Summer Student Lecture Program 2021

# Electronics, Trigger and Data Acquisition. 1/3

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Credits:

SSLP ETD lecture series by R.Ferrari (2019) and earlier ones

EM: lectures on DAQ/Trigger at U.Padua 2018-2020 ISOTDAQ: International School of Trigger and DAQ https://indico.cern.ch/event/928767/

Material from various papers and books (bibliography at the end)

- Trigger and DAQ system concepts
- From signal to physics through examples
- Timing
- Data transport, links, buses
- Queues and Event building
- On-line data processing

## A quick tour – menu

- Introduction:
  - Sensors, detectors, experiments, historical perspective
  - Acquiring data from sensors
- Basics of analog signal processing
- From analog to digital
- Measuring time
- Trigger
- Role of CPU and data buses
- Event building

### Examples taken from nuclear and (mostly) particle physics Feel free to bring your own example/problems from other areas for discussion in the Q&A



### Sensors

- In modern parlance, "a device, module, machine, or subsystem whose purpose is to detect events or changes in its environment and send the information to other <u>electronics</u>"
- In practice, any device that detects or measures a physical event or quantity and transforms this event or quantity into another that is "easier" to perceive and/or measure – sensors and transducers can be often exchanged/confused
- In most cases today, the final quantity is an electrical signal (either steady or transient)
- In what follows, we will be dealing mostly with transducers that produce an electrical signal, in most cases a pulse



## **Detectors**

In real life, we often deal with a complex of (one or more) sensors or transducers, not necessarily homogeneous. We refer to this complex as a "detector" and this often includes the electronics used to read out and process the information about the physical quantity or event. In NP and HEP, when we say "detector", we almost always mean "ionizing particle detector"

Sometimes we mean a whole experiment ("the CMS detector")





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## **Detectors and Signals**

- Sometimes, somewhere, something happens → in some short time, several particles interact within our detector
- Even a single particle interaction is composed of many different probabilistic (quantum-mechanics) processes → fluctuations are built-in
- Practically all modern detectors, at the end, generate "electrical" signals at their output terminals. This signals:
  - a) have different characteristics (size, arrival time, duration, ...)
  - b) carry different (normally independent) information
  - c) require some **electronics** in order to become "usable" to measure a physical quantity



## A bit of history...

- Experiments of the past often used analog (or sensorial) means to measure/register a phenomenon
  - Visual or aural observation
    - Often involving counting the occurrences of some phenomenon
    - ...and taking note (recording)
  - (analog) photography was often used for more complex observations (emulsion experiments, bubble chambers)
- Counting and recording information are all things a computer does better **once the information is in digital form**



## **Rutherford Scattering**





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## Role of Electronics, Trigger and DAQ

Process the signals generated in a detector and save (only) the interesting information on a permanent storage medium **Modern DAQ is all about digital information** 

However, physics is not digital...sensors produce analog signals that **must be treated and interpreted** before being **digitized** Most of the "real" physics analysis (the one that gets you the Nobel prize) happens "offline" (what "offline" means has changed over time) however:

 $\rightarrow$ There is a lot of physics (and math, and technology) that you only learn in DAQ and trigger  $\leftarrow$ 





## A modern Trigger/DAQ looks like this





## (Probably) the first DAQ

All the main components constituting an experiment, its acquisition **and trigger** are there.

Can you recognize them ? The detector (E) is connected to a programmable trigger (HR) and analog front-end electronics (G) is acquired (R) and recorded on "digital" media by means of (P')

## ON THE AUTOMATIC REGISTRATION OF $\alpha$ -PARTICLES, $\beta$ -PARTICLES AND $\gamma$ -RAY AND X-RAY PULSES



Alois F. Kovarik Sheffield Scientific School Yale University New Haven, Conn. January 25, 1919



## Coincidence (trigger)

- Bruno Rossi (Nature, 1930):
- "Method of Registering Multiple Simultaneous Impulses of Several Geiger Counters"
- $\rightarrow$  online coincidence of 3 signals!



Fig. 17 – Il circuito di Rossi per rivelare coincidenze di raggi cosmici che arrivano sui contatori Geiger (i rettangoli in alto dello schema)<sup>19</sup>.



Fig. 18 – L'uso del circuito di Rossi per rivelare una coincidenza tripla che, nella disposizione in figura dei tre contatori, mostra la produzione di una radiazione secondaria (linea tratteggiata) da parte della radiazione primaria (linea continua)<sup>20</sup>.



## A recent one

### L1-Trigger/HLT/DAQ

#### hTracks in L1-Trigger at 40 MHz

- PFlow-like selection 750 kHz output
- HLT output 7.5 kHz

#### **Barrel Calorimeters**

- · ECAL crystal granularity readout at 40 MHz with precise timing for e/y at 30 GeV
- ECAL and HCAL new Back-End boards

#### Muon systems

- DT & CSC new FE/BE readout
- **RPC** back-end electronics
- New GEM/RPC 1.6 < n < 2.4 •
- Extended coverage to n ≃ 3



- 3D showers and precise timing
- Si, Scint+SiPM in Pb/W-SS

### Tracker

- Si-Strip and Pixels increased granularity
- Design for tracking in L1-Trigger
- Extended coverage to n ~ 3.8

### **MIP Timing Detector**

Precision timing with:

- Barrel layer: Crystals + SiPMs
- Endcap layer: Low Gain Avalanche Diode

Detector + Back-End Boards + ATCA Back plane TTC/TTS TCDS2 DAQ and TTC Hub (DTH) TTT TTTI 30 m 185 m Single-Mode Fiber 30 m Data Concentration Network RU/BL 400 GbE 400 GbE **RoCE Event Builder** Event Back 3on Cluster Storage Storage Manager HLT TOR 000 CDR

Detector Front-Ends (FE)

up to 100 BGTs x BE board

b

Level1 Trigger

EVM



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Beam Radiation Instr. and

and Infrastructure

Luminosity, and Common Systems

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## Not only for colliders...

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## A quick tour of the DAQ/Trigger Chain

## Signals from a detector: amplification

Detectors may be electrically represented as a capacitor  $C_d$  (more realistic schemes will include other contributions) Interactions of passing particle  $\rightarrow$  energy release E  $\rightarrow$  short current pulse  $i_s$ Weak signals require amplification: • adapts it to next stages

• avoids Signal-to-Noise-Ratio (SNR) degradation





 $E \bigotimes Q_s = \int i_s(t) dt$ 

A current-sensitive amplifier provides a signal That **does not depend on**  $C_d$  – more on this later if time



## **Op-amp**





(a)



The gain of an op-amp (K) ranges from  $10^4$  to  $10^7$  with a typical value of  $10^5$ . To amplify the difference between the input signals, the op-amp draws power from an electrical power supply. If  $V_+$  and  $V_-$  denote the positive and negative voltages provided by the power supply, the output of the op-amp cannot exceed these limits and therefore saturates at these levels, as shown in the figure on the left.



## Signals from a detector: shaping



Reduce signal bandwidth (low-pass filter)

→ improve SNR fast rising signals have large bandwidth shaper broadens signals

Limit pulse width (high-pass filter) → avoid overlap of successive pulses increase maximum signal rate **at the cost of more noise** 









## Analog signal treatment

- Many other aspects
  - Real-life amplifiers
  - Charge-sensitive amplifiers, integration
  - Gate generation, delay
  - Signal transmission: reflection, impedance matching
  - Response function of an apparatus
- Many good textbooks to go in depth
- We will now move on to digital
  - We will start with analog-to-digital conversion



## Analog to digital conversion (sampling)



Reminder: we need an Analog to Digital Converter (**ADC**) to turn our voltage pulse into a binary number for processing and storage



In its simplest form, an ADC compares the signal with M fractions of a reference voltage (the unit ticks on the ruler) In a nutshell, this is the working principle of the **FlashADC** 



## Digitizing a voltage pulse: flash ADC





## Voltage comparator, op amps

 $v_O = V_{OL}$  for  $v_P < v_N$  $v_O = V_{OH}$  for  $v_P > v_N$ 













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Voltage amplifiers with: a) 2 (differential) inputs b) high gain (A+ = A- =  $\infty$ ) c) high input impedance (Zin =  $\infty$ ) d) low output impedance (Zout = 0)







### Thermometer or one-hot





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## Thermometer to binary: priority encoder

Digital Inputs								Binary Output		
D <sub>7</sub>	D <sub>6</sub>	$D_5$	D <sub>4</sub>	D <sub>3</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>	Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	х	0	0	1
0	0	0	0	0	1	х	х	0	1	0
0	0	0	0	1	х	х	х	0	1	1
0	0	0	1	х	х	х	х	1	0	0
0	0	1	х	х	х	х	х	1	0	1
0	1	х	х	х	х	х	х	1	1	0
1	х	х	х	х	х	х	х	1	1	1

Exercise: use the truth table to design the 8 to 3 priority encoder How do things change for the one-hot case ? Do you see any advantages/disadvantages in one or the other ?



## Flash ADC: the simplest (and fastest)

- Resolution (Least Significant Bit), the ruler unit:
  - V<sub>max</sub> /2N
  - E.g. 8bit,  $1V \rightarrow LSB=3.9mV$
- Quantization error
  - because of finite size of the ruler unit: ± LSB/2
- Dynamic range: Vmax /LSB
  - N for linear (flash) ADC
  - >N for non-linear ADC
- Flash ADC has constant relative resolution on the valid input range





## **Measuring Time**

## **Time-to-digital Converter**



- TDC principle is quite simple: count regular pulses from a start to a stop signal
- Resolution: 1/f
- Dynamic range: N
- Single hit TDC



e.g. a noise spike comes just before the signal  $\rightarrow$  measure is lost

### Counter

1 ---

1 - - -

Frequency =  $f \div 2$ 

0

D

EN -





## Multi-hit TDC



- Gate resets and starts the counter. It also provides the measurement period. It must be smaller than 2<sup>N</sup>/f
- Each "hit" (i.e. signal) forces the FIFO to load the current value of the counter, that is the delay after the gate start
  - In order to distinguish between hits belonging to different gates, some additional logic is needed to tag the data
- Common-start configuration
- This is e.g. a typical configuration FOR A COLLIDER EXPERIMENT:



## Trigger

## Trigger





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## Sampling with a periodic clock





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## What if...

Trigger

- Events asynchronous and unpredictable
  - E.g.: beta decay studies
- A physics trigger is needed
  - Discriminator: generates an output signal only if amplitude of input pulse is greater than a given threshold



 delay introduced to compensate for the trigger latency





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## Sampling a physics process



What happens if New trigger arrives while system is busy ? a) Each new trigger is accepted and "restarts" the process -> paralysable b) No new trigger is accepted until the process is complete -> non-paralysable



ms

## DAQ and Trigger with busy logic

- Busy logic avoids triggers while the system is busy in processing
  - E.g.: AND port and a latch
- Latch (flip-flop):
  - a bistable circuit that changes state (Q) by signals applied to the control inputs (SET, CLEAR)




### Deadtime

- Which (average) DAQ rate can we achieve now?
  - Reminder: w/ a clock trigger and  $\tau = 1$  ms the limit is 1 kHz
- Definitions



- f: average rate of physics phenomenon (input)
- $\nu$ : average rate of DAQ (output)
- τ: deadtime, the time the system requires to process an event, without being able to handle other triggers
- probabilities: P[busy] =  $v \tau$ ; P[free] = 1  $v \tau$
- Therefore:

$$v = f P[free] \Rightarrow v = f(1 - v\tau) \Rightarrow v = \frac{f}{1 + f\tau}$$



# Derandomization





# A bit of queueing theory



- Efficiency vs traffic intensity ( $\rho = \tau / \lambda$ ) for different queue depths
  - $\rho > 1$ : the system is overloaded ( $\tau > \lambda$ )
  - $\rho \ll 1$ : the output is over-designed ( $\tau \ll \lambda$ )
  - $\rho \sim 1$ : using a queue, high efficiency obtained even w/ moderate depth



# **CPUs and data buses**

# **Deadtime and Efficiency**

• Due to stochastic fluctuations

- DAQ rate always < physics rate 
$$v = \frac{f}{1+f\tau} < f$$

- Efficiency always < 100% 
$$\epsilon = \frac{N_{saved}}{N_{tot}} = \frac{1}{1+f\tau} < 100\%$$

 So, in our specific example

$$\begin{array}{c|c} f = 1 \, kHz \\ \tau = 1 \, ms \end{array} \rightarrow \begin{array}{c|c} \nu = 500 \, Hz \\ \epsilon = 50 \, \% \end{array}$$





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# The CPU does the rest...



### Microprocessor architecture





# CPU (a simplified view)

- Does computation on words
  - Sequences of bits (32,64,128...)
- Fetches instructions from memory (the "program")
- Instructions are bit codes corresponding to an operation (part of an "instruction set")
  - They are first decoded and then fed to an Arithmetic and Logical Unit (ALU) that performs the operation on data contained in registers (e.g. add, complement, compare, shift, move...)
  - A **program counter** keeps track of the current location in the program being executed
- Data (as instructions) are fetched (usually) from memory over a bus
  - A **bus controller** handles the communication with memory and other I/O peripherals (such as a DAQ board, for example)





# CPU (a simplified view)

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 Modern CPUs have a more or less large "cache" – fast memory that contains recently or frequently accessed data for quick retrieval (not requiring access to the memory bus)

The **stack** is a portion of memory that works like a LIFO: there are two fundamental instructions PUSH and POP to move data to and from the stack, and a **stack pointer** always pointing at the top – more on this (maybe) later This simplified view is common (give or take few parts) to many different architectures, **including** those specific to DAQ and trigger (the CPU could be just an "intelligent bus master") 4 July 2021



# Back to DAQ

- In complex experiments, many channels are received by multiple electronic boards interconnected by a data bus – a computer may not be the most convenient form factor for this, we use modular electronics
  - In an architecture similar to the one discussed before at least one particular element on the BUS is a CPU
  - It is common to use the bus (e.g. VME) to collect data from multiple boards in a single portion of memory



Parallel (e.g. VME) Shared lines



Serial (e.g. PCle) Point-to-point connections



# A note about parallel vs. serial

Parallel Buses Are Dead! (RT magazine, 2006)

What is wrong about "parallel"?

- You need lots of pins on the chips and wires on the PCBs
- The skew between lines limits the maximum speed

What is wrong about "bus"?

- Speed is a function of the length (impedance) of the lines
- Communication is limited to one master/slave pair at a time (no scalability)
- The handshake may slow down the maximum speed

•

#### All parallel buses are dead. All? No!

- There is lots of legacy equipment
- VMEbus is still used heavily (military / research)







	VME64x Pl Connector								
Pin	Signal Name	Signal Name	Signal Name	Signal Name	Signal Name				
	Row z	Row A	Row B	Row C	Row d				
				$\frown$					
1	MPR	D00	BBSY*	D08	V PC				
2	GND	D01	BCLR*	D09	GND				
3	MCLK	D02	ACFAIL*	D10	+1 V				
4	GND	D03	BG0IN*	D11	+∀2				
5	MSD	D04	BG0OUT*	D12	RsvU				
6	GND	D05	BG1IN*	D13	- V 1				
7	MMD	D06	BG1OUT*	D14	- V 2				
8	GND	D07	BG2IN*	D15	RsvU				
9	MCTL	GND	BG2OUT*	GND	G AP*				
10	GND	SYSCLK	BG3IN*	SYSFAIL*	G A0				
11	RESP*	GND	BG3OUT*	BERR*	GA1				
12	GND	DS1*	BR0*	SYSREST*	+3.3v				
13	RsvBus	DS0*	BR1*	LWORD*	GA2*				
14	GND	WRITE*	BR2*	AM5	+3.3♥				
15	RsvBus	GND	BR3*	A23	GA3*				
16	GND	DTACK*	AM0	A22	+3.3♥				
17	RsvBus	GND	AM1	A21	GA4*				
18	GND	AS*	AM2	A20	+3.3♥				
19	RsvBus	GND	AM3	A19	RsvBus				
20	GND	IACK*	GND	A18	+3.3V				
21	RsvBus	IACKIN*	SERCLK	A17	RsvBus				
22	GND	IACKOUT*	SERDAT*	A16	+3.3♥				
23	RsvBus	AM4	GND	A1.5	RsvBus				
24	GND	A07	IRQ7*	A14	+3.3₹				
25	RsvBus	A06	IRQ6*	A13	RsvBus				
26	GND	A0.5	IRQ5*	A12	+3.3V				
27	RsvBus	A04	IRQ4*	A11	LI/I*				
28	GND	A03	IRQ3*	A10	+3.3♥				
29	RsvBus	A02	IRQ2*	A09	LI/O*				
30	GND	A01	IRQ1*	A08	+3.3♥				
31	RsvBus	-12V	+5V Standby	+12∀	GND				
32	GND	+5V	+5v	+5V	V PC				

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L. Davis

# Serial: PCIe (aka PCI Express)

- Not a bus any more but a point-to-point link
- Data not transferred on parallel lines but on one or several serial lanes
  - Lane: One pair of LVDS lines per direction
  - Clock rate: 2.5 GHz (PCIe2.0: 5 GHz, PCIe 3.0: 8 GHz, PCIe 4.0: 16 GHz)
  - 8b/10b encoding (from PCIe3.0: 128/130b encoding)

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- 250 MB/s (PCIe 1.0) raw transfer rate per lane
- Devices can support up to 32 lanes





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# Example: VME write

#### Example: (Simplified) write cycle



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# **Event Building**

- In large experiments consisting of many different sub-detectors, read out is performed by different boards and by multiple CPUs (for example in multiple VME crates)
  - We want to combine all the portions corresponding to the same "event" in the memory of a single CPU for processing and eventually storage
- Need a mechanism to associate all data corresponding to the same event (e.g. a bunch crossing in a collider, a gamma ray shower in a telescope array...)
  - This process is called event building





# **Event Building**

• Event Building used to be performed on the bus itself







- But the bus forces the process to be sequential (only one board can "speak" at a time)
- It is also not infinitely extendible (does not "scale")
- In all LHC experiments event building is performed by distributed processes through a **switched network**





# **Ethernet switch**

Layer-2 device

- Switches frames to their destination using the MAC address
- · Learns the address associated to each port and stores it in a table



Port	MAC Address
2	00:01:02:ab:cd:ef(John)
4	01:02:03:a1:b2:c3(Matt)



# **Event Building**





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# Questions?

# **Additional topics**

# DAQ-with-a-scope

# Real-life example: measuring energy



Measure the energy deposited by a particle traversing a (special) medium The (detector) medium is a <u>scintillator</u>  $\rightarrow$  The molecules, excited by the passing particle, relax emitting light The amount of light is proportional to the deposited energy The light is then collected, using dedicated optical means (light guide), and fed into a photodetector  $\rightarrow$  photomultiplier



# Photomultiplier Tube (PMT)





*Photo cathode*: photon to electron conversion via photo-electric effect Typical quantum efficiency  $\approx$ 1-10% (depends on material and wavelength) *Dynodes*: electrodes that amplify the number of electrons thanks to secondary emission Typical overall gain  $\approx 10^6$ Dark current: current flowing in the PMT without light  $\rightarrow$  noise



# **Pulse signals**

- Sensors and signals the analog way
- Acquiring data from sensors
- Practical introduction with real life example
  - Historical Perspective
  - Examples taken from nuclear and (mostly) particle physics
     Feel free to bring your own example/problems for discussion
- From analog to digital: introductory notions

Since most sensor produce an electrical pulse in response to the phenomenon they measure, it is important to define and understand a certain number of concepts concerning a pulse signal



#### • Approximate Q measurement using oscilloscope

- Linear approximation of a exponential decay





• Approximate Q measurement using oscilloscope

- Linear approximation of a exponential decay





• Approximate Q measurement using oscilloscope

- Linear approximation of a exponential decay





### Approximate Q measurement using oscilloscope

Linear approximation of a exponential decay Easy, but...

Deadtime 5 min, ~ 3 10<sup>-3</sup> measurements per second (if you are good) Necessary to encode data into some sort of electronic format by hand **It would be much more convenient to have a direct electronic measurement: a data acquisition system** Save data in some digital format, fill a histogram on-line, etc ...

N.B.: the oscilloscope method is still fundamental it allows for the validation of your DAQ yes, you should never trust it a priori! Always remember the oscilloscope also has limitations – we will discuss them later





# Memory access



### Programmed data transfer

A software routine residing in memory requests the peripheral device for data transfer. Programmed data transfers are generally used when a small amount of data is transferred with relatively slow I/O devices.

### Direct Memory Access (DMA) or "block" transfer

In this mode, the data transfer is controlled by the peripheral device. DMA transfer is used when a large block of data is to be transferred.



# Addressing memory (simplified view)



 $\begin{array}{c} & & & \\ \hline Data ln & & \\ \hline \\ R/\overline{W} & \\ \hline \\ R/\overline{W} & \\ \hline \end{array}$ 

urce: Heuring - Jordan: Computer Systems Architecture and Design

The address space is the total number of unique addresses available to the CPU. Different addressable devices are "mapped" to different unique address ranges for access over the bus

Source: Heuring – Jordan: Computer Systems Architecture and Design



# **Decoding addresses**



Name	Value	0 ns	200 ns	400 ns	600 ns
🕨 號 a[1:0]	11	00	01 10		11
🕨 📲 b[3:0]	0001	0001	0010 0100		1000 0001

library IEEE; use IEEE.STD\_LOGIC\_1164.all;

```
entity decoder is
  port(
    a : in STD_LOGIC_VECTOR(1 downto 0);
    b : out STD_LOGIC_VECTOR(3 downto 0)
    );
end decoder;
architecture bhv of decoder is
begin
```



end bhv;



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# Mapping addresses: example

The processor can usually address a memory space that is much larger than the memory space covered by an individual device or memory chip.

In order to splice a device into the address space of the processor, decoding is necessary.

For example, the Intel 8088 had 20bit addresses for a total of 1MB of memory address space.

However, the BIOS on a 2716 EPROM had only 2KB of memory and 11 address pins

A decoder was used to decode the additional 9 address pins and allow the EPROM to be placed in any 2KB section of the 1MB address space





### Modernized Geiger-Marsden Experiment





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# Modernized Geiger-Marsden Experiment



- Can you identify the elements in the diagram ?
- Scaler in nuclear instrumentation is another word for "<u>counter</u>"
- Counting pulses over some threshold (or in a certain window of amplitude) is a common need
- We will discuss counters in the context of digital logics
- The anticoincidence A not B is a form of trigger





# Counting pulses: discriminator



Combining signals from multiple discriminators may require compensating for the different length of the signal paths



Trigger signal
## Important aspects of discriminators

### Pulse width

- Limits the effective hit rate
- Must be adapted to the desired trigger rate

### Time walk

- ▶ The threshold-crossing time depends on the signal amplitude
- Must be **minimal** good trigger systems



Time walk can be suppressed by triggering on total signal fraction

- ► Applicable on same-shape input signals with different amplitude
- Scintillator detectors and photomultipliers





## **Constant fraction discriminator**

Signals with the same rising time, at a fraction **f** 

$$\Delta t_f = t(f \cdot A_0) - t(A_0) = \text{const.}$$
  
 
$$A(t)/f - \cdot A(t - \Delta t) = 0 \text{ at } t = t_{CDF}$$



- Attenuation + configurable delay
  - applied before the discrimination determines *t*<sub>CFD</sub>
- If delay too short, the unit works as a normal discriminator
  - the output of the normal discriminator fires later than the CFD part









# Possibly useful information

## Analog and digital: basic concepts





represented by

waveform A

.



A 3-digit binary number is replaced for each octal digit, and vice versa. The 3-digit terms are then grouped (or octal terms are grouped). A 4-digit binary number is replaced for each hex digit, and vice versa. The 4-digit terms are then grouped (or hex terms are grouped).



## **Negative integers**

In 2's complement representation a negative number is represented by a binary which results in zero when added to its corresponding positive

Decimal to 2's complement	DECIMAL	SIGN-MAGNITUDE	COMPLEMENT
	+7	0000 0111	0000 0111
+ $4_{10}$ to 2's complement	+6	0000 0110	0000 0110
	+5	0000 0101	0000 0101
true binary = 0010 1001	+4	0000 0100	0000 0100
2's comp = 0010 1001	+3	0000 0011	0000 0011
	+2	0000 0010	0000 0010
	+1	0000 0001	0000 0001
$-41_{10}$ to 2's complement	0	0000 0000	0000 0000
	-1	1000 0001	1111 1111
true binary $= 0010\ 1001$	-2	1000 0010	1111 1110
1's comp = 1101 0110	-3	1000 0011	1111 1101
Add 1 = $+1$	-4	1000 0100	1111 1100
2's comp = 1101 0111	-5	1000 0101	1111 1011
	6	1000 0110	1111 1010
	-7	1000 0111	1111 1001
	-8	1000 1000	1111 1000

#### Subtracting



the long way

In 2's complement representation the procedures for adding and subtracting positive and negative numbers are the same



## More on response, power spectra, Bode diagram

- When discussing the response of any kind of circuit or device, the range of frequencies delimited by the points at which the response falls by 3 dB is defined as the bandwidth and represents the range of accepted frequencies.
- When referring to measurements of power quantities, a ratio can be expressed as a level in decibels by evaluating ten times the base-10 logarithm of the ratio of the measured quantity to the reference value. Thus, the ratio of P (measured power) to P0 (reference power) is represented by LP, that ratio expressed in decibels: the base-10 logarithm of the ratio of the measured quantity to the reference value.

