Quantum Computing for Color Reconnection

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The team

- This talk is based on ongoing research by
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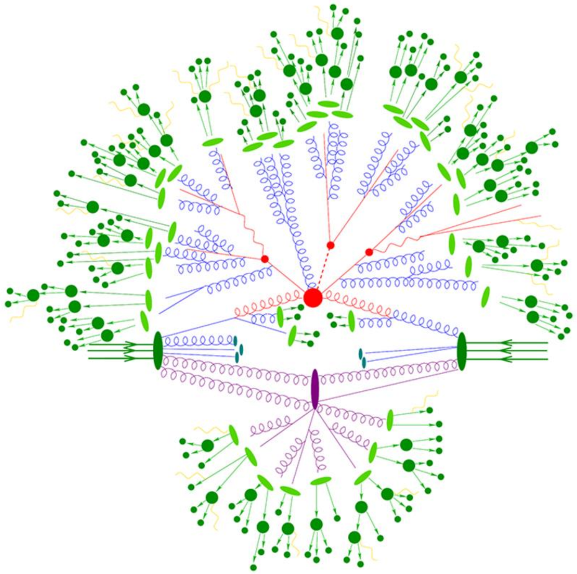
Physics motivation

- Data analysis at HEP colliders typically proceeds via the comparison of real experimental data to data simulated under various theory models.
- One of the stages of the simulation pipeline is the simulation of the non-perturbative QCD effects.
- Heuristic, phenomenological models are used for non-perturbative QCD.



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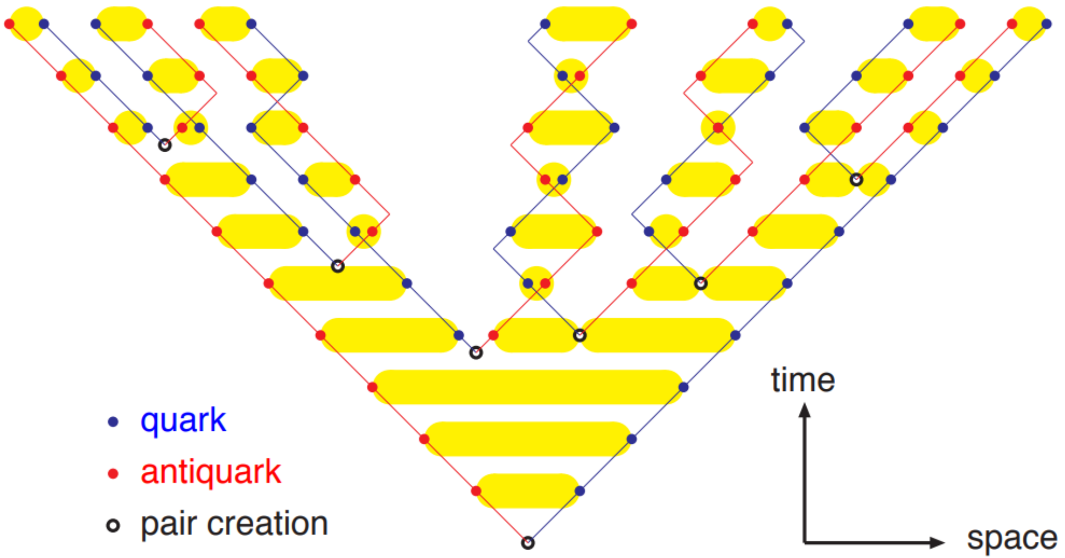
Physics motivation: Lund string model

- The Lund string model can be used to simulate the production of hadrons (observable particles) from bare partons produced by the hard scattering process.



Physics motivation: Lund string model

- Lund strings of the form $q - ng - \overline{q}$ are produced, which subsequently dissolve to produce hadrons (again heuristically)

Physics motivation: Color reconnection

- The model doesn't fit certain aspects of the data well.
- Allowing a scrambling of the color connections (to reduce an energy measure) improved the modeling. This is known as color **reconnection**.
- Which brings us to the problem we are trying solve using quantum computers…
- Given:
  - n quarks, n anti-quarks, and m gluons.
  - Weights w_{ij} between all the pairs of particles.
- Task:
  - Find a minimum energy valid connection of the quarks and gluons: $q_i - g_{a_1} \ldots - \overline{q_j}$
  - Energy of the configuration: sum of used edge weights

Improving the QCD model can improve the sensitivity of the experimental analyses

The Quantum Computing connection

- Step 1: Translate the task into a Quadratic Unconstrained Binary Optimization (QUBO) problem

$$F(\vec{x}) = \sum_{ij} A_{ij} x_i x_j + \sum_i B_i x_i + C\,, \qquad x_i \in \{0, 1\}$$

- Solve using Quantum techniques to minimize the Ising Hamiltonian.
  - D-wave
  - Quantum Approximate Optimization Algorithm (QAOA)

Converting to a QUBO problem

- One variable for each edge in the problem.
- The variables are indexed by (i, j)
  - $x_{ij} = 0$ (or 1) $\Rightarrow$ The edge between i and j is inactive (or active)
- Energy function is simple:

$$E = \sum_{ij} w_{ij} x_{ij}$$

- We need some constraints to avoid invalid assignments:
  - Each quark and anti-quark is used exactly once. Each gluon is used exactly twice.

$$\sum_{j} x_{ij} = 1, \qquad \text{for } i \in \text{quarks and anti−quarks}$$

$$\sum_{j} x_{ij} = 2, \qquad \text{for } i \in \text{gluons}$$

Incorporate constraints into the energy function

- Introduce penalty terms to incorporate the constraints…

$$E = \sum_{ij} w_{ij} x_{ij} + \lambda \sum_{i \in Q \cup A} \left[1 - \sum_j x_{ij}\right]^2 + \lambda \sum_{i \in G} \left[2 - \sum_j x_{ij}\right]^2$$

- This still doesn't avoid "subtours"

- It is not very efficient to introduce constraints to eliminate all the subtours.
  - Instead, we can simply minimize E.
  - If the minimizing state has a subtour, we can introduce a constraint for that subtour and try again…

Resource requirements for QAOA…

- Need one qubit/variable. One variable/edge.
- Number of edges:
  - $\frac{m(m-1)}{2}$ gluon – gluon edges
  - nm quark – gluon edges
  - nm anti-quark – gluon edges
  - n^2 quark – anti-quark edges

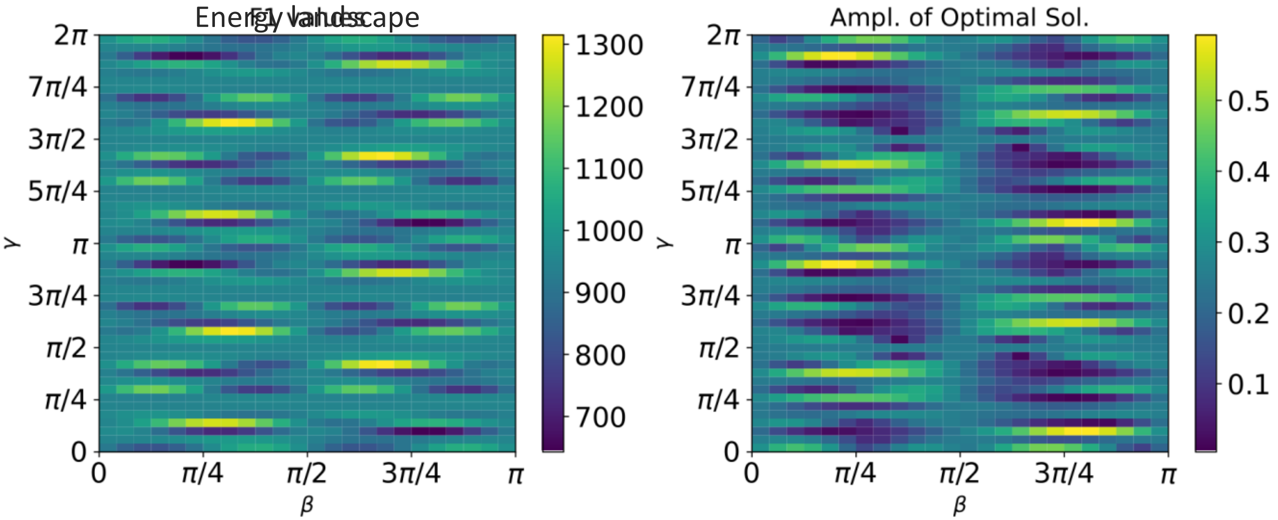
  - $O(n^2, m^2, nm)$ qubits
- Grows fast. The qubit count becomes large even for small toy problems
  - Even 2 quarks/anti-quarks, 6 gluons require 43 qubits
  - Typical color-reconnection problems in simulations involve hundreds of particles.
- **So, where do we go from here?**

Use classical techniques…

- Presently, in Pythia (an event generator program) color-reconnection is performed greedily. Start with the original color connection and move gluons between chains to reduce the energy. Terminate when no more energy reduce moves are available.
- Now, we are incorporating full optimization of the color-reconnection problem using powerful classical optimizers. This change to the simulators leads to tangible effects in the simulated data which can be experimentally validated/invalidated.
- This is a “quantum inspired” change to the simulation model
  (in that it is a result of trying to find a HEP application for quantum computing)

Solve small toy problems using simulated QAOA

- QAOA is performed by minimizing the energy of a parameterized state $|\vec{\gamma}, \vec{\beta}\rangle$.
- QAOA is performed in two stages:
  - Find the set of angles $\vec{\gamma}, \vec{\beta}$ which minimize the energy.
  - Take shots of the state repeatedly to find low-energy solutions.



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Use the opportunity to study QAOA. Ongoing work…

- There are several classical simulation techniques to
  - Find the energy of QAOA states for large problems (with low connectivity)
  - Approximately simulate and sample bitstrings from QAOA states
  - Reduce the problem size using the properties of the graph
- These are usually studied and benchmarked in the context of Max-Cut problem.
- We are studying the application of such techniques to our problem (highly connected graphs with real-valued weights). "HEP for Quantum Computing"

Thank you!

Acknowledgements

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