Lattice QCD for semileptonic form factors: $B_s^0 \to \phi\mu^+\mu^-$ and $B_s^0 \to K^-\mu^+\nu_\mu$

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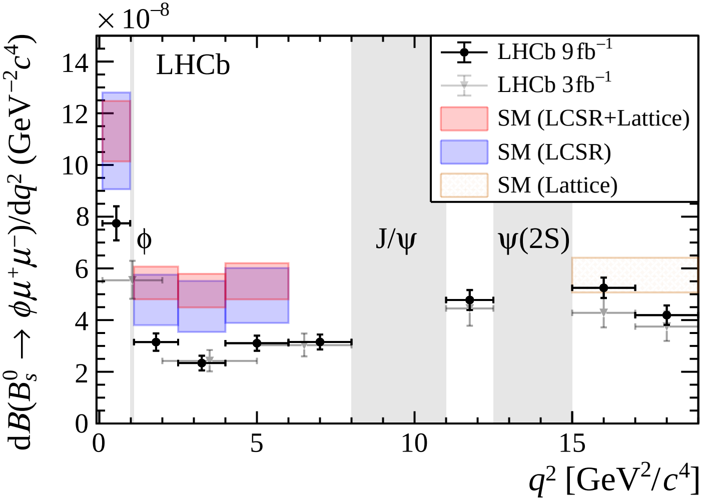
$B_s^0 \to \phi\mu^+\mu^-$



[LHCb 2021[1]]

Local matrix elements

$$H_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} (C_i O_i + C_i' O_i')$$

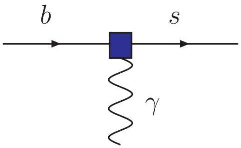
$O_{1,\ldots,6}^{(\prime)}$ are 4-quark operators, $O_8^{(\prime)}$ contains gluon field strength

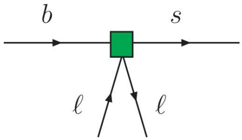
$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b \, F_{\mu\nu}$$

$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \ell$$

$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

O_i' from O_i by $L \leftrightarrow R$ and C_i' suppressed in SM





Local matrix elements from 7, 9, 10 operators, hence lattice matrix elements

$$\langle \phi | \left\{ \begin{matrix} \bar{s}\gamma^\mu b \\ \bar{s}\gamma^\mu \gamma_5 b \\ \bar{s}\sigma^{\mu\nu} b \\ \bar{s}\sigma^{\mu\nu}\gamma_5 b \end{matrix} \right\} | B_s^0 \rangle$$

Seven form factors: $V, A_0, A_1, T_1, T_2, A_{12}, T_{23}$

Nonlocal contributions

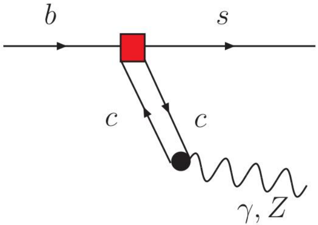
Nonlocal matrix elements of terms in $H_{\text{eff}}^{b\to s}$ with electromagetic current

$$\int d^4x\, e^{iq\cdot x}\langle\phi|\mathrm{T}J^\mu(x)O_i(0)|B_s^0\rangle$$

especially

$$O_1^c = (\bar{s}_i\gamma_\mu P_L c_j)(\bar{c}_j\gamma^\mu P_L b_i)$$
$$O_2^c = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b)$$

OPE for high-q^2 leads to matrix elements of local operators $O_{7,9}$ at leading order:
$C_{7,9} \longrightarrow C_{7,9}^{\text{eff}}$ [Grinstein and Pirjol[2], Beylich et al[3]]

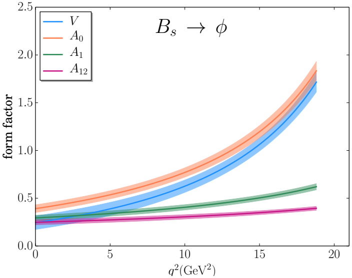


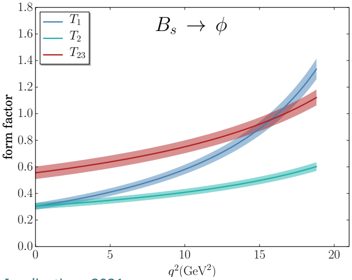
- Above charmonium region, but how big are remaining effects?
- Prospects to calculate directly?
  - LD contributions to $K^+ \to \pi^+\ell^+\ell^-$ are being calculated [Christ et al[4,5]]
  - Some first steps for $B \to K\ell^+\ell^-$ taken [Nakayama, Ishikawa and Hashimoto[6]]

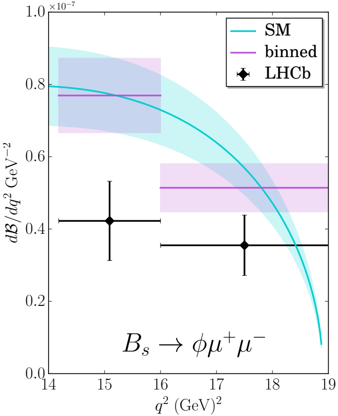
Unstable ϕ

- LQCD calculations so far treated ϕ (and K^*) as stable
- Formalism developed to relate FV LQCD calculations to infinite volume matrix elements with multiple hadrons in (initial or) final state
  - $1 \longrightarrow 2$ processes, with application to $B^0 \longrightarrow K^* \ell^+ \ell^- \longrightarrow \pi K \ell^+ \ell^-$, considered by Briceño, Hansen and Walker-Loud [Briceño et al 2015[7]]

$B_s \longrightarrow \phi$ results

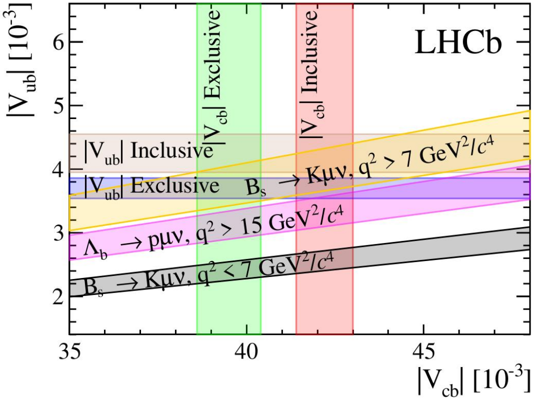


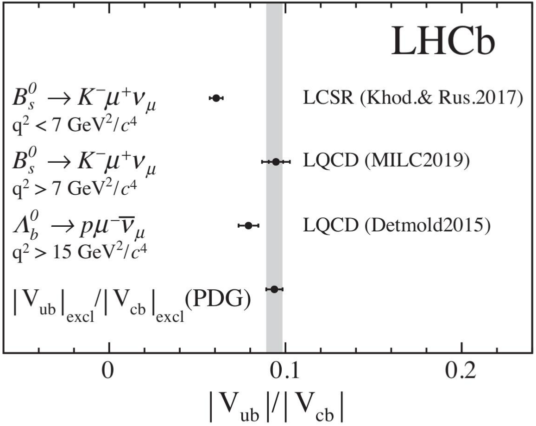




- This slide: form factors and SM $d\mathcal{B}/dq^2$ prediction (with leading non-local contribution via OPE) [Horgan et al 2015 [8–10]] vs LHCb 2013 [Aaij et al [11]]
- Preliminary results for form factors from RBC/UKQCD [RBC/UKQCD 2016 [12,13]]

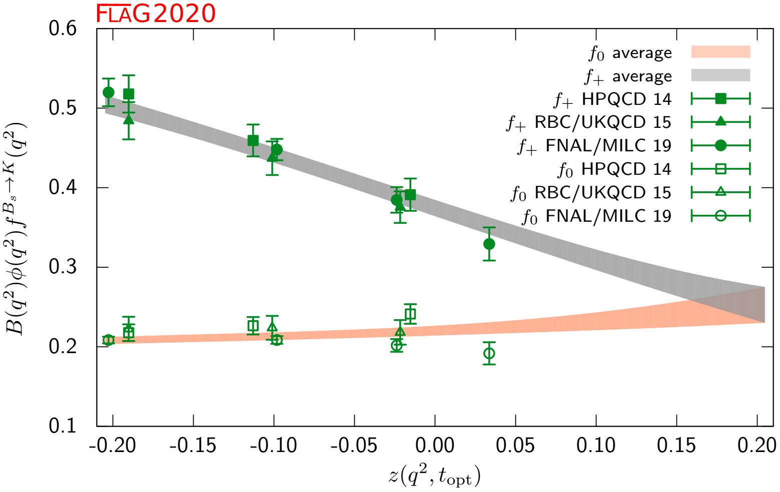
$B_s \longrightarrow K\ell\nu$





[LHCb 2021[14]]

$B_s \to K \ell \nu$ form factors



FLAG Dec 2020 web update[15] with results from HPQCD[16], RBC/UKQCD[17], FNAL/MILC[18]

$B_s \to K\ell\nu$ outlook

- RBC/UKQCD updated results [Flynn Lattice2021] ▸ χ-ctm ▸ errors ▸ z-fits
- Results in FLAG Dec 2020 all use an effective action for b quarks
  - HPQCD now have all-HISQ approach: all quarks using same action and extrapolation for heavy quarks from charm-like masses to m_b. [Bouchard Latice2021]
  - FNAL/MILC also doing all-HISQ with heavy-quark extrapolation to m_b. [Lytle Lattice2021]
- Expect $\mathrm{FF}_K/\mathrm{FF}_{D_s}$ ratios of partially integrated decay rates (minus CKM factors) in high-q^2 region for comparison to LHCb [LHCB Aaij et al 2021[14]]
  - HPQCD have earlier correlated study of $B_s \to K$ and $B_s \to D$ [HPQCD 2018[19]]
- Ongoing ALPHA computations with HQET up to $1/m_b$ [Bahr et al[20,21]]

R ratios for LFU tests

$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\ell\bar{\nu}_\ell)}{dq^2}} \qquad R^{new}(P) = \frac{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q^2_{min}}^{q^2_{max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)}\to P\ell\bar{\nu}_\ell)}{dq^2}}$$

- Adopt idea proposed for $B_{(s)} \longrightarrow V$ decays [Isidori–Sumensari[22]]
    - Common integration range; $q^2_{min} \geq m_\tau^2$ [Freytsis et al[23], Bernlochner et al[24], Soni[25]]
    - *Same* weights for vector parts in integrands for τ and ℓ
- Write

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \Phi(q^2)\omega_\ell(q^2)\left[F_V^2+(F_S^\ell)^2\right]$$

$$\Phi(q^2) = \eta \frac{G_F^2|V_{xb}|^2}{24\pi^3}|\vec{k}|$$

$$\omega_\ell(q^2) = \left(1 - \frac{m_\ell^2}{q^2}\right)^2\left(1 + \frac{m_\ell^2}{2q^2}\right)$$

$$F_V^2 = \vec{k}^2|f_+(q^2)|^2$$

$$(F_S^\ell)^2 = \frac{3}{4}\frac{m_\ell^2}{m_\ell^2 + 2q^2}\frac{(M^2 - m^2)^2}{M^2}|f_0(q^2)|^2$$

R ratios for LFU tests

$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\ell\bar{\nu}_\ell)}{dq^2}}$$

$$R^{\text{new}}(P) = \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)}\to P\ell\bar{\nu}_\ell)}{dq^2}}$$

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  - *Same* weights for vector parts in integrands for τ and ℓ
- Write

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \Phi(q^2)\omega_\ell(q^2)\left[F_V^2 + (F_S^\ell)^2\right]$$

- If drop scalar contribution, $(F_S^\ell)^2$, in denominator ($m_\ell^2/2q^2 \leq m_\mu^2/2m_\tau^2 = 0.002$) expect

$$R^{\text{new,SM}}(P) = 1 + \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \Phi(q^2)\omega_\tau(q^2)(F_S^\tau)^2}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \Phi(q^2)\omega_\tau(q^2)F_V^2}$$

Dispersive bounds revived

Bourrely, Machet, de Rafael[26]	1981	for K_{l3} decays
Lellouch[27]	1996	for $B \longrightarrow \pi l \nu$
Di Carlo, Martinelli, Naviglio, Sanfilippo, Simula, Vittorio[28–31]	2021	revived

disperion relation for current-current correlator

+

2-ptcl piece of complete set of states

+

z-transformation

↓

$$\frac{1}{2\pi i}\oint \frac{dz}{z}\,|\phi(z)f(z)|^2 \leq \chi$$

- $\oint$ ← unit circle
- $\phi(z)$ ← outer fn and Blaschke factors
- $f(z)$ ← form factor
- χ ← from current-current correlator

$$z(t;t_*,t_0) = \frac{\sqrt{t_*-t}-\sqrt{t_*-t_0}}{\sqrt{t_*-t}+\sqrt{t_*-t_0}}$$

$$t = q^2$$

→ **z-fits**

$\frac{\phi(z)f(z)}{\sqrt{\chi}}$ analytic inside unit circle

$$\frac{\phi(z)f(z)}{\sqrt{\chi}} = \sum_{n=0}^{\infty} a_n z^n \quad \leftarrow \text{BGL}$$

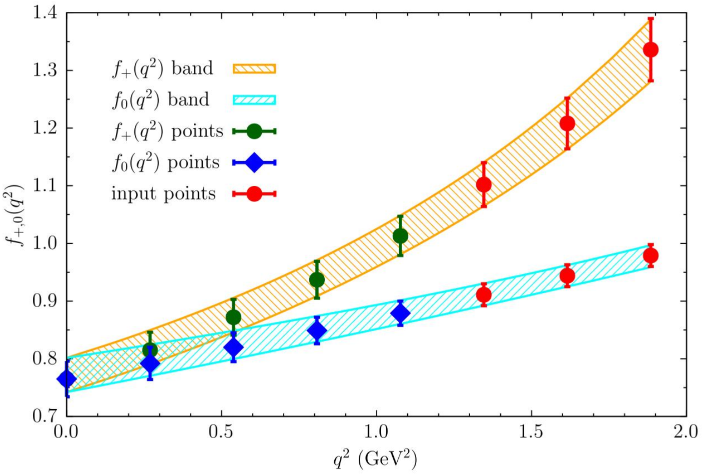
with $\sum_{n=0}^{\infty} |a_n|^2 \leq 1$

→ **dispersive bounds**

combine constraint with known values of form factor

$$f(q_i^2) = f_i \qquad i = 1,2,\ldots,N$$

Example: dispersive bounds for $D \longrightarrow K$



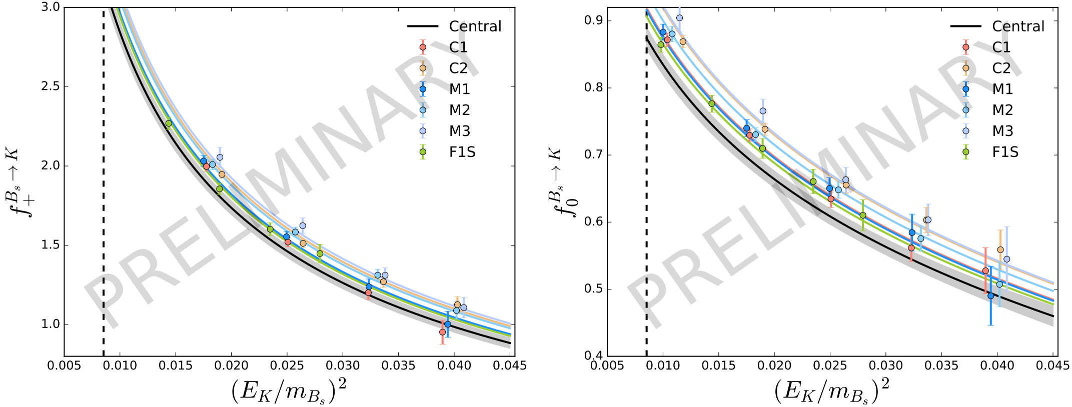
- Points from z-fit to LQCD data for $D \longrightarrow K$ SL decays [ETMC 2017[32]]
- Bands are dispersive bounds using red points as inputs [Di Carlo et al[28]]
- Used form factors and susceptibilities calculated on same ensembles
- Consider also for $B \longrightarrow D^{(*)}$ decays [Martinelli et al[30,31]] decays and for $B \longrightarrow \pi$, $B_s \longrightarrow K, \ldots$

Summary

- Lots of activity in heavy quark decays on the lattice for range of processes
- Technology for multi-hadron states established
- Calculations of long-distance contributions (?)

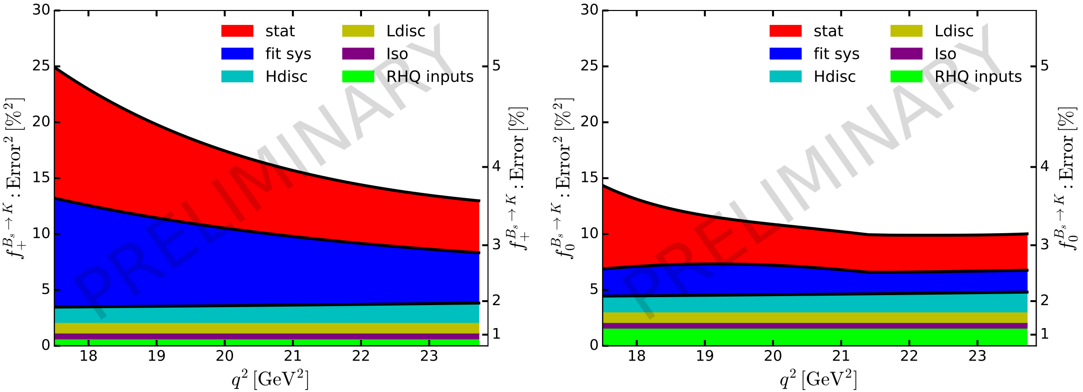
Additional slides

$B_s \to K$ chiral-continuum extrapolation

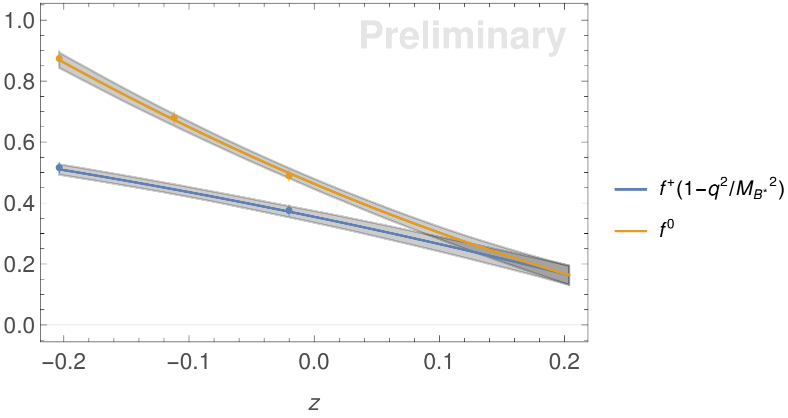


$$f^{B_s \to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K + \Delta}\left[c_0\left(1 + \frac{\delta f(M_\pi^s) - \delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_1 \frac{\Delta M_\pi^2}{\Lambda^2} + c_2 \frac{E_K}{\Lambda} + c_3 \frac{E_K^2}{\Lambda^2} + c_4 (a\Lambda)^2\right]$$

$B_s \longrightarrow K$ cumulative error budget



$B_s \to K$ z-fits



- Do z-fits after χ-ctm extrapolation
- Use BGL[26,27,33–37] and BCL[38]

- Example shown is BCL fit for $B_s \to K$, with $f^+(q^2)(1 - q^2/m^2_{B^*})$ (lower) and $f^0(q^2)$ (upper) plotted

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