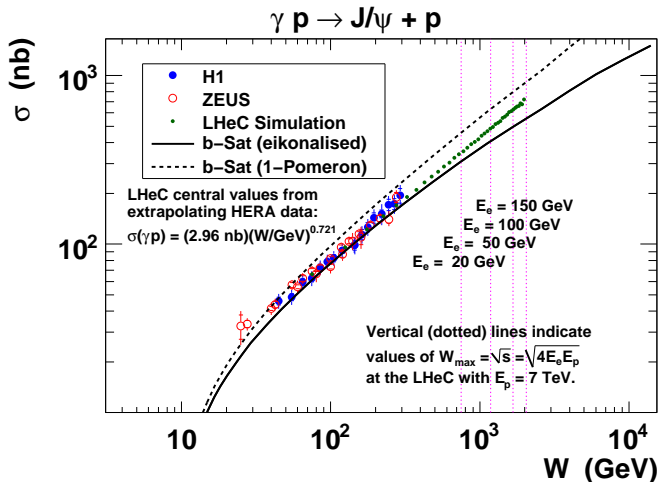


BG  $\Psi_V$  with b-Sat or b-CGC



⇒ Use **b-Sat model** with **BG  $\Psi_V$**  for extrapolation to LHeC.

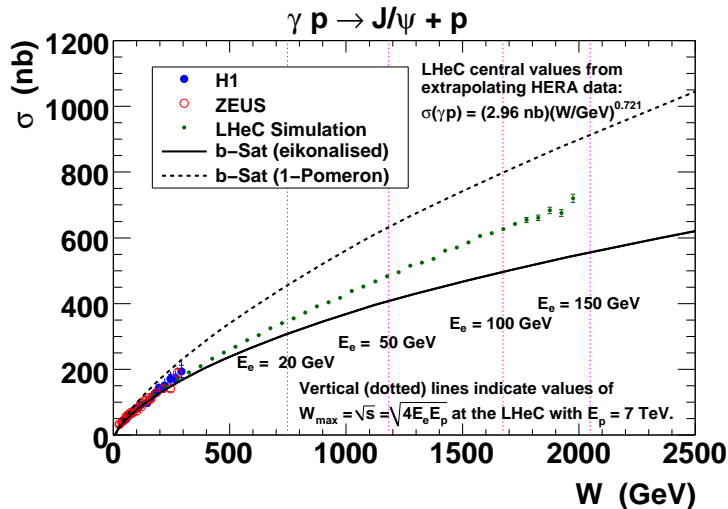
# Extrapolation to LHeC energies for $J/\psi$ (log-log plot)



- LHeC Simulation:** pseudo-data points with **only statistical errors** ( $E_e = 150 \text{ GeV}$ ,  $1^\circ$  angular acceptance,  $L = 1 \text{ fb}^{-1}$ ) [P. Newman].

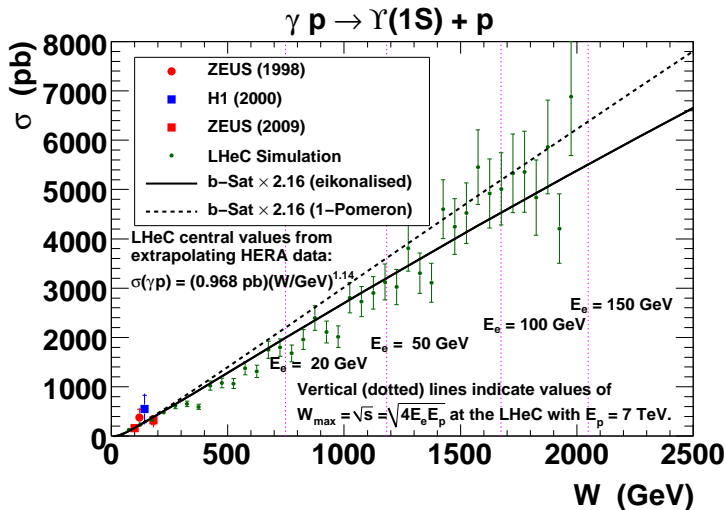


# Extrapolation to LHeC energies for $J/\psi$ (linear-linear plot)



- Significant unitarity corrections with increasing energy  $W$ .

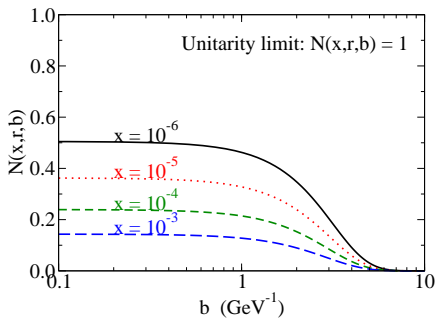
# Extrapolation to LHeC energies for $\Upsilon$ (linear-linear plot)



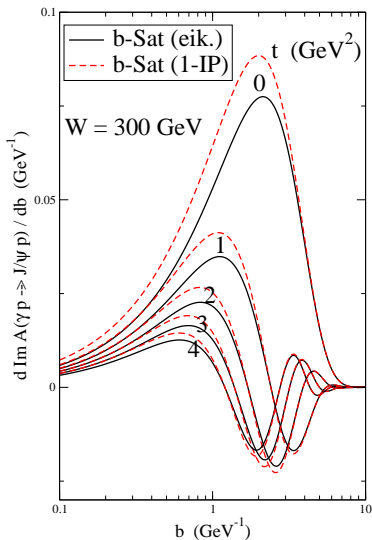
- **Unitarity corrections smaller** due to larger scale  $m_b = 4.5 \text{ GeV}$ .

# Probe smaller impact parameter $b$ by going to larger $|t|$

"b-Sat" dipole scattering amplitude with  $r = 1 \text{ GeV}^{-1}$



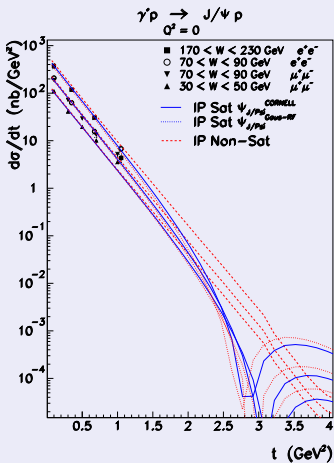
- Recall impact parameter  $\vec{b}$  is Fourier conjugate variable to  $\vec{\Delta}$ , where  $t = -|\vec{\Delta}|^2$ .
- $\Rightarrow$  Central  $b$  region enhanced at **large momentum transfer  $|t|$** .



# “Diffractive dips” in elastic $d\sigma/dt$ at large $|t|$

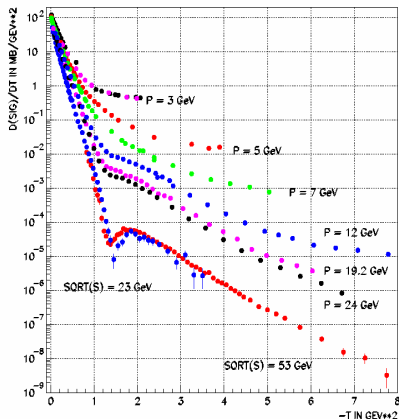
$\gamma p \rightarrow J/\psi + p$  in b-Sat

[Kowalski, Teaney, hep-ph/0304189]

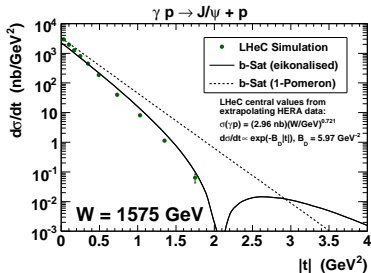
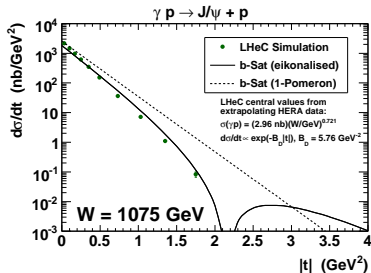
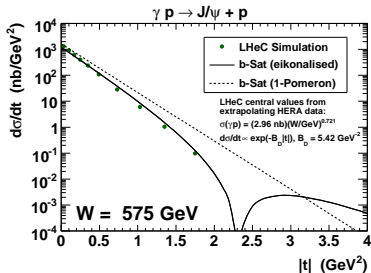
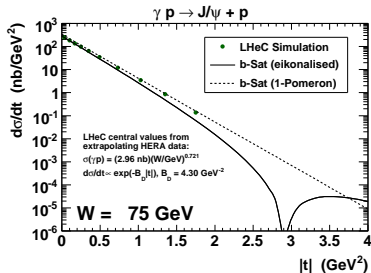


cf.  $pp$  elastic cross section data

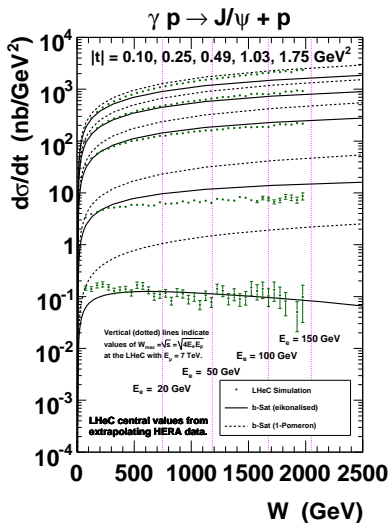
[Arneodo, Diehl, hep-ph/0511047]



# “Diffractive dips” for exclusive $J/\psi$ at the LHeC



# $d\sigma/dt$ versus $W$ for different $|t|$ at the LHeC



## HERA extrapolation

- Fit both H1 and ZEUS data.
- **Power-law fit of  $\sigma$  vs.  $W$ :**  

$$\sigma = (2.96 \text{ nb})(W/\text{GeV})^{0.721}$$
- Parameterise:  

$$d\sigma/dt = \sigma \cdot B_D \exp(-B_D |t|)$$
- **Linear fit to  $B_D$  vs.  $W$ :**  

$$B_D = B_0 + 4 \alpha'_P \ln(W/90 \text{ GeV}),$$
 with  $B_0 = 4.400 \text{ GeV}^{-2}$   
 and  $\alpha'_P = 0.137 \text{ GeV}^{-2}$ .
- Largest  $|t| = 1.75 \text{ GeV}^2$  close to the “diffractive dip” at large energies  $W$ .

# Summary

## Impact-parameter-dependent saturation (“b-Sat”) model

- Eikonalised gluon density with DGLAP evolution.
- Inclusion of Gaussian impact-parameter dependence.
- Validated against a wide range of HERA data.
- Simple estimate of size of unitarity corrections via comparison of “eikonalised” versus “1-Pomeron” predictions.

## Exclusive diffractive $J/\psi$ photoproduction at the LHeC

- Ideal observable for investigating **unitarity effects** at a **perturbative scale** provided by the charm-quark mass.
- Extrapolation of  $W$  dependence indicates **sizeable unitarity corrections at large  $W$**  beyond the reach of HERA.
- **Large  $|t|$**  provides access to *central* impact parameters  $b$ : “diffractive dips” measurable at LHeC (proton taggers)?