SMASH hadronic transport

Dmytro (Dima) Oliinychenko

Institute for Nuclear Theory, University of Washington
oliiny@uw.edu

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Physical idea: separation of hard and soft physics

- Do you know what hard and soft $\equiv$ bulk physics means in context of heavy ion collisions?
  Press yes/no. Take 60 seconds to write it, after 60 seconds post in the chat.

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Imagine a fountain of water (jet) shooting through a dense fog (bulk) or a jet in jacuzzi

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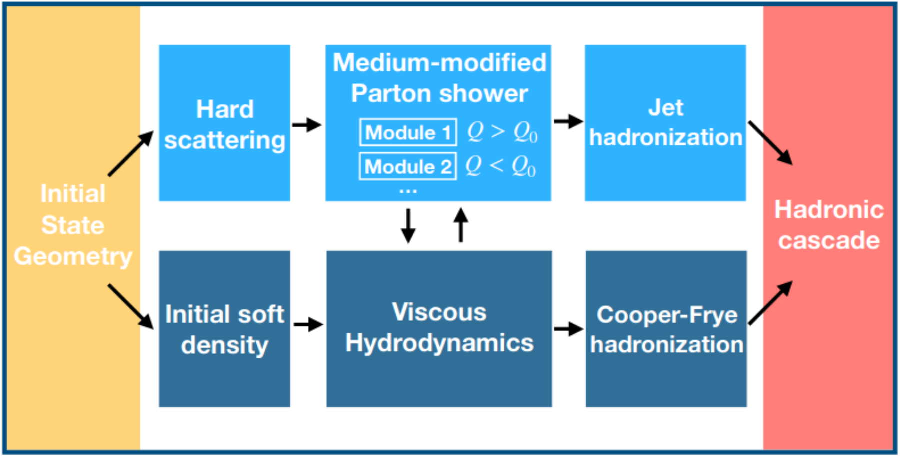
JETSCAPE

Hard and soft / bulk physics separated explicitly



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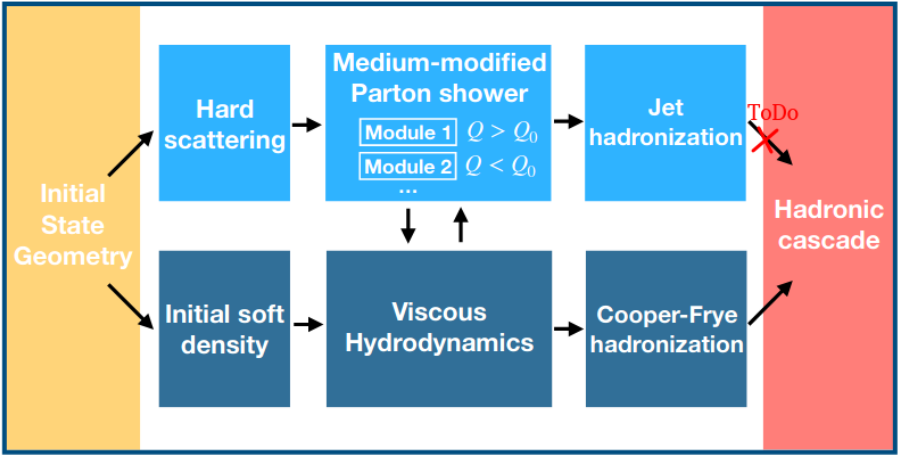

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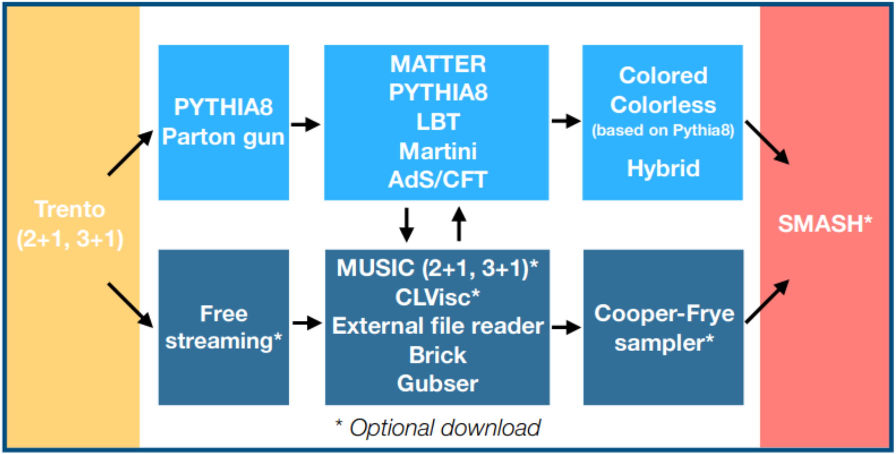

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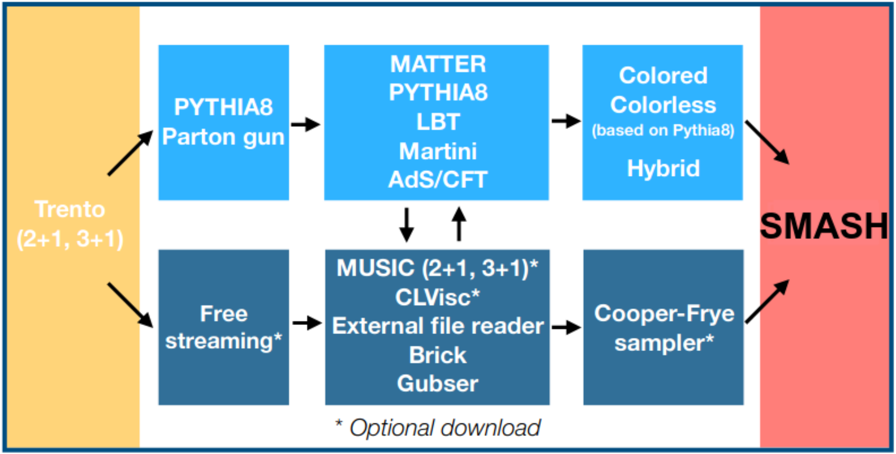

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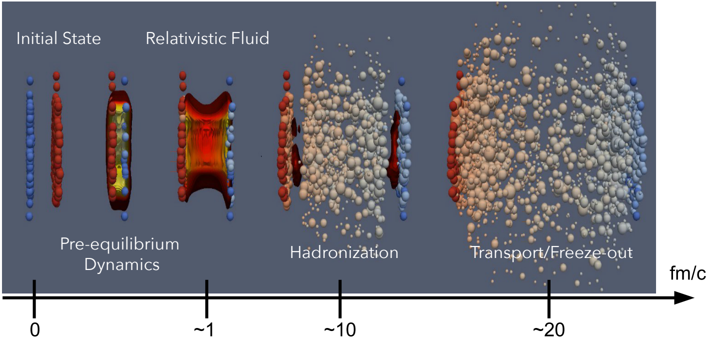
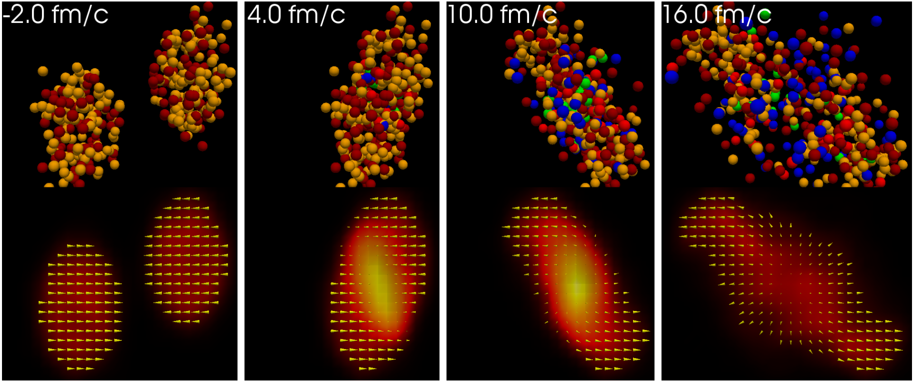
Soft physics: hydrodynamics + transport approach



- Hydrodynamics: local thermal equilibrium, $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^\mu = 0$, EoS, boundary conditions
  Applicability: mean free path $\ll$ system size $\implies$ high density
- Transport: Monte-Carlo solution of Boltzmann equations
  Applicability: mean free path $\gg$ $\lambda_{Compton}$ $\implies$ low density
- Hybrid: hydro at high density + transport at low density

Transport is simple: particles propagate, collide, decay

Au+Au, $\sqrt{s_{NN}} = 3$ GeV, $b = 5$ fm



...but the devil is in the details

Applications of hadronic transport

- Full simulations of ion collisions at lower and intermediate energies ($\sqrt{s} \lesssim 20$ GeV, e.g. SMASH, UrQMD, HSD, IQMD), at higher energy they tend to undershoot v_2
- With some partonic part: full simulations at any energies (e.g. PHSD, AMPT)
- Higher energies ion collisions ($\sqrt{s} \gtrsim 20$ GeV) as a hadronic afterburner after hydrodynamics
- $e + A$, $\nu + A$, e.g. GiBUU
- Participate in simulations of air-shower from cosmic rays, e.g. UrQMD

Some theoretical foundations

- Conceptually transport codes rely on Vlasov and Boltzmann equations
- Have you heard about Vlasov and Boltzmann equations before? Press yes/no. If yes, take 60 seconds to write 1-2 random facts you know about them. After 60 seconds post it in the slack chat.

Vlasov equation (non-relativistic version)

Motion of particles in self-generated mean field

$$\frac{\partial}{\partial t} f(t, \vec{r}, \vec{p}) + \frac{\vec{p}}{m} \nabla_{\vec{r}} f(t, \vec{r}, \vec{p}) - \nabla_{\vec{r}} U(\vec{r}) \nabla_{\vec{p}} f(t, \vec{r}, \vec{p}) = 0$$

$$U(\vec{r}) = \int d^3 r' \, d^3 p \, V(\vec{r} - \vec{r'}) f(t, \vec{r'}, \vec{p})$$

Classical single-particle equations of motion:

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m}$$

$$\frac{d\vec{p}}{dt} = -\nabla_{\vec{r}} U(\vec{r})$$

$$\frac{df(t, \vec{r}, \vec{p})}{dt} = \left(\frac{\partial}{\partial t} + \dot{\vec{r}} \nabla_{\vec{r}} + \dot{\vec{p}} \nabla_{\vec{p}} \right) f = 0$$

Easy way to think: f is number of particles per $d^3x \, d^3p$. Conserving number of particles and phase space volume (Liouville theorem).

Boltzmann equation (non-relativistic)

Neglect quantum effects like interference, assume 2-body correlations are local in space-time (space and time span of collisions $\ll$ mean free path). Same left side as for Vlasov equation, but there is right side responsible for collisions.

$$\frac{df(t,\vec{r},\vec{p})}{dt} = \frac{\partial}{\partial t} f(t,\vec{r},\vec{p}) + \frac{\vec{p}}{m}\nabla_{\vec{r}} f(t,\vec{r},\vec{p}) - \nabla_{\vec{r}} U(\vec{r})\nabla_{\vec{p}} f(t,\vec{r},\vec{p}) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

$$I_{coll} \equiv \left(\frac{\partial f}{\partial t}\right)_{coll} = \left(\frac{\partial f}{\partial t}\right)_{gain} - \left(\frac{\partial f}{\partial t}\right)_{loss}$$

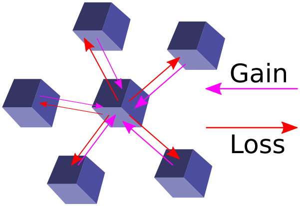
Boltzmann equation: gain and loss terms

Number of particles $dN(t,r,p)$ in the phase-space cell $d^3\vec{r}\,d^3\vec{p}$:

$$\frac{d}{dt}N(t,r,p) = dN_{coll}(p', \dots \to p, \dots) - dN_{coll}(p, \dots \to p', \dots)$$

$$dN(t,r,p) = f(t,r,p)d^3\vec{r}\,d^3\vec{p}$$

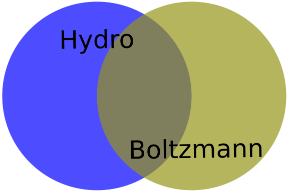
- Assumptions to calculate dN_{coll}:
  - only $2 \to 2$ scattering, neglect many-particle scatterings as rare
  - incoming particles uncorrelated
  - separation between long- and short-range interactions



$$I_{coll} = \int \frac{d^3p_2}{E_2}\frac{d^3p_1'}{E_1}\frac{d^3p_2'}{E_2'} \times W(p,p_2 \to p_1',p_2') \times (f(p_1')f(p_2') - f(p)f(p_2))$$

Ideal hydro follows from equilibrated Boltzmann

Regions of applicability for hydro and Boltzmann



Regardless of cross sections if one waits long enough then entropy reaches maximum (H-theorem). With corresponding equilibrium distribution

$$f_0(r,p) = exp((-p^\mu u_\mu + \mu(r))/T(r))$$

right hand side of Boltzmann equation vanishes.

From $p^\mu \frac{\partial f_0}{\partial x^\mu} = 0$ follows $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu j^\mu = 0$, where

$T^{\mu\nu} = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^\mu p^\nu}{p^0} f(p)$ and $j^\mu = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^\mu}{p^0} f(p)$.

Solving coupled Boltzmann equations in practice

- All (or almost all) hadron species known: π, ρ, K, a_2, f_1, ϕ, $\dots$, N, $\Delta(1232)$, $N(1440)$, $\dots$: more than 100 species without accounting for charges
- Solve coupled equations, $D \equiv \frac{d}{dt}$:

$$\begin{aligned} Df_\pi &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\ Df_N &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\ Df_\Delta &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\ &\dots \end{aligned}$$

- Left hand side: testparticle ansatz

$$f \sim \frac{1}{N_{test}} \sum_i^{N_{test}} \delta^4(x - x_i)\delta^4(p - p_i)\delta(p^\mu p_\mu = m^2)$$

- Collision integrals: Monte-Carlo approach

Treatment of potentials in transport codes

Have you heard terms QMD and BUU before? Press yes/no.

Treatment of potentials in transport codes

- Boltzmann - Ühling - Uhlenbeck (BUU) approach
  - Mean-field potentials depend on densities: $U = U(\rho(\{\vec{r}_1, \vec{r}_2, \dots\}))$
  - Utilize testparticle ansatz: $N \to N \cdot N_{test}$, $\sigma \to \sigma / N_{test}$
  - Precise energy and momentum conservation only in the limit $N_{test} \to \infty$
  - Solve Boltzmann equations in the limit of $N_{test} \to \infty$
  - No correlations in the limit $N_{test} \to \infty$
- Quantum Molecular Dynamics (QMD) approach
  - Pairwise potentials depend on coordinates $U(r_{12})$
  - Energy and momentum conserved exactly event-by-event
  - Does not solve any particular equation for distribution function

Treatment of collisions in transport codes

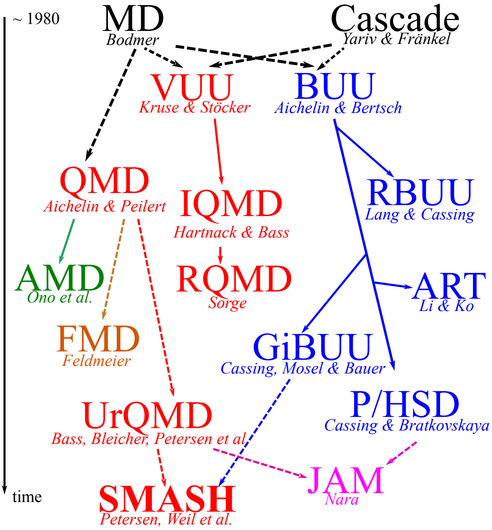
- A) Geometrical criterion: $d_{ij} \leq \sqrt{\sigma/\pi}$ (d_{ij} in the CM frame of colliding paricles)
  - Only allows $2 \to 2$, not $3 \to 2$ or $3 \to 1$: detailed balance violation
  - Collision time: time of closest approach
  - Sort by collision time, perform the earliest
  - Time sorting depends on frame, problems with Lorentz-invariance
  - Kodama criterion: smaller Lorentz-invariance troubles
- B) Stochastic rates: choose two random particles in cell and collide with some probability

  Cassing, NPA 700, 618 (2002); Xu and Greiner, PRC 71, 064901 (2005)
  - No problems with Lorentz-invariance
  - Allows $3 \to 2$ or $3 \to 1$ collisions
  - Not applicable with QMD
  - Needs care concerning cell size and timestep
  - So far only in BAMPS and (P)HSD, recently implemented in SMASH

String models: $\sqrt{s} > 3 - 4$ GeV

- Pick up $q\bar{q}$ or quark-diquark from colliding hadrons
- They form a string, which undergoes a sequence of decays: "string fragmentation"
- Different models of string fragmentation: FRITIOF (Lund), PYTHIA (Lund), HERWIG
- In Lund string: tunnelling through QCD potential
  $$\mathcal{P} \sim exp\left(\frac{-\pi p_\perp^2}{\kappa}\right) exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$
- Flavor composition, longitudinal momenta $\rightarrow\simeq$ 10 parameters

SMASH and its ancestors



thanks to Steffen Bass

- SMASH : **S**imulating **M**any **A**ccelerated **S**trongly-interacting **H**adrons
- first C++ code in this historical chain
- written from scratch
- started coding in 2013
- under *git* version control from the very beginning

SMASH: general properties J. Weil *et al.*, Phys.Rev. C94 (2016) no.5, 054905

- Monte-Carlo solver of relativistic Boltzmann equations
  BUU type approach, testparticles ansatz: $N \to N \cdot N_{test}, \sigma \to \sigma / N_{test}$
- Degrees of freedom
  - most of established hadrons from PDG up to mass 2.5 GeV
  - strings: do not propagate, only form and decay to hadrons
  - leptons and photons production, decoupled from hadronic evolution
- Propagate from action to action (timesteps only for potentials)
  action $\equiv$ collision, decay, wall crossing
- Geometrical collision criterion: $d_{ij} \leq \sqrt{\sigma/\pi}$
- Interactions: $2 \leftrightarrow 2$ and $2 \to 1$ collisions, decays, potentials, string formation (soft - SMASH, hard - Pythia 8) and fragmentation via Pythia 8
- C++ code, git version control, public on github

SMASH: initialization

- “collider” - elementary or AA reactions, $E_{beam} \gtrsim 0.5$ *A* GeV
- “box” - infinite matter simulations
  detailed balance tests, computing transport coefficients, thermodynamics of hadron gas
  Rose et al., PRC 97 (2018) no.5, 055204
- “sphere” - expanding system
  testing collision term via comparison to analytical solution of Boltzmann equation,
  Tindall et al., Phys.Lett. B770 (2017) 532-538
- “list” - hadronic afterburner after hydrodynamics

SMASH: degrees of freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\ 1370}$	$f'_{2\ 1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\phi_{3\ 1850}$	K^*_{1410}
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^*_{1680}
N_{1710}		Λ_{1820}	Σ_{2030}			η_{1475}	ϕ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N_{1900}		Λ_{2100}					$h_{1\ 1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N_{2080}		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N_{2190}						ρ_{1700}				
N_{2220}							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N_{2250}						ω_{783}				
						ω_{1420}		$\omega_{3\ 1670}$		
						ω_{1650}				

- Isospin symmetry
- Perturbative treatment of non-hadronic particles (photons, dileptons)

Hadrons and decay modes configurable via human-readable files

Interactions in SMASH

- Resonance formation and decay
  Ex. $\pi\pi \to \rho \to \pi\pi$, quasi-inlastic scattering
  $\pi\pi \to f_2 \to \rho\rho \to \pi\pi\pi\pi$
- (In)elastic $2 \to 2$ scattering
  parametrized cross-sections $\sigma(\sqrt{s}, t)$ or
  isospin-dependent matrix elements $|M|^2(\sqrt{s}, I)$
- String formation/fragmentation
  $2 \to n$ processes

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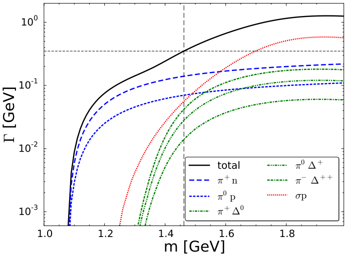
$N^*(1440)^+$



For every resonance:

- Breit-Wigner spectral function

$$\mathcal{A}(m) = \frac{2\mathcal{N}}{\pi} \frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}$$

- Mass dependent partial widths $\Gamma_i(m)$
  Manley formalism for off-shell width Manley and Saleski, Phys. Rev. D 45, 4002 (1992)
  Total width $\Gamma(m) = \sum_i \Gamma_i(m)$

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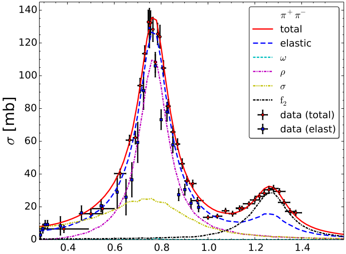

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  Total width $\Gamma(m) = \sum_i \Gamma_i(m)$
- $2 \to 1$ cross-sections from detailed balance relations

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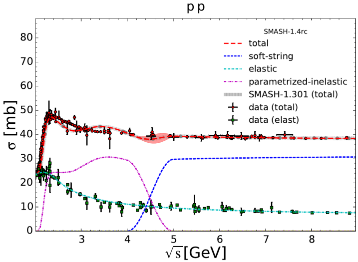

- $NN \to NN^*$, $NN \to N\Delta^*$, $NN \to \Delta\Delta$, $NN \to \Delta N^*$, $NN \to \Delta\Delta^*$

  angular dependencies of $NN \to XX$ cross-sections implemented

- Strangeness exchange $KN \to K\Delta$, $KN \to \Lambda\pi$, $KN \to \Sigma\pi$

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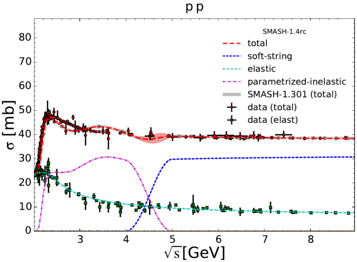

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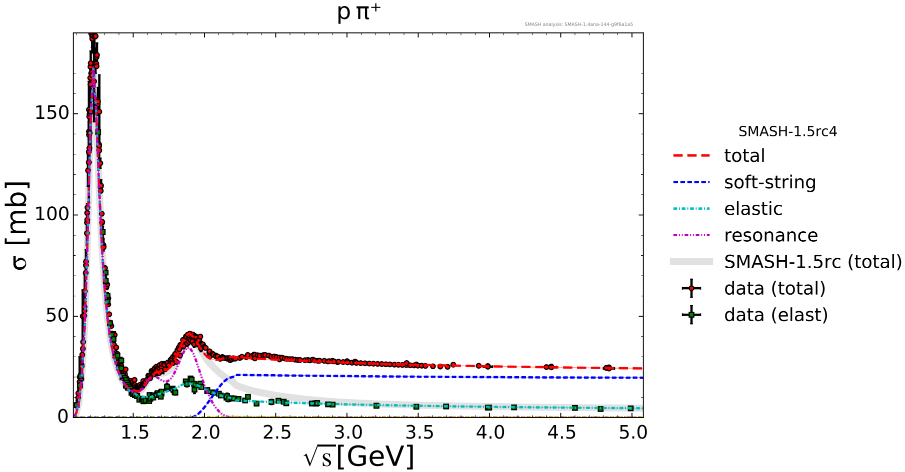
string model parameters tuned to NA61 pp

Mohs et al., J.Phys.G 47 (2020) 6, 065101

- String (soft or hard) fragmentation: always via Pythia 8
- Hard scattering and string formation: Pythia
- Soft string formation: SMASH
  - single/double diffractive
  - $B\bar{B}$ annihilation
  - non-diffractive

Cross-sections

Resonances at lower energies, string models at $\sqrt{s} \gtrsim 3 - 4$ GeV



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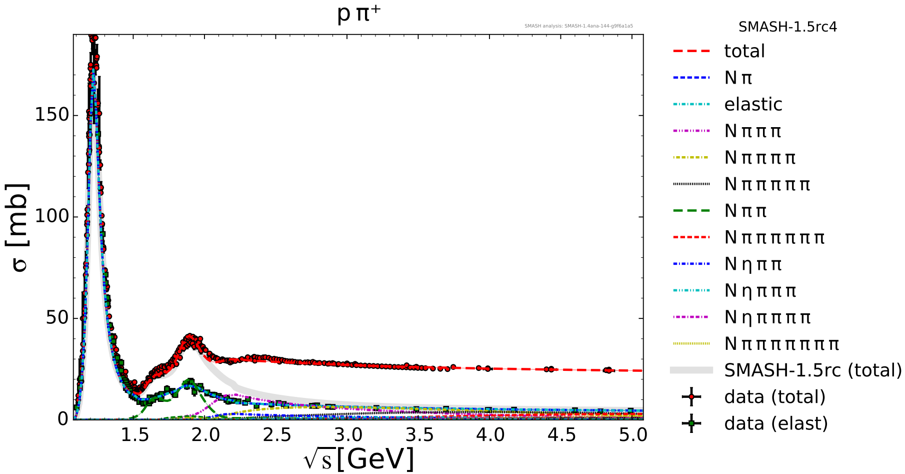

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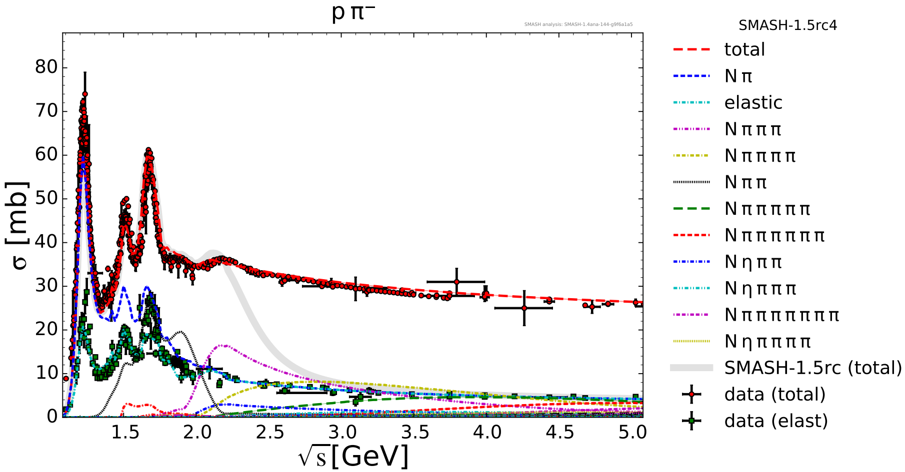

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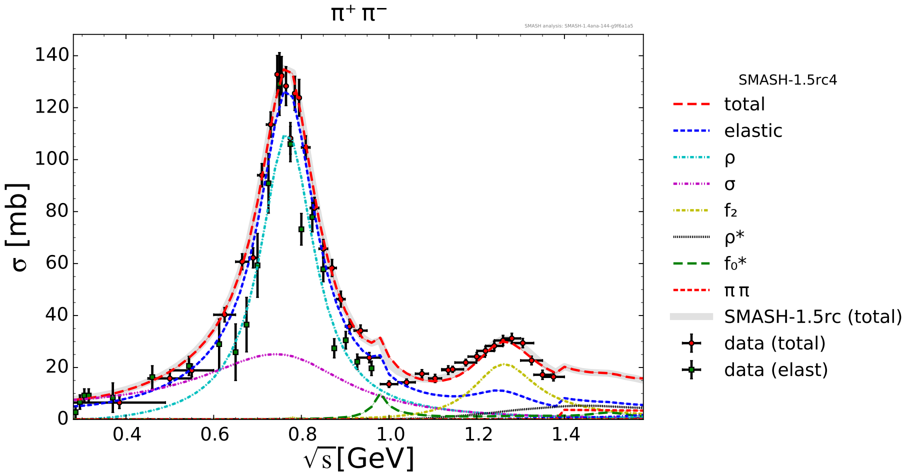

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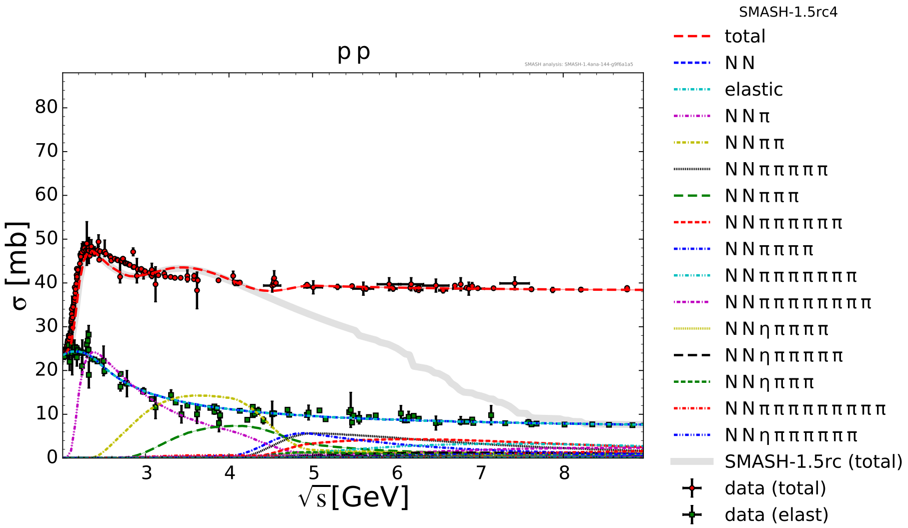

SMASH: analysis suite

SMASH analysis suite

Quick test

Did you learn anything from this lecture? Press yes/no.
If yes then write 1-2 random things you learned in the chat.