Parton Showers and Jet Rates

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- e^+e^- k_t-jet rates
  - 4- and 5-jet rates
  - resummation
- angular and p_t ordering
  - colour structure
- anti-k_t jet rates

Parton showers

- A convenient way to resum enhanced terms to all orders
  - Should reproduce correct jet rates
- k_t-jet rates:

$$y_{ij} = 2\min\{E_i^2, E_j^2\}(1-\cos\theta_{ij})/Q^2 > y_{\rm cut}$$

Brown & Stirling, Z.Phys.C53:629-636,1992.

Catani et al., Phys.Lett.B269:432-438,1991.

k_t jet rates

Table 1. Jet fractions in $e^+e^- \to$ hadrons to NLL order in $L = \ln(1/y_{\rm cut})$, expanded to third order in $a = \alpha_S/\pi$.

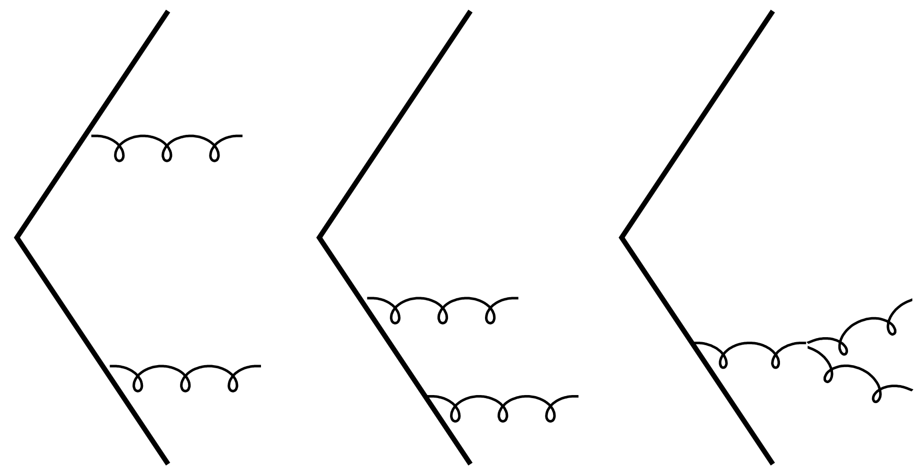
$$
\begin{aligned}
R_2 &= 1 + a(R_{21}L + R_{22}L^2) + a^2(R_{23}L^3 + R_{24}L^4) + a^3(R_{25}L^5 + R_{26}L^6) + \ldots \\
R_{21} &= 3C_F/2 \\
R_{22} &= -C_F/2 \\
R_{23} &= -3C_F^2/4 - 11C_FC_A/36 + C_FN_f/18 \\
R_{24} &= C_F^2/8 \\
R_{25} &= 3C_F^3/16 + 11C_F^2C_A/72 - C_F^2N_f/36 \\
R_{26} &= -C_F^3/48
\end{aligned}
$$

$$
\begin{aligned}
R_3 &= a(R_{31}L + R_{32}L^2) + a^2(R_{33}L^3 + R_{34}L^4) + a^3(R_{35}L^5 + R_{36}L^6) + \ldots \\
R_{31} &= -3C_F/2 \\
R_{32} &= C_F/2 \\
R_{33} &= 3C_F^2/2 + 7C_FC_A/12 - C_FN_f/12 \\
R_{34} &= -C_F^2/4 - C_FC_A/48 \\
R_{35} &= -9C_F^3/16 - 137C_F^2C_A/288 - 7C_A^2C_F/160 + 5C_F^2N_f/72 + C_FC_AN_f/160 \\
R_{36} &= C_F^3/16 + C_F^2C_A/96 + C_FC_A^2/960
\end{aligned}
$$

$$
\begin{aligned}
R_4 &= a^2(R_{43}L^3 + R_{44}L^4) + a^3(R_{45}L^5 + R_{46}L^6) + \ldots \\
R_{43} &= -3C_F^2/4 - 5C_FC_A/18 + C_FN_f/36 \\
R_{44} &= C_F^2/8 + C_FC_A/48 \\
R_{45} &= 9C_F^3/16 + 71C_F^2C_A/144 + 217C_FC_A^2/2880 - 41C_F^2N_f/720 - C_FC_AN_f/120 \\
R_{46} &= -C_F^3/16 - C_F^2C_A/48 - 7C_FC_A^2/2880
\end{aligned}
$$

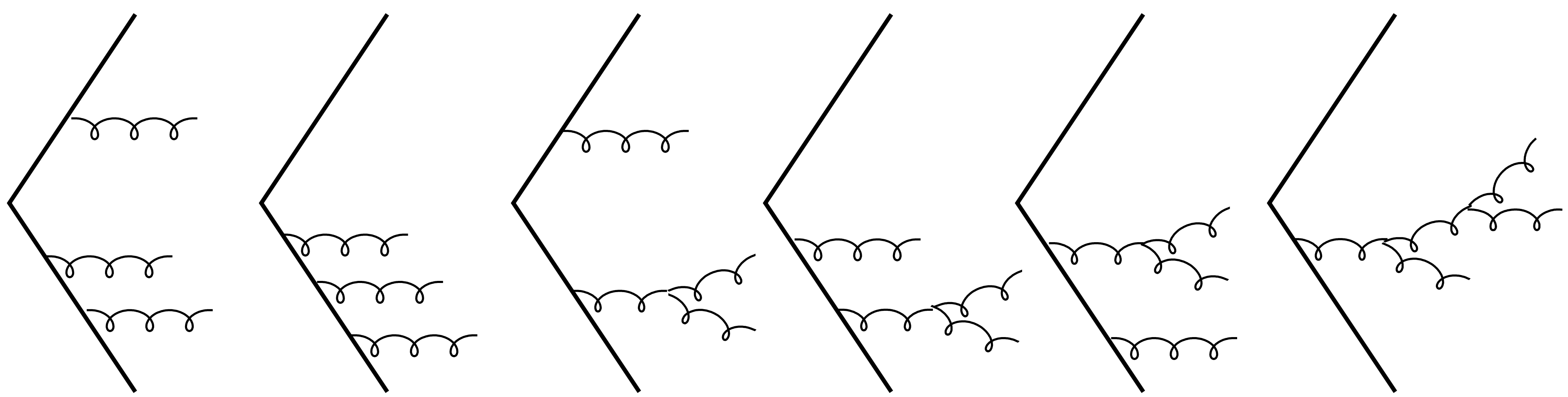
$$
\begin{aligned}
R_5 &= a^3(R_{55}L^5 + R_{56}L^6) + \ldots \\
R_{55} &= -3C_F^3/16 - 49C_F^2C_A/288 - 91C_FC_A^2/2880 + 11C_F^2N_f/720 + C_FC_AN_f/480 \\
R_{56} &= C_F^3/48 + C_F^2C_A/96 + C_FC_A^2/720
\end{aligned}
$$

4- and 5-jet rates



(a) (b) (c)

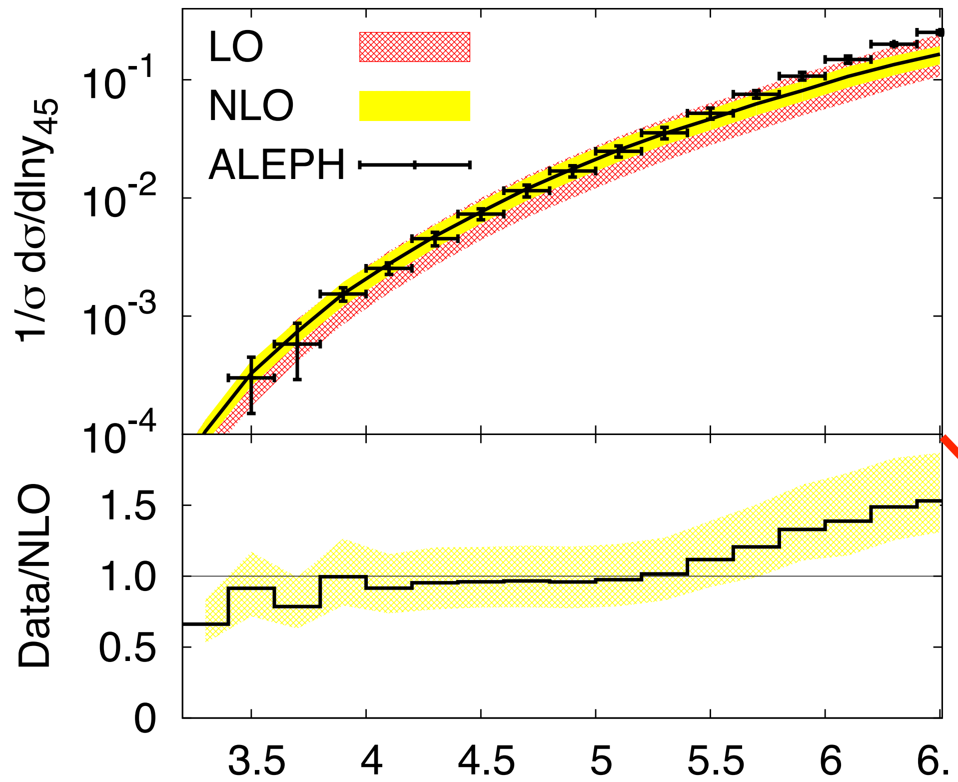
$$\Rightarrow \quad R_4 \sim a^2 L^4 (C_F^2/8 + C_F C_A/48)$$



(a) (b) (c) (d) (e) (f)

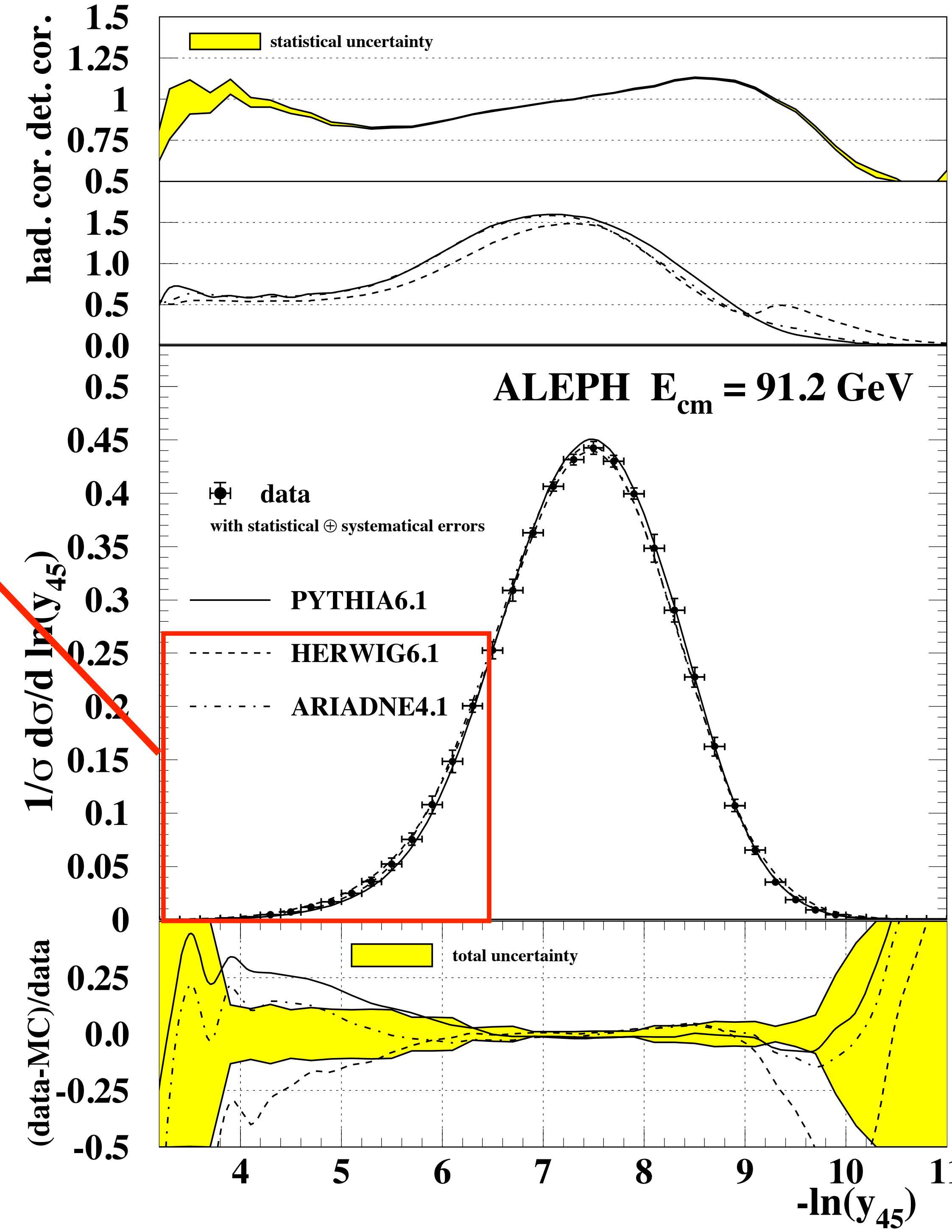
$$\Rightarrow \quad R_5 \sim a^3 L^6 (C_F^3/48 + C_F^2 C_A/96 + C_F C_A^2/720)$$

5-jet differential rate



NLO: Frederix et al., arXiv:1008.5313 (2010)

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dy_{45}} = -\sum_{n=5}^{\infty}\frac{dR_n}{dy_{\rm cut}}\bigg|_{y_{\rm cut}=y_{45}}$$



ALEPH: Heister et al., Eur.J.Phys.C35,457 (2004)

➡ Fixed order breaks down for $-\ln(y_{45})>6$

Resummation of jet rates

- Leading abelian $(C_F)^n$ terms exponentiate

$$R_{n+2}^{(\mathrm{ab})} \sim \frac{1}{n!}\left(\frac{1}{2}aC_F L^2\right)^n \exp\left(-\frac{1}{2}aC_F L^2\right)$$

  - NLL and non-abelian terms complicated
  - Can use parton shower to resum:
    - sequential 1→2 branching process
    - branching probability $\frac{dq}{q}\frac{\alpha_S}{\pi}P(z)\,dz$

Parton showers

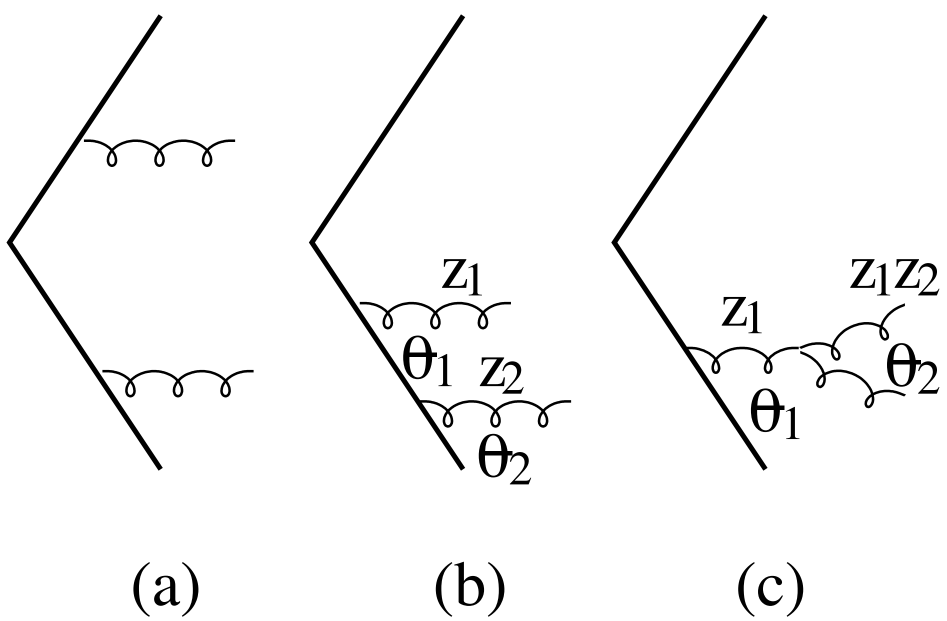
- Angular ordered shower gives correct rates

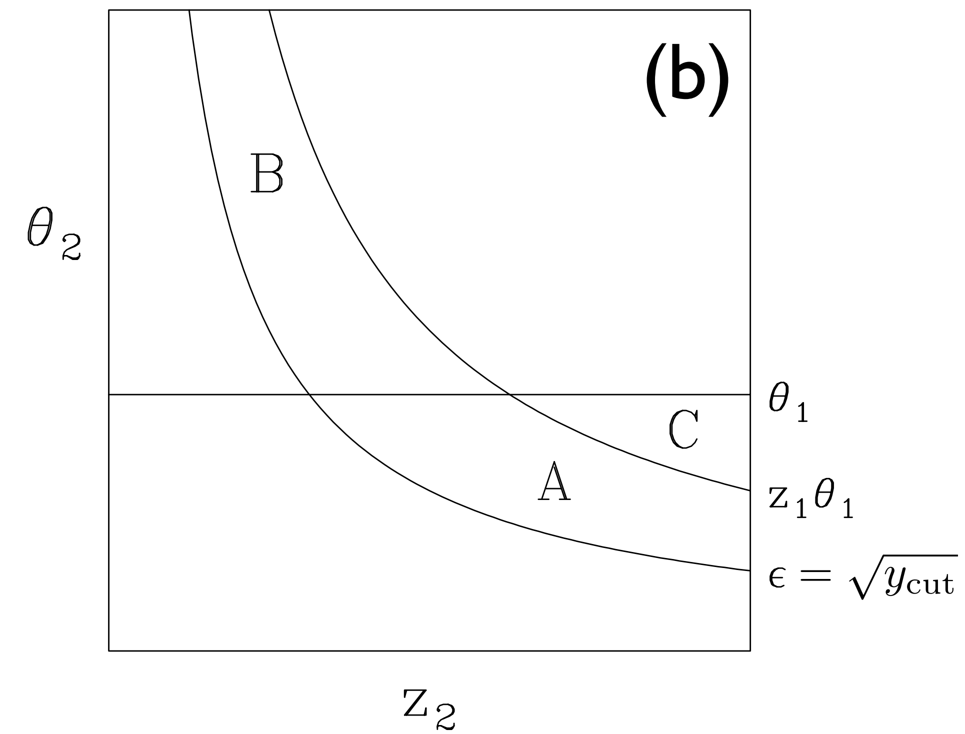
$$\begin{aligned} R_4 &\sim a^2L^2(C_F^2/8 + C_F C_A/48) \\ R_5 &\sim a^2L^2(C_F^3/48 + C_F^2 C_A/96 + C_F C_A^2/720) \end{aligned}$$

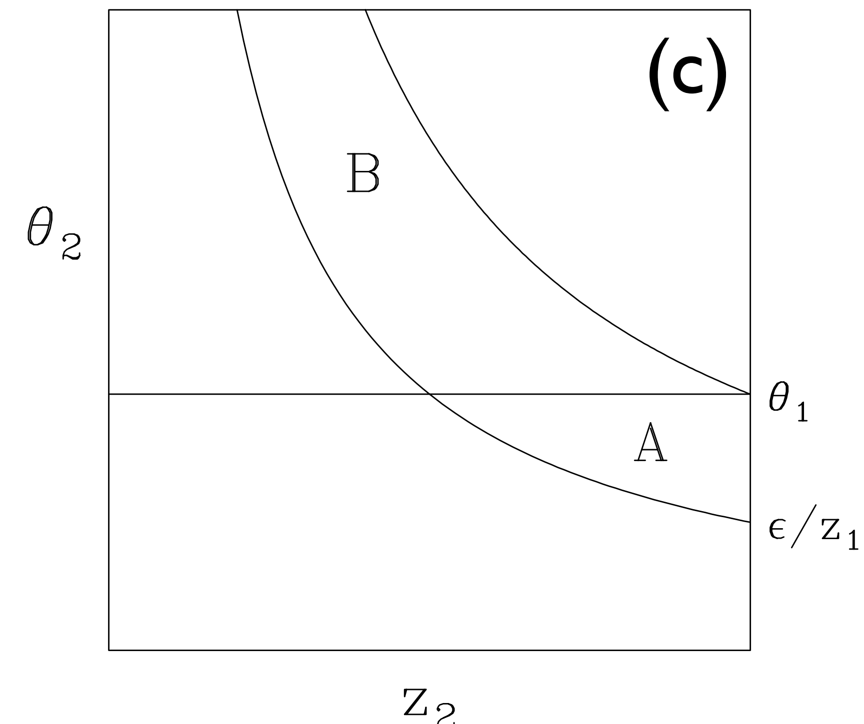
- p_t-ordered shower more convenient
  - hardest emissions come first
  - easier NLO improvement (POWHEG,...)
- But jet rates are wrong

$$\begin{aligned} R_4^{(p_t)} &\sim a^2L^2(C_F^2/8 + C_F C_A/24) \\ R_5^{(p_t)} &\sim a^2L^2(C_F^3/48 + C_F^2 C_A/48 + C_F C_A^2 13/2880) \end{aligned}$$

Angular vs p_t ordering



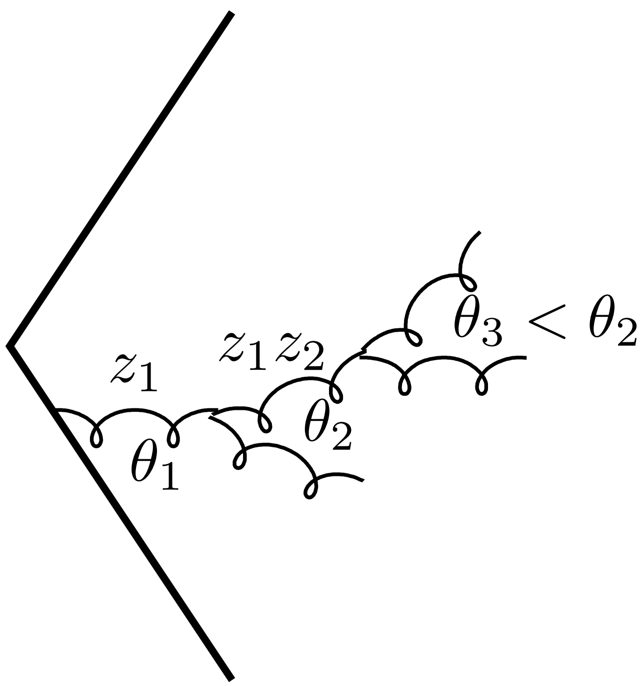


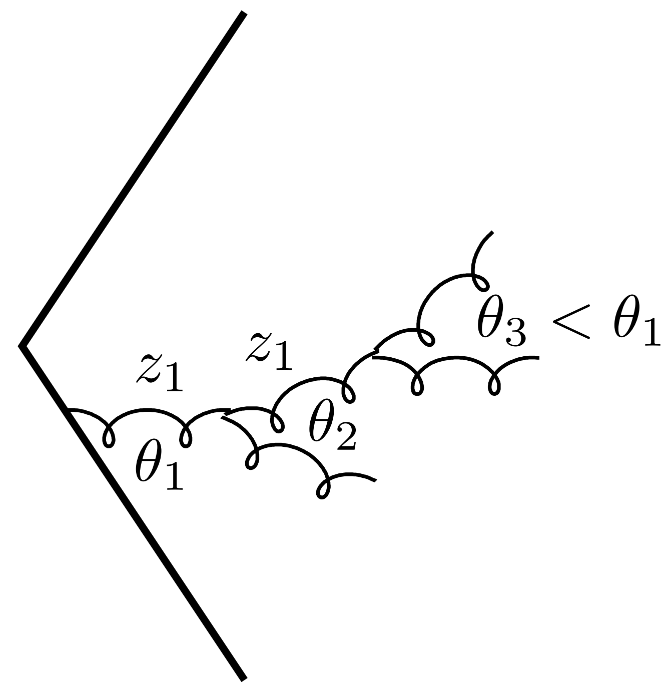


- Angular: regions A (+C)
- p_t : regions A+B
  - (b) B~C ➔ logs OK
  - (c) no C ➔ overestimate
  - should angle-order (c) only

p_t + soft-angle ordering

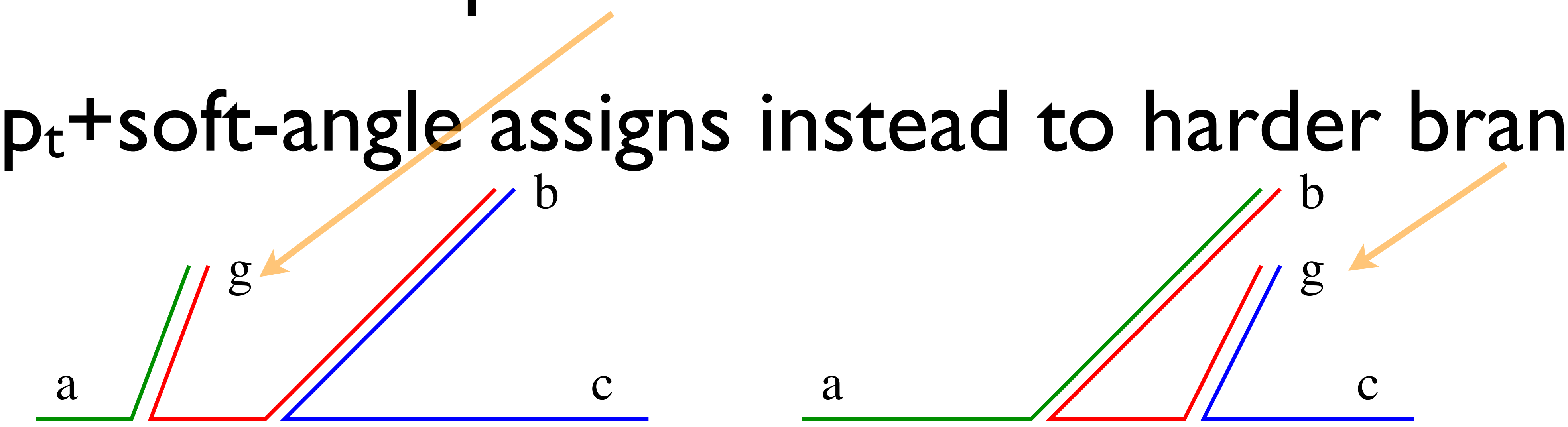
- p_t ordering gives correct jet rates if we veto angles disordered w.r.t. "creation" vertex
  - "created" parton is the softer one





CKKW: Catani et al., JHEP11(2001)063

- This gives correct parton-level jet rates and distributions but wrong colour structure
  - Angular ordering assigns coherent emission from b+c to parent a
  - p_t+soft-angle assigns instead to harder branch



(a) (b)

- Rearrange colour structure for hadronization
  - or do p_t ordering plus "truncated showers"

POWHEG: Nason, JHEP11(2004)040

Anti-k_t Jet Rates

- Anti-k_t for e^+e^-:
- Define $\epsilon_{ij} = \min\{Q/E_i, Q/E_j\}\theta_{ij}$, $\epsilon_i = \epsilon\, Q/E_i$
  - Combine i,j if ϵ_{ij} smallest,
  - Else if ϵ_i smallest, then
    - If $\epsilon_i < 1$ keep i as a jet
    - Else throw i away
- Resum $L = \ln(1/\epsilon^2)$

Anti-k_t: Cacciari, Salam & Soyez, JHEP04(2008)063

Anti-k_t Jet Rates

$$R_4^{\text{anti}} \sim a^2 L^4 (C_F^2/2 + C_F C_A/8)$$

$$R_5^{\text{anti}} \sim a^3 L^6 (C_F^3/6 + C_F^2 C_A/8 + C_F C_A^2/48)$$

- LL abelian terms exponentiate again

$$R_{n+2}^{(\text{anti,ab})} \sim \frac{1}{n!} \left(a C_F L^2\right)^n \exp\left(-a C_F L^2\right)$$

- Resum NLL and non-abelian?

Conclusions

- Parton showers
  - angular ordering needed
  - p_t^+ soft angle OK for jet rates
    - but needs colour rearrangement
- Anti-k_t algorithm
  - different pattern of leading logs
  - resummation?