Bayesian analysis in heavy-ion collisions: bulk physics, July 28-29

JETSCAPE Online Summer School 2021

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Schedule

July 28 (Weiyao Ke)

- Hands-on exercises for the Gaussian emulator. Notebook
- Apply emulator-assisted Bayesian analysis to a toy model of bulk physics + pseudodata. Notebook

July 29 (Dan Liyanage)

- Application to JETSCAPE medium simulations + pseudodata. Notebook
- Application to JETSCAPE simulations + real data (homework). Notebook

Goals:

- Training and assessing the quality of emulators.
- Understand the importance of validation.
- Handle multi-dimensional model input / output and how to understand high-dimensional posterior.

Recap the Bayes theorem

The problem:

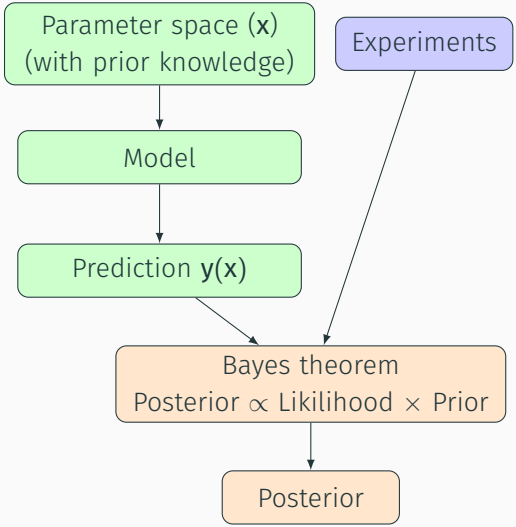
1. A model $\mathcal{M}$: predict observables $\mathbf{y}$ at given input parameters $\mathbf{x}$.
2. A prior belief of the distribution of true values of $\mathbf{x}$: $P_0(\mathbf{x}_{\mathrm{true}})$
3. Make the measurement $\mathbf{y}_{\mathrm{exp}}$, and update the knowledge: $P_0 \to P(\mathbf{x}_{\mathrm{true}})$.

$$\text{Bayes' theorem:} \quad \underbrace{P(\mathbf{x}_{\mathrm{true}}|\mathcal{M}, \mathbf{y}_{\mathrm{exp}})}_{\text{Posterior}} = \frac{\overbrace{L(\mathbf{y}_{\mathrm{exp}}|\mathcal{M}, \mathbf{x}_{\mathrm{true}})}^{\text{Likelihood}} \overbrace{P_0(\mathbf{x}_{\mathrm{true}})}^{\text{Prior}}}{\underbrace{\int L(\mathbf{x}) P_0(\mathbf{x}) d\mathbf{x}}_{\text{Normalization (evidence)}}}$$

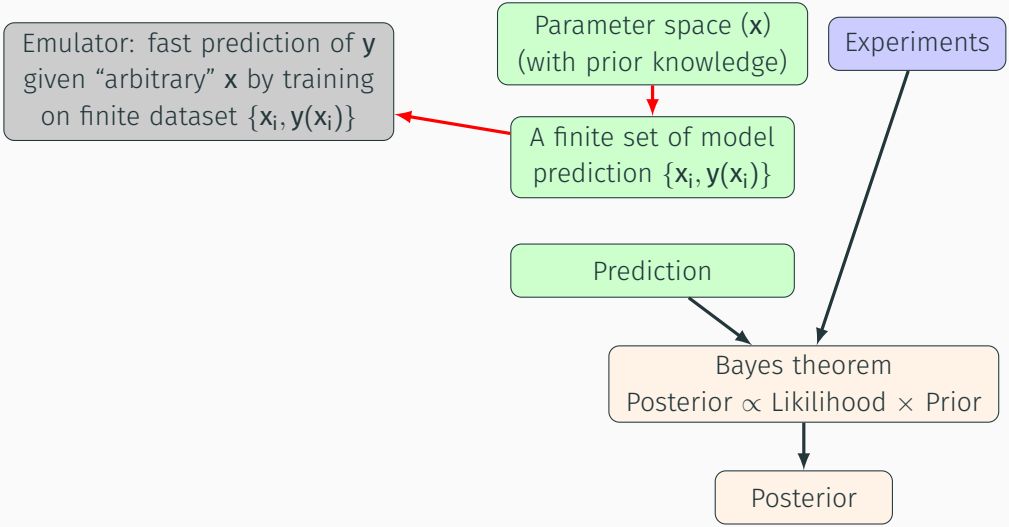
L is often unknown. Commonly assumed to take the form of a multivariate Gaussian:

$$\ln L = \frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma| - \frac{1}{2} \Delta\mathbf{y} \Sigma^{-1} \Delta\mathbf{y}^T, \quad \Delta\mathbf{y} = \mathbf{y}_{\mathrm{exp}} - \mathbf{y}(\mathbf{x}; \mathcal{M})$$

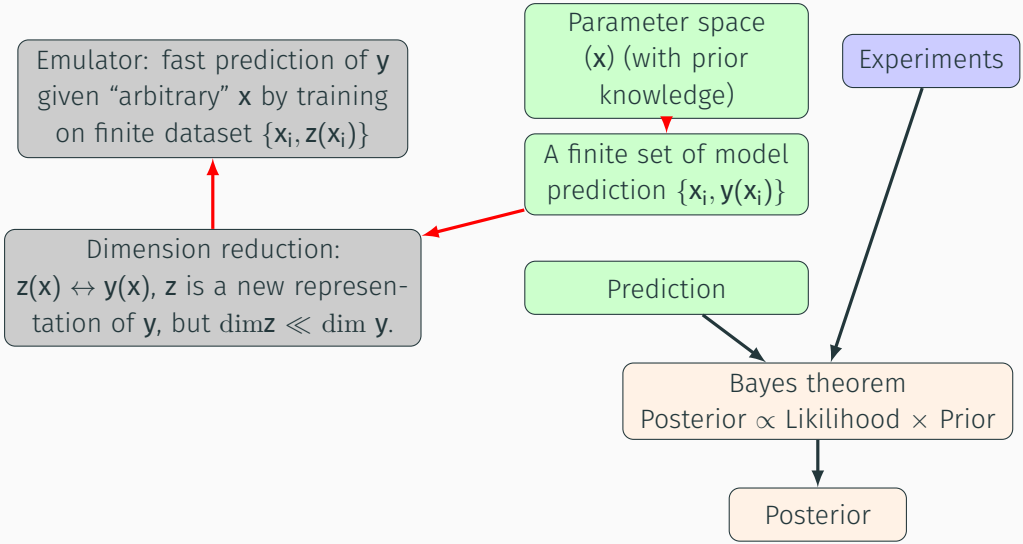
For simple models that $y(x)$ is easy to compute:



For computationally intensive model, such as those for heavy-ion collisions



For computational intensive models + high-dimensional output



Finally, the workflow of the emulator-assisted Bayesian analysis