Issues and Opportunities in Studies of ν-Oxygen Interactions with NINJA

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Outline

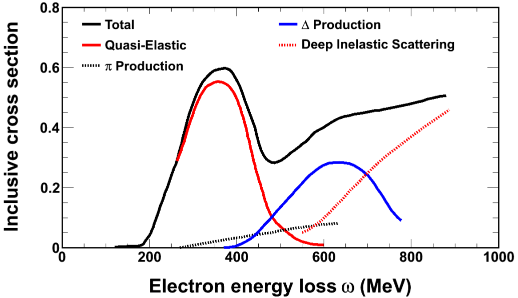
- The trouble with measured neutrino-nucleus cross sections
- The factorisation ansatz
  - Initial state
  - Interaction vertex
  - Final state
- Testing the model with electron-scattering data
  - Inclusive processes
  - Semi-Inclusive processes
- Application to neutrino interactions
- Summary & outlook
- Potential impact of NINJA

The Trouble with Flux Average

- The energy-transfer dependence of the cross section of the process

$$e + A \to e' + X$$

at fixed beam energy and electron scattering angle displays a complex landscape



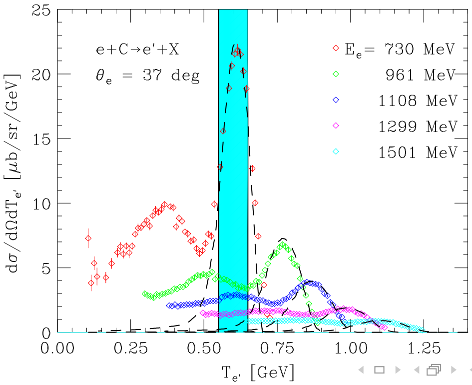
- The contributions of the main reaction mechanisms—involving both nuclear and nucleon structure—can be clearly identified

- In neutrino interactions, e.g.

$$\nu_\mu + A \to \mu^- + X$$

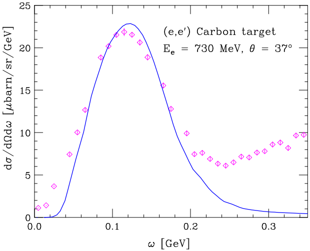
the energy of the beam particle is spread over a broad distribution

- different reaction mechanisms contribute to the cross section at fixed muon energy and emission angle
- This feature clearly emerges from the analysis of the available electron-scattering data

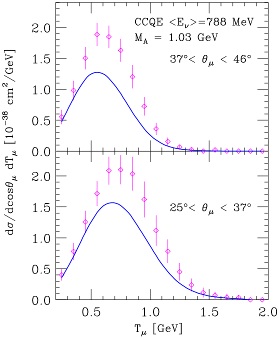


COMPARING e- AND ν_μ-CARBON 0π CROSS SECTIONS

- ▷ Electron scattering



- ▷ MiniBooNe CCQE cross section



- Theoretical calculations carried out using the same formalism
- Owing to flux average, reaction mechanisms other than single-nucleon knock out contribute to the neutrino cross section

The New Paradigm

- ⋆ To achieve a firm understanding of the measured neutrino-nucleus cross sections we need to

  - develop a comprehensive model of neutrino-nucleus interactions, capable to pin down all relevant reaction mechanisms within a consistent framework. This is the basic requirement for a reliable neutrino energy reconstruction, needed for the oscillation analysis

  - exploit the available and forthcoming electron scattering data to assess the accuracy of the model in each channel

  - increase the data base of measured neutrino-nucleus cross section, including both inclusive and semi-inclusive data

WHERE WE ARE

- Over the two decades since the first NuINT Workshop—that we may characterise as the post-Fermi-gas age—a number of more advanced models of neutrino-nucleus interactions have been developed
- Several models have achieved the degree of maturity required to compare their predictions to the data. However, models based on different assumptions turn out to provide very similar result
- Accurate results have been also obtained from *ab initio* Monte Carlo calculations. However, this technique is inherently non relativistic, and its applicability is limited to light nuclear targets
- Arguably, the approach based on the factorisation *ansatz* and the Green's function formalism is emerging as a viable option for the development of a unified description of neutrino-nucleus interactions

The Lepton-Nucleus x-Section

⋆ Consider, for example, the cross section of the process

$$\ell + A \to \ell' + X$$

at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

- $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
- The determination of the nuclear response tensor

$$W_A^{\mu\nu} = \sum_X \langle 0 | J_A^{\mu\,\dagger} | X \rangle \langle X | J_A^\nu | 0 \rangle \delta^{(4)}(P_0 + k - P_X - k')$$

involves

- the ground state of the target nucleus, $|0\rangle$
- all relevant hadronic final states, $|X\rangle$
- the nuclear current operator

$$J_A^\mu = \sum_i j_i^\mu + \sum_{j>i} j_{ij}^\mu$$

THE NON RELATIVISTIC REGIME

- In the low-energy regime quasi elastic scattering leading to final states involving nucleons only, i.e.

$$|X\rangle = |(A-1)^\star\, p\rangle \,, |(A-2)^\star\, pp\rangle \ldots$$

  is the dominant reaction mechanism

- At low to moderate momentum transfer, typically in the range $|\mathbf{q}| \lesssim 500$ MeV, the non relativistic approximation can be employed to carry out highly accurate *ab initio* calculations, based on realistic nuclear Hamiltonians strongly constrained by phenomenology

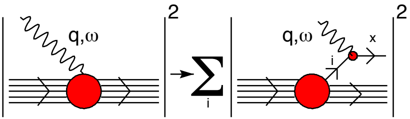
$$H = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk} \;,$$

  and consistent nuclear current operators J_A^μ

- The non relativistic approach has been widely employed to describe the electromagnetic and weak responses of *isoscalar* nuclei with $A \leq 12$

The Impulse Approximation (IA) Regime

- at large momentum transfer, the final state and the current operator can no longer be described within the non relativistic approximation
- for $\lambda \sim 1/|\mathbf{q}| \ll d_{\rm NN} \sim 1.6$ fm, the average nucleon-nucleon distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



- Basic assumptions
  - $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$: single-nucleon coupling
  - $|X\rangle \to |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$: factorization of the final state
- As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected (more on this later)

THE IA CROSS SECTION

- Factorisation allows to rewrite the nuclear transition amplitude as

$$\langle X | J_A^\mu | 0 \rangle \to \sum_i \int d^3k \, M_n(\mathbf{k}) \langle \mathbf{k} + \mathbf{q} | j_i^\mu | \mathbf{k} \rangle$$

  - The nuclear amplitude $M_n = \langle n | a_{\mathbf{k}} | 0 \rangle$ describes initial sate properties, independent of momentum transfer. It can be obtained from non relativistic nuclear many-body theory
  - The matrix element of the current between free-nucleon states can be computed exactly using the fully relativistic expression

- Nuclear x-section

$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P(\mathbf{k}, E)$$

- The spectral function $P(\mathbf{k}, E) = \text{Im } G(\mathbf{k}, E)/\pi$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the nuclear ground state, leaving the residual system with excitation energy E

- The lepton-nucleon cross section $d\sigma_N$ can be obtained—at least in principle—from proton and deuteron data

Nuclear Spectral Function

- The analytic structure of the two-point Green's function—dictated by the Källèn-Lehman representations—is reflected by the spectral function

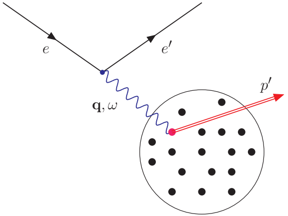
$$P(\mathbf{k}, E) = \sum_{h \in \{F\}} Z_h |M_h(\mathbf{k})|^2 F_h(E - e_h) + P_B(\mathbf{k}, E)$$

- According to the mean field approximation underlying the independent-particle model
  - Momentum dependence $M_h(\mathbf{k}) \to \phi_h(\mathbf{k})$, the momentum-space wave function of the single-particle state h
  - Spectroscopic factors $Z_h \to 1$, normalisation of $\phi_h(\mathbf{k})$
  - Energy distribution $F_h(E - e_h) \to \delta(E - e_h)$
  - Smooth contribution $P_B(\mathbf{k}, E) \to 0$
- The spectral functions of several nuclei—including oxygen and iron—have been obtained within the local density approximation (LDA), combining electron scattering data and the results of theoretical nuclear matter calculations

The $(e, e'p)$ Reaction

- Consider the process

$$e + A \to e' + p + (A-1)$$



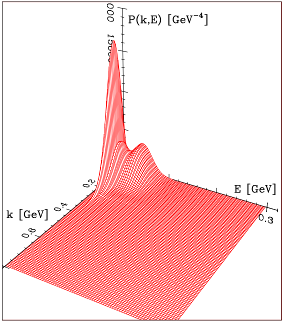
in which both the outgoing electron and the proton, carrying momentum p', are detected in coincidence, and the recoiling nucleus can be left in a any (bound or continuum) state $|n\rangle$ with energy E_n

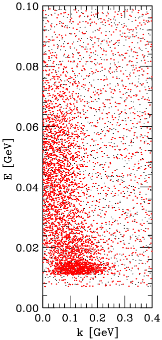
- In the absence of final state interactions (FSI)—which can be taken into acount as corrections—the the *measured* missing momentum and missing energy can be identified with the momentum of the knocked out nucleon and the excitation energy of the recoiling nucleus, $E_n - E_0$

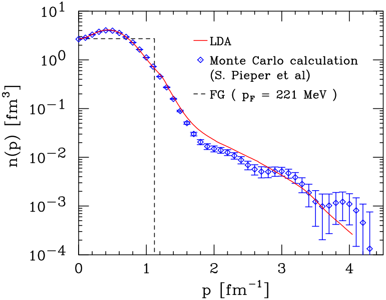
$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q} \quad , \quad E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

Spectral Function and Momentum Distribution of ^{16}O

- $\star$ $n(k) = \int dE\, P(k,E)$







- $\star$ the shell model accounts for $\sim 80\%$ of the strenght
- $\star$ the remaining $\sim 20\%$, arising from NN correlations, is located mainly at high momentum and large removal energy

Comparison to Inclusive Electron Scattering Data

- ⋆ elastic and inelastic processes consistently taken into account
- ⋆ no adjustable parameters needed

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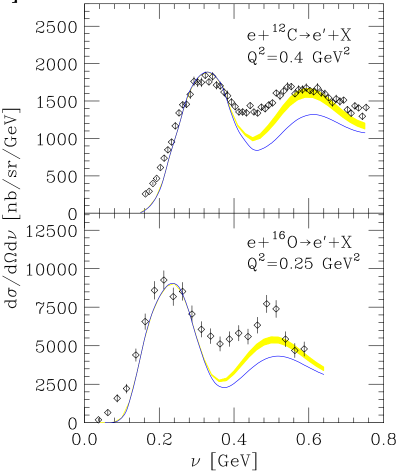
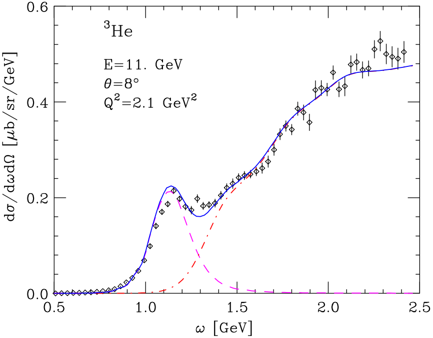
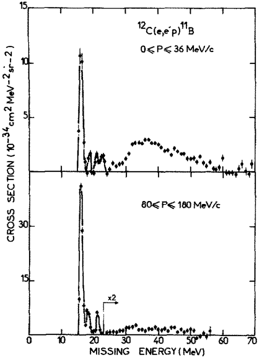




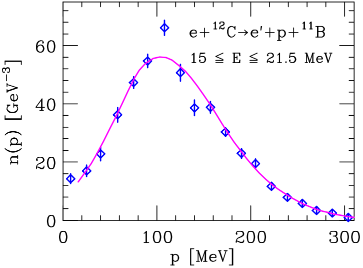
PINNING DOWN THE 1P1H CONTRIBUTION

⋆ $C(e, e'p)$ at Moderate Missing Energy: $e + {}^{12}C \to e' + p + {}^{11}B^*$

- Missing energy spectrum measured at Saclay (Mougey et al, 1976)



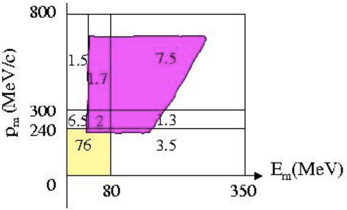
- P- state momentum distribution

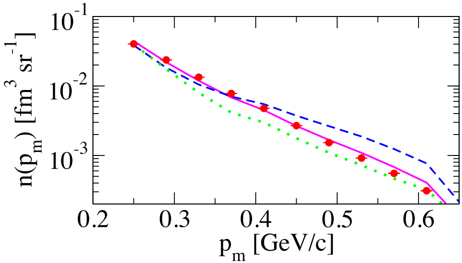


⋆ The measured spectroscopic factors are significantly lower than the predictions of the independent particle model

Comparison to the Measured Correlation Strength

- ⋆ The correlation strength in carbon has been investigated in JLab Hall C by the E97-006 Collaboration



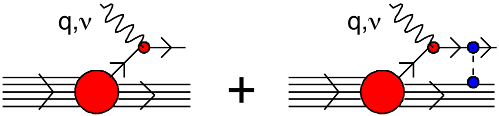


- ⋆ Measured correlation strength (Rohe et al, 2005)

Experiment	0.61 ±0.06
Greens function theory [3]	0.46
CBF theory [2]	0.64
SCGF theory [4]	0.61

Corrections to the IA: Final State Interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the occurrence of significant FSI effects



$$d\sigma_A = \int d^3k dE \; d\sigma_N \; P(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing

$$P_p(|\mathbf{k}+\mathbf{q}|, \omega - E) \to \delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

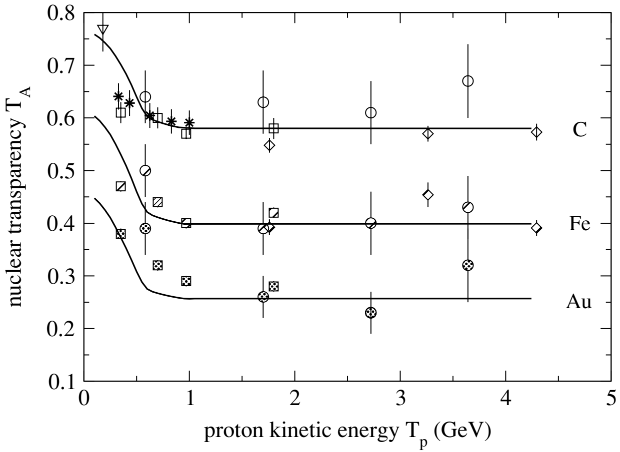
- effects of FSI on the inclusive cross section
  - shift in energy transfer due to the mean field of the spectator nucleons
  - redistributions of the strength due to the occurrence of rescattering of the knocked out nucleon
- high energy (eikonal) approximation
  - the struck nucleon moves along a straight trajectory with constant velocity
  - the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers

$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T_{|\mathbf{k}+\mathbf{q}|}}\,\delta(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$
$$+(1 - \sqrt{T_{|\mathbf{k}+\mathbf{q}|}})f(\widetilde{\omega} - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

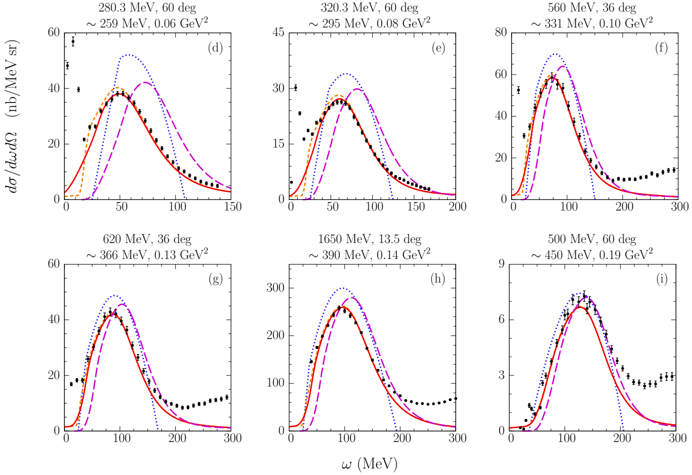
- the nuclear transparency T is measured by $(e, e'p)$ experiments, and the folding function f can be computed within nuclear many-body theory using as input nucleon-nucleon scattering data
- complex pattern of significant medium effects

Gauging FSI: Nuclear Transparency from $(e, e'p)$

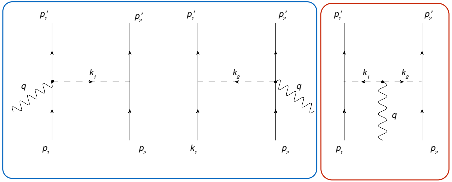
- Nuclear transparency, measured by the ratio $\sigma_{\text{exp}}/\sigma_{\text{IA}}$



⋆ $e + {}^{12}\mathrm{C} \to e' + X$ quasi elastic cross section computed within the IA including FSI. The predictions of the Relativistic Fermi Gas Model (RFGM) are also shown for comparison.

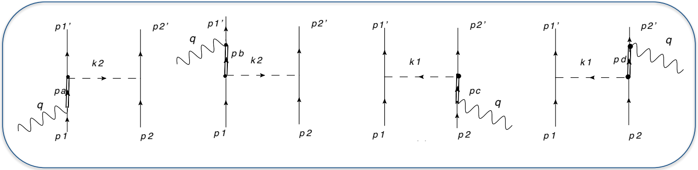


CORRECTIONS TO THE IA: MESON-EXCHANGE CURRENTS



Seagull
or
contact
term

Pion
in
flight
term



THE EXTENDED FACTORISATION *ansatz*

- ⋆ Highly accurate and consistent calculations of processes involving MEC can be carried out in the non relativistic regime
- ⋆ Fully relativistic MEC used mainly within the Fermi gas model
- ⋆ Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to two-nucleon emission amplitudes
  - Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)},\mathbf{p},\mathbf{p}'\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k},\mathbf{k}') \langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle \, \delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

The amplitude

$$M_n(\mathbf{k},\mathbf{k}') = \langle n_{(A-2)},\mathbf{k},\mathbf{k}'|0\rangle$$

is independent of q , and can be obtained from non relativistic many-body theory

TWO-NUCLEON SPECTRAL FUNCTION

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

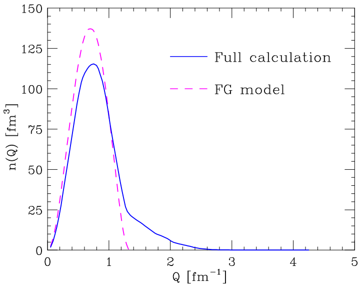
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE \, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

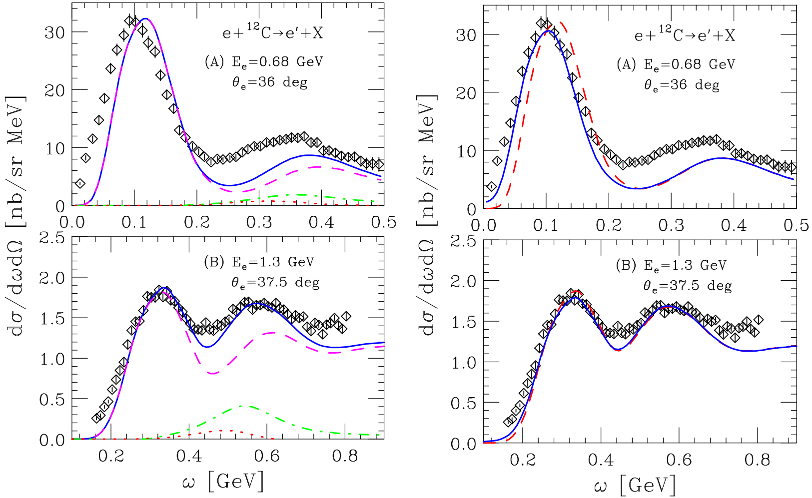
- ⋆ Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi|\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \ \ , \ \ \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$

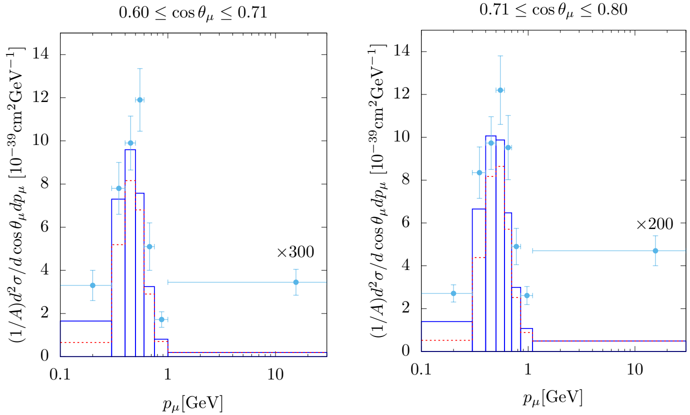


Pinning down FSI & MEC



Application to Neutrino Interactions

- ⋆ Comparison with the inclusive flux=integrated ν_μ-Carbon CC cross section measured by the T2K collabortion [PRD **98**. 012004 (2018)]. Inelastic structure functions provided by T. Sato, No MEC, no FSI.



More on FSI: the High-Energy Approximation

- At high energy, the propagation of the knocked out nucleon can be described within the eikonal approximation assuming that the spectator particles be frozen, that is, behave as fixed scattering centres

$$H_{\rm A} = H_{\rm A-1} + T_1 + \sum_{j=2}^{\rm A} v_{1j} = H_{\rm A-1} + T_1 + H_{\rm FSI}$$

- A-particle evolution operator within the frozen-spectators approximation

$$\mathrm{e}^{-iH_A t} = \mathrm{e}^{-i(H_{\rm A-1}+T_1+H_{FSI})t} \to \mathrm{e}^{-iH_{\rm A-1}t}\mathrm{e}^{-i(T_1+H_{FSI})t}$$

- In eikonal approximation

$$\langle x_1^\prime | \mathrm{e}^{-i(T_1+H_{FSI})t} | x_1 \rangle = \langle x_1^\prime | \mathrm{e}^{-iT_1 t} | x_1 \rangle \, \Omega_p(\boldsymbol{x}_1, t)$$

- Propagation of the knocked out particle driven by

$$\Omega_p(\boldsymbol{x},t) = \frac{1}{\rho_A(\boldsymbol{x})} \langle 0| \frac{1}{A} \sum_{i=1}^{\mathrm{A}} \mathcal{P}_z \prod_{j\neq i} [1 - \Gamma_p(\boldsymbol{x}_i + \boldsymbol{v}t - \boldsymbol{x}_j)] \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_i)|0\rangle$$

$$\Gamma_p(\boldsymbol{x}) = \theta(z)\gamma_p(\boldsymbol{x}_\perp) \quad , \quad \gamma_p(\boldsymbol{x}_\perp) = -\frac{i}{2} \int \frac{d^2 k_\perp}{(2\pi)^2} f_p(\boldsymbol{k}_\perp) \mathrm{e}^{i\boldsymbol{x}_\perp \cdot \boldsymbol{x}_\perp}$$

  where $f_p(\boldsymbol{k}_\perp)$ is the measured NN scattering amplitude, corrected to take into account medium effects

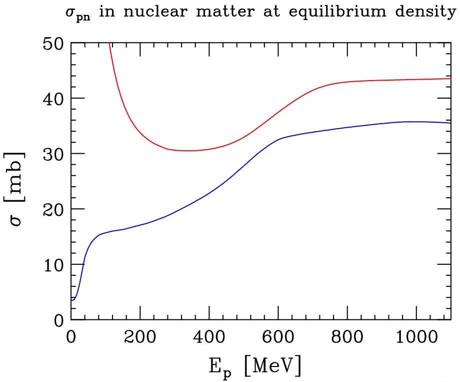
- Final State

$$\langle x_1, \ldots, x_\mathrm{A}|\mathrm{F}\rangle = \langle x_2, \ldots, x_\mathrm{A}|\mathrm{n}\rangle \otimes \Omega(\boldsymbol{x}_1, t) \frac{1}{\sqrt{V}} \mathrm{e}^{i\mathbf{p}\cdot\boldsymbol{x}_1}$$

- The leading contribution to $\Omega(\boldsymbol{x}_1, t)$ corresponds to the IA, in which FSI are neglected

⋆ the NN scattering amplitude is written in terms of three parameters, extracted from fits to the data: σ (total x-section), β (slope), and α (ratio between real and imaginary part of)

⋆ medium modifications of NN scattering are significant, and must be taken into account



⋆ the averaged scattering operator

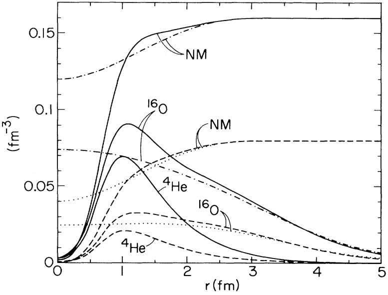
$$\bar{\Omega}(z) = \frac{1}{\mathrm{A}} \int d^3x \rho_{\mathrm{A}}(\boldsymbol{x}_1)\, \Omega(\boldsymbol{x}_1, z)\delta(z - z_1)$$

is strongly affected by correlation effects

⋆ to see this, consider that the probablity of NN rescattering depends upon the joint probability of finding the the struck particle at position $\boldsymbol{x}_1$ and a spectator at position $\boldsymbol{x}_2$

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho_{\mathrm{A}}(\boldsymbol{x}_1)\rho_{\mathrm{A}}(\boldsymbol{x}_2)\, g(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

⋆ FSI are strongly suppressed at $|\boldsymbol{x}_1 - \boldsymbol{x}_2| \lesssim 1$ fm



Summary & Outlook

- ⋆ In spite of the complexity arising from the flux average, a consistent description of the neutrino-nucleus cross section—in both elastic and inelastic channels—appears to be possible within the approach based on factorisation.
- ⋆ Initial state physics and the weak interaction vertices are largely under control
- ⋆ The present development of the treatment of FSI, while being adequate to describe inclusive processes, in which only the charged final-state lepton is detected, need to be improved to describe exclusive processes and pin down the relevant reaction mechanisms
- ⋆ The formalism based on the eikonal propagator of the primary final state nucleon, supplemented by an accurate cascade simulation taking into account rescattering against the spectators, is potentially capable to provide a reliable description of the occurrence of multinucleon final states
- ⋆ Long-range effects and the breakdown of factorisation at low momentum transfer need to be carefully investigated

The potential impact of NINJA

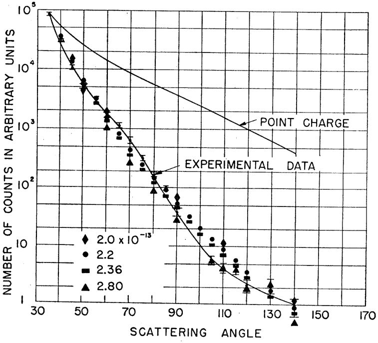
- ⋆ The availability of an accurate model of the oxygen ground state—including the spectral function and the ground-state wave function obtained from Monte Carlo calculations—will be essential to generate initial and final state variables relevant to NINJA events
- ⋆ A systematic study of the measured multitrack events will provide unprecedented access to information needed to test the theoretical calculations of FSI effects

Backup slides

ELASTIC SCATTERING: $e + A \to e' + A$, $\lambda \gg R_A \sim A^{1/3}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{eA} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2 \,,$$

- The Mott x-section described the electromagnetic interaction of a relativistic electron with a point target



Hofstadter et al, A.D. 1953 Gold target, $E_e = 125$ MeV

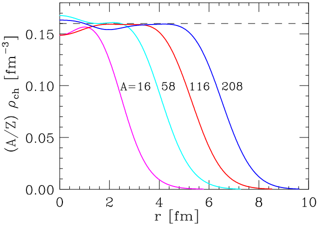
FROM NUCLEAR SYSTEMATICS TO MICROSCOPIC DYNAMICS

- The deviations from the Mott x-section provide information on target size and shape

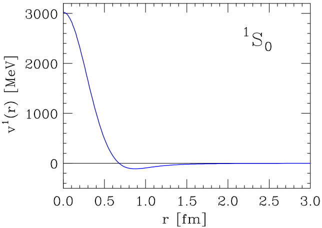
$$F(\mathbf{q}) = \int d^3r \; \rho_{\rm ch}(\mathbf{r})$$

- The observation that the central density of atomic nuclei is largely A-independent for $A > 16$, indicates that nuclear forces are strongly repulsive at short range

Nuclear charge densities



Nucleon-nucleon potential



- The repulsive core is a prominent feature of the nucleon-nucleon potential, giving rise to strong short-range correlations

- Within the factorization *ansatz* underlying the IA, the target response reduces to

$$W_A^{\mu\nu} = N \int d^3k \, dE \, \frac{m}{E_k} P(\mathbf{k}, E) w^{\mu\nu}$$

$$w^{\mu\nu} = \sum_x \int d^3p_x \langle \mathbf{k}, n | j^\mu | x, \mathbf{p}_x \rangle \langle \mathbf{p}_x, x | j^\nu | n, \mathbf{k} \rangle \delta^{(4)}(k + \tilde{q} - p_x)$$

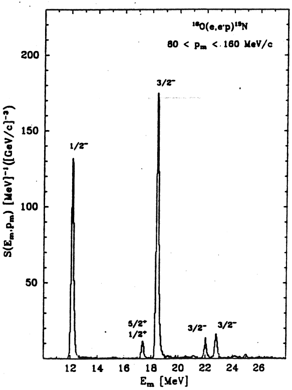
- $w^{\mu\nu}$ is the tensor describing the interaction of a free neutron of momentum $\mathbf{k}$ at four momentum transfer

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \quad , \quad \tilde{\omega} = \omega + M_A - E_R - E_k$$

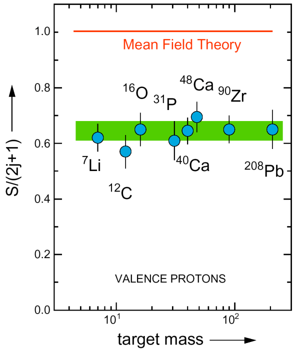
- The substitution $\omega \to \tilde{\omega} < \omega$ accounts the fact that an amount $\delta\omega = \omega - \tilde{\omega}$ of the energy transfer goes into excitation energy of the residual system.
- The spectral function $P(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the target nucleus, leaving the residual system with excitation energy E

$(e, e'p)$ Cross Section at Moderate p_m and E_m

- ⋆ The spectroscopic lines corresponding to the energies of the shell model states are clearly seen in the missing energy spectra

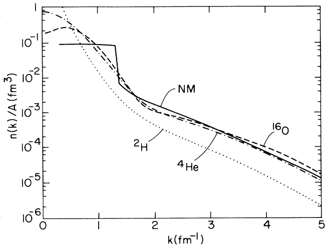


- ⋆ The integrated strengths yielding their normalisations are significantly below unity



THE LOCAL DENSITY APPROXIMATION (LDA)

★ Bottom line: accurate theoretical calculations show that the tail of the momentum distribution, arising from the continuum contribution to the spectral function, turns out to be largely A-independent for $A > 2$



★ Spectral functions of nuclei can be obtained within the Local Density Approximation (LDA)

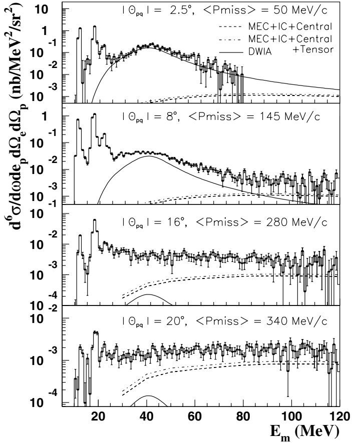
$$P_{\rm LDA}(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + \int d^3r\; \rho_A(r)\; P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

using the Mean Field (MF), or shell model, contributions obtained from $(e, e'p)$ data

★ The continuum contribution $P^{NM}_{corr}(\mathbf{k}, E)$ is computed for uniform nuclear matter at different densities using accurate theoretical approaches

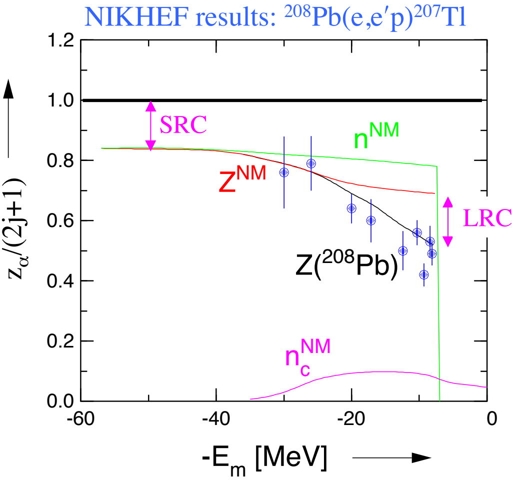
LARGE $|\mathbf{p}_m|$ AND E_m STRENGTH IN OXYGEN

- $|\mathbf{p}_m|$-evolution of missing energy spectrum in Oxygen. JLab data



- The interpretation of the data at large missing energy and missing momentum is hindered by significant Final State Interactions (FSI) and Meson-Exchange-Currents (MEC) effects

SPECTROSCOPIC FACTORS OF ^{208}Pb



⋆ Deeply bound states are largely unaffected by finite size and shell effects

FINAL STATE INTERACTIONS IN $(e, e'p)$

- In the presence of FSI, the distorted spectral function describing the mean field region can be written in the form

$$P^D_{MF}(\mathbf{p}_m, \mathbf{p}, E_m) = \sum_\alpha Z_\alpha |\phi^D_\alpha(\mathbf{p}_m, \mathbf{p})|^2 F_\alpha(E_m - E_\alpha)$$

with

$$\sqrt{Z_\alpha}\, \phi^D_\alpha(\mathbf{p}_m, \mathbf{p}) = \int d^3p_i\, \chi^\star_p(\mathbf{p}_i + \mathbf{q})\phi(\mathbf{p}_i)$$

where $\chi^\star_p(\mathbf{p}_i + \mathbf{q})$ describes the distortion arising from FSI effects

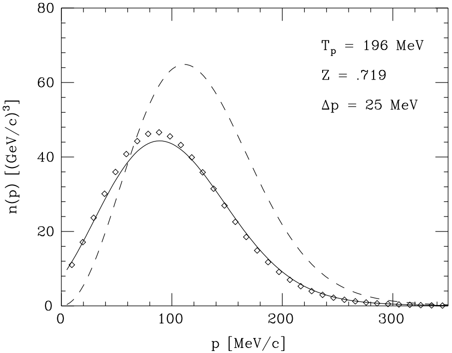
- The large body of existing work on $(e, e'p)$ data suggests that the effects of FSI can be strongly reduced measuring the cross section in *parallel kinematics*, that is with $\mathbf{p} \parallel \mathbf{q}$.
- in parallel kinematics, the distorted momentum distribution at fixed $|\mathbf{p}|$ becomes a function of missing momentum only

$$n^D_\alpha(p_m) = Z_\alpha\, |\phi^D(p_m)|^2 \,,$$

and the effects FSI can be easily identified.

Distorted Momentum Distribution

- Knock out of a P-shell protons in oxygen. Proton energy $T_p = 196$ MeV
- Distortion described by a complex optical potential (OP)



- FSI lead to a shift in missing momentum (real part of the OP), and a significant quenching, typically by a factor ~ 0.7 (imaginary part of the OP).

EFFECTS OF LONG-RANGE CORRELATIONS

- $|\mathbf{q}|$-evolution of the density-response of isospsin-symmetric nuclear matter. Calculation carried out within CBF using a realistic nuclear Hamiltonian.

$|\mathbf{q}| \approx 480\,\mathrm{MeV}$

$|\mathbf{q}| \approx 300\,\mathrm{MeV}$

$|\mathbf{q}| \approx 60\,\mathrm{MeV}$