

# $\gamma$ measurement and combination at LHCb

Federico Betti on behalf  
of the LHCb collaboration



Implications of LHCb measurements  
and future prospects

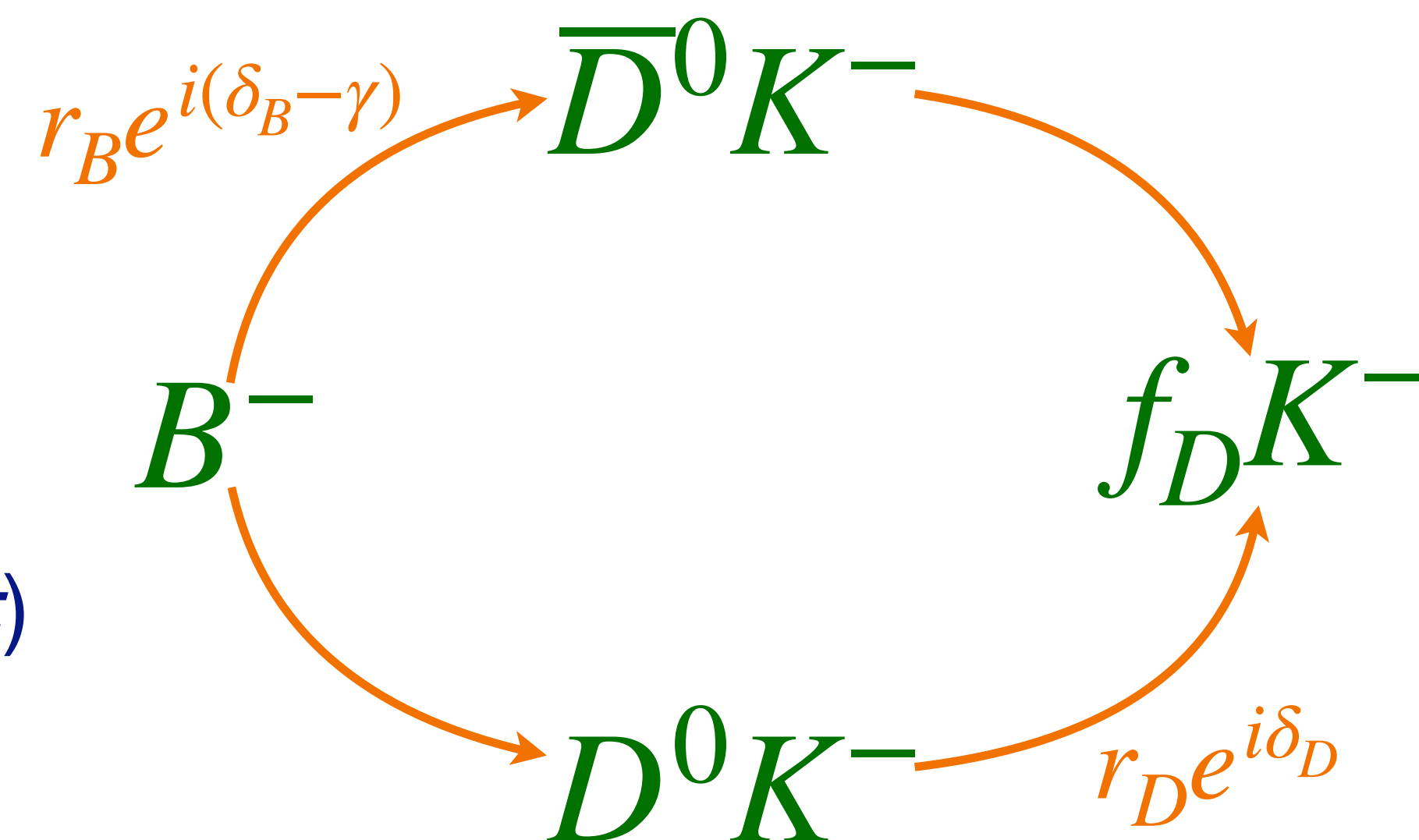
CERN — 22 October 2021



$$\gamma \equiv \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

- $\gamma$  standard **candle** of SM
- Comparison between direct and indirect  $\gamma$  determinations is a test of SM
- $\gamma$  measured in **tree-level**  $B$  decays that are theoretically **clean**
- **SM uncertainties** negligible  $\sim (10^{-5})^\circ$  [JHEP 01 \(2014\) 051](#)
- **New Physics** can cause  $\mathcal{O}(10^\circ)$  shift to  $\gamma$  [PRD 92 \(2015\) 033002](#)  
[JHEP 07 \(2020\) 177](#)

- $\gamma$  measured in **tree-level** decays sensitive to interference between  $b \rightarrow cW$  and  $b \rightarrow uW$  transition amplitudes
- Golden modes:  $B^\pm \rightarrow DK^\pm$
- Various methods according to  $D$  final state
  - ▶ **GLW**:  $CP$  eigenstates (e.g.  $KK, \pi\pi$ )
  - ▶ **ADS**: interference of CF and DCS transitions (e.g.  $K\pi$ )
  - ▶ **BPGGSZ**: 3-body final states (e.g.  $K_s^0\pi\pi$ )
- **Time-dependent** analyses of  $B_{(s)}^0$  are also used



# Previous status of $\gamma$

Unitarity constrained results:

CKMfitter

$$\gamma = (65.7^{+0.9}_{-2.7})^\circ$$

UTFit

$$\gamma = (65.8 \pm 2.2)^\circ$$

LHCb combination (2018):

LHCb-CONF-2018-002

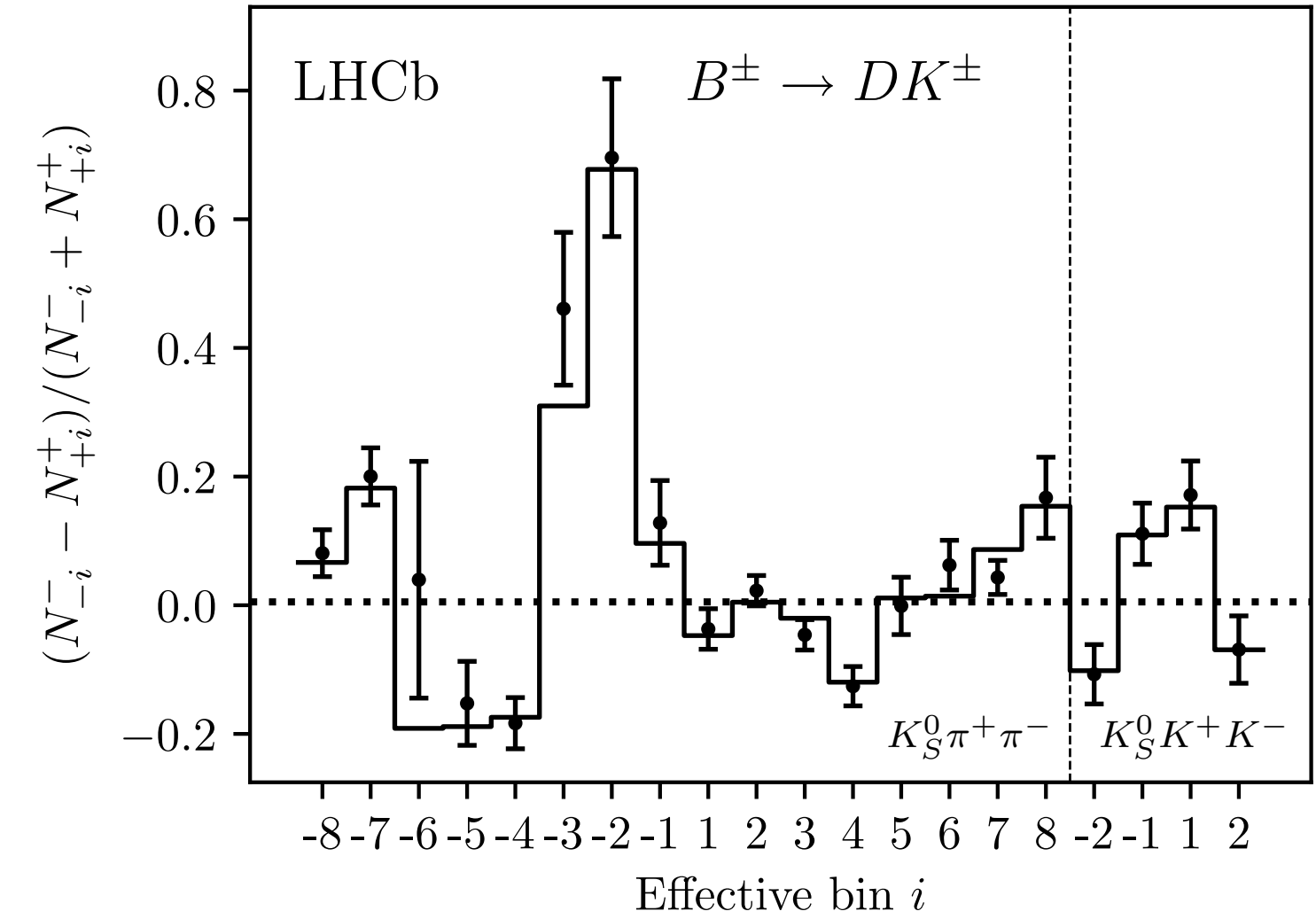
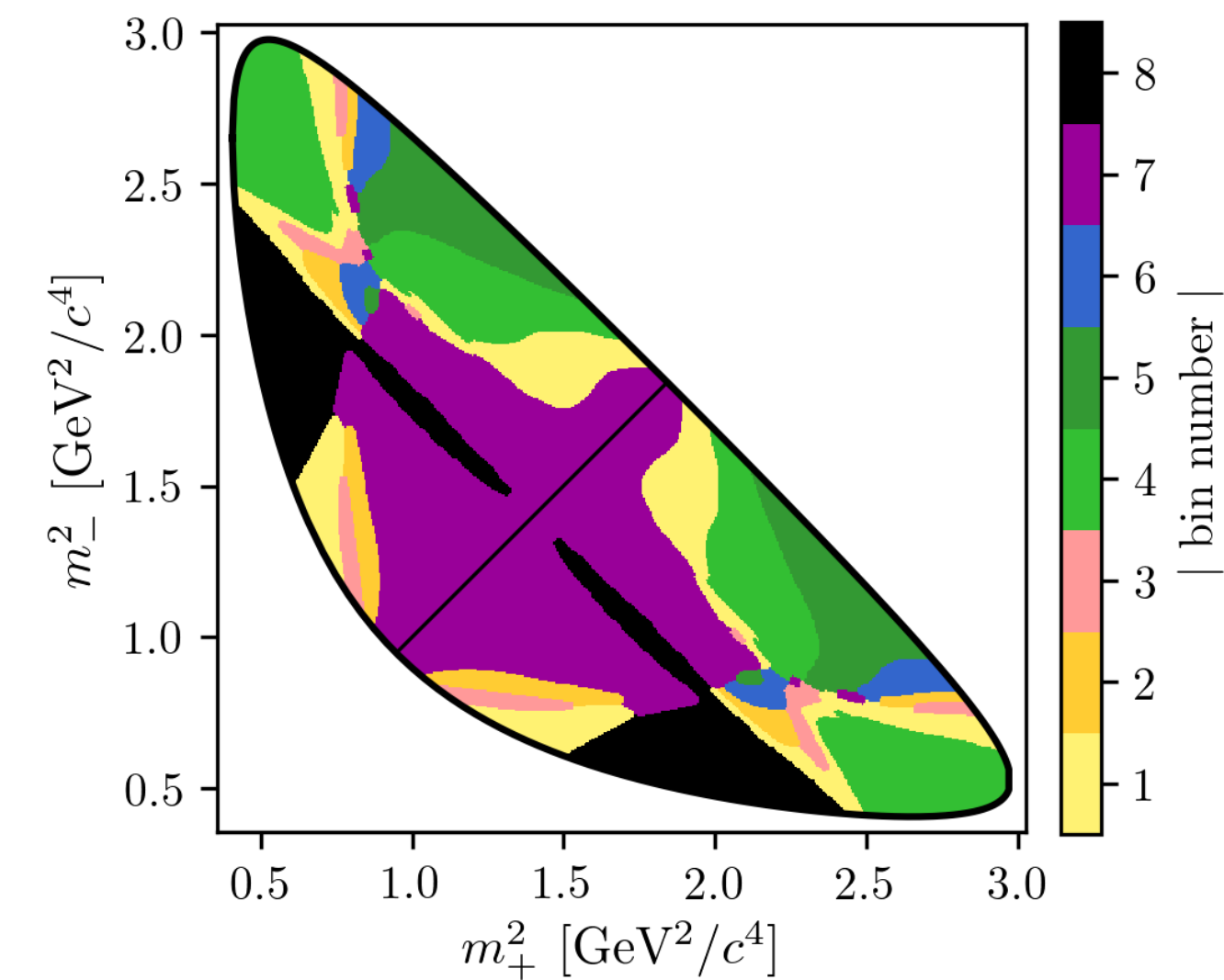
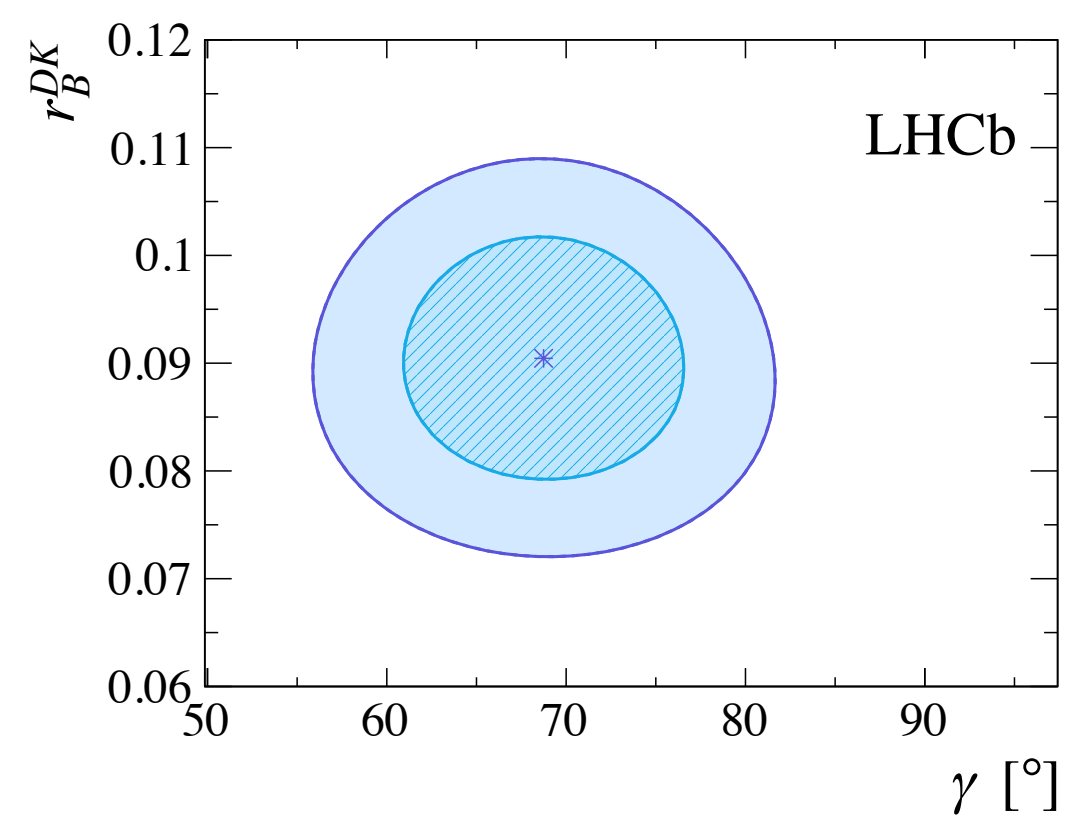
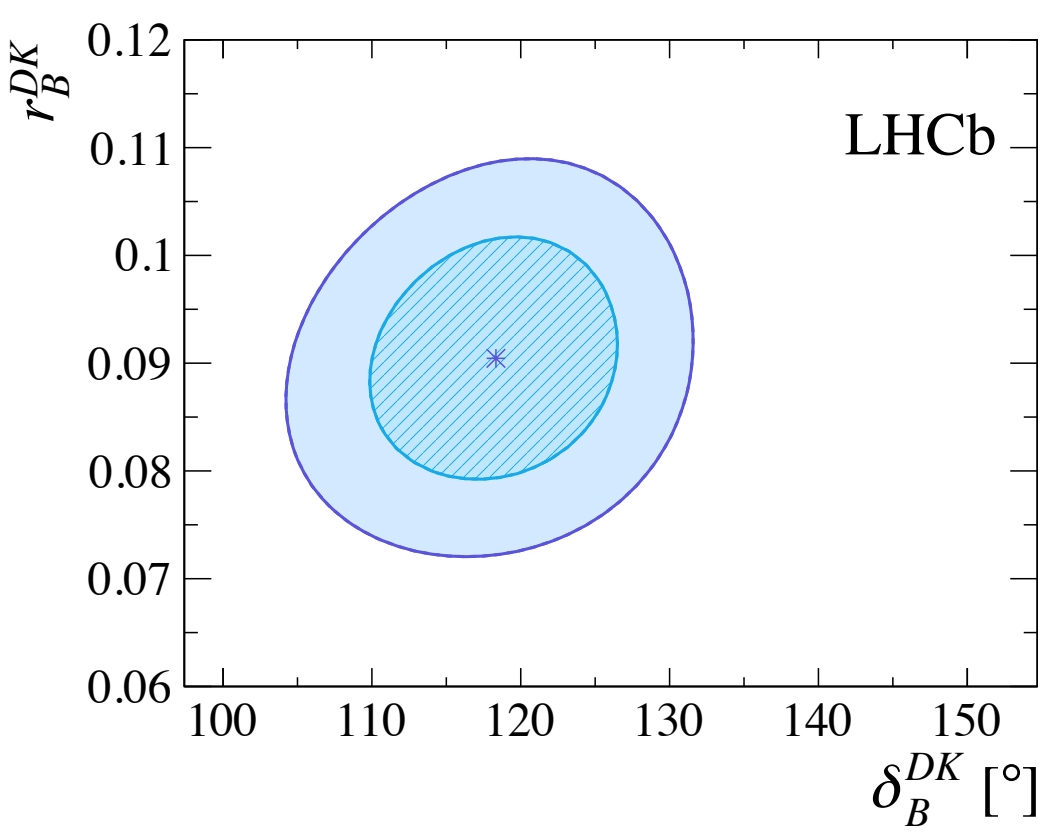
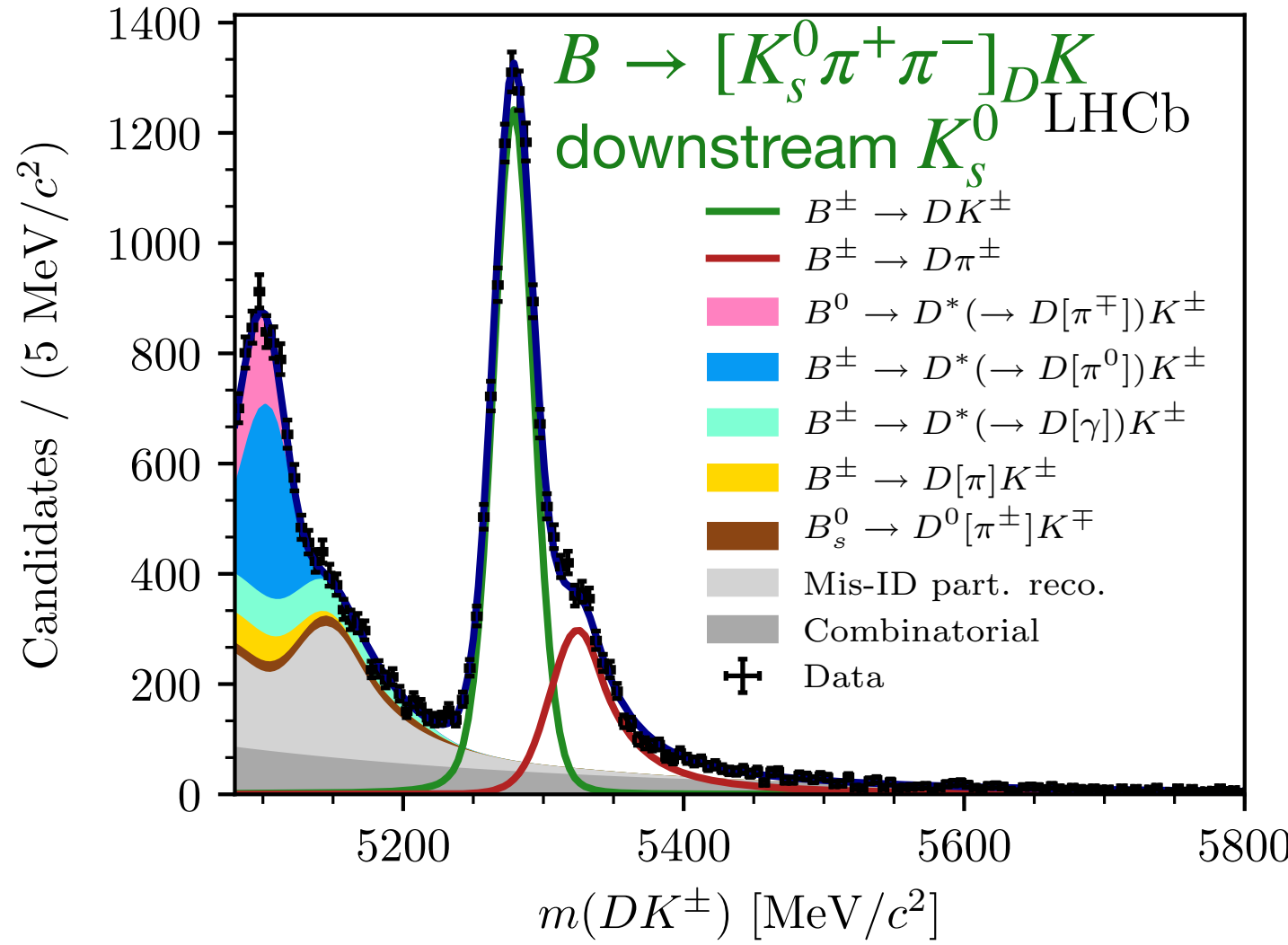
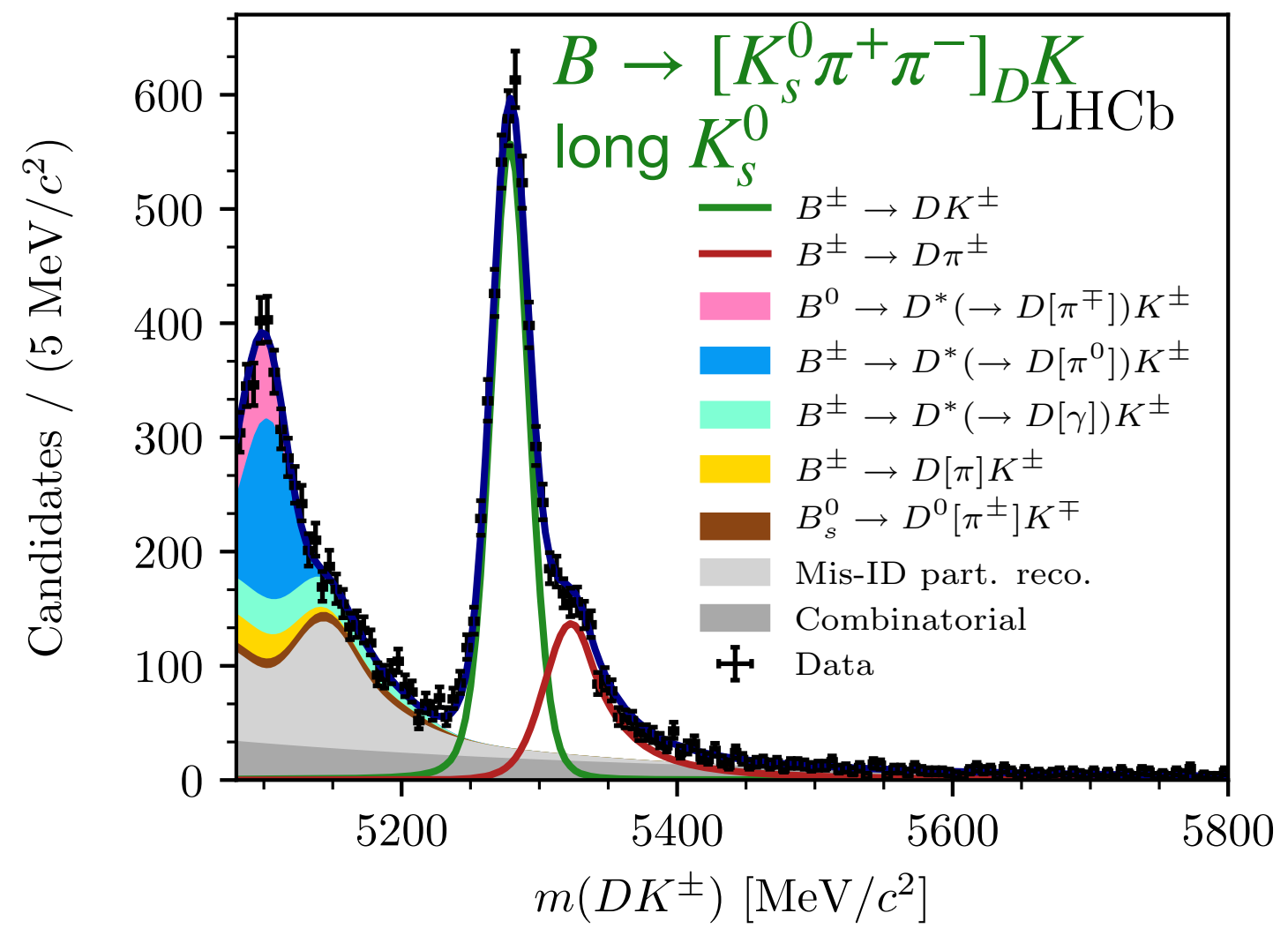
$$\gamma = (74.4^{+5.0}_{-5.8})^\circ$$



# $B^- \rightarrow Dh^-, D \rightarrow K_s^0 h^+ h^-$

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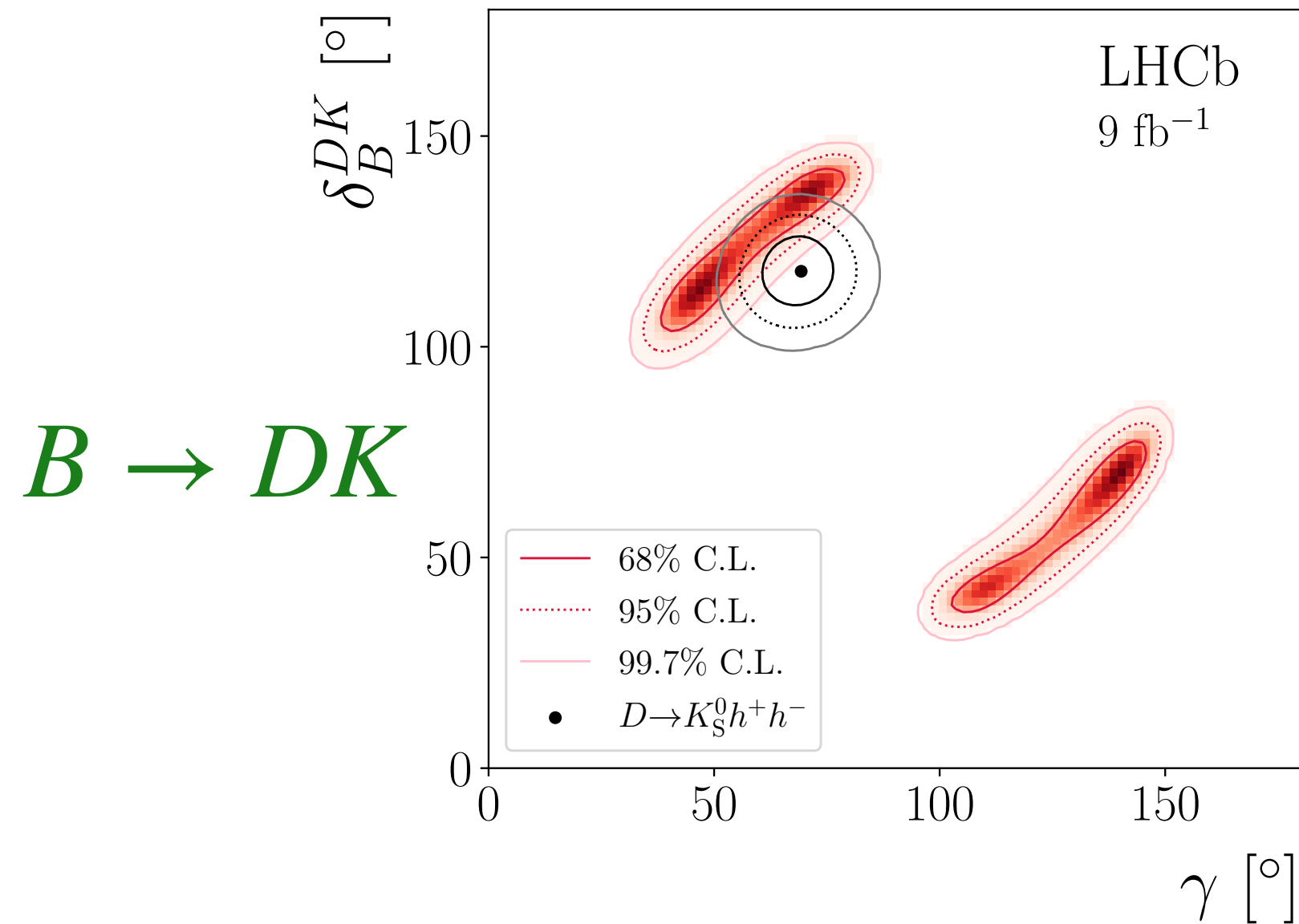
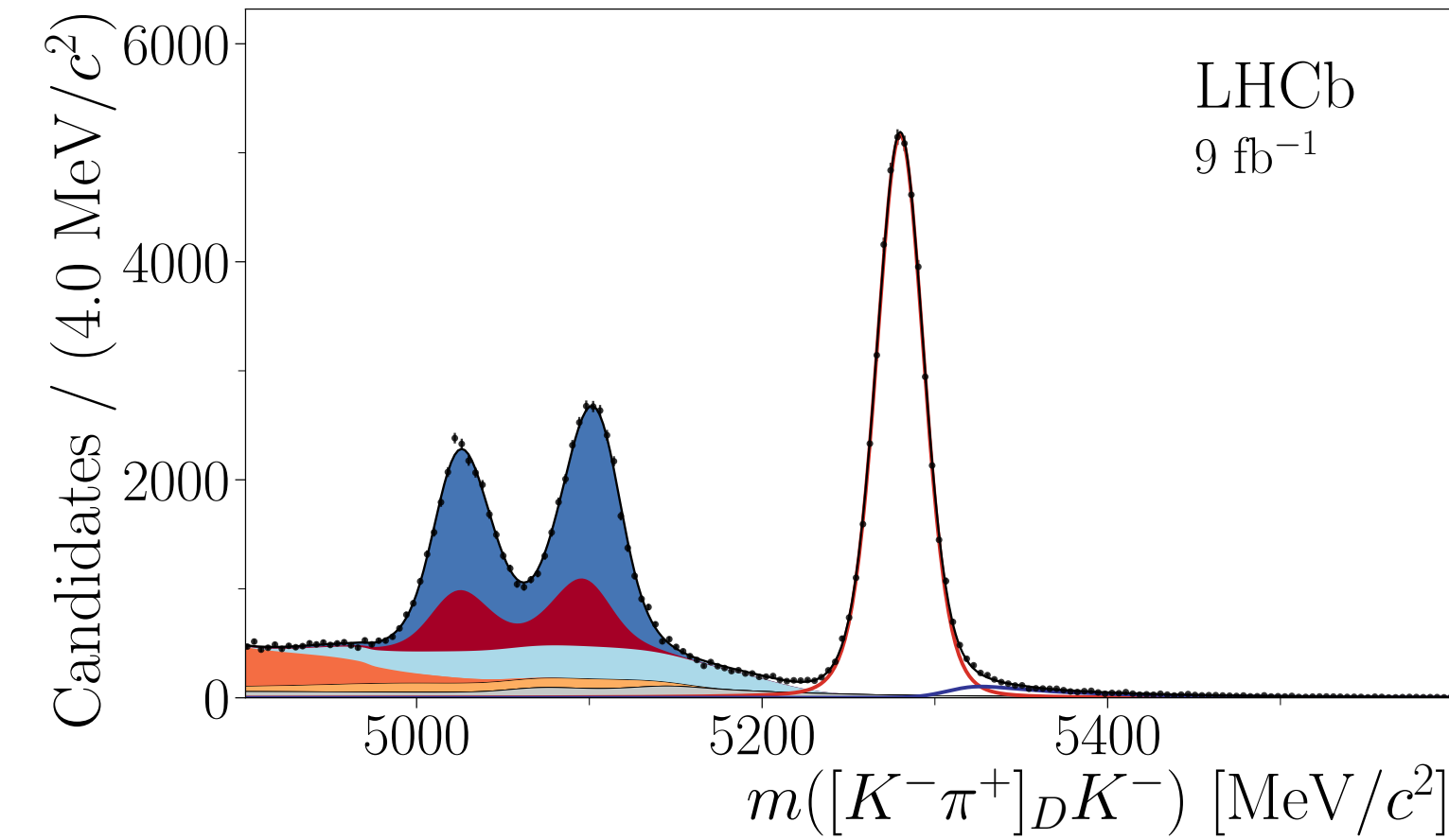
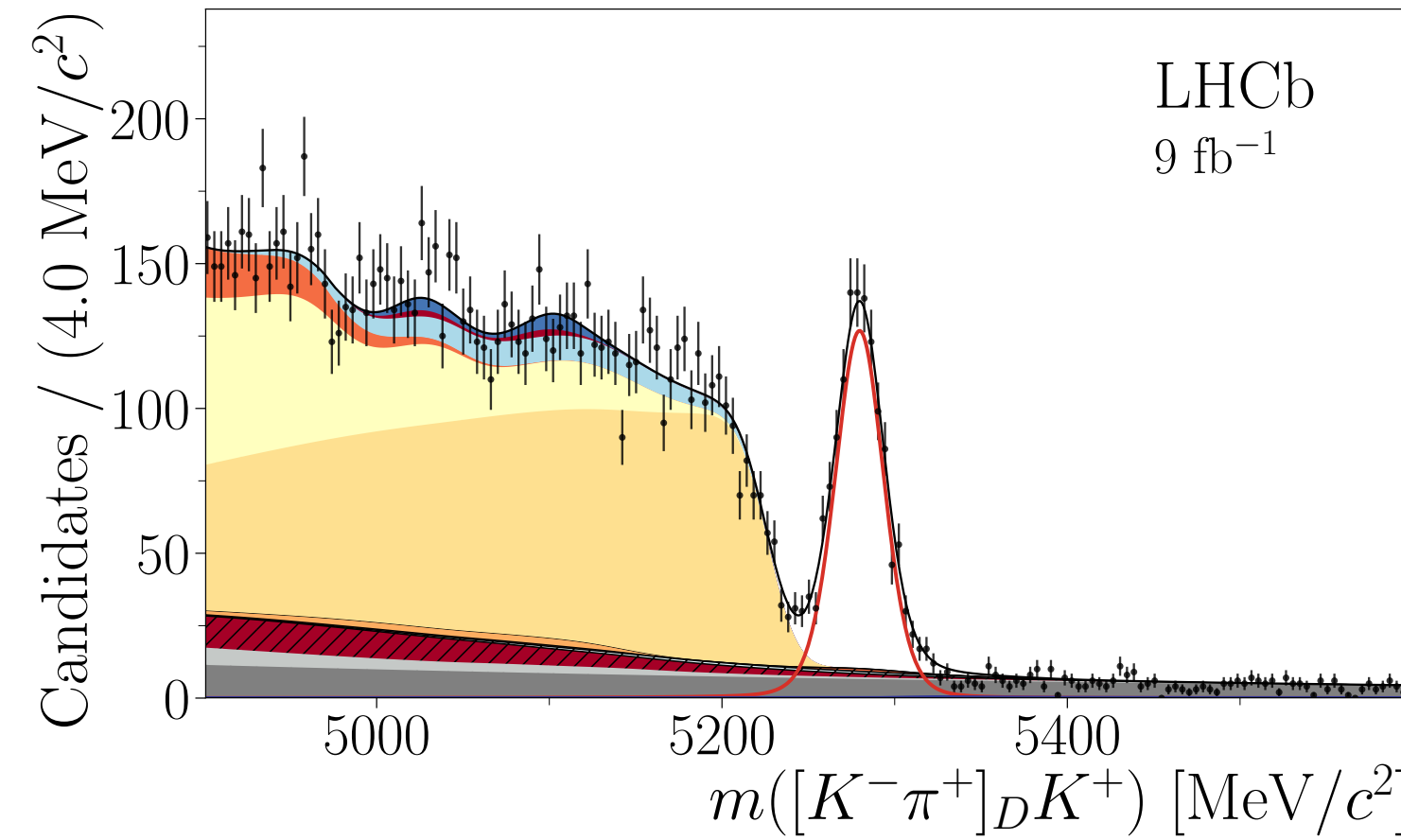
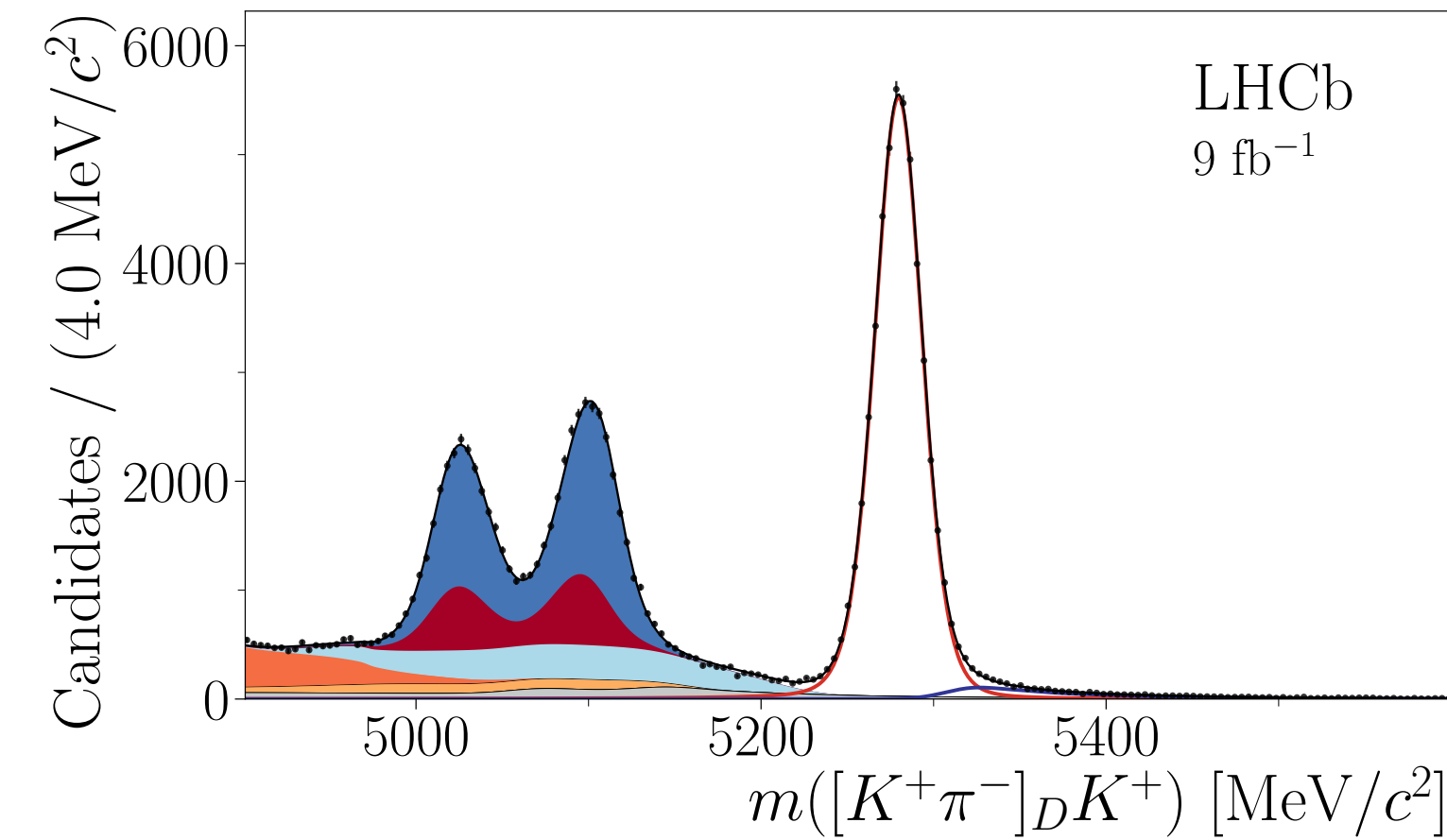
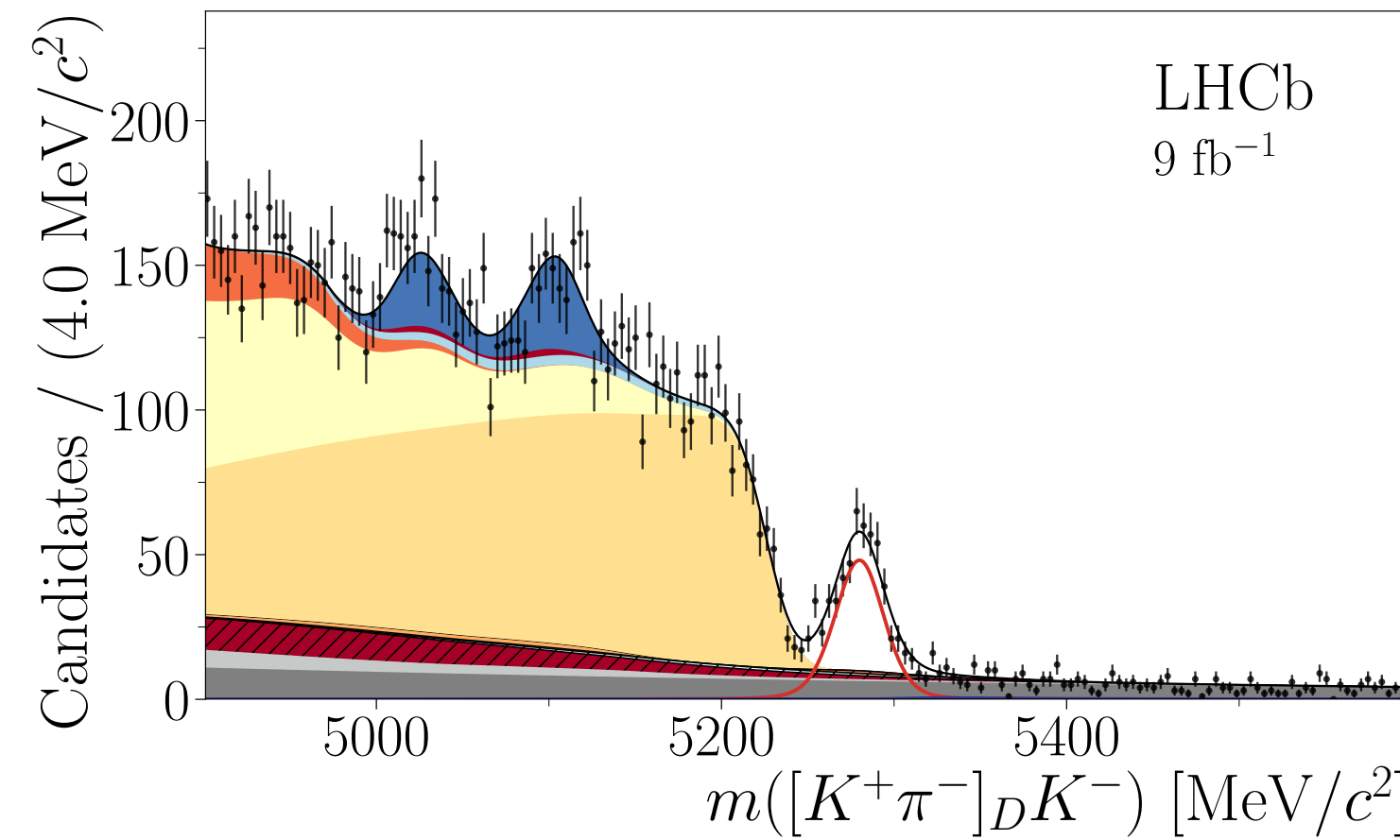
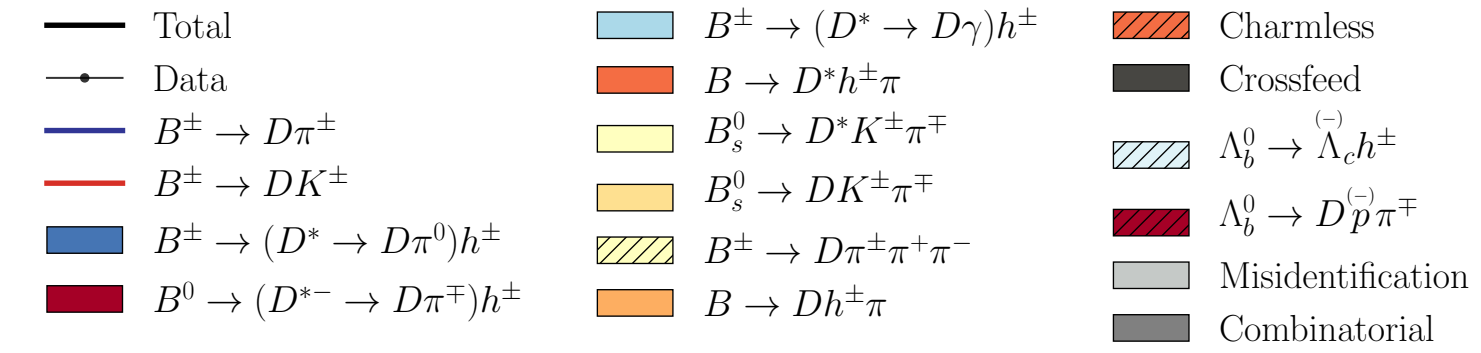
- Update with **full** Run 1 + Run 2
- **Model-independent**: analysis performed in Dalitz bins, using inputs from CLEO and BESIII
- $\gamma = (68.7^{+5.2}_{-5.1})^\circ$



# $B^- \rightarrow D^{(*)}h^-, D \rightarrow h^+h'^-$

- Update with **full** Run 1 + Run 2
- Partial reconstruction** method for  $D^* \rightarrow D^0\pi^0, D^0\gamma$
- First observation** of ADS  $B^- \rightarrow (D\pi^0)_{D^*}\pi^-$
- $\gamma = (61.80 \pm 4.0)^\circ$  when combined with  $K_s^0 h^+ h^-$  result

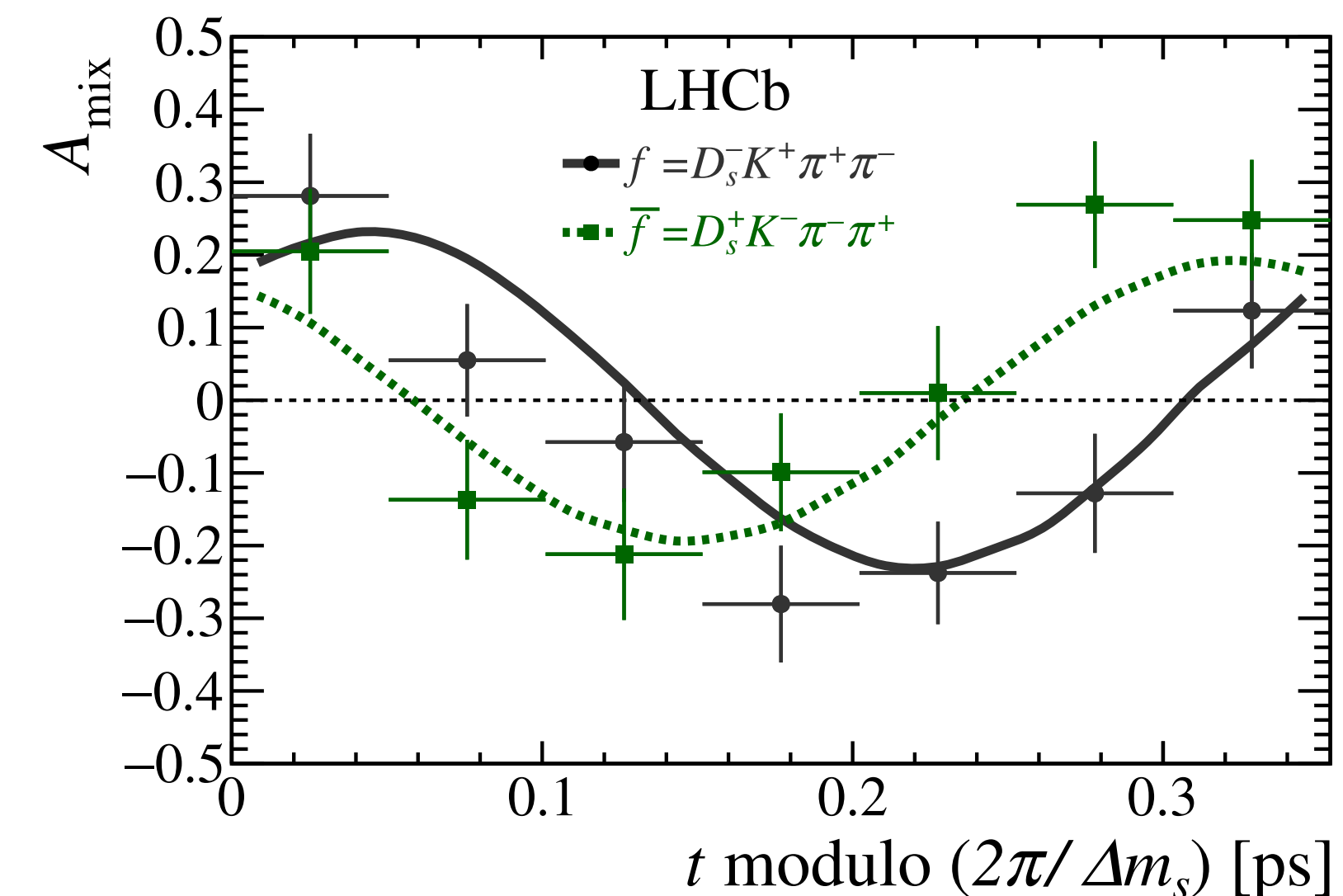
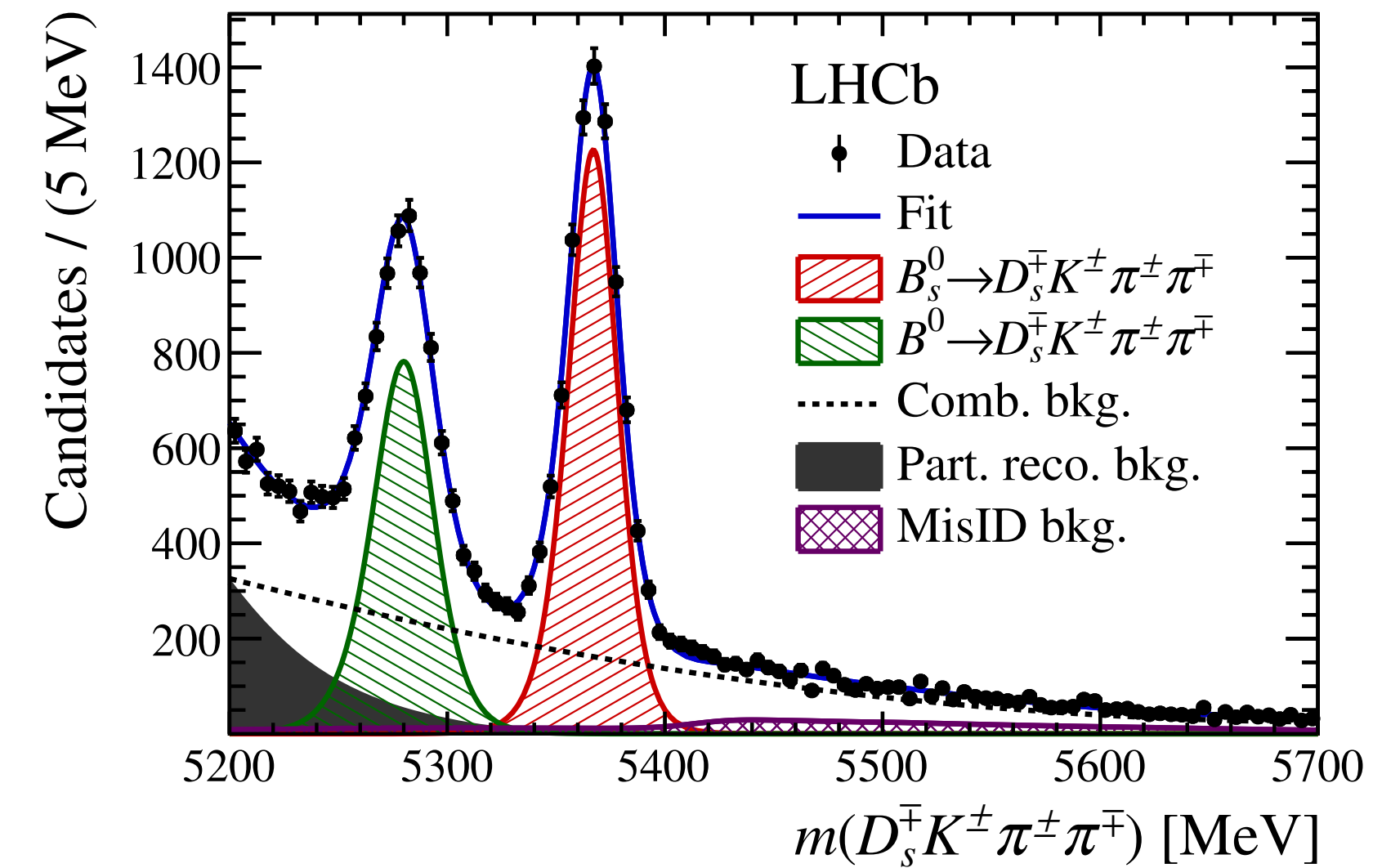
[JHEP 04 \(2021\) 081](#)



$$B_s^0 \rightarrow D_s^\mp K^\pm \pi^+ \pi^-, D_s^+ \rightarrow h^+ h^- \pi^+$$

[JHEP 03 \(2021\) 137](#)

- **New** measurement with **full** Run 1 + Run 2
- **Time-dependent amplitude** analysis, takes  $\phi_s$  as external input
- Less precise **model-independent** analysis
- $B_s^0 \rightarrow D_s^- \pi^+ \pi^+ \pi^-$  used as calibration
- $\gamma = (44_{-13}^{+20})^\circ$  modulo  $180^\circ$  with model-independent approach
- $\gamma = (44 \pm 12)^\circ$  modulo  $180^\circ$  with time-dependent amplitude analysis

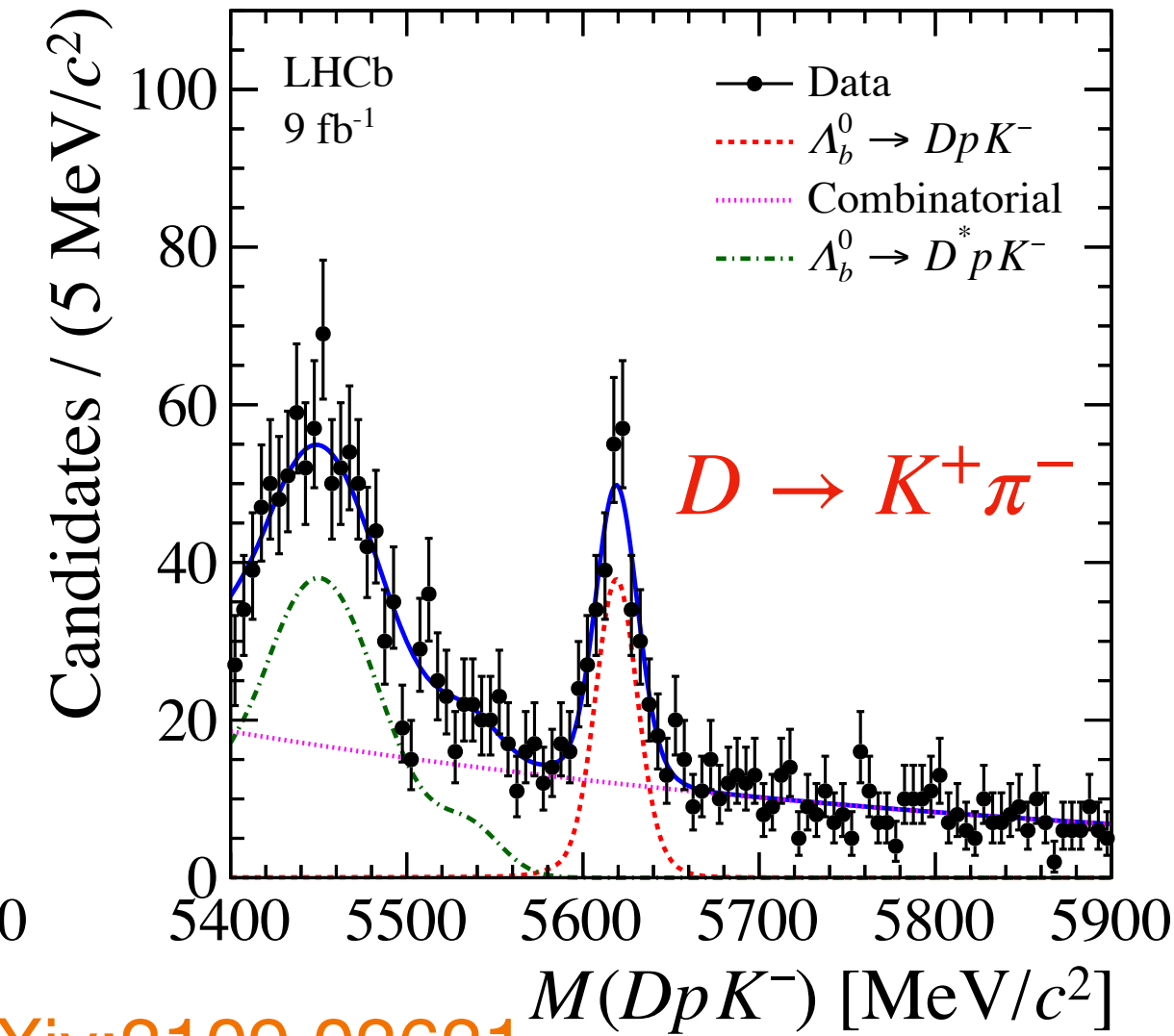
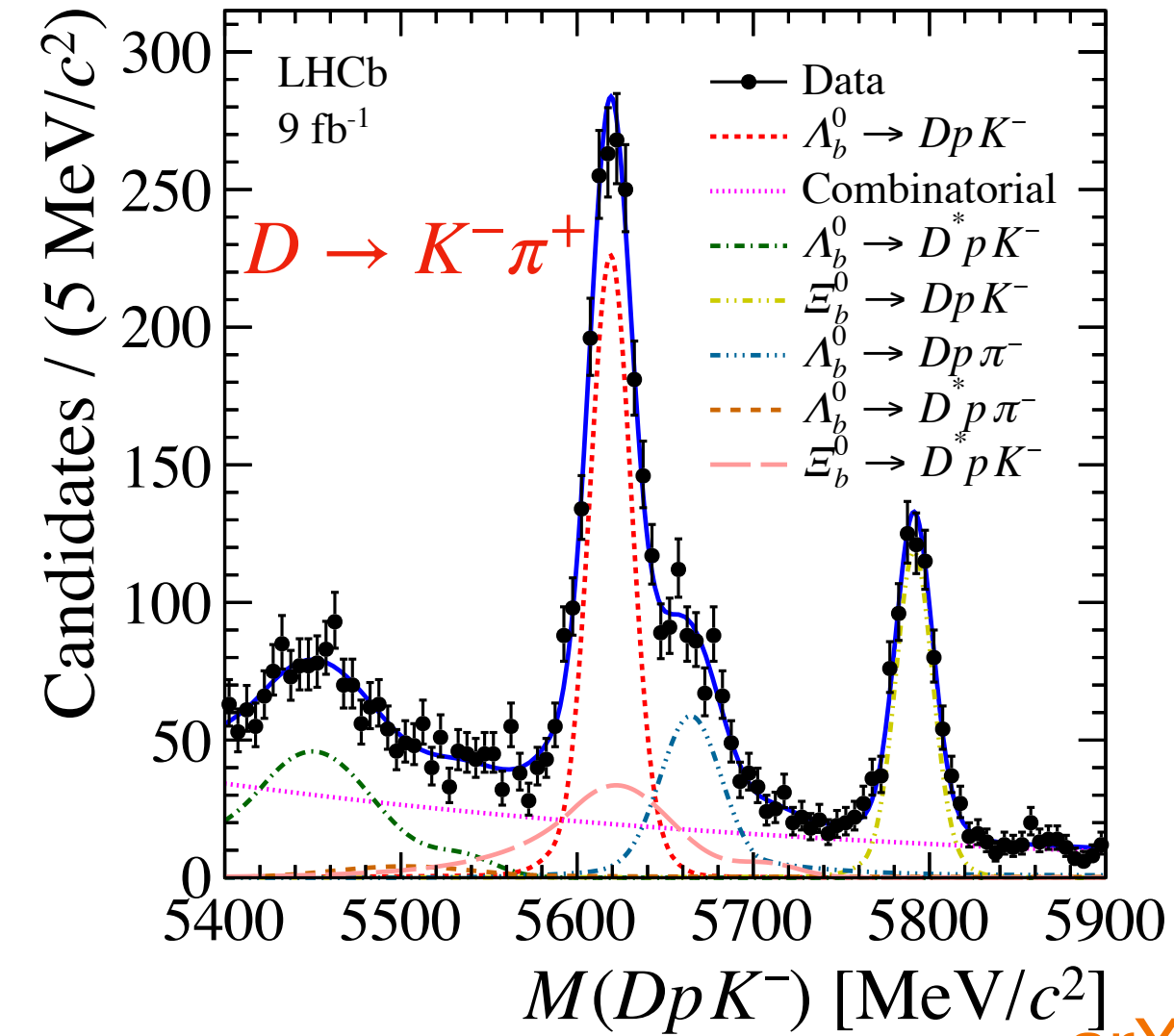
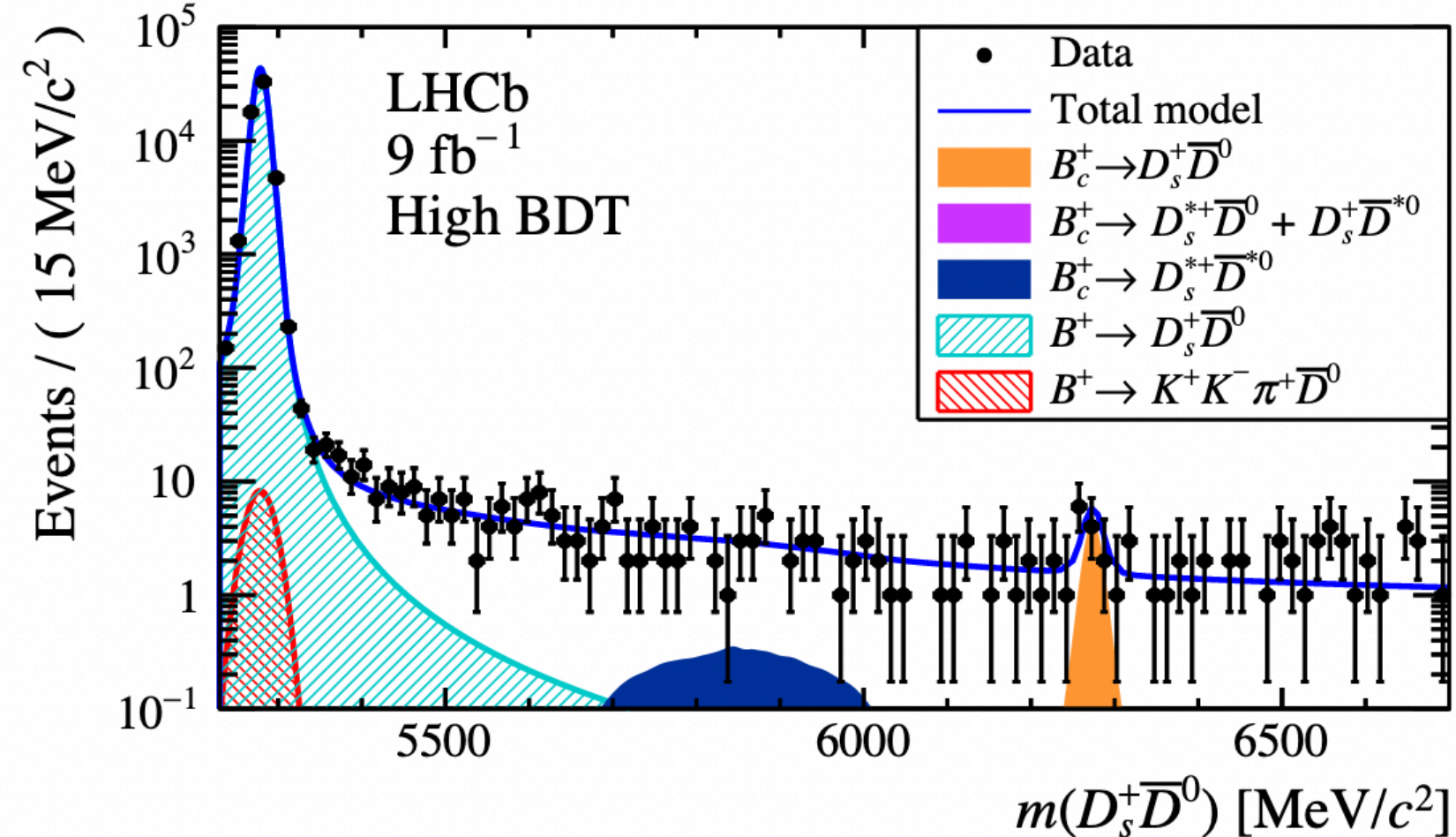




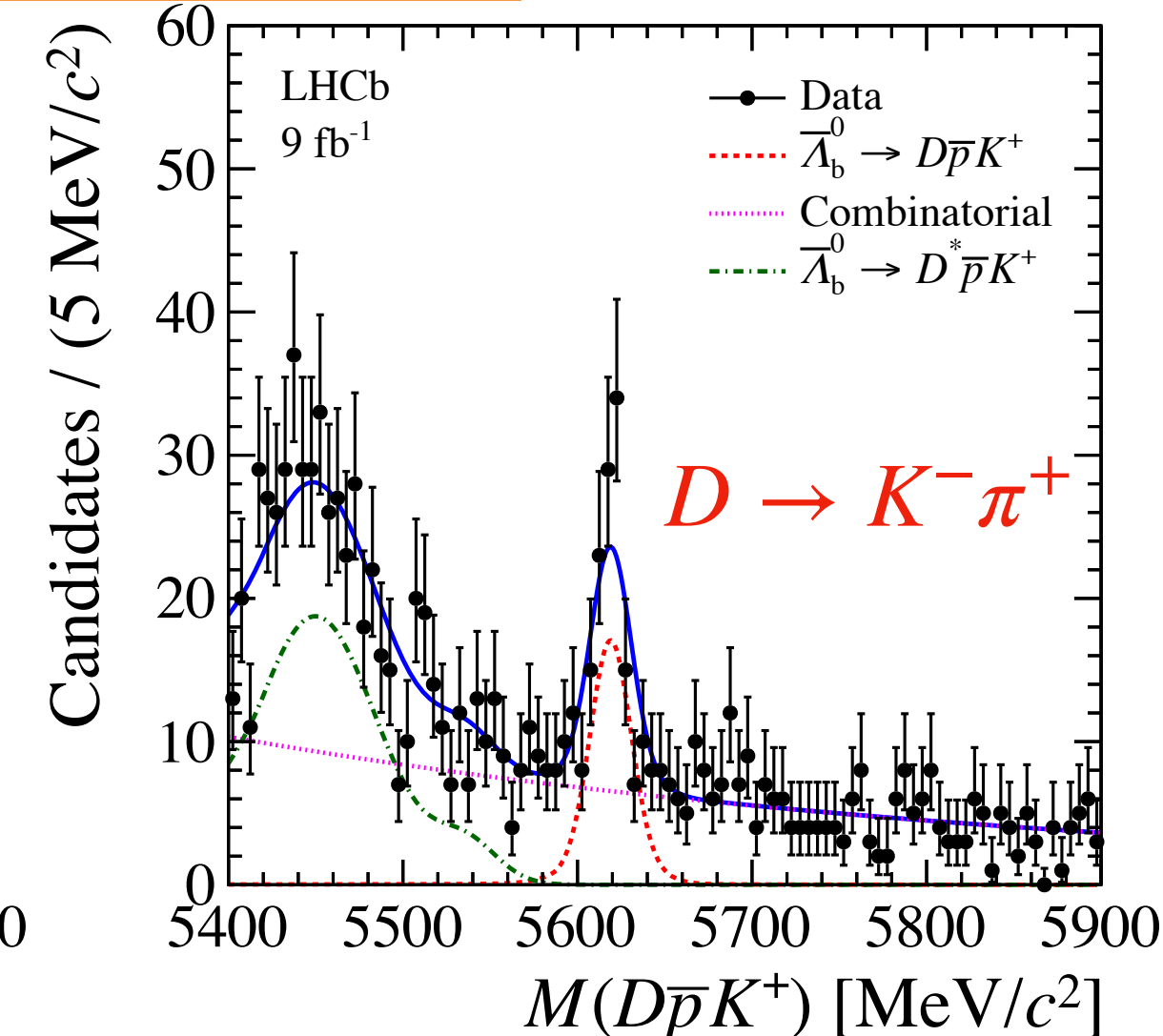
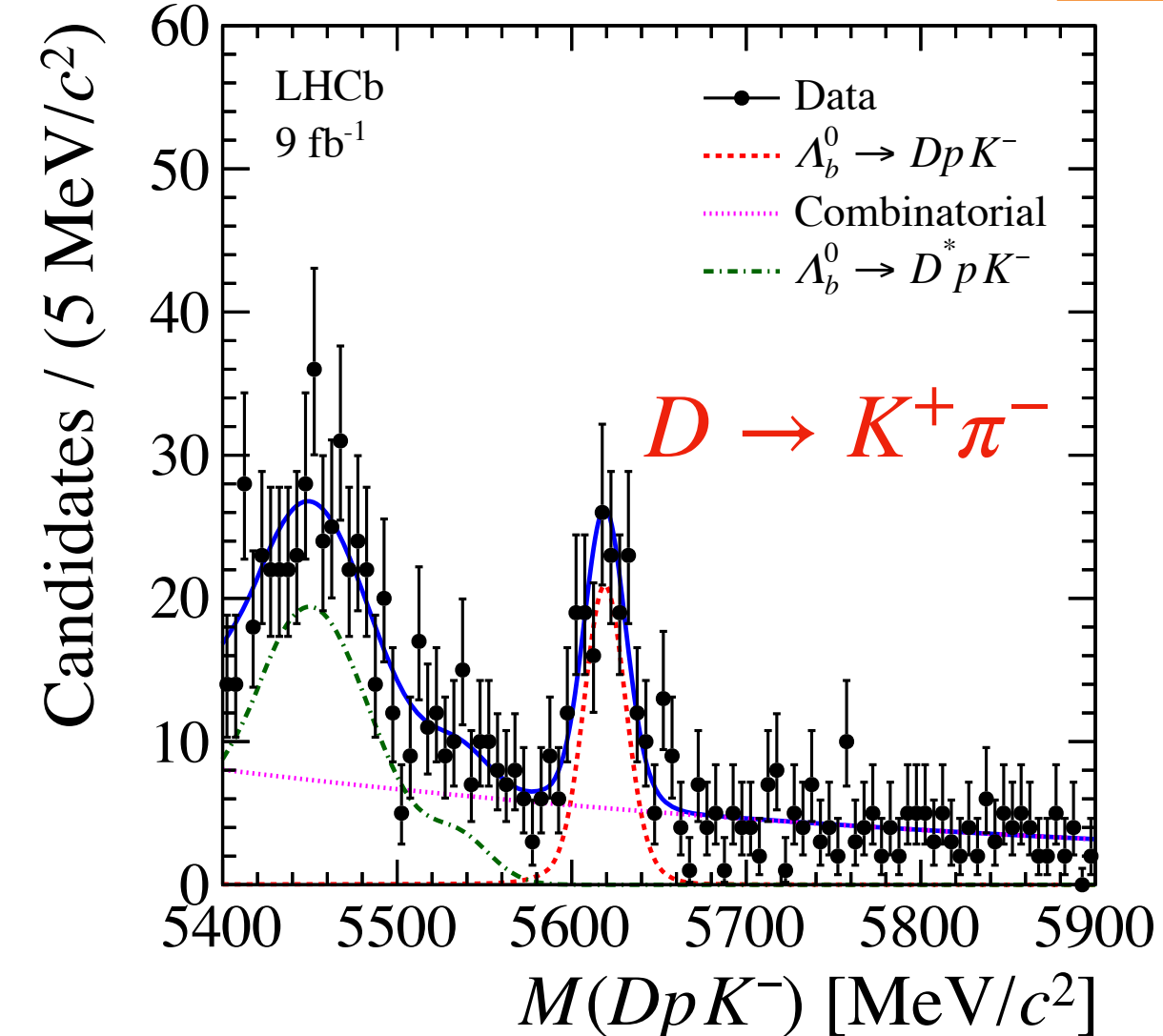
# Other decay modes

- Recent measurements with full Run 1 + Run 2:
  - $A_{CP}$  of  $\Lambda_b^0 \rightarrow [K\pi]_D p K^-$
  - Search for  $B_c^+ \rightarrow D_{(s)}^{(*)+} D^{(*)}$
- Potential to be sensitive to  $\gamma$  with future runs

[arXiv:2109.00488](https://arxiv.org/abs/2109.00488)



[arXiv:2109.02621](https://arxiv.org/abs/2109.02621)



- In the past, charm parameters ( $x_D, y_D, \delta_D^{K\pi}, r_D^{K\pi}$ ) have been taken as **external input** from HFLAV
- $B^\pm \rightarrow DK^\pm$  with  $D \rightarrow K^\pm \pi^\mp$  are sensitive to  $\delta_D^{K\pi}$ , if  $x_D, y_D$  and  $r_D^{K\pi}$  are taken as input

$$A_{\text{fav}}^{K\pi} = \frac{2r_B^K r_D^{K\pi} \sin(\delta_B^K - \delta_D^{K\pi}) \sin \gamma}{1 + (r_B^K r_D^{K\pi})^2 + 2r_B^K r_D^{K\pi} \cos(\delta_B^K - \delta_D^{K\pi}) \cos \gamma}$$

⇒ best option is to combine **LHCb charm+beauty** results to constrain all four parameters

⇒ 8 charm measurements are included

$$\frac{\Gamma(D^0 \rightarrow K^+ \pi^-, t)}{\Gamma(D^0 \rightarrow K^- \pi^+, t)} \approx (r_D^{K\pi})^2 + r_D^{K\pi} (x_D \sin \delta_D^{K\pi} - y_D \cos \delta_D^{K\pi}) \frac{t}{\tau}$$

- In charm measurements of  $D^0 \rightarrow K^+ \pi^-$  to  $D^0 \rightarrow K^- \pi^+$  ratio,  $y_D$  and  $\delta_D^{K\pi}$  are highly correlated  $\Rightarrow$  the charm+beauty combination improves precision on  $y_D$
- Further motivation to include charm measurements:  $D$  mixing has significant effects on  $B^\pm \rightarrow D\pi^\pm$



# LHCb $\gamma$ + charm combination



- 7 new/updated beauty measurements
- 151 observables
- 52 free parameters
- Frequentist treatment (*Plugin* method)
- Cross-checked with Bayesian framework

[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)

<i>B</i> decay	<i>D</i> decay	Ref.	Dataset	Status since Ref. [24]
$B^\pm \rightarrow Dh^\pm$	$D \rightarrow h^+h^-$	[27]	Run 1&2	<b>Updated</b>
$B^\pm \rightarrow Dh^\pm$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	[28]	Run 1	As before
$B^\pm \rightarrow Dh^\pm$	$D \rightarrow h^+h^-\pi^0$	[29]	Run 1	As before
$B^\pm \rightarrow Dh^\pm$	$D \rightarrow K_S^0 h^+ h^-$	[26]	Run 1&2	<b>Updated</b>
$B^\pm \rightarrow Dh^\pm$	$D \rightarrow K_S^0 K^\pm \pi^\mp$	[30]	Run 1&2	<b>Updated</b>
$B^\pm \rightarrow D^* h^\pm$	$D \rightarrow h^+ h^-$	[27]	Run 1&2	<b>Updated</b>
$B^\pm \rightarrow DK^{*\pm}$	$D \rightarrow h^+ h^-$	[31]	Run 1&2(*)	As before
$B^\pm \rightarrow DK^{*\pm}$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	[31]	Run 1&2(*)	As before
$B^\pm \rightarrow Dh^\pm \pi^+ \pi^-$	$D \rightarrow h^+ h^-$	[32]	Run 1	As before
$B^0 \rightarrow DK^{*0}$	$D \rightarrow h^+ h^-$	[33]	Run 1&2(*)	<b>Updated</b>
$B^0 \rightarrow DK^{*0}$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	[33]	Run 1&2(*)	<b>New</b>
$B^0 \rightarrow DK^{*0}$	$D \rightarrow K_S^0 \pi^+ \pi^-$	[34]	Run 1	As before
$B^0 \rightarrow D^\mp \pi^\pm$	$D^+ \rightarrow K^- \pi^+ \pi^+$	[35]	Run 1	As before
$B_s^0 \rightarrow D_s^\mp K^\pm$	$D_s^+ \rightarrow h^+ h^- \pi^+$	[36]	Run 1	As before
$B_s^0 \rightarrow D_s^\mp K^\pm \pi^+ \pi^-$	$D_s^+ \rightarrow h^+ h^- \pi^+$	[37]	Run 1&2	<b>New</b>
–	$D^0 \rightarrow h^+ h^-$	[38] [40]	Run 1&2	<b>New</b>
–	$D^0 \rightarrow h^+ h^-$	[41]	Run 1	<b>New</b>
–	$D^0 \rightarrow h^+ h^-$	[42] [45]	Run 1&2	<b>New</b>
–	$D^0 \rightarrow K^+ \pi^-$	[46]	Run 1	<b>New</b>
–	$D^0 \rightarrow K^+ \pi^-$	[47]	Run 1&2(*)	<b>New</b>
–	$D^0 \rightarrow K^\pm \pi^\mp \pi^+ \pi^-$	[48]	Run 1	<b>New</b>
–	$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	[49] [50]	Run 1&2	<b>New</b>
–	$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	[51]	Run 1	<b>New</b>



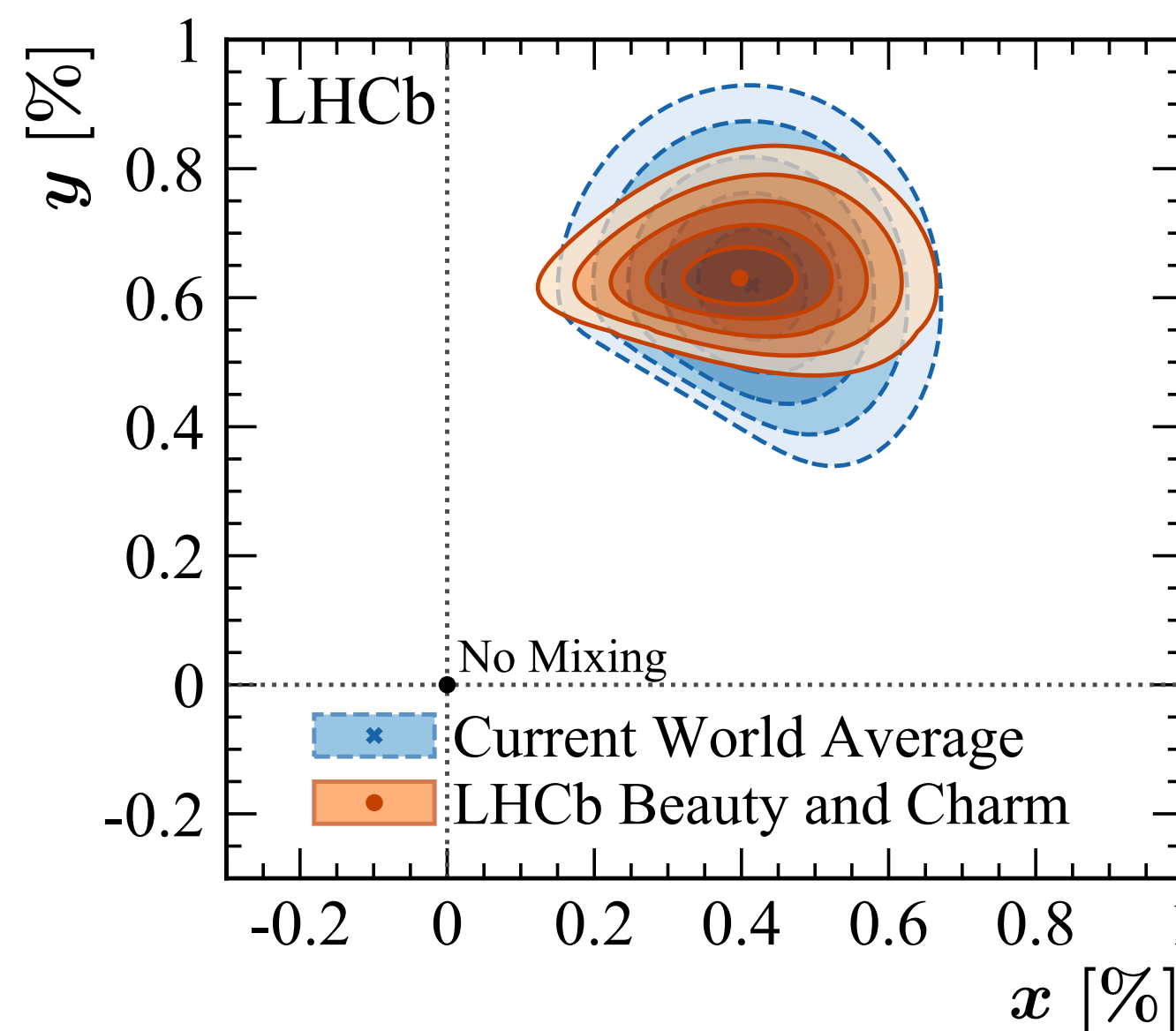
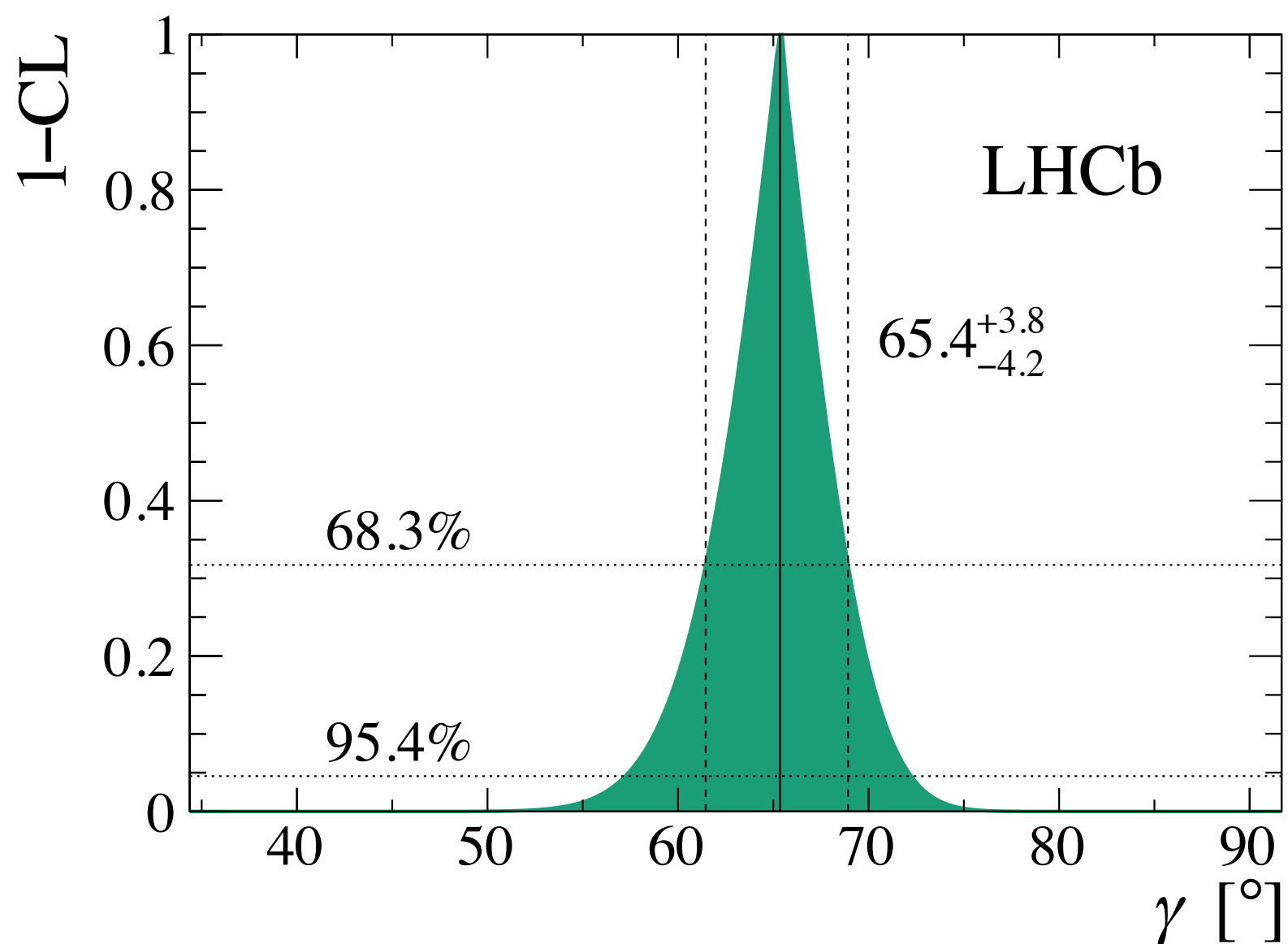
# Auxiliary inputs

External constraints needed for hadronic parameters and coherence factors in multibody  $B$  and  $D$  decays

[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)

Decay	Parameters	Source	Ref.	Status since Ref. [24]
$B^\pm \rightarrow DK^{*\pm}$	$\kappa_{B^\pm}^{DK^{*\pm}}$	LHCb	[31]	As before
$B^0 \rightarrow DK^{*0}$	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[52]	As before
$B^0 \rightarrow D^\mp \pi^\pm$	$\beta$	HFLAV	[18]	Updated
$B_s^0 \rightarrow D_s^\mp K^\pm(\pi\pi)$	$\phi_s$	HFLAV	[18]	Updated
$D \rightarrow h^+ h^- \pi^0$	$F_{\pi\pi\pi^0}^+, F_{K\pi\pi^0}^+$	CLEO-c	[53]	As before
$D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$	$F_{4\pi}^+$	CLEO-c	[53]	As before
$D \rightarrow K^+ \pi^- \pi^0$	$r_D^{K\pi\pi^0}, \delta_D^{K\pi\pi^0}, \kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[48, 54, 56]	Updated
$D \rightarrow K^\pm \pi^\mp \pi^+ \pi^-$	$r_D^{K3\pi}, \delta_D^{K3\pi}, \kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[48, 54, 56]	Updated
$D \rightarrow K_S^0 K^\pm \pi^\mp$	$r_D^{K_S^0 K\pi}, \delta_D^{K_S^0 K\pi}, \kappa_D^{K_S^0 K\pi}$	CLEO	[57]	As before
$D \rightarrow K_S^0 K^\pm \pi^\mp$	$r_D^{K_S^0 K\pi}$	LHCb	[58]	As before

$$\gamma = (65.4^{+3.8}_{-4.2})^\circ$$



[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)

$$x_D = (4.00^{+0.52}_{-0.53}) \times 10^{-3}$$

$$y_D = (6.30^{+0.33}_{-0.30}) \times 10^{-3}$$

$$\delta_D^{K\pi} = (190.0^{+4.2}_{-4.1})^\circ$$

$$r_D^{K\pi} = (58.67 \pm 0.15) \times 10^{-3}$$

- Excellent **precision** on  $\gamma$

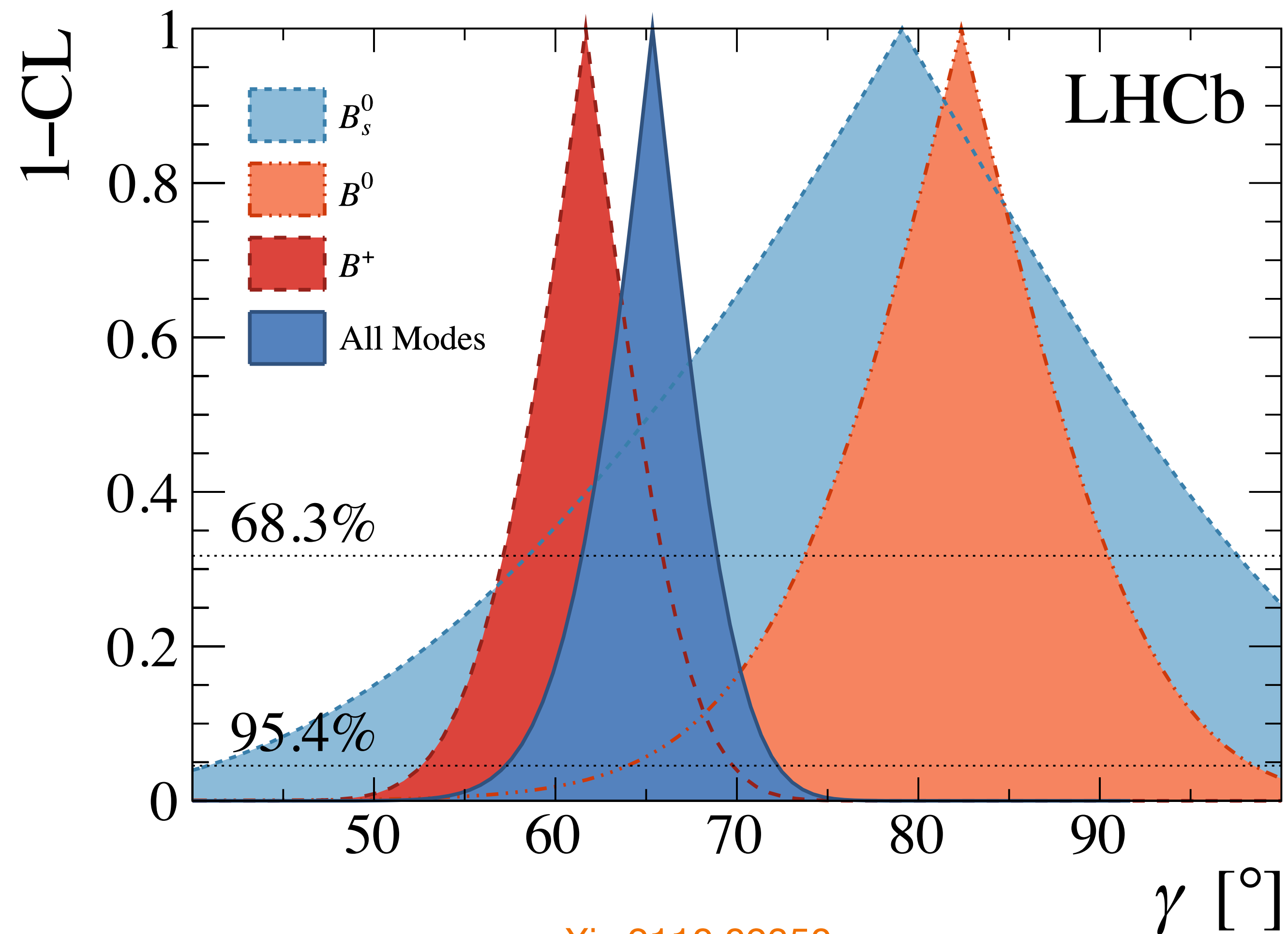
- Agreement with **indirect** global CKM fitters:  $\gamma = (65.7^{+0.9}_{-2.7})^\circ$  CKMfitter  $\gamma = (65.8 \pm 2.2)^\circ$  UTFit

New world average:  $\gamma = (66.2^{+3.4}_{-3.6})^\circ$  HFLAV



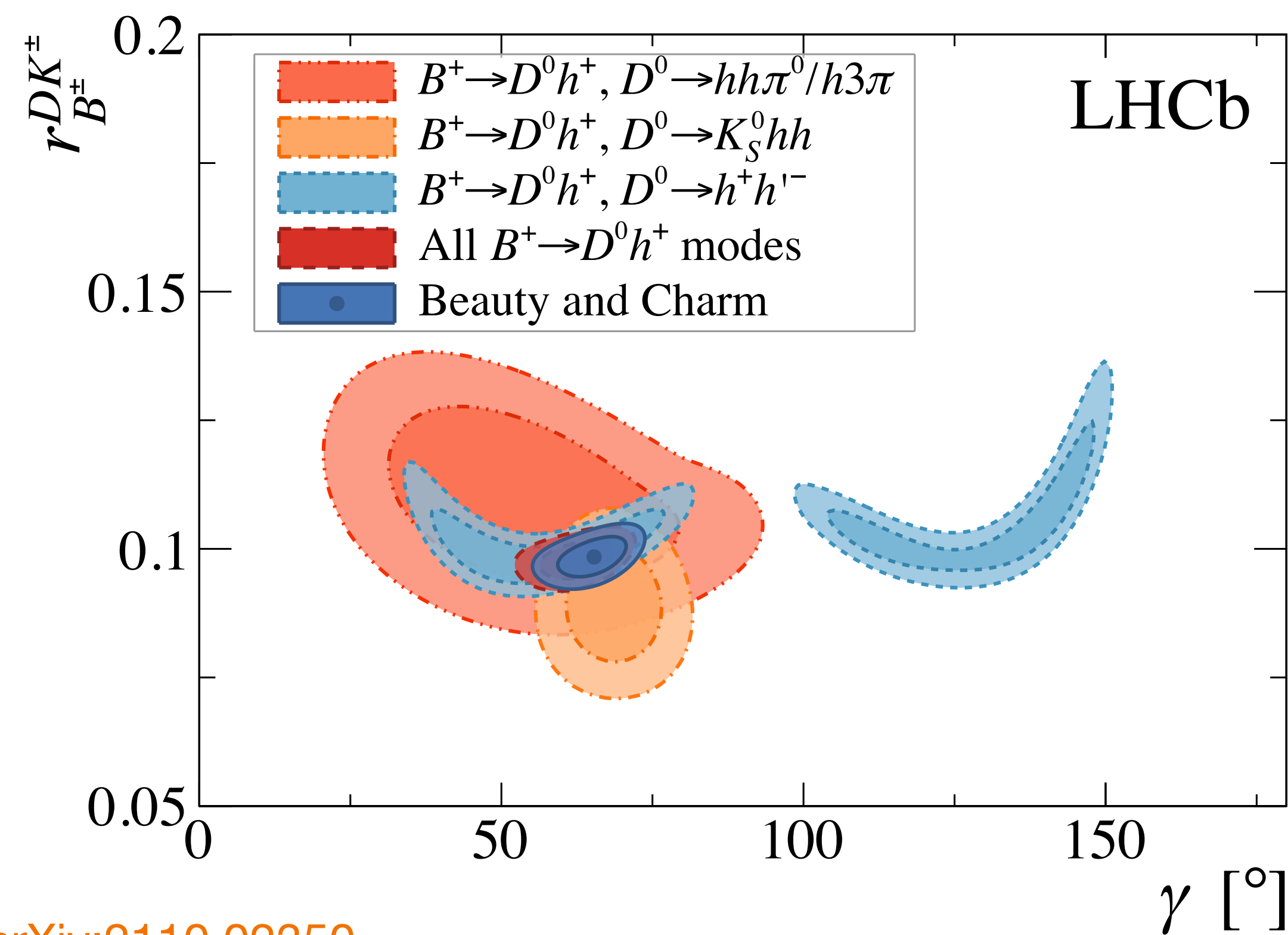
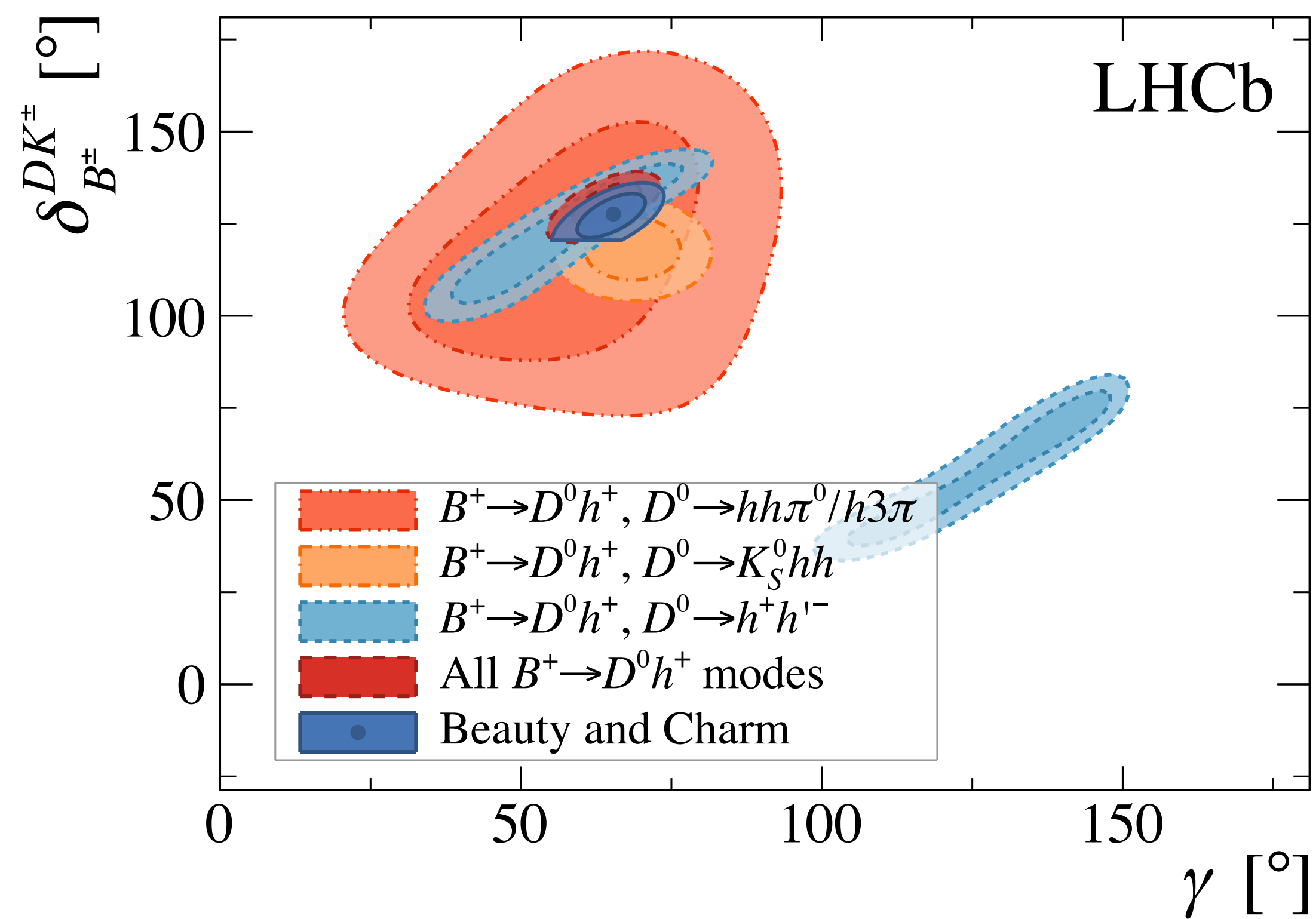
## Combinations for different *B* species

Species	Value [°]	68.3% CL	
		Uncertainty	Interval
$B^+$	61.7	+4.4 -4.8	[56.9, 66.1]
$B^0$	82.0	+8.1 -8.8	[73.2, 90.1]
$B_s^0$	79	+21 -24	[55, 100]



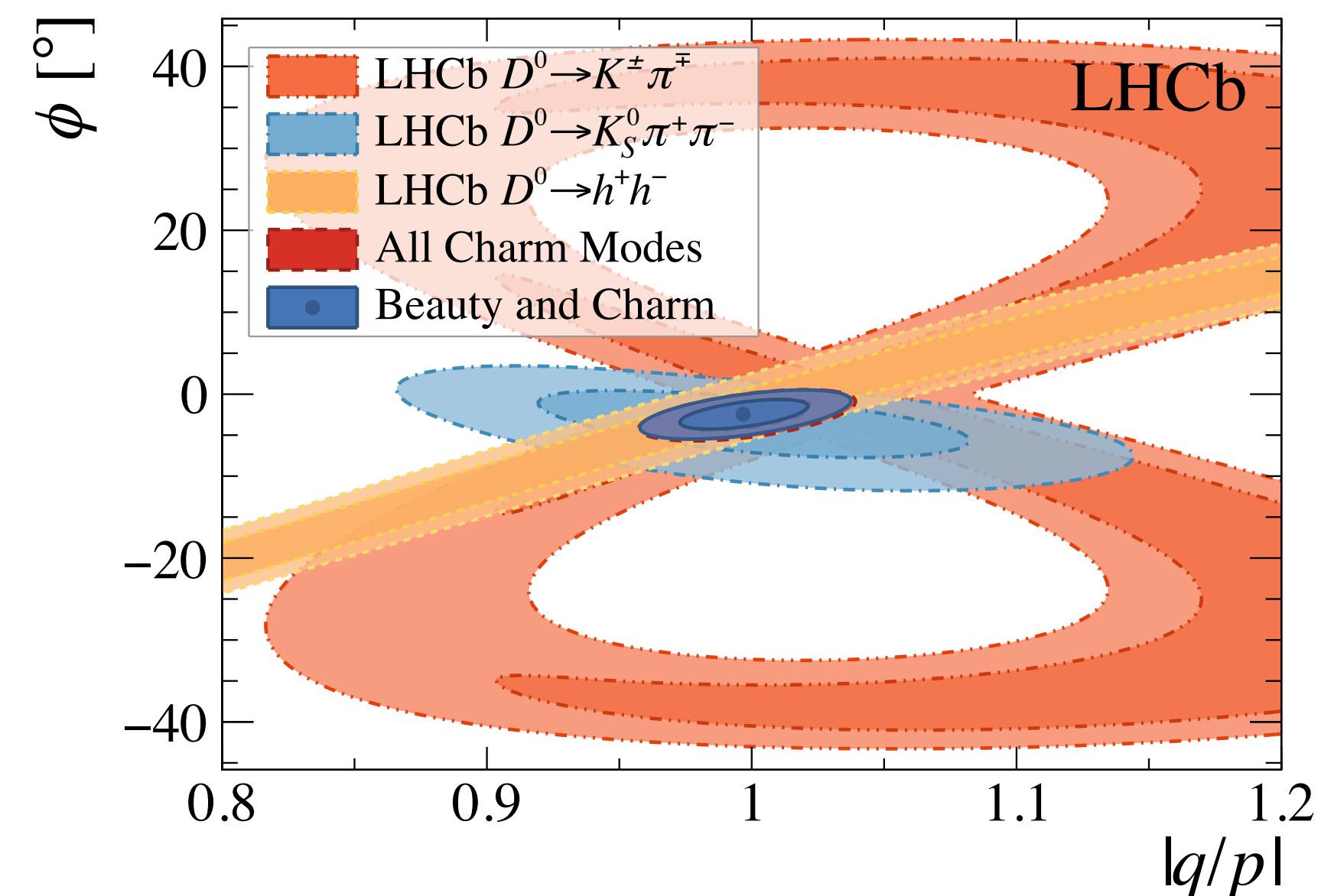
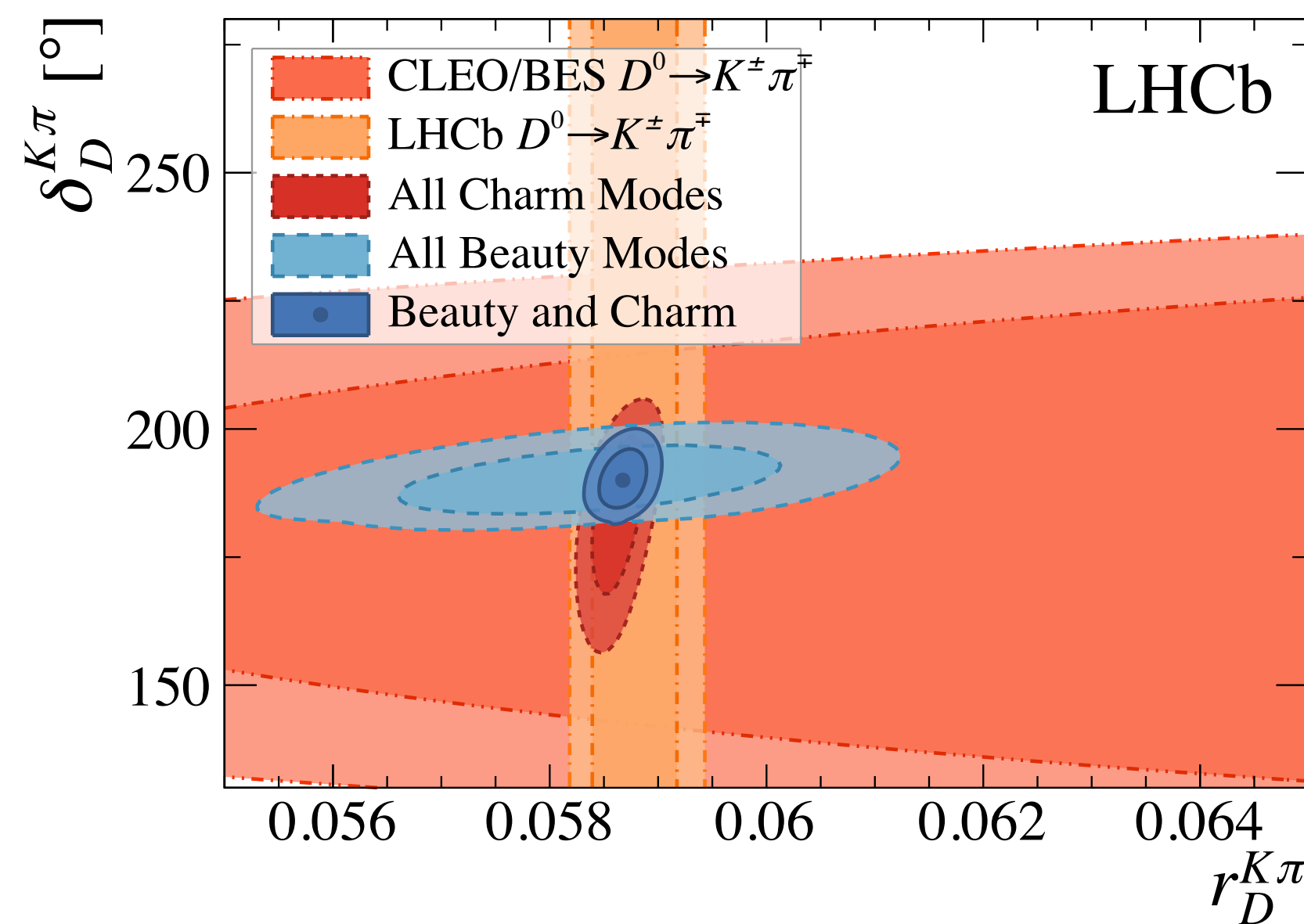
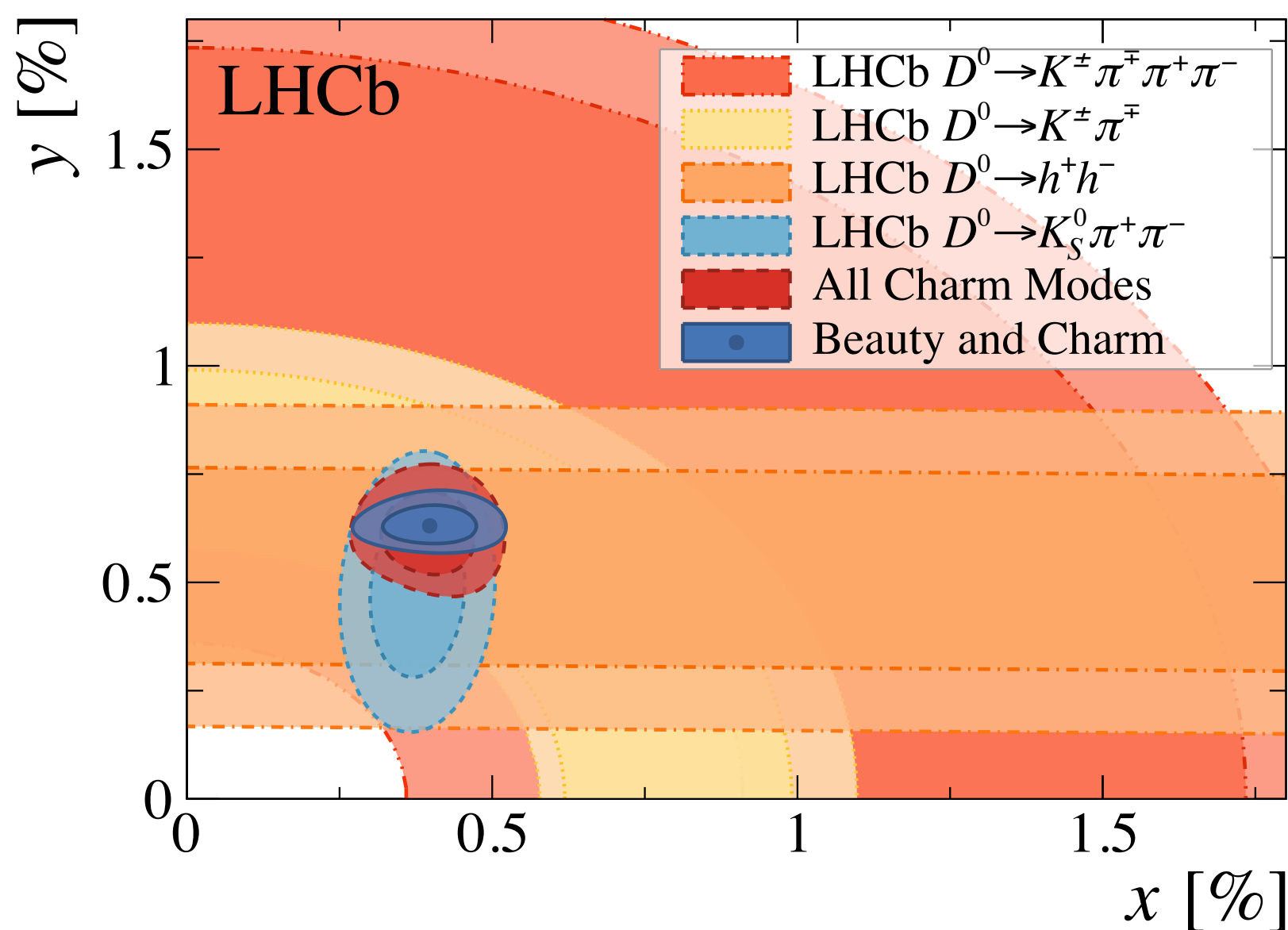
[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)

## Combinations for different $D$ decays for most sensitive modes



[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)

## Complementarity of beauty and charm on charm parameters



Precision on  $\delta_D^{K\pi}$  and  $y_D$  improved by almost a factor 2

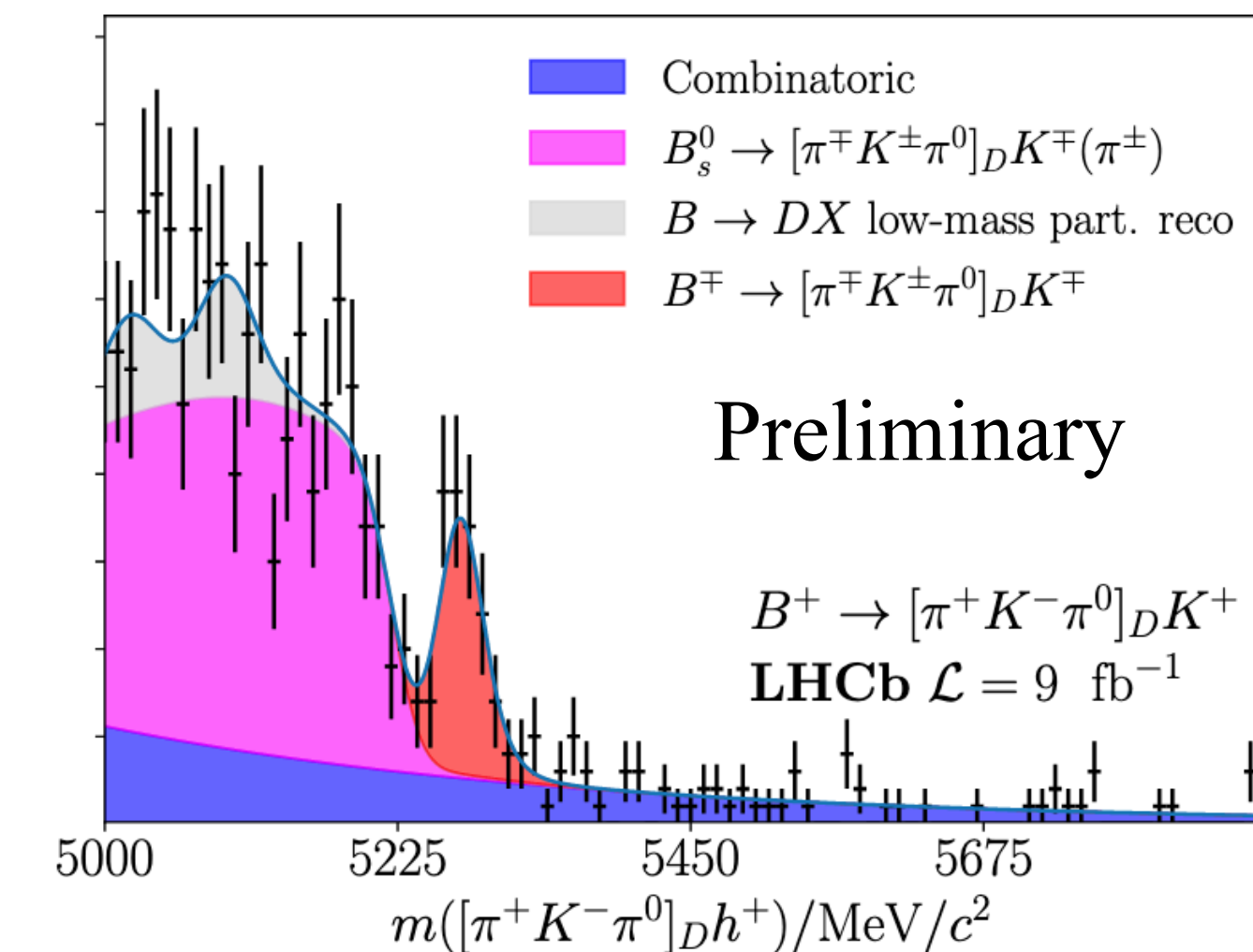
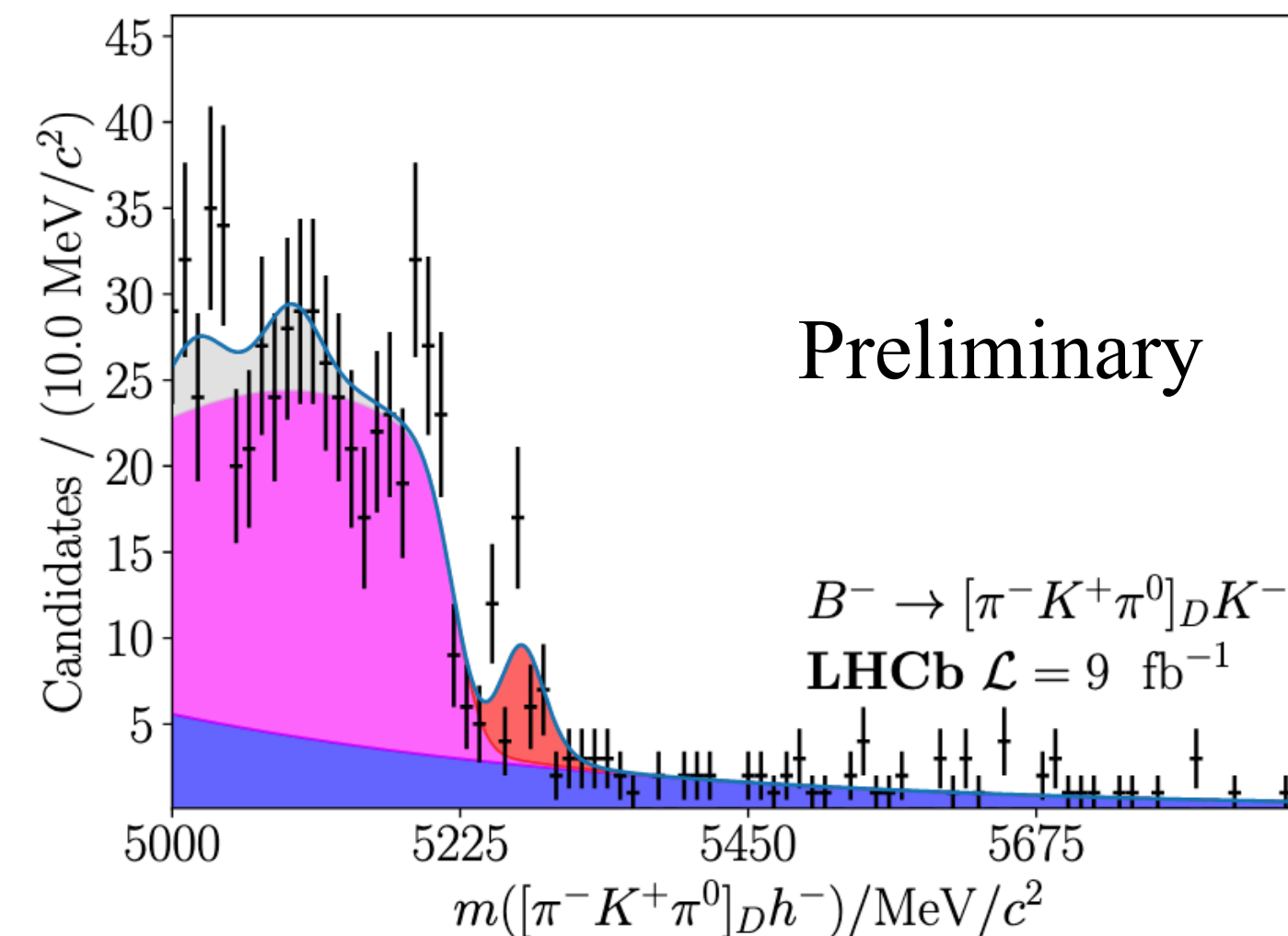
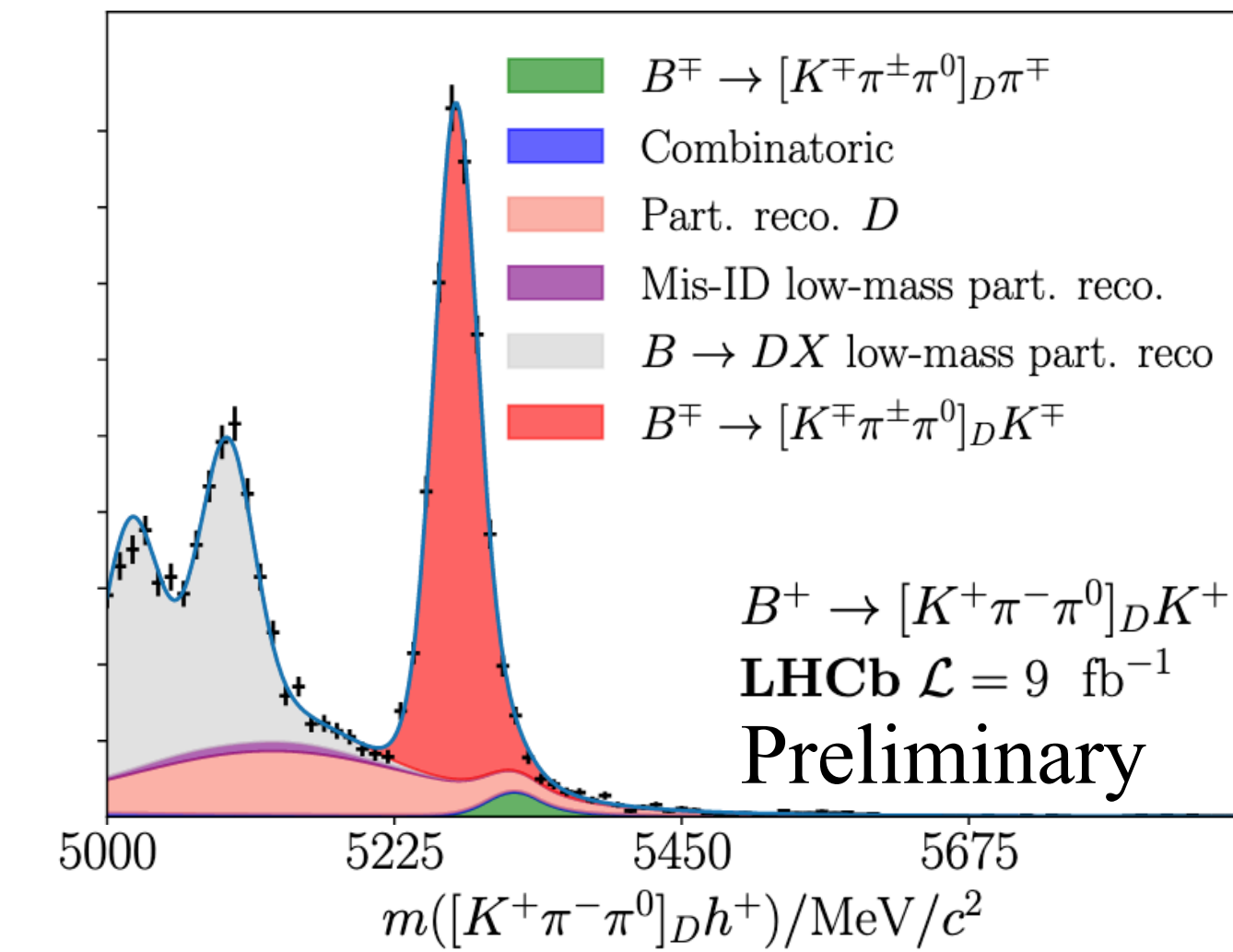
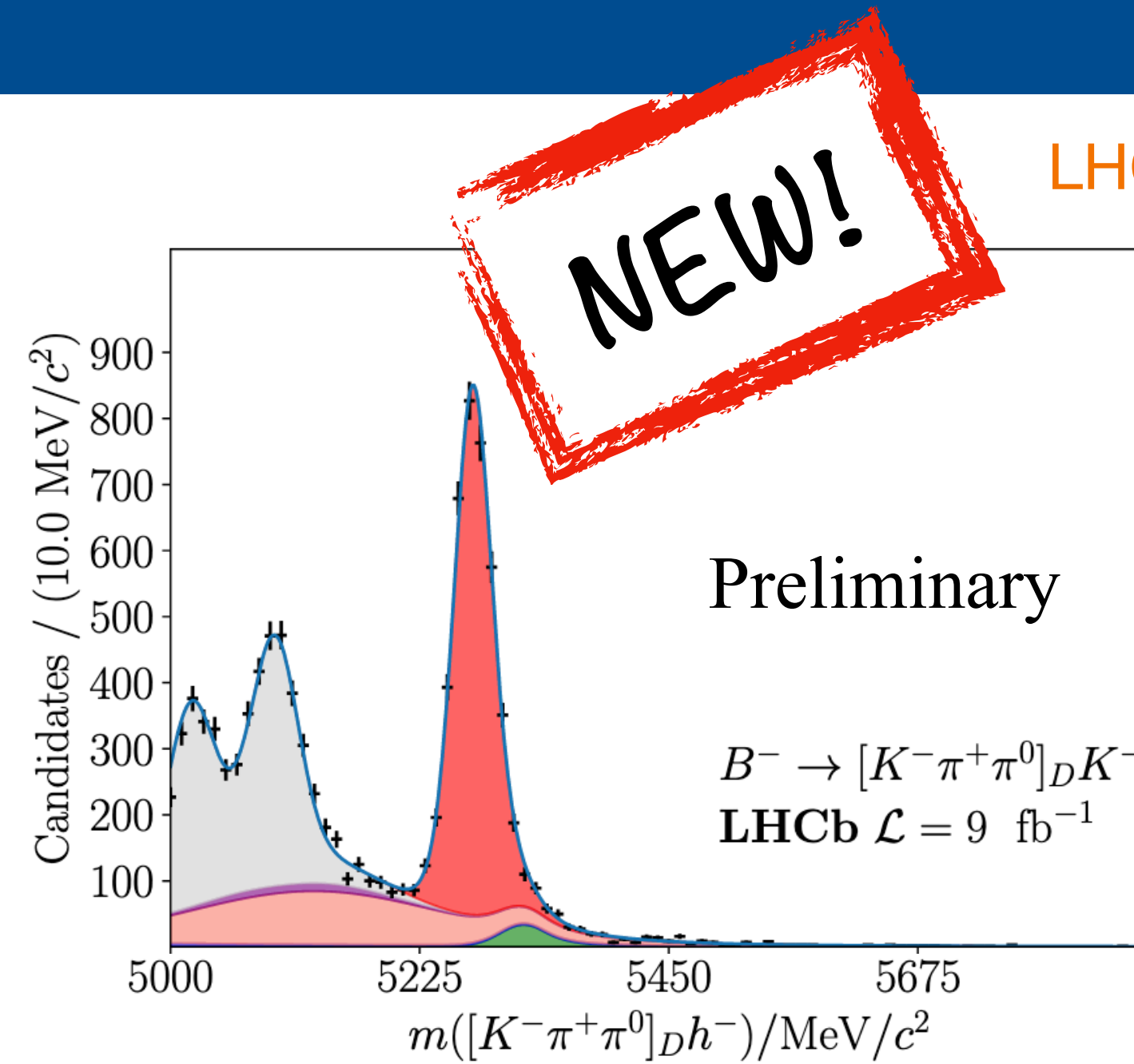
[arXiv:2110.02350](https://arxiv.org/abs/2110.02350)



# $B^- \rightarrow Dh^-, D \rightarrow h^+h^-\pi^0$

- Update with **full** Run 1 + Run 2
- **Quasi-ADS** and **quasi-GLW** modes  $\Rightarrow$  use dilution factors taken from external inputs
- **Observation** of suppressed  $B^- \rightarrow [\pi^-K^+\pi^0]_DK^-$  with more than  $7\sigma$  significance

LHCb-PAPER-2021-036 (in preparation)



# $B^- \rightarrow Dh^-, D \rightarrow h^+h^-\pi^0$

- Global minimum at  $\gamma = (145^{+9}_{-39})^\circ$
- Second solution consistent with combination

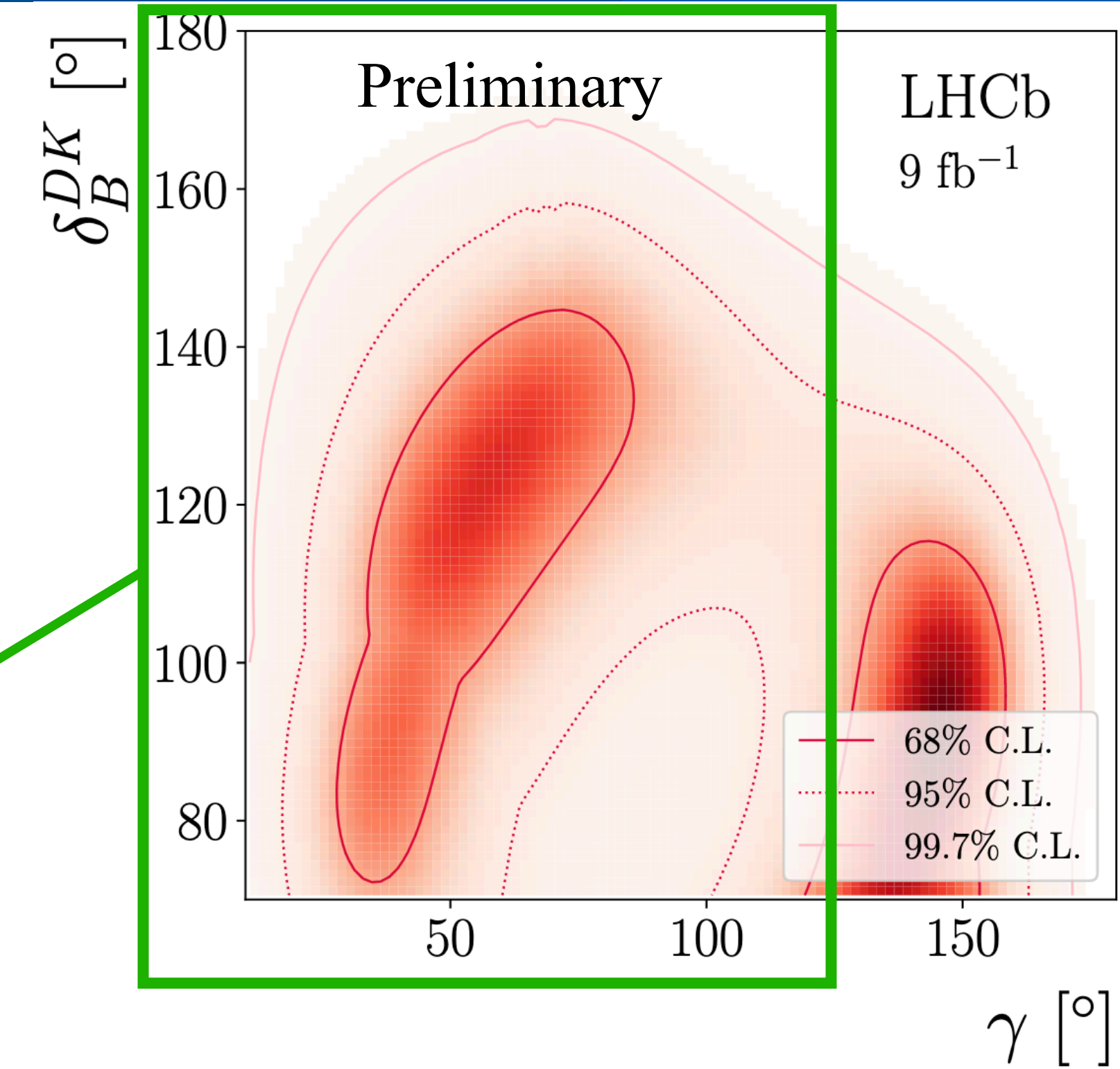
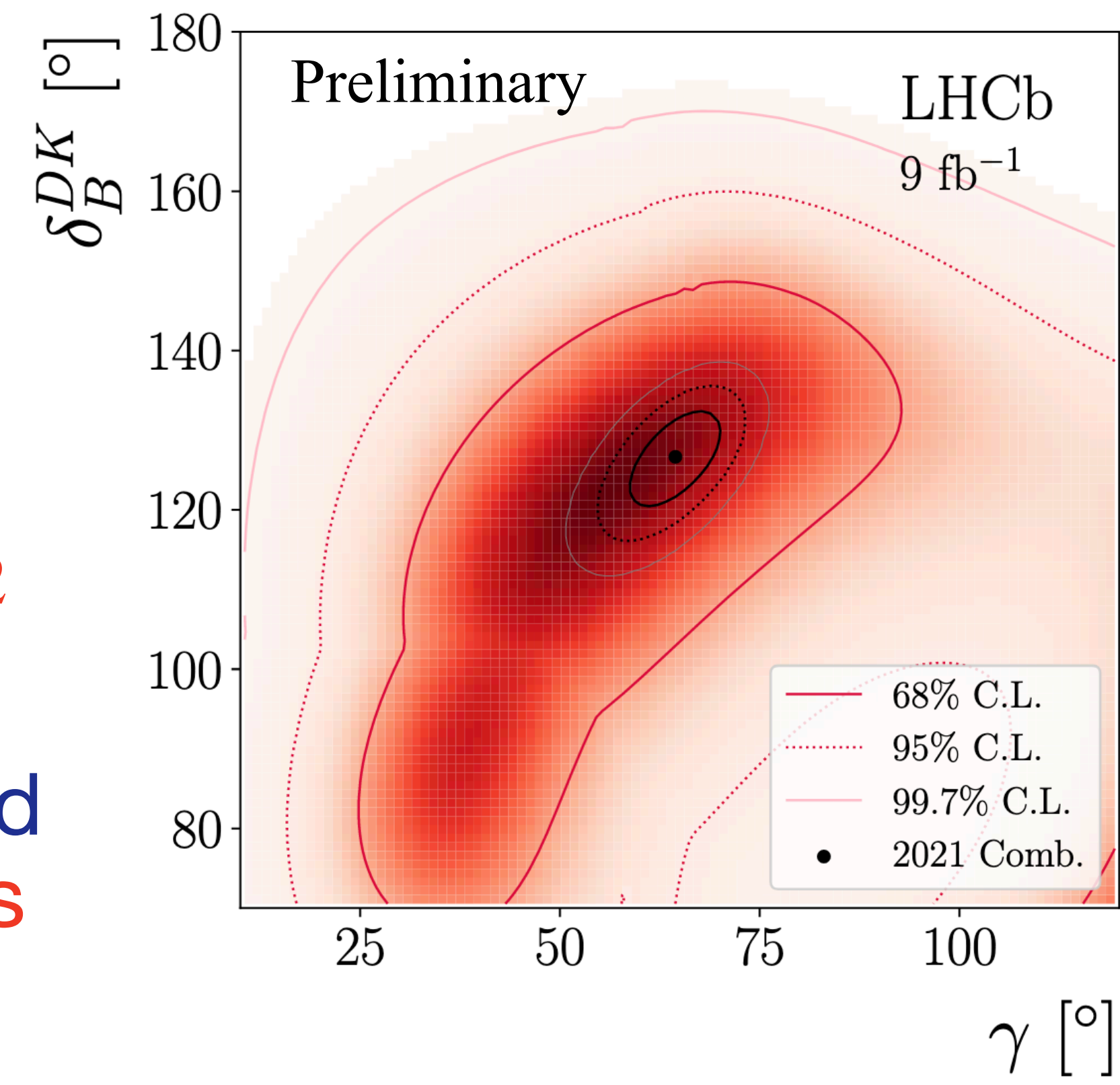
**NEW!**

$\gamma = (56^{+24}_{-19})^\circ$

$\delta_B = (122^{+19}_{-23})^\circ$

$r_B = (9.25^{+1.04}_{-0.85}) \times 10^{-2}$

- This result will be used in future combinations

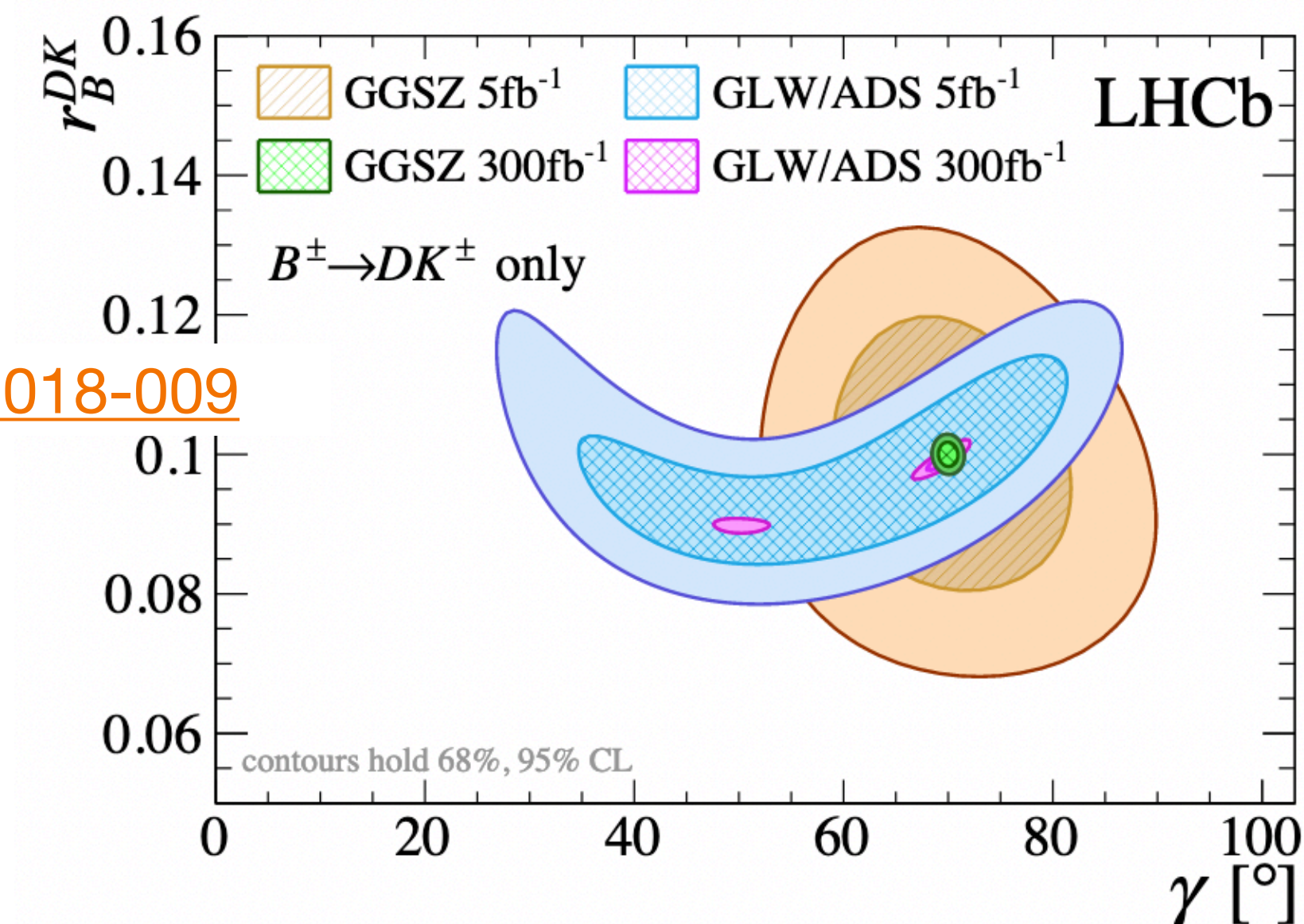
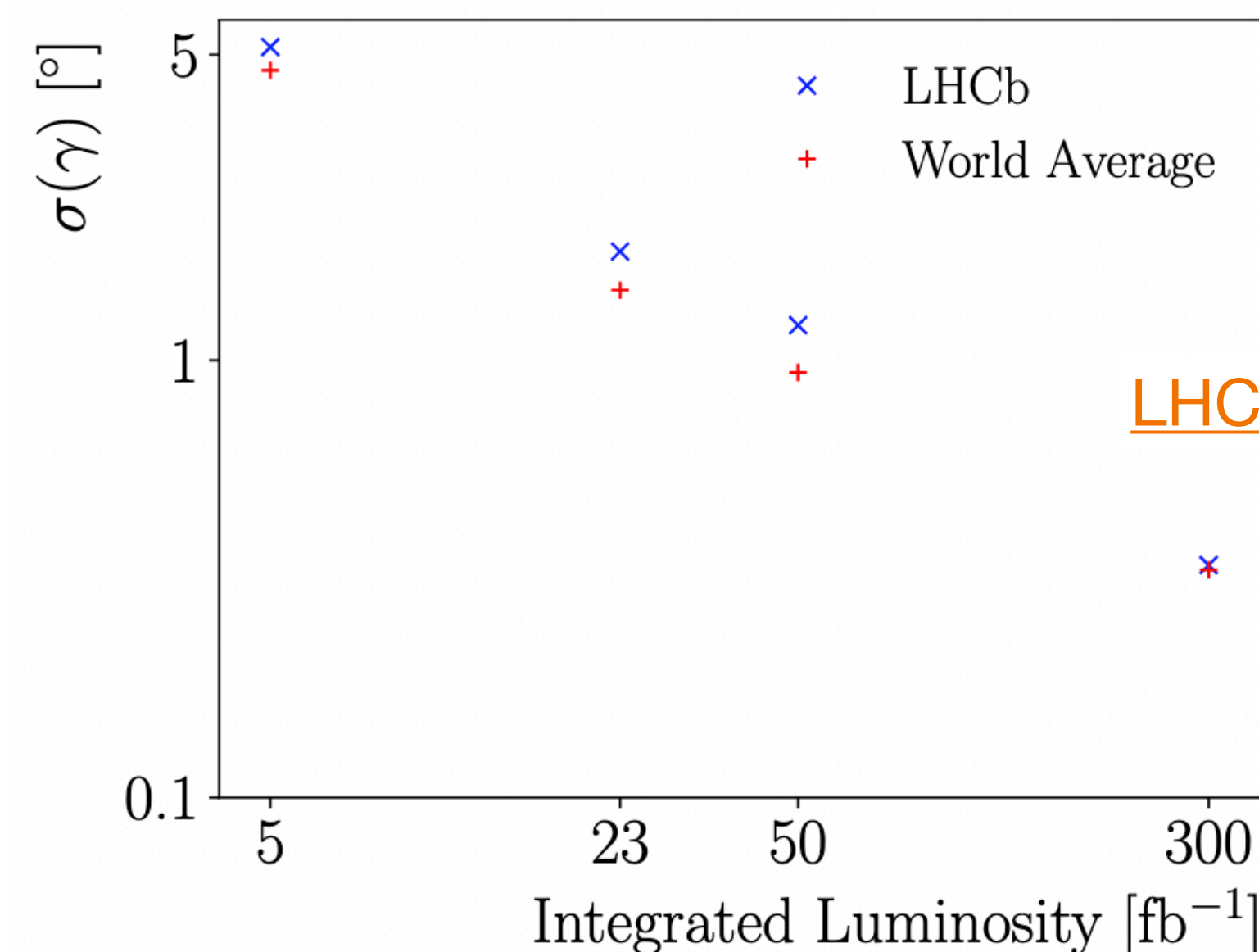


LHCb-PAPER-2021-036 (in preparation)



# Future prospects

- Other  $B \rightarrow Dh$  measurements ongoing with **Run 2** data (e.g.  $B \rightarrow [K\pi\pi]_D h$ )
- Sensitivity of  $B^0 \rightarrow [K_s^0 h^+ h^-]_D K^+ \pi^-$  and  $B_s^0 \rightarrow D_s^{\mp} K^{\pm}$  is expected to increase by a **factor 2** with Run 2 data
- **Upgrades** will be fundamental to increase precision on  $\gamma$



- Many **recent results** by LHCb with full data sample to determine  $\gamma$
- LHCb combination:  $\gamma = (65.4^{+3.8}_{-4.2})^\circ$
- New measurements and updates with Run 2 data still **ongoing**
- LHCb upgrades aim for  $< 1^\circ$  precision on  $\gamma$

**Backup**

- $K^0 - \bar{K}^0$  mixing
  - For  $D \rightarrow K_s^0 h^+ h^-$ :  $\mathcal{O}(10^{-3})$  relative shift on  $\gamma$ , also taken into account in systematics
  - For  $D \rightarrow K_s^0 K^\pm \pi^\mp$ : larger effect, but sensitivity of the related measurement to  $\gamma$  is low
- Assume  $U$ -spin symmetry when considering  $CP$  violation in  $D \rightarrow h^+ h^-$  in the beauty sector:  $0.3^\circ$  effect
- Neglect  $CP$  violation in charm mixing in beauty sector:  $0.1^\circ$  effect
- Correlation of systematics between measurements: negligible effect because the combination is statistically dominated ( $3.3^\circ$  out of  $3.6^\circ$ )



Quantity	Value	68.3% CL		95.4% CL	
		Uncertainty	Interval	Uncertainty	Interval
$\gamma [^\circ]$	65.4	+3.8 -4.2	[61.2, 69.2]	+7.5 -8.7	[56.7, 72.9]
$\tau_{B^\pm}^{DK^\pm}$	0.0984	+0.0027 -0.0026	[0.0958, 0.1011]	+0.0056 -0.0052	[0.0932, 0.1040]
$\delta_{B^\pm}^{DK^\pm} [^\circ]$	127.6	+4.0 -4.2	[123.4, 131.6]	+7.8 -9.2	[118.4, 135.4]
$\tau_{B^\pm}^{D\pi^\pm}$	0.00480	+0.00070 -0.00056	[0.00424, 0.00550]	+0.0017 -0.0011	[0.0037, 0.0065]
$\delta_{B^\pm}^{D\pi^\pm} [^\circ]$	288	+14 -15	[273, 302]	+26 -31	[257, 314]
$\tau_{B^\pm}^{D^*K^\pm}$	0.099	+0.016 -0.019	[0.080, 0.115]	+0.030 -0.038	[0.061, 0.129]
$\delta_{B^\pm}^{D^*K^\pm} [^\circ]$	310	+12 -23	[287, 322]	+20 -71	[239, 330]
$\tau_{B^\pm}^{D^*\pi^\pm}$	0.0095	+0.0085 -0.0061	[0.0034, 0.0180]	+0.017 -0.0089	[0.0006, 0.026]
$\delta_{B^\pm}^{D^*\pi^\pm} [^\circ]$	139	+22 -86	[53, 161]	+32 -129	[10, 171]
$\tau_{B^\pm}^{DK^{*\pm}}$	0.106	+0.017 -0.019	[0.087, 0.123]	+0.031 -0.040	[0.066, 0.137]
$\delta_{B^\pm}^{DK^{*\pm}} [^\circ]$	35	+20 -15	[20, 55]	+57 -28	[7, 92]
$\tau_{B^0}^{DK^{*0}}$	0.250	+0.023 -0.024	[0.226, 0.273]	+0.044 -0.052	[0.198, 0.294]
$\delta_{B^0}^{DK^{*0}} [^\circ]$	197	+10 -9.3	[187.7, 207]	+24 -18	[179, 221]
$\tau_{B_s^0}^{D_s^\mp K^\pm}$	0.310	+0.098 -0.092	[0.218, 0.408]	+0.20 -0.21	[0.10, 0.51]



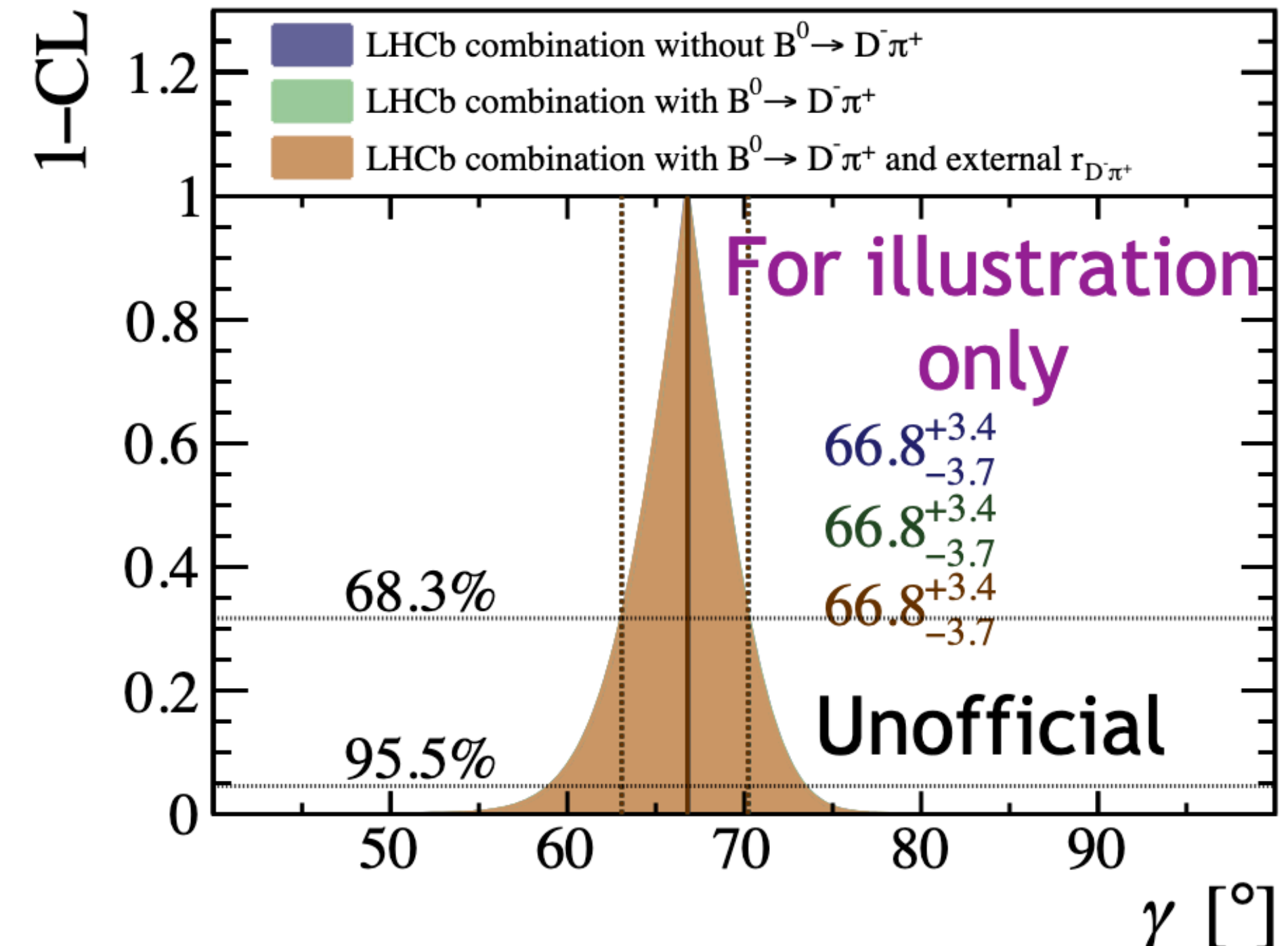
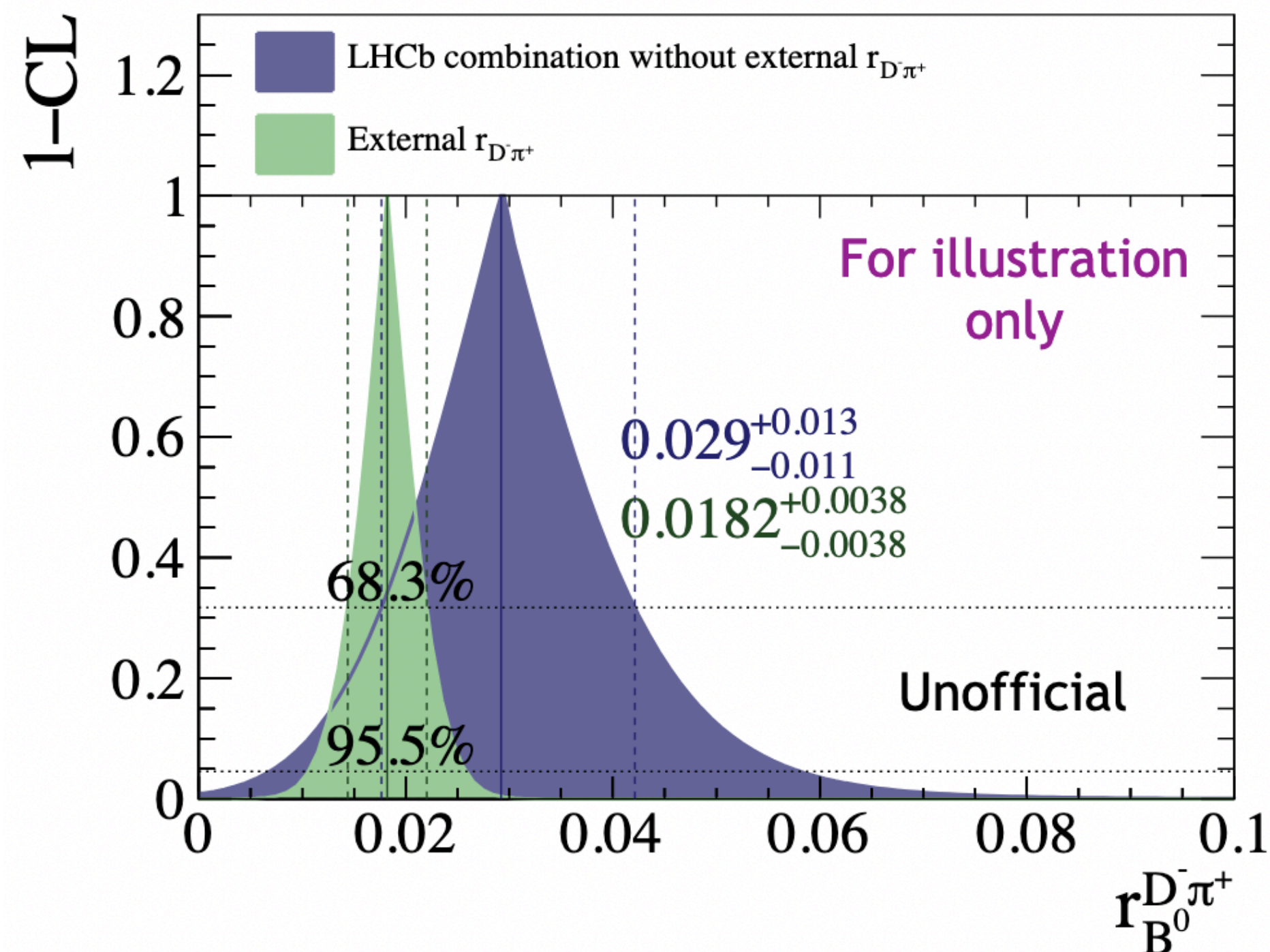
# Results

Quantity	Value	68.3% CL		95.4% CL	
		Uncertainty	Interval	Uncertainty	Interval
$\delta_{B^0}^{D_s^\mp K^\pm} [^\circ]$	356	+19 -18	[338, 375]	+39 -39	[317, 395]
$r_{B^0}^{D_s^\mp K^\pm \pi^+ \pi^-}$	0.460	+0.081 -0.084	[0.376, 0.541]	+0.16 -0.17	[0.29, 0.62]
$\delta_{B^0}^{D_s^\mp K^\pm \pi^+ \pi^-} [^\circ]$	345	+13 -12	[333, 358]	+26 -25	[320, 371]
$r_{B^0}^{D^\mp \pi^\pm}$	0.030	+0.014 -0.012	[0.018, 0.044]	+0.036 -0.028	[0.002, 0.066]
$\delta_{B^0}^{D^\mp \pi^\pm} [^\circ]$	30	+26 -37	[-7, 56]	+45 -81	[-51, 75]
$r_{B^\pm}^{DK^\pm \pi^+ \pi^-}$	0.079	+0.028 -0.034	[0.045, 0.107]	+0.050 -0.079	[0.000, 0.129]*
$r_{B^\pm}^{D\pi^\pm \pi^+ \pi^-}$	0.067	+0.025 -0.029	[0.038, 0.092]	+0.040 -0.067	[0.000, 0.107]*
$x [\%]$	0.400	+0.052 -0.053	[0.347, 0.452]	+0.10 -0.11	[0.29, 0.50]
$y [\%]$	0.630	+0.033 -0.030	[0.600, 0.663]	+0.069 -0.058	[0.572, 0.699]
$r_D^{K\pi}$	0.05867	+0.00015 -0.00015	[0.05852, 0.05882]	+0.00031 -0.00030	[0.05837, 0.05898]
$\delta_D^{K\pi} [^\circ]$	190.0	+4.2 -4.1	[185.9, 194.2]	+8.6 -8.3	[181.7, 198.6]
$ q/p $	0.997	+0.016 -0.016	[0.981, 1.013]	+0.033 -0.033	[0.964, 1.030]
$\phi [^\circ]$	-2.4	$\pm 1.2$	[-3.6, -1.2]	$\pm 2.5$	[-4.9, 0.1]
$\Delta A_{CP}$	-0.00152	$\pm 0.00029$	[-0.00181, -0.00123]	$\pm 0.00058$	[-0.00210, -0.00094]



# Input for $B^0 \rightarrow D^- \pi^+$

- Previously  $r_B^{D^- \pi^+}$  was taken as external input from theory
- Now  $r_B^{D^- \pi^+}$  is determined by the combination
- $r_B^{D^- \pi^+}$  found to be consistent with theoretical value
- $\gamma$  totally unaffected by how  $r_B^{D^- \pi^+}$  is treated





# $B^- \rightarrow Dh^-, D \rightarrow h^+h^-\pi^0$

**NEW!**

LHCb-PAPER-2021-036 (in preparation)

$R_{F^+}^{KK\pi^0}$	=	1.021	± 0.079	± 0.005
$R_{F^+}^{\pi\pi\pi^0}$	=	0.902	± 0.041	± 0.004
$A_K^{K\pi\pi^0}$	=	-0.024	± 0.013	± 0.002
$A_K^{KK\pi^0}$	=	0.067	± 0.073	± 0.003
$A_K^{\pi\pi\pi^0}$	=	0.109	± 0.043	± 0.003
$A_\pi^{KK\pi^0}$	=	-0.001	± 0.019	± 0.002
$A_\pi^{\pi\pi\pi^0}$	=	0.001	± 0.010	± 0.002
$R_K^+$	=	0.0179	± 0.0024	± 0.0003
$R_K^-$	=	0.0085	± 0.0020	± 0.0004
$R_\pi^+$	=	0.00188	± 0.00027	± 0.00005
$R_\pi^-$	=	0.00227	± 0.00028	± 0.00004
$R_{\text{ADS}(K)}$	=	0.0127	± 0.0016	± 0.0002
$A_{\text{ADS}(K)}$	=	-0.38	± 0.12	± 0.02
$R_{\text{ADS}(\pi)}$	=	0.00207	± 0.00020	± 0.00003
$A_{\text{ADS}(\pi)}$	=	0.069	± 0.094	± 0.016,

$$R_{F^+}^{KK\pi^0} \approx R_{K/\pi}^{KK\pi^0} / R_{K/\pi}^{K\pi\pi^0}, \quad R_{F^+}^{\pi\pi\pi^0} \approx R_{K/\pi}^{\pi\pi\pi^0} / R_{K/\pi}^{K\pi\pi^0}$$

$$A_K^{K\pi\pi^0} = \frac{\Gamma(B^- \rightarrow [K^- \pi^+ \pi^0]_D K^-) - \Gamma(B^+ \rightarrow [K^+ \pi^- \pi^0]_D K^+)}{\Gamma(B^- \rightarrow [K^- \pi^+ \pi^0]_D K^-) + \Gamma(B^+ \rightarrow [K^+ \pi^- \pi^0]_D K^+)}$$

$$A_{F^+(K)}^{hh\pi^0} = \frac{\Gamma(B^- \rightarrow [hh\pi^0]_D K^-) - \Gamma(B^+ \rightarrow [hh\pi^0]_D K^+)}{\Gamma(B^- \rightarrow [hh\pi^0]_D K^-) + \Gamma(B^+ \rightarrow [hh\pi^0]_D K^+)}$$

$$A_{F^+(\pi)}^{hh\pi^0} = \frac{\Gamma(B^- \rightarrow [hh\pi^0]_D \pi^-) - \Gamma(B^+ \rightarrow [hh\pi^0]_D \pi^+)}{\Gamma(B^- \rightarrow [hh\pi^0]_D \pi^-) + \Gamma(B^+ \rightarrow [hh\pi^0]_D \pi^+)}$$

$$R_K^\mp = \frac{\Gamma(B^\mp \rightarrow [\pi^\mp K^\pm \pi^0]_D K^\mp)}{\Gamma(B^\mp \rightarrow [K^\mp \pi^\pm \pi^0]_D K^\mp)}$$

$$R_\pi^\mp = \frac{\Gamma(B^\mp \rightarrow [\pi^\mp K^\pm \pi^0]_D \pi^\mp)}{\Gamma(B^\mp \rightarrow [K^\mp \pi^\pm \pi^0]_D \pi^\mp)}$$

$$R_{\text{ADS}(h)} = \frac{\Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-) + \Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-)}{\Gamma(B^- \rightarrow [K^- \pi^+ \pi^0]_D h^-) + \Gamma(B^- \rightarrow [K^- \pi^+ \pi^0]_D h^-)}$$

$$A_{\text{ADS}(h)} = \frac{\Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-) - \Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-)}{\Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-) + \Gamma(B^- \rightarrow [\pi^- K^+ \pi^0]_D h^-)}$$

# History of LHCb combinations

