GEANT4 simulation of X-ray transition radiation at small angles

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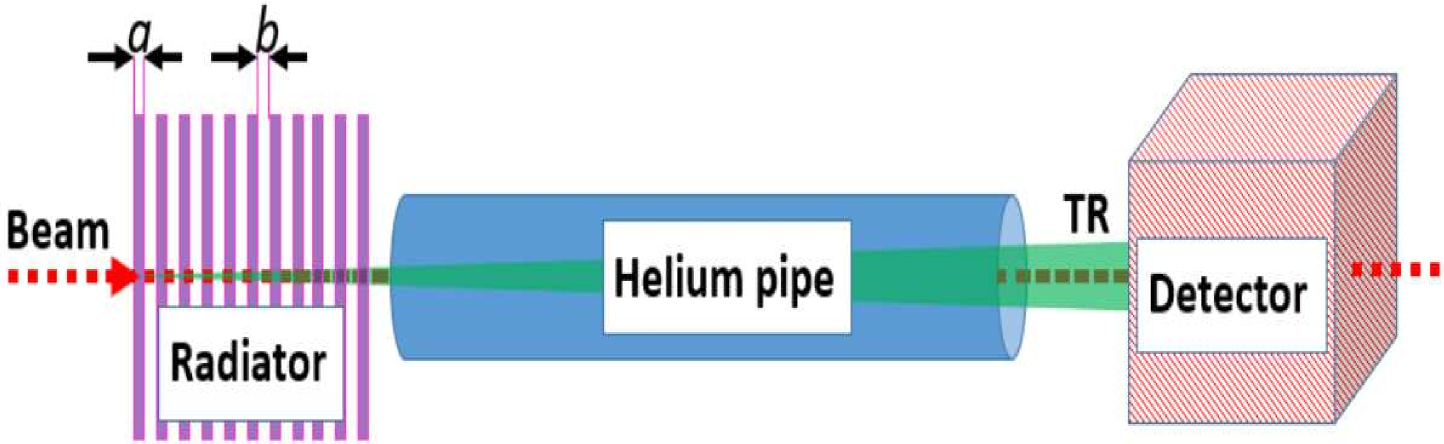
Abstract

GEANT4 simulation of X-ray transition radiation at small angles (less than 1 mrad) is discussed taking into account recent experimental data from the ATLAS TRT test beam (electrons 20 GeV, regular radiator thirty 50 μm thick mylar foils separated by $\sim$2.96 mm air gaps).

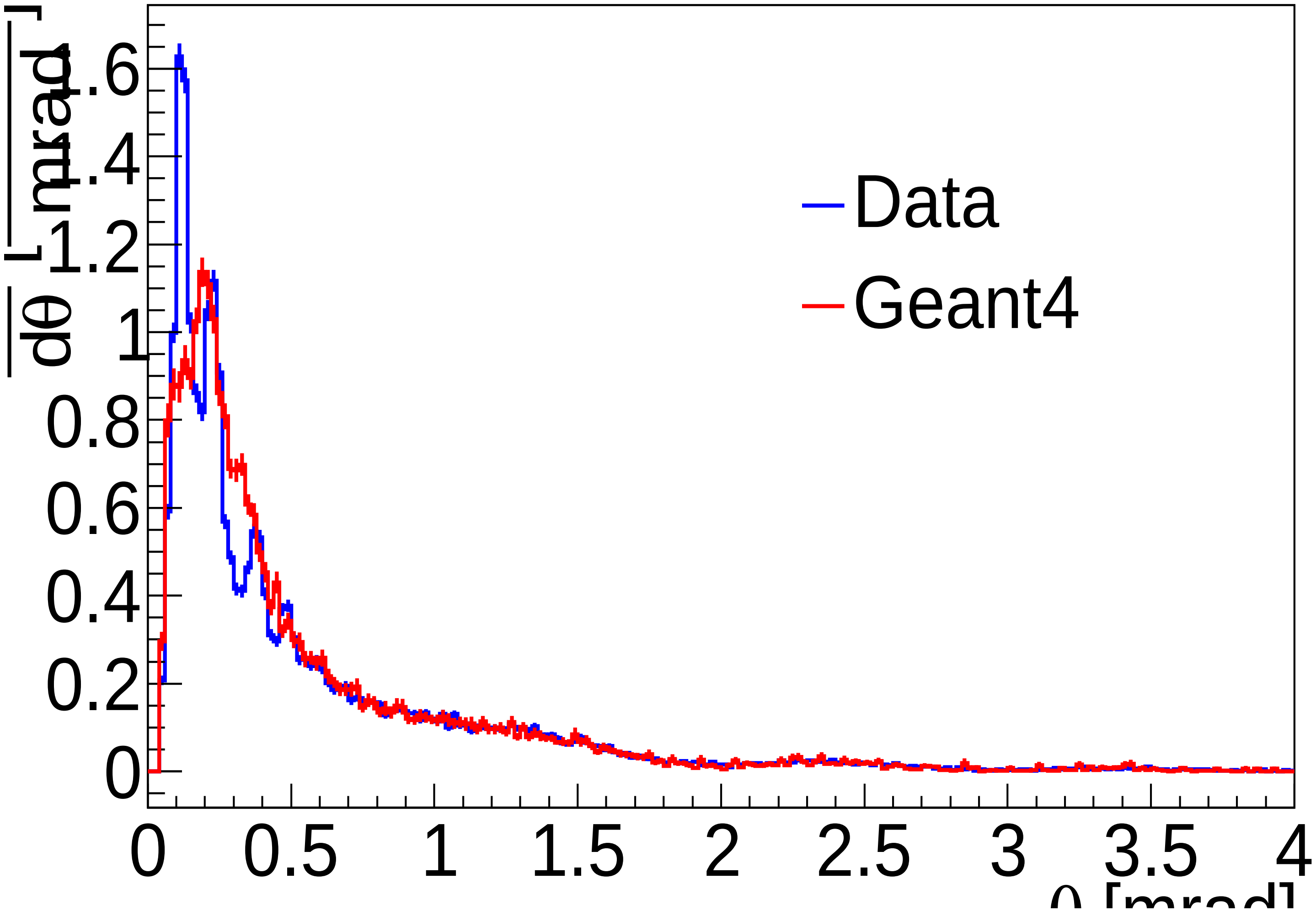
1 Motivation

1. The ATLAS TRT team based on the test beam results required to improve the X-ray transition radiation (XTR) angle distribution for the angles less than milliradian. A way how to improve the resolution was proposed by the TRT team.

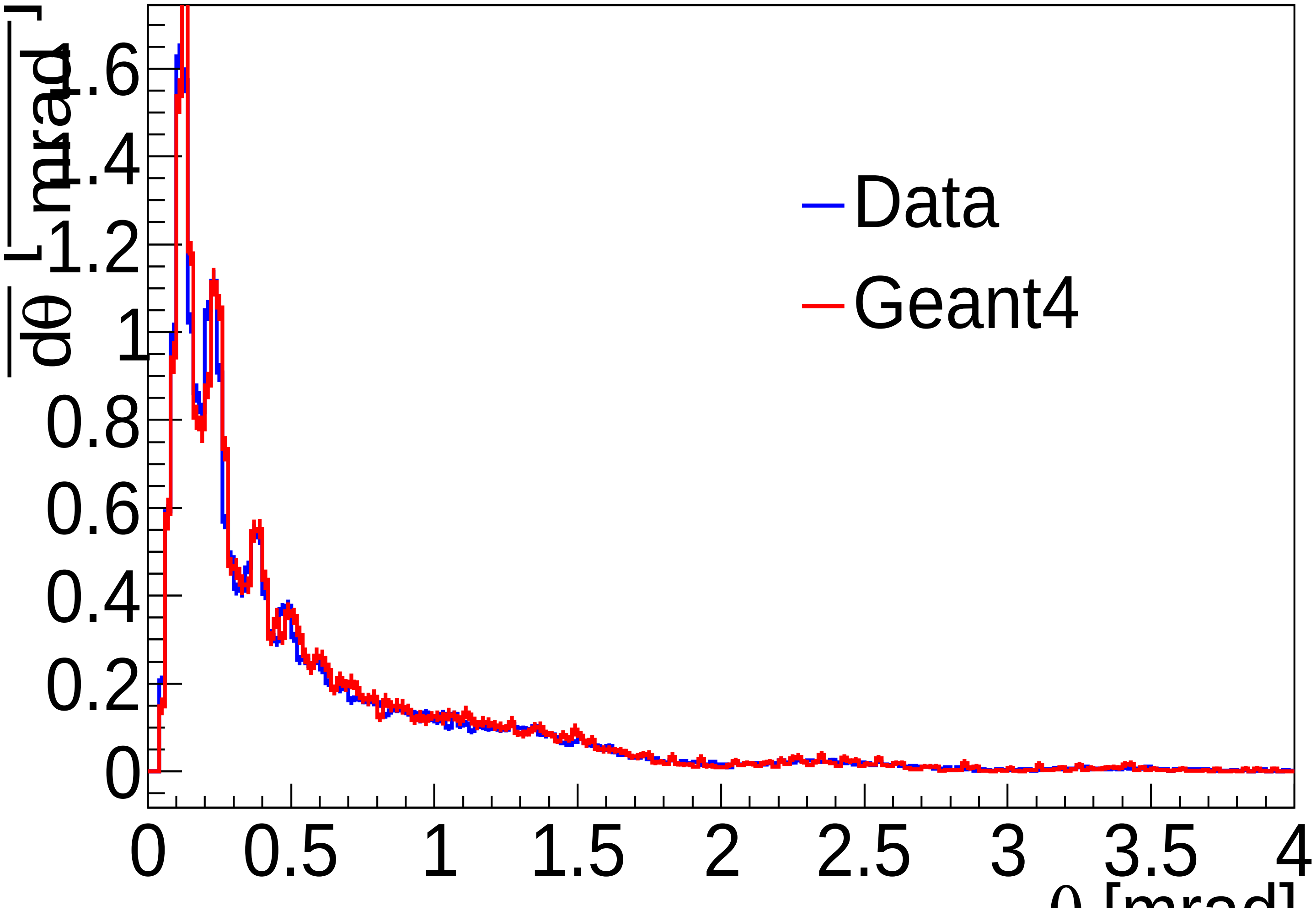
2. The TRT team kindly sent us the GEANT4 test beam example (based on TestEm10) and experimental data with background.



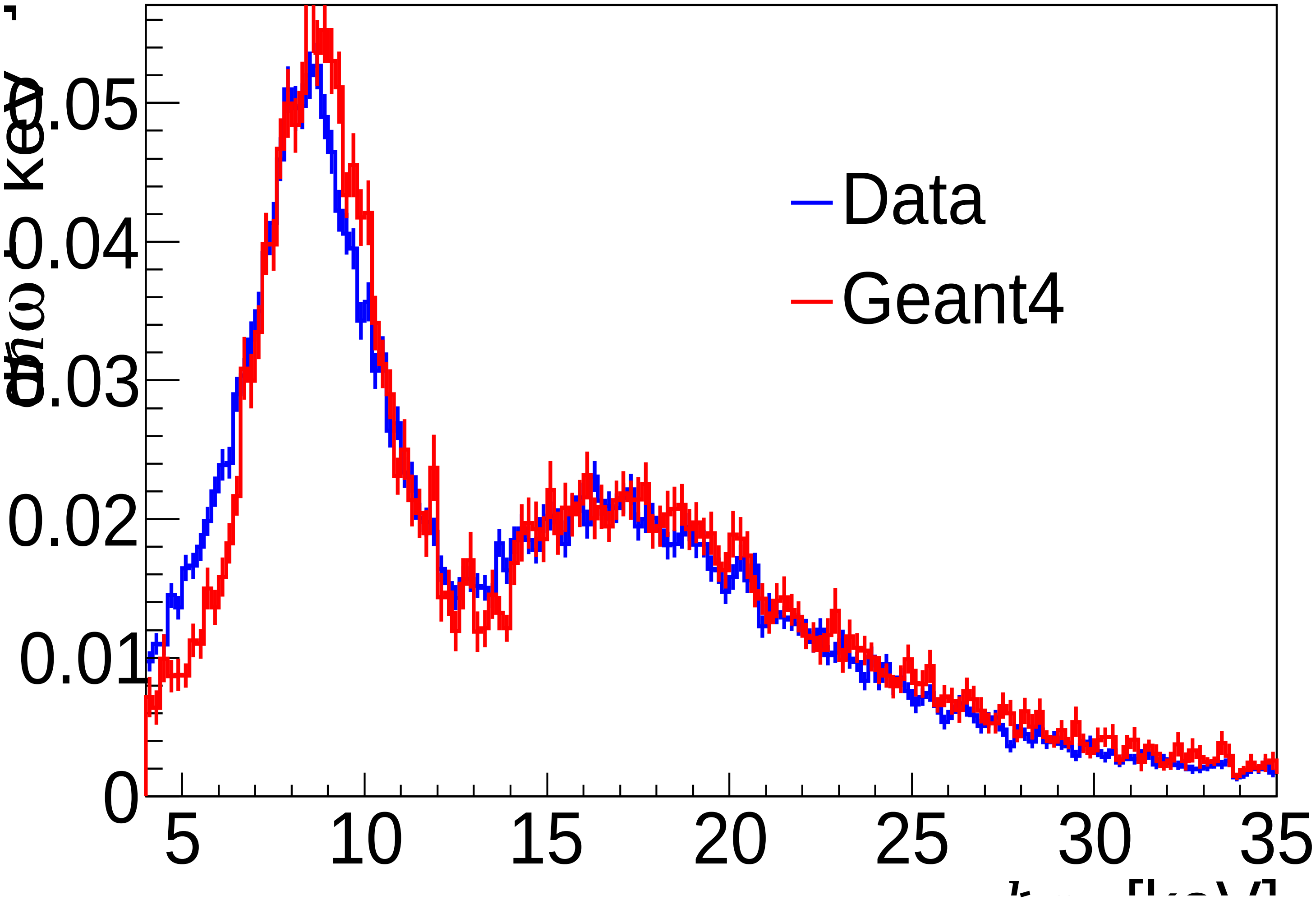
ATLAS test beam layout for XTR detecting at small angles (20 GeV e^-, thirty 50μm thick mylar foils, 2.96 mm air gaps, 2 m He pipe, pixel Si)



GEANT4 simulation of the XTR small angles (the XTR angle spectrum)



ATLAS TRT simulation of the XTR small angles (the XTR angle spectrum)



GEANT4 simulation of the XTR small angles (the XTR energy spectrum)

2 GEANT4 formalism for regular radiators

The integration of the double (angle, frequency) differential XTR spectrum:

$$\frac{d^2\bar{N}_{in}}{d\omega\, d\theta^2} = \frac{2\alpha}{\pi c^2}\omega\theta^2 \mathrm{Re}\left\{\langle R^{(n)}\rangle\right\},$$

in respect to θ^2 can be simplified for the case of regular radiator ($\nu_{1,2} \to \infty$) with media transparent for XTR photons, and $n \gg 1$. Then the radiator factor $\langle R^{(n)}\rangle$ reads:

$$\langle R^{(n)}\rangle = 2\sin^2\varphi_1 \frac{\sin^2 n\varphi}{\sin^2\varphi} \xrightarrow{n\gg 1} 2n\sin^2\varphi_1 \sum_k \delta(\frac{\varphi}{\pi} - k),$$

$$\varphi = \varphi_1 + \varphi_2, \quad \varphi_j = \frac{t_j}{4L_j} \quad L_j = \frac{c}{\omega}\left[\gamma^{-2} + \frac{\omega_j^2}{\omega^2} + \theta^2\right]^{-1}.$$

The energy spectrum of emitted XTR photons can be expressed as a finite sum:

$$\frac{d\bar{N}_{in}}{\hbar d\omega} = \int_0^{\theta^2_{max}} d\theta^2 \frac{d^2\bar{N}_{in}}{\hbar d\omega\, d\theta^2} \simeq \frac{4\alpha n}{\pi\hbar\omega}(C_1+C_2)^2 \cdot \sum_{k=k_{min}}^{k_{max}} \frac{(k - C_{min})}{(k - C_1)^2(k + C_2)^2}\sin^2\left[\frac{\pi t_1(k + C_2)}{t_1 + t_2}\right],$$

$$C_{1,2} = \frac{t_{1,2}(\omega_1^2 - \omega_2^2)}{4\pi c\omega}, \quad C_{min} = \frac{1}{4\pi c}\left[\frac{\omega(t_1 + t_2)}{\gamma^2} + \frac{t_1\omega_1^2 + t_2\omega_2^2}{\omega}\right],$$

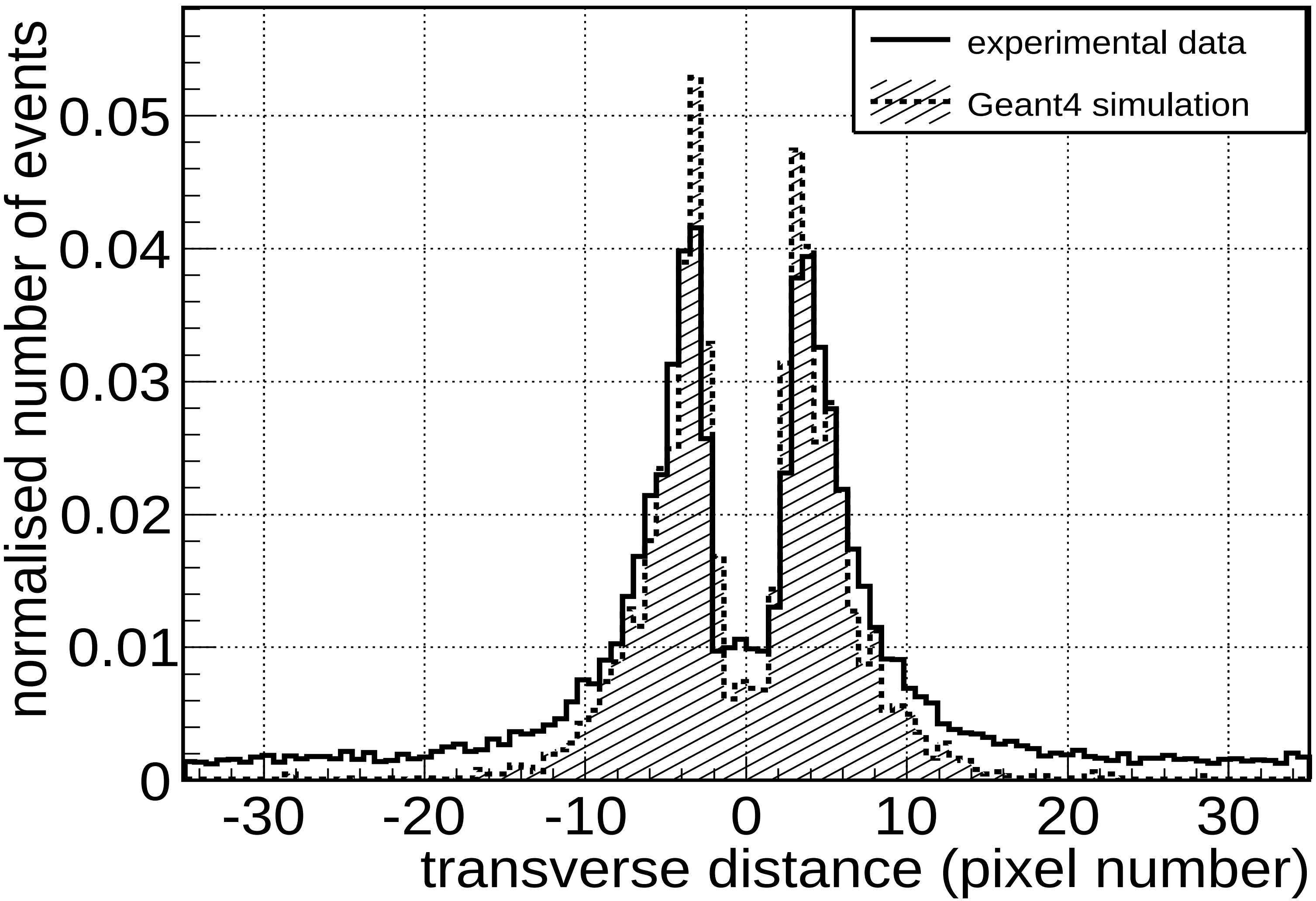
where $\hbar$ is the Planck's constant. The sum in $d\bar{N}_{in}/\hbar d\omega$ is defined by the terms with integer $k \gtrsim k_{min}$ corresponding to the region of $\theta \gtrsim 0$. Therefore k_{min} should be the nearest to C_{min} integer $k_{min} \geq C_{min}$. Each term in the sum of spectrum $d\bar{N}_{in}/\hbar d\omega$ corresponds to the emitting angle θ_k^2:

$$\theta_k^2 = \frac{4\pi c}{\omega(t_1 + t_2)}|k - C_{min}|.$$

Then the angle spacing is:

$$\Delta\theta^2 = \theta_{k+1}^2 - \theta_k^2 = \frac{4\pi c}{\omega(t_1 + t_2)}.$$

This value corresponds to the angle bin ($\sqrt{\Delta\theta^2}$) of about 0.3 mrad for the typical XTR energy of 10 keV generated by charged particle with the Lorentz factor $\sim 10^4$ in the typical radiator with $t_1 + t_2 \sim 0.3$ mm. It is not enough for measurements with high angle resolution.



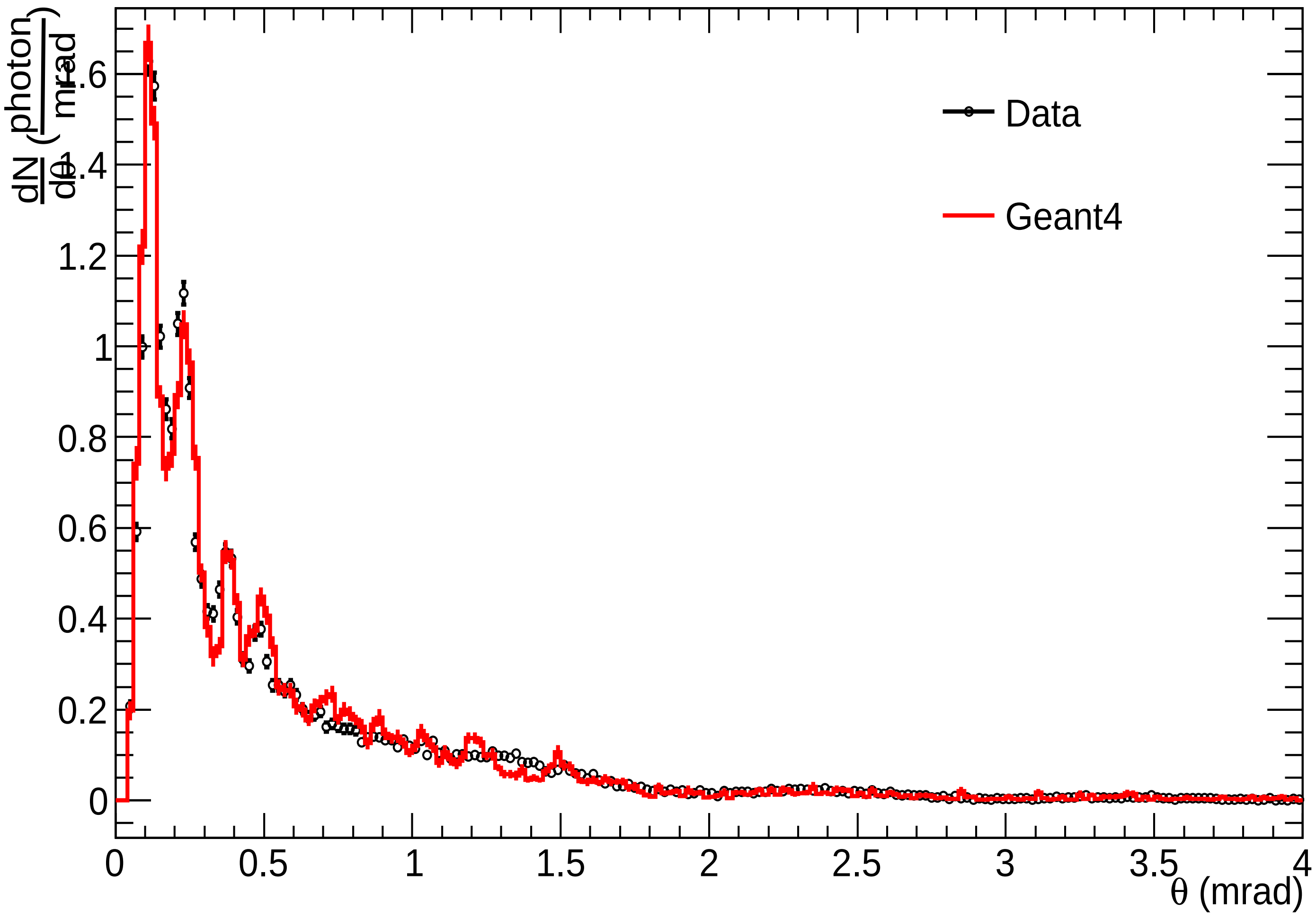
GEANT4 simulation of XTR for the DESY test beam (pixel∼1 mrad) data (∼ 2011).

3 XTR angle distribution in sub-milliradian range

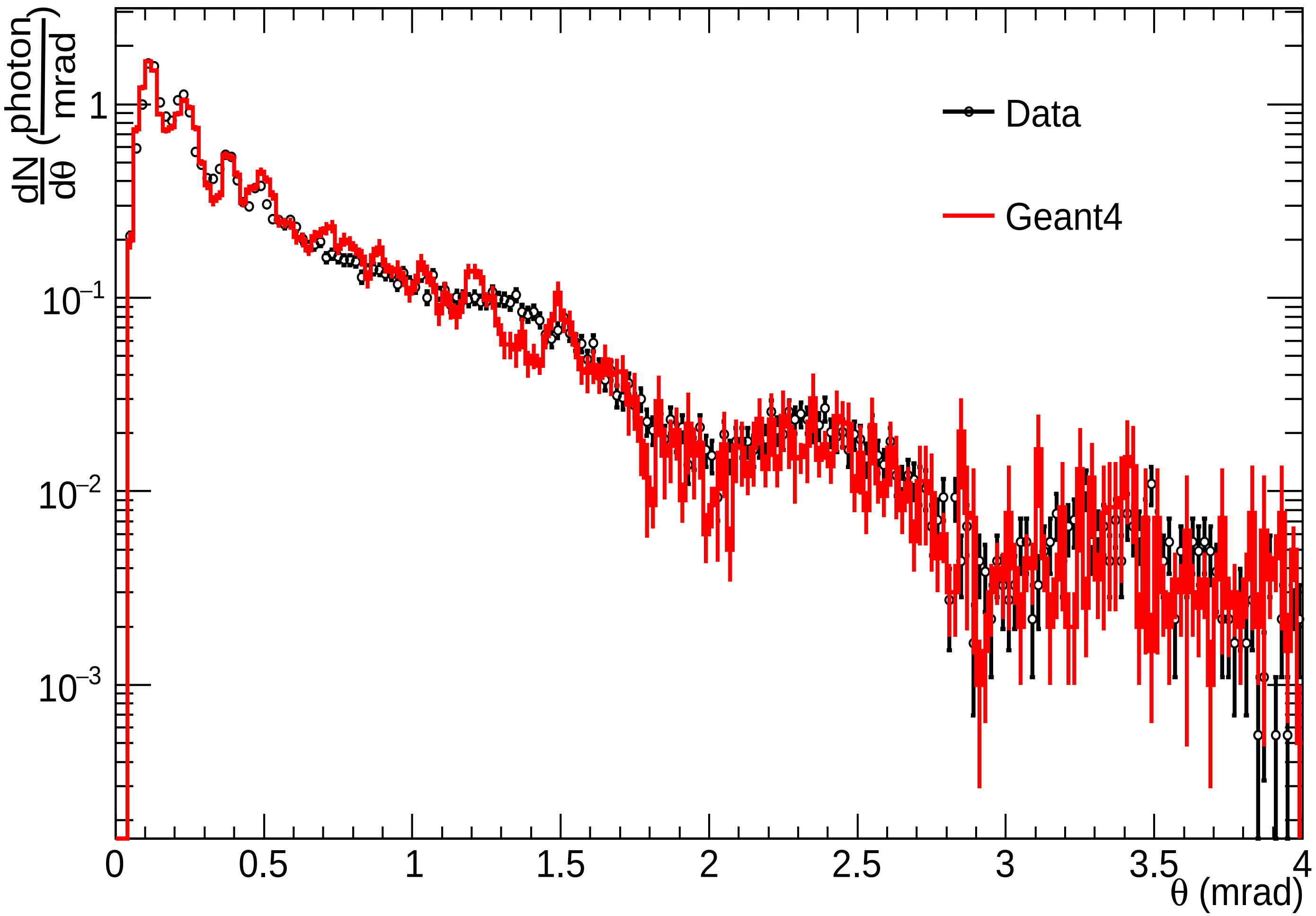
GEANT4 XTR library, even for regular radiators, allows a user to integrate over θ^2 by numerical methods, since it utilizes the radiator medium absorption and the relations for the XTR yield don't have singularities.

The point is the selection of the number of bins and their thickness. It results in, however, worse perfomance, if one use models with Γ-distributed gap thicknesses, while the transparent regular model produces additional peaks.

The optimization for the angle resolution versus perfomance is under investigation ...



Updated GEANT4 simulation of the XTR small angles (the XTR angle spectrum, the transparent regular radiator)



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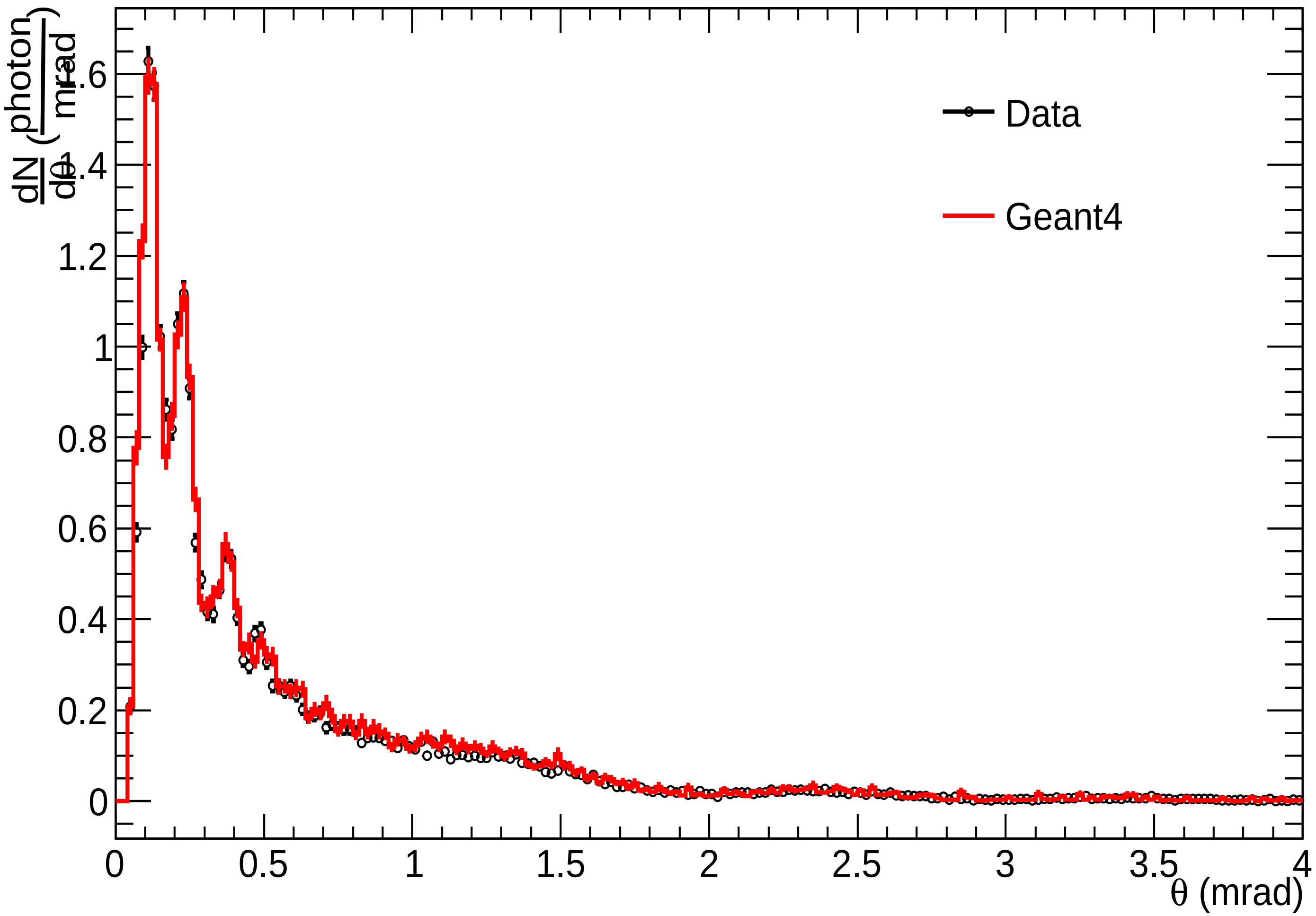
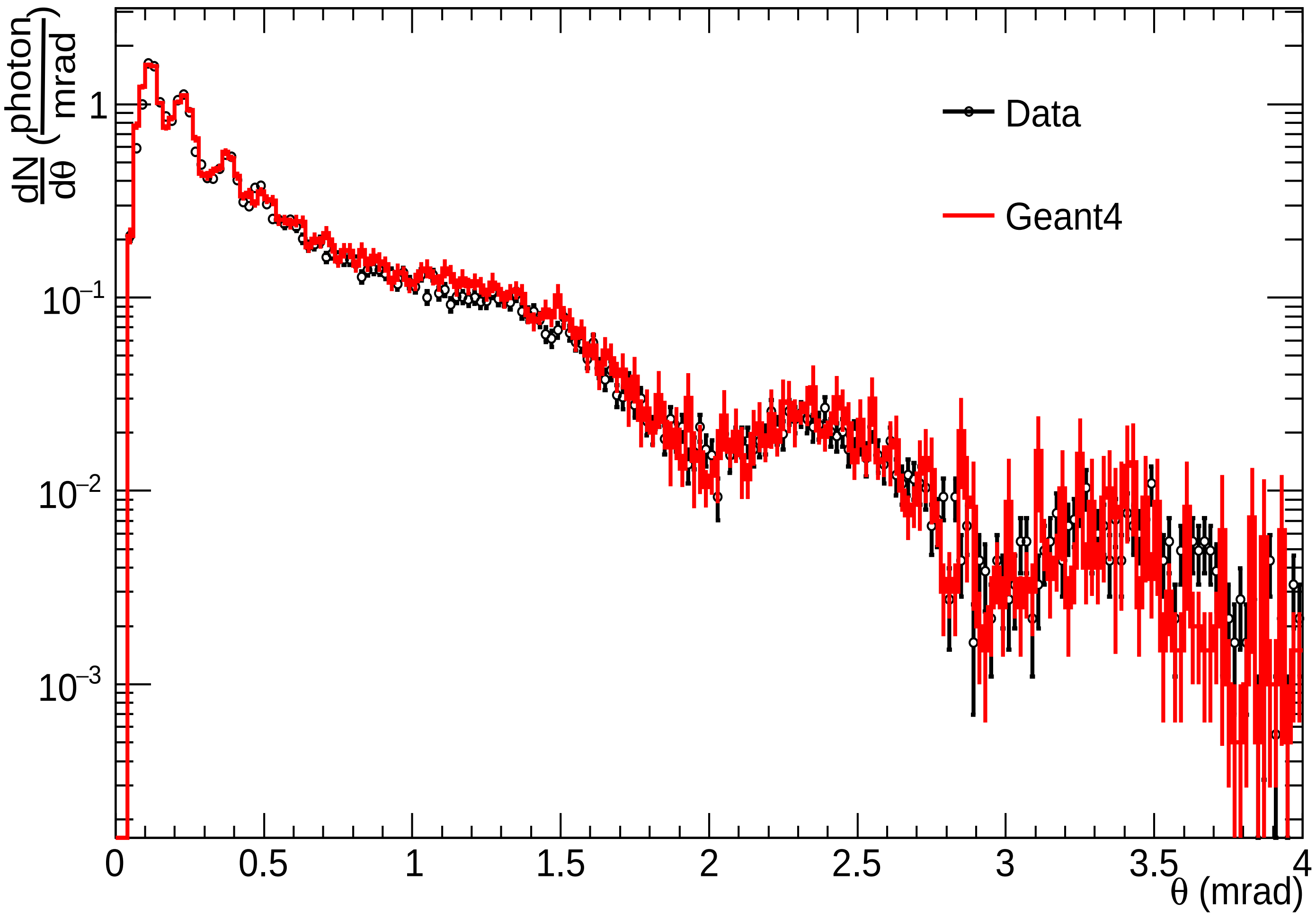


Updated GEANT4 simulation of the XTR small angles (the XTR angle spectrum, the gamma 1000/1000 radiator). Irregularity, $\sim$3%



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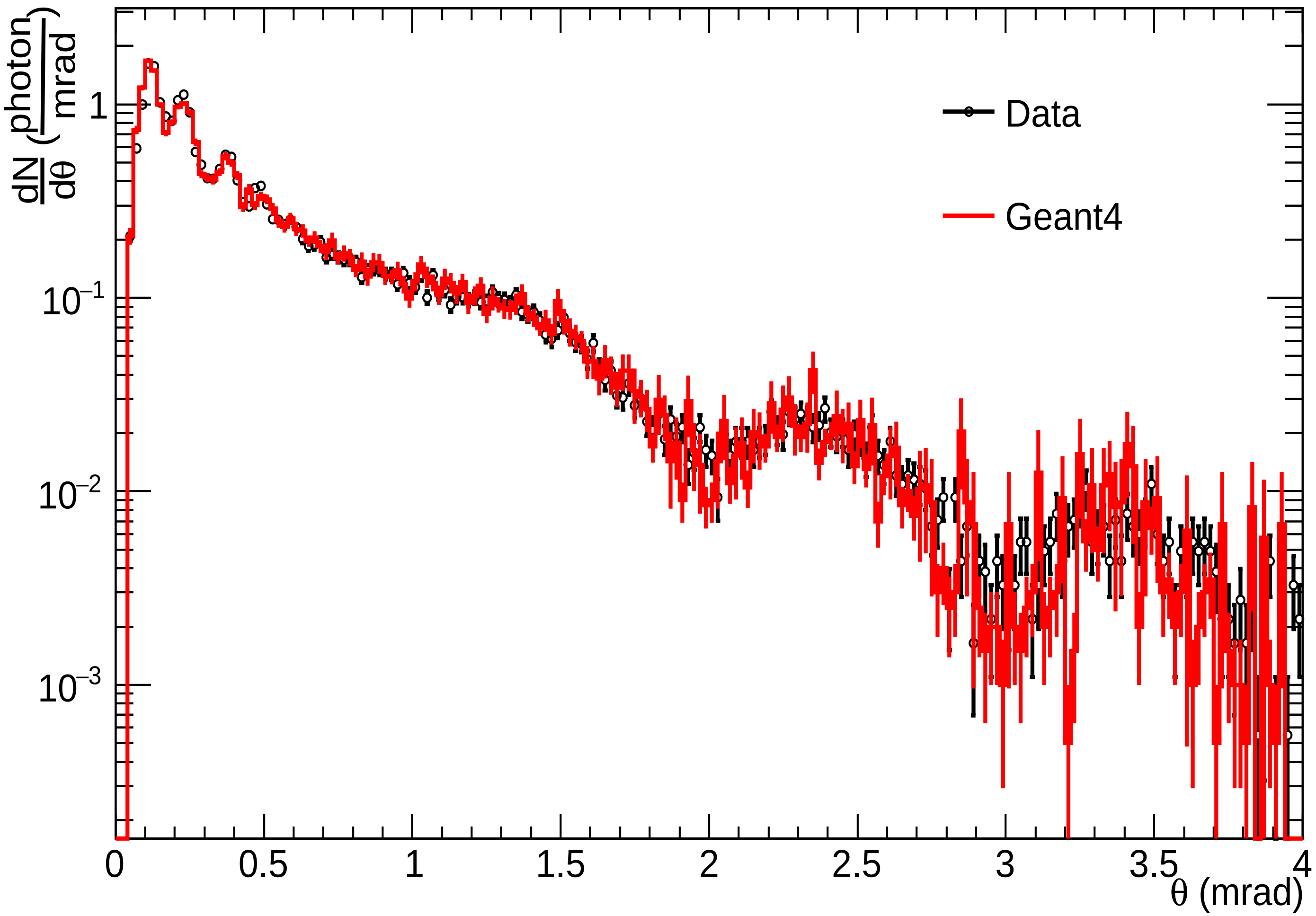
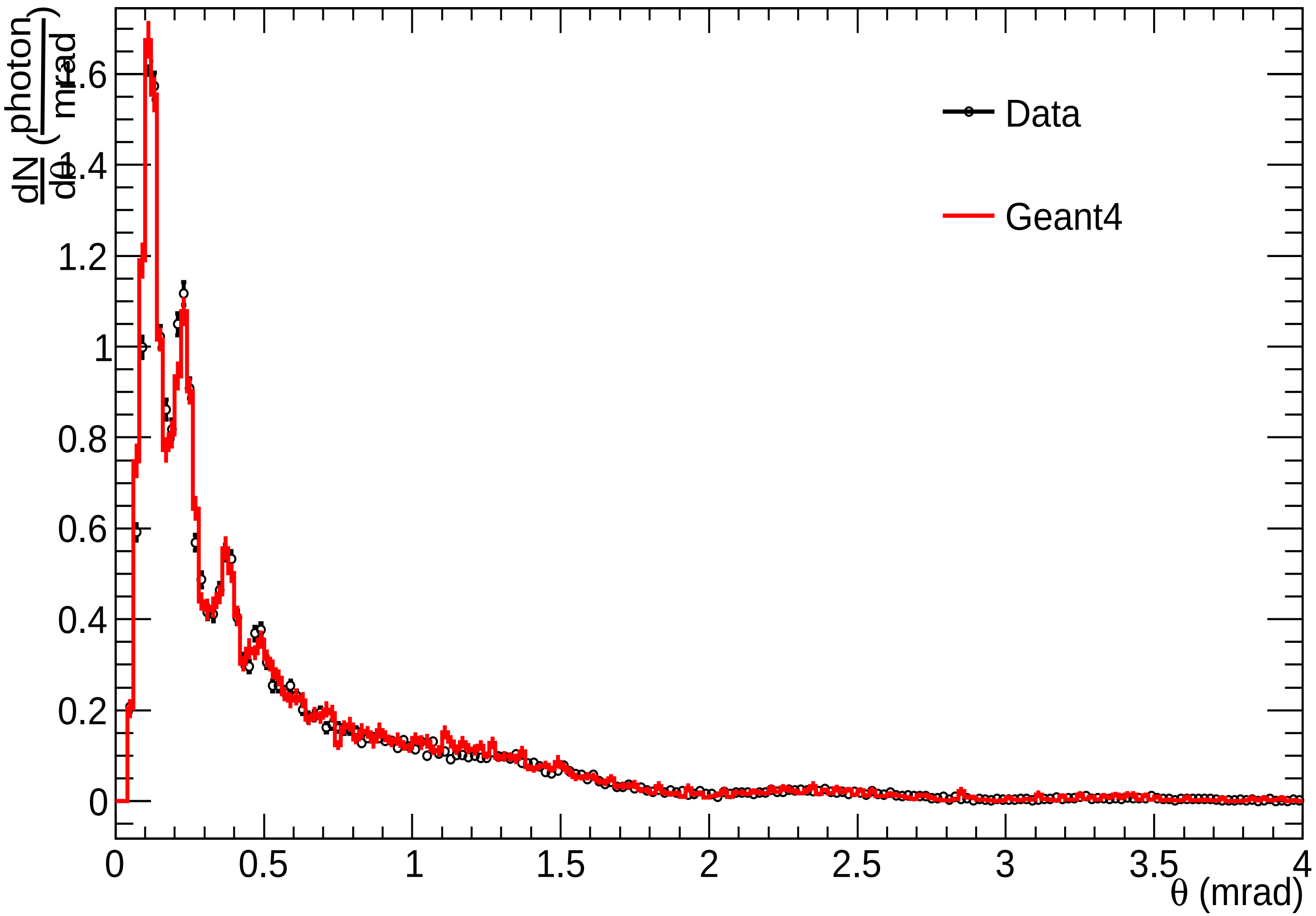


Updated GEANT4 simulation of XTR angle spectrum. Transparent regular radiator with Gauss fluctuating gas gaps (plate∼1%, gas∼2.5%).



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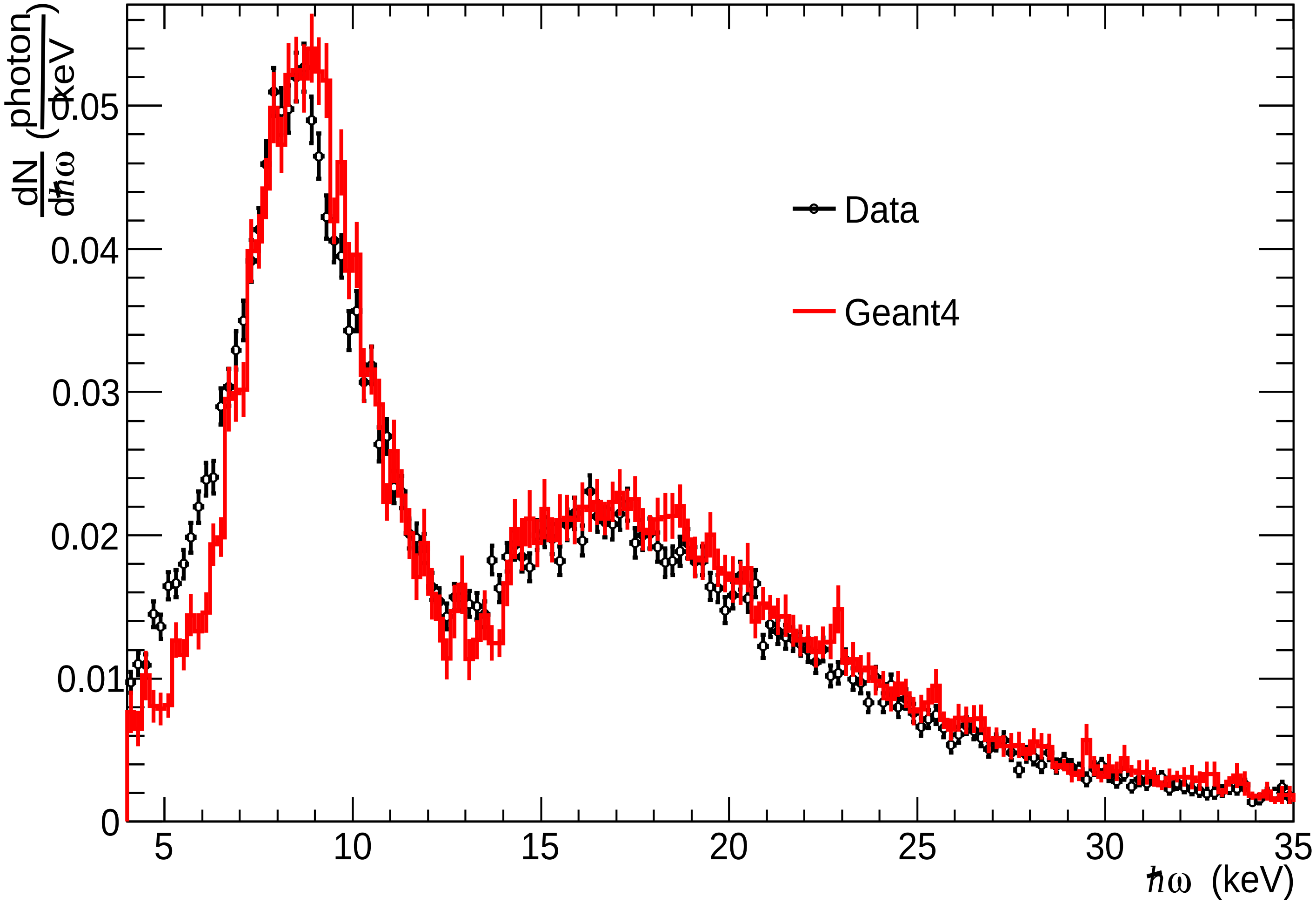

Updated GEANT4 simulation of XTR small angles (XTR energy spectrum).
Transparent regular radiator with Gauss fluctuating gas gaps.

4 Conclusions

1. The accuracy for the X-ray transition radiation angle distribution in the sub-milliradian range was improved, utilizing the existing numerical integration with smaller bins.

2. The flexibility of the GEANT4 XTR library allows a user to play with the accuracy requirements in different ranges of angles and energies.

3. G4TransparentRegXTRadiator class updated includes both the gas and plate fluctuations according the Gauss distribution with RMS $\ll$ thicknesses, $\sigma_{gas} \ll t_{gas}, \quad \sigma_{plate} \ll t_{plate}$, ($\sim$few %).