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Are $N_c = 3 \neq \infty$ effects important in parton showers?

- Why N_c suppressed terms may be important
- Sources of N_c suppressed terms
- A basis for treating color structure
- "Color amplitude shower"
- Results from a toy shower
- Conclusion and future plans

Work in progress, JHEP 0909:087

Cambridge
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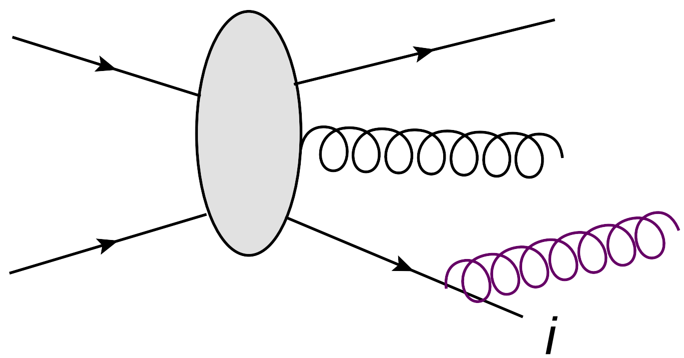
A first generation parton shower

- A parton can be seen as emitted from one other parton using pure $1 \longrightarrow 2$ splitting (JETSET)



- Resums the collinear splitting probability using DGLAP splitting functions

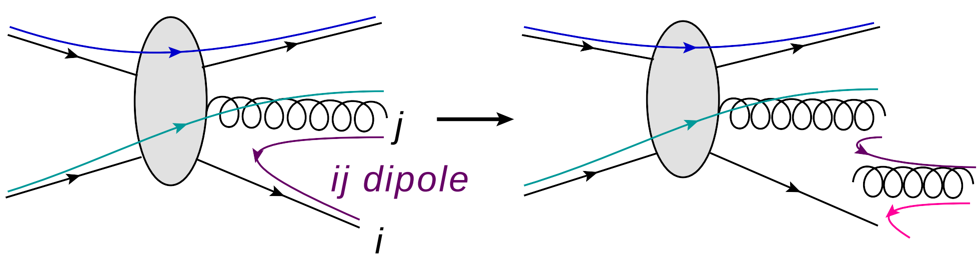
$$\Delta_k(t) = \exp(-\sum_i \int_{t_0}^{t} \frac{dt'}{t'} \alpha_{\mathrm{s}}(t') \int \frac{dz}{2\pi} P_{ik})$$

A second generation parton shower

- Resums also the softly enhanced radiation probabilities in the $N_c \to \infty$ limit

$$\Delta_k(t) = \exp(-\frac{2}{\pi} \sum_{\text{dipoles i,j(i)}} \int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int \frac{dy d\phi}{2\pi} \frac{k_T^2 p_i . p_j}{2\, p_i . k\; p_j . k})$$

- In the soft limit a parton can be seen as being emitted coherently from a pair of color connected partons, "dipole shower"



Why worry about N_c suppressed terms?

- " Non-leading color terms are suppressed by 1/ N_c^2 "
  (not quite true, true for LO gluon only processes)
- The number of suppressed terms grows $\sim (N_{\text{partons}}!)^2$
  The number of non-suppressed terms grows just like $\sim (N_{\text{partons}}!)$
- If non-leading terms always were N_c^2 suppressed, the relative importance can grow like $\sim (N_{\text{partons}}!)/N_c^2$
  (slower with random averaging)

Different sources of N_c-suppressed terms

- In a tree level parton shower (no virtual exchange, only emission), N_c-suppressed terms are ignored $\rightarrow$ one source of ignored $1/N_c^{(2)}$
- At loop level, another source of suppressed terms comes from virtual gluon exchanges which rearrange the color structure $\rightarrow$ **exponentiation** has to be done at the **amplitude level** $\rightarrow$ basis needed

$$\exp(-\int_{t_0}^{t}\frac{dt'}{t'}\alpha_s(t)\int ...\mathbf{Matrix})|\text{state}>$$

The color rearranging terms tend to be suppressed, but in

$$\exp(-\int_{t_0}^{t}\frac{dt'}{t'}(\text{moderate}+\text{small}))$$

the small number is not irrelevant when $\int_{t_0}^{t}\frac{dt'}{t'}$ is large!
$\rightarrow$ different source of N_c-suppressed terms

A basis for the color space

- The color space is a finite dimensional vector space equipped with a (real) scalar product

$$< A, B >= \sum_{a,b,c,\ldots} A_{a,b,c,\ldots} (B_{a,b,c,\ldots})^*$$

Example: If $A = t^g_{ab} t^g_{cd}$, then $< A|A >= \sum_{a,b,c,d,g,f} t^g_{ab} t^g_{cd} t^f_{ba} t^f_{dc}$

- Individual colors are not observed, we always sum or average over them $\longrightarrow$ We don't need a basis caring about red green or blue, we only need a basis caring about how the color of various partons are related to each other!

A basis for the color space

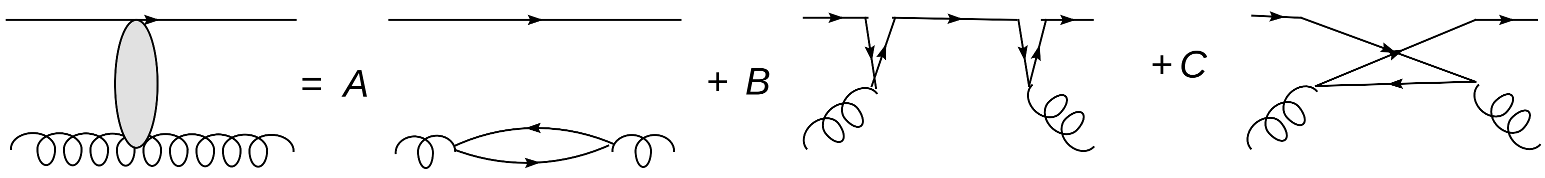
One way of constructing a complete basis for any fixed number of external (colored or uncolored) particles is to:

- Decompose all gluons into $q\bar{q}$-pairs
- Connect quarks and anti-quarks in all possible ways, such that the $q\bar{q}$ pairs corresponding the the same gluon are not connected (SU(3) generators are traceless)
- If only gluons, make sure the internal quarks and anti-quarks enter on equal footing

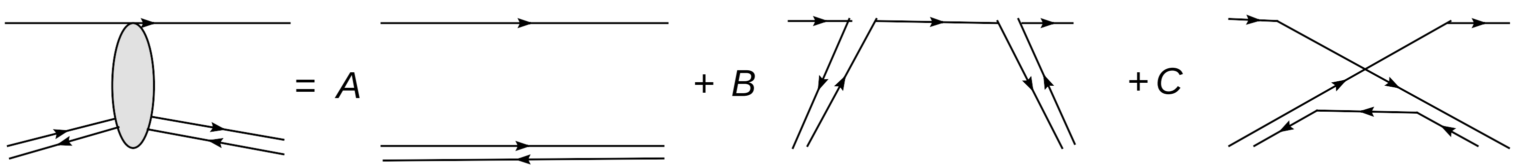
A basis for the color space

Example: qg $\longrightarrow$ qg

- Split gluons into $q\bar{q}$ pairs and connect lines in all possible ways



- In the $N_c \to \infty$ limit



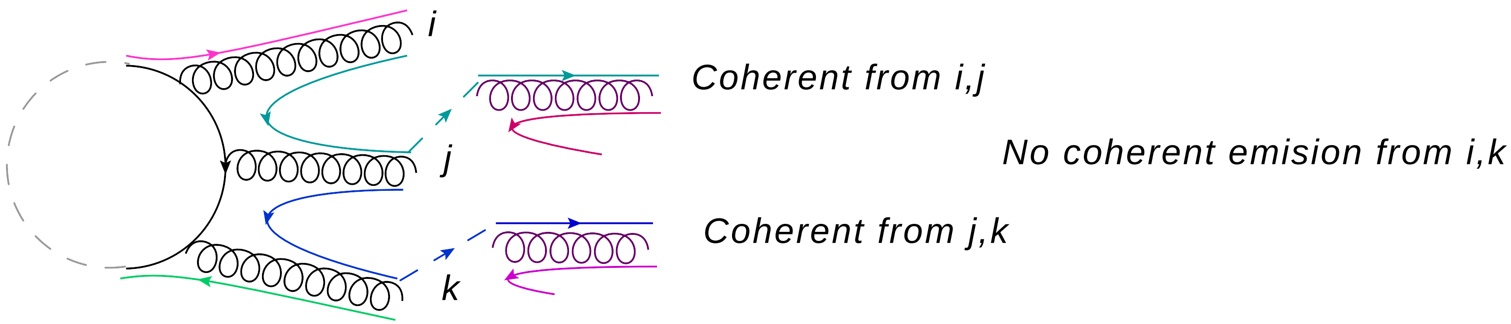
A basis for the color space

- The basis constructed in this way is complete
- Overcomplete for $N_c \neq \infty$ (and more than a few partons)
- The basis states are orthogonal only when $N_c \to \infty$, otherwise their scalar products are suppressed by $1/N_c$ or higher powers
- For $N_\mathrm{q} = N_{\bar{\mathrm{q}}}$ and $N_g = 0$, there are precisely $N_\mathrm{q}!$ basis states
- The number of basis tensors grows roughly factorially as $(N_g + N_\mathrm{q})!$ for $N_\mathrm{q} + N_{\bar{\mathrm{q}}} + N_g$ partons
- Hence the naive importance of suppressed terms $\sim (N_{\text{partons}}!)^2/N_c^2$ from the number of terms in

$$< A, A >= \sum_{n,m} c_m c_n^* < C^m | C^n >$$

A dipole shower in this basis

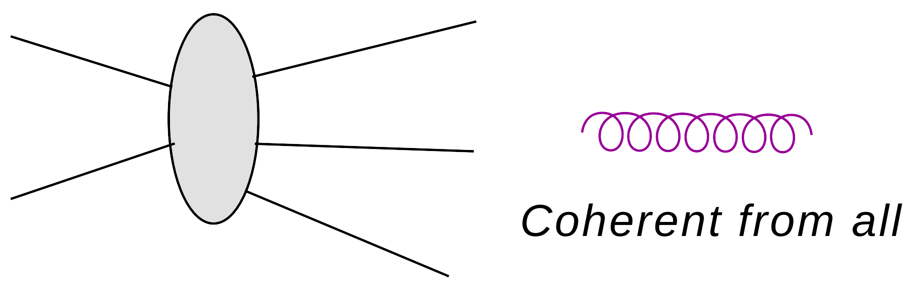
- Can be thought of in the language of the $N_c \to \infty$ limit of the above basis (apart from C_F...)



- Also, it is easy to see that in this limit only "color neighbors" radiate, i.e. only neighboring partons on the quark-lines in the basis
  $\longrightarrow$ above basis well suited for comparing to parton showers

An amplitude color shower

- Emit one parton at the time (= imagine an evolution time)
  → the 5th gluon cannot interfere with the 3rd, this is intrinsic to the shower approximation
  → "amplitude shower < all Feynman diagrams calculation"
- Keep *all* contributions to the emission n → n+1



- In this sense "amplitude shower > shower"
- "shower < amplitude shower < all Feynman diagrams calculation"

An amplitude color shower

- First step: do this at the tree level only
  (no virtual color rearranging gluons)

$$\Delta_k(t) = \exp(-\frac{2}{\pi}\sum_{\text{pairs i,j}}\sum_{mn}\int_{t_0}^{t}\frac{dt'}{t'}\alpha_s(t')\int\frac{dy d\phi}{2\pi}\ldots$$

$$\frac{1}{\text{Norm}}c_n c_m^* < C^n|\frac{k_T^2 p_i\cdot p_j}{2\,p_i\cdot k\;p_j\cdot k}T^{(i)}\cdot T^{(j)}|C^m >)$$

$\longrightarrow$ ugly color factors and bad scaling, but doable

Let each pair “compete” with its own scale

- Also like to keep the virtual color rearranging terms
  $\longrightarrow$ rotate state in color space as well $\longrightarrow$ matrix exponentiation needed

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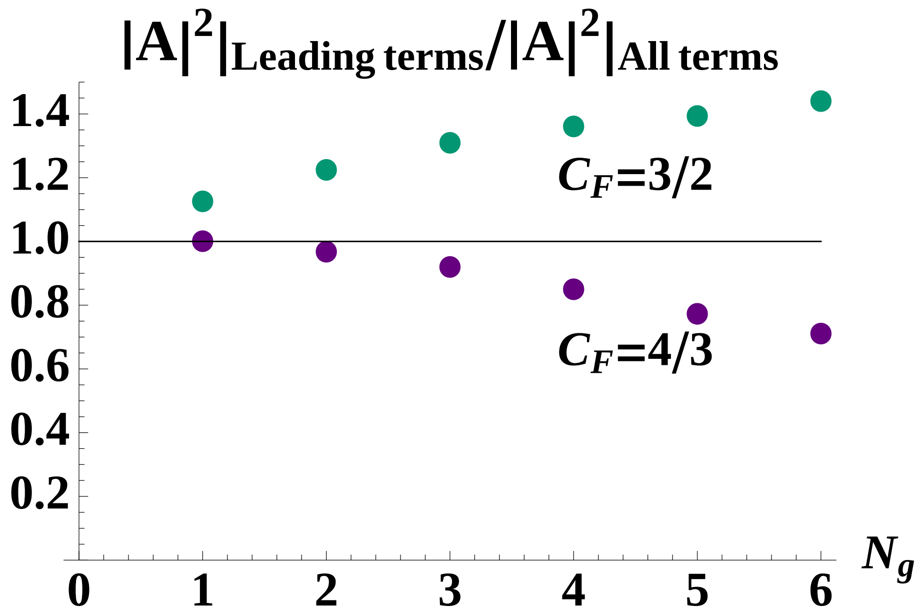
A toy amplitude color shower

- Treat: $N_c = 3$ color $\otimes$ random number ($\in$ [-0.5,0.5])
  (to be replaced by $N_c = 3$ color $\otimes$ other quantum numbers)
- Start with $\mathrm{q\bar{q}}$ and radiate N_g gluons, compare the results when all, respectively only some (leading), terms are kept
- Will the ratio

$$\frac{|A(N_{\text{partons}})|^2|_{\text{Leding terms}}}{|A(N_{\text{partons}})|^2|_{\text{All terms}}} \neq 1?$$

- So far, only tree level parton shower, no virtual gluon exchange!
- Throw away all sub-leading terms, take $C_F = N_c/2 = 3/2$ (true limit)
- Keep powers of $C_F = (N_c^2 - 1)/(2N_c) = 4/3$, but ignore other suppressed terms (parton shower like)

Results from a toy QCD shower



- Importance of suppressed terms does grow, but not like N_{partons}!
  $\longrightarrow$ Random averaging, shower structure, more?
- Treatment of C_F is very important

Conclusion and future plans

- No one knows if suppressed terms are important
- Good reasons to expect that they are $\longrightarrow$ well worth to study

For this reason:

- Incorporate color code in existing tree level parton shower (probably in cooperation with Simon Plätzer)
- See if we find anything interesting
- Add virtual gluon exchange (rearrange the color without emission)
- Speed up program by saving intermediate results

An ordinary parton shower

- Treats:

$$p \quad \otimes \quad f \otimes \text{spin average} \otimes N_c \to \infty \text{ color}$$

- Emits one particle at the time
- Assumes an ordering variable like
  $k_\perp$, Ariadne, Sherpa
  virtuality, Pythia
  angle, Herwig

$$\Delta(t) = \exp(-\int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int ...)$$

- Resums large logs which compensate for the smallness of α_s
- is a Markov process at the $|A|^2$ level
  (the next step depend on the state, but not on the history)

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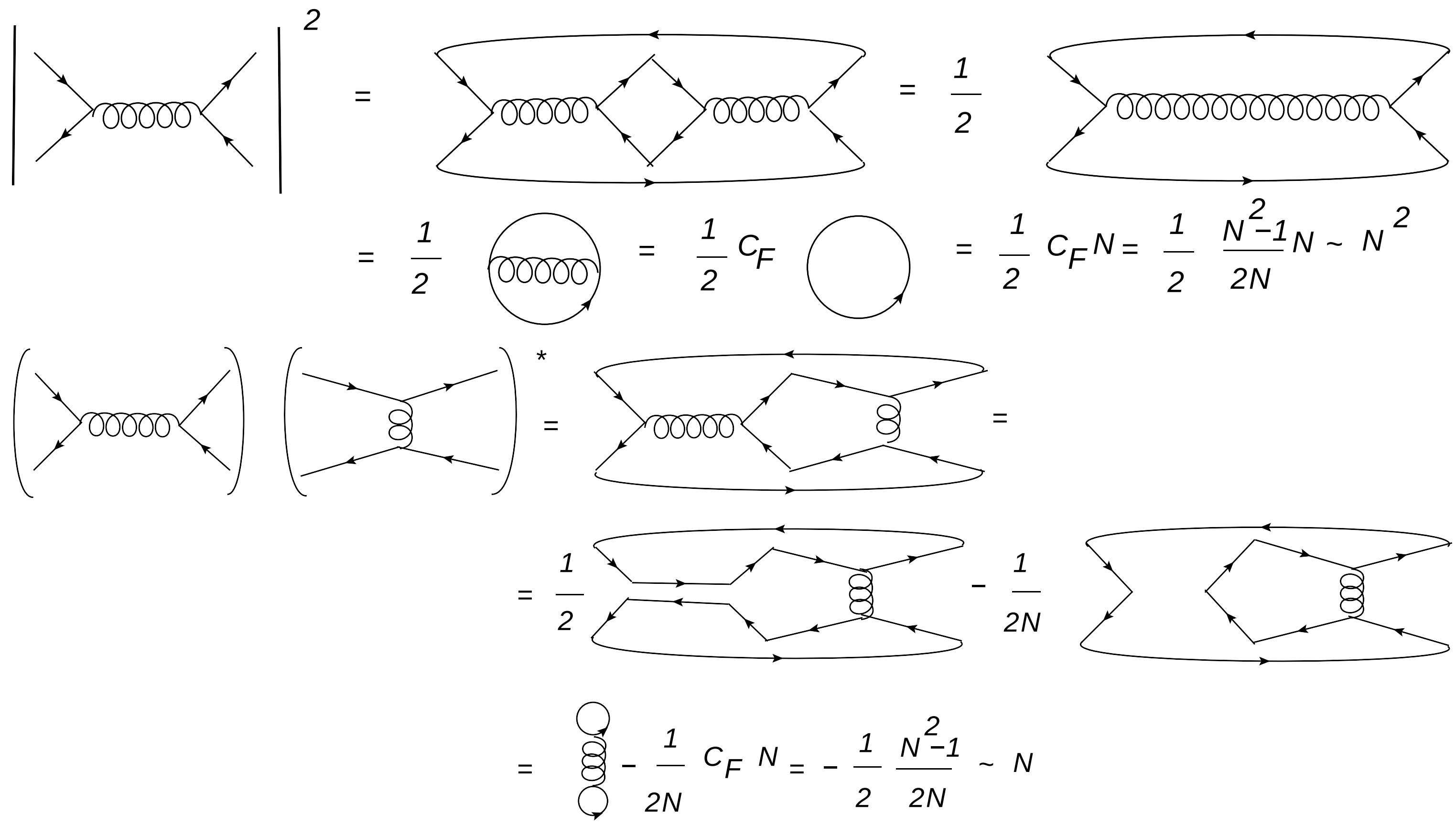
Why worry about N_c suppressed terms?

" Non-leading color terms are suppressed by $1/N_c^2$ ": a counter example



Why worry?

- " Non-leading color terms **tend to be** suppressed by $1/N_c^2$ "
  counter examples exist
- Is true for same order $\alpha_{\rm s}$ diagrams with only gluons ('t Hooft 1973)
- A parton shower is an all order (Sudakov) exponentiation

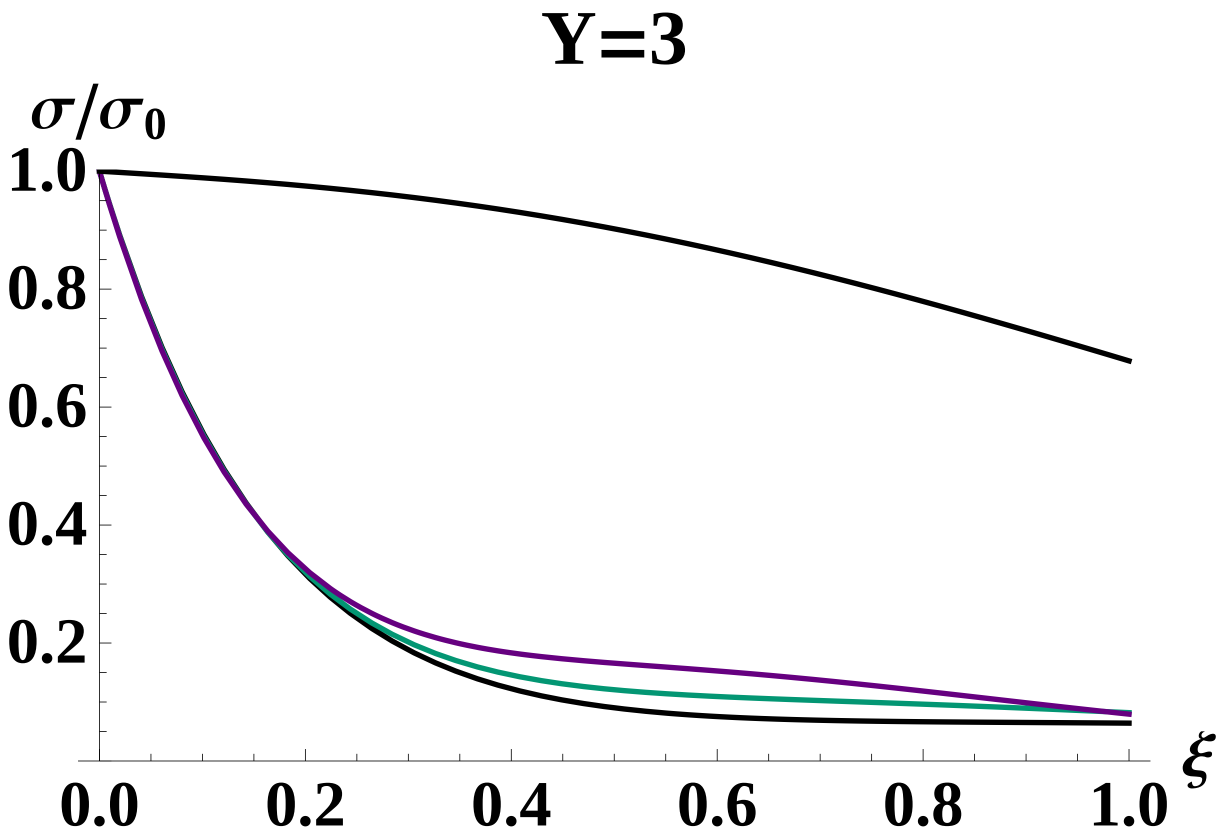
$$\Delta(t) = \exp(-\int_{t_0}^{t} \frac{dt'}{t'} \alpha_{\rm s}(t) \int ...)$$

- Certainly not only one power in $\alpha_{\rm s}$ is needed

Some rescuing mechanisms?

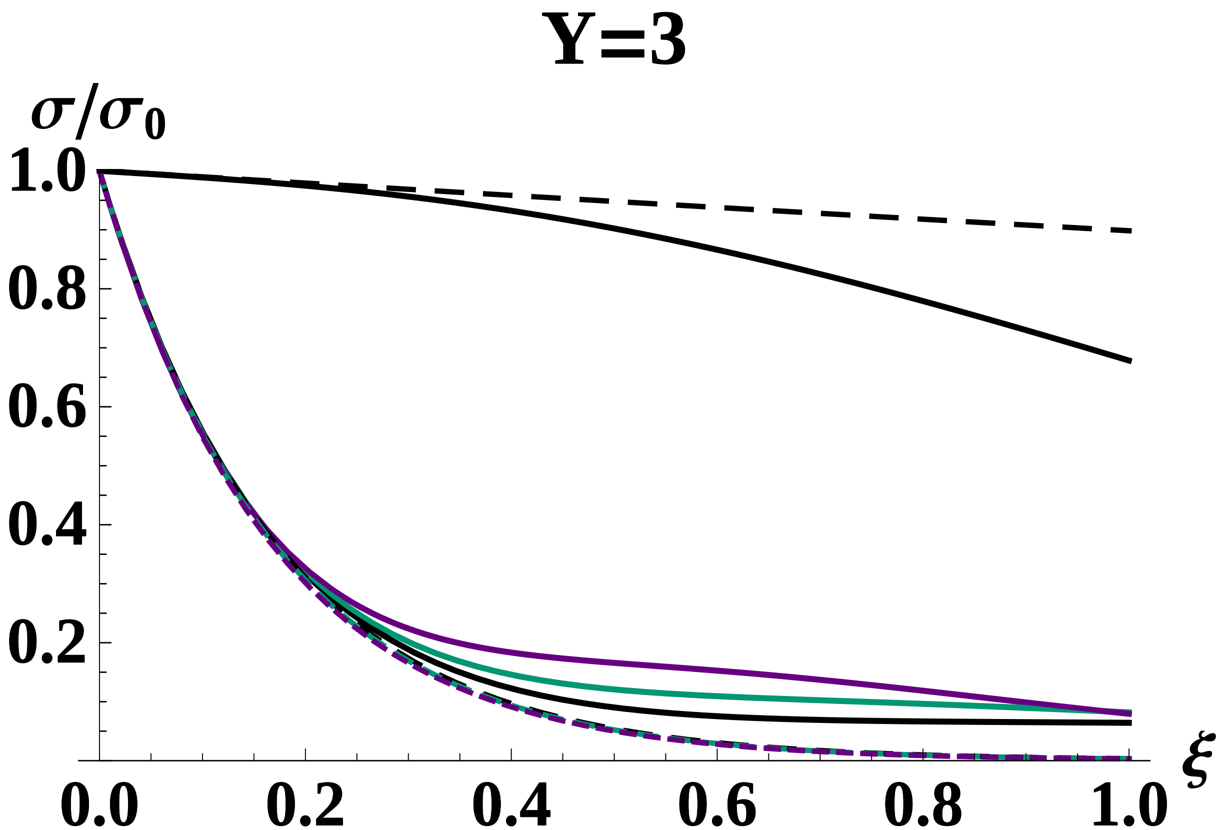
- Current parton showers actually do work quite well, this is a reason for believing that there is a suppression mechanism, but:
  Suppressed terms will be more important at LHC as more space in $\log(k_T)$
- In the collinearly (rather than softly) enhanced regions, the emitted parton can be seen as coming from only one parton and the color structure is trivial $\longrightarrow$ no need for N_c suppressed terms
- Random averaging:
  The suppressed terms sometimes contributes positively to the cross section, and sometimes negatively. Perhaps they tend to cancel quicker than expected? (Via correlations in emission?)
- α_s suppression: $1/N_c^1$ suppressed terms tend to also be associated with powers of α_s^2, but remember:
  Large logs accompany α_s, this is why we need resummation

Gap survival: qq → qq, qg → qg and gg → gg



The gap survival probability for **quarks**, **gluons** and **one quark and one gluon**, **dashed** lines without color rearranging gluons. $\xi = \int_{Q_0}^{Q} \alpha_s(q_T)\frac{dq_T}{q_T}$ (Cone radius 1).

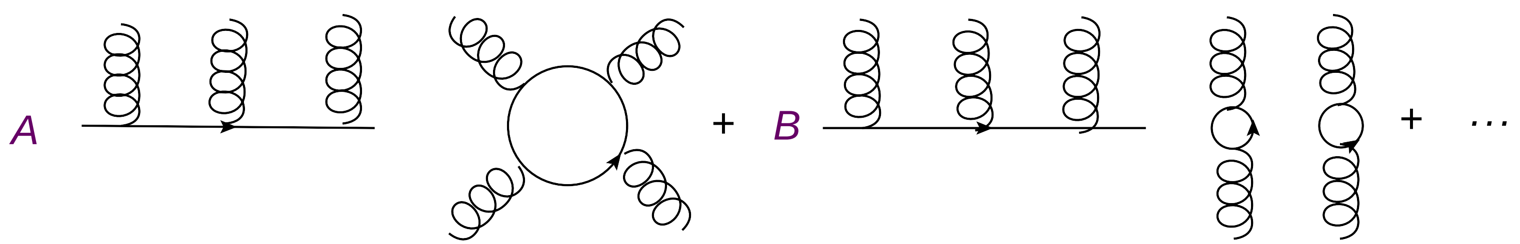
Gap survival: qq $\to$ qq, qg $\to$ qg and gg $\to$ gg



The gap survival probability for **quarks**, **gluons** and **one quark and one gluon**, **dashed** lines without color rearranging gluons. $\xi = \int_{Q_0}^{Q} \alpha_s(q_T) \frac{dq_T}{q_T}$ (Cone radius 1).

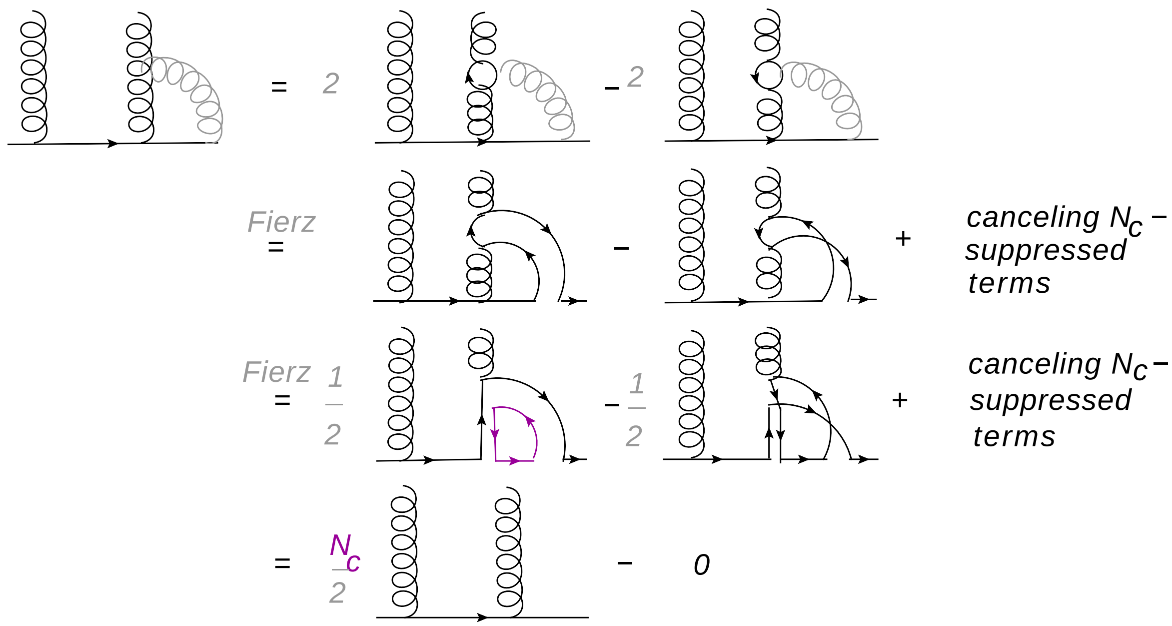
A basis for the color space

- In general an amplitude is a linear combination of different color structures, for example...



A basis for the color space

A gluon exchange in this basis "directly" i.e. without using scalar products gives back a linear combination of (at most 4) basis tensors



- N_c-enhancement possible only for near by partons
  $\longrightarrow$ only "color neighbors" radiate in the $N_c \to \infty$ limit