Precise Standard Model predictions for $\boldsymbol{B}$ decays

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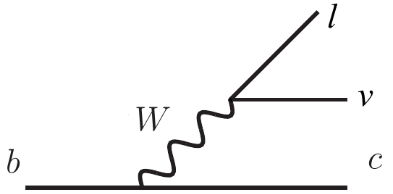
Introduction

Flavour changing currents

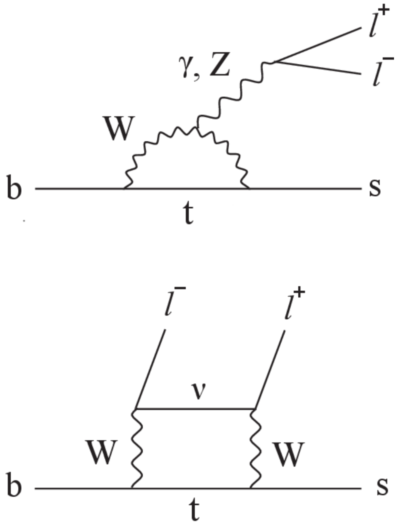
flavour changing charged currents (FCCC) occur at tree level (mediated by $W^{\pm}$) in the SM



FCCC

flavour changing neutral currents (FCNC) absent at tree level in the SM
FCNC are loop, GIM and CKM **suppressed in the SM**

FCNC **sensitive to new physics** contributions
probe the SM through **indirect searches**



FCNC

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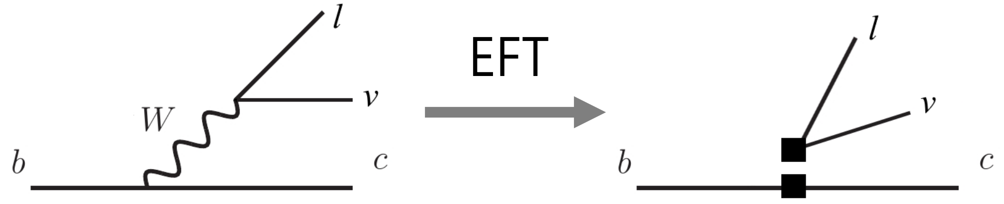
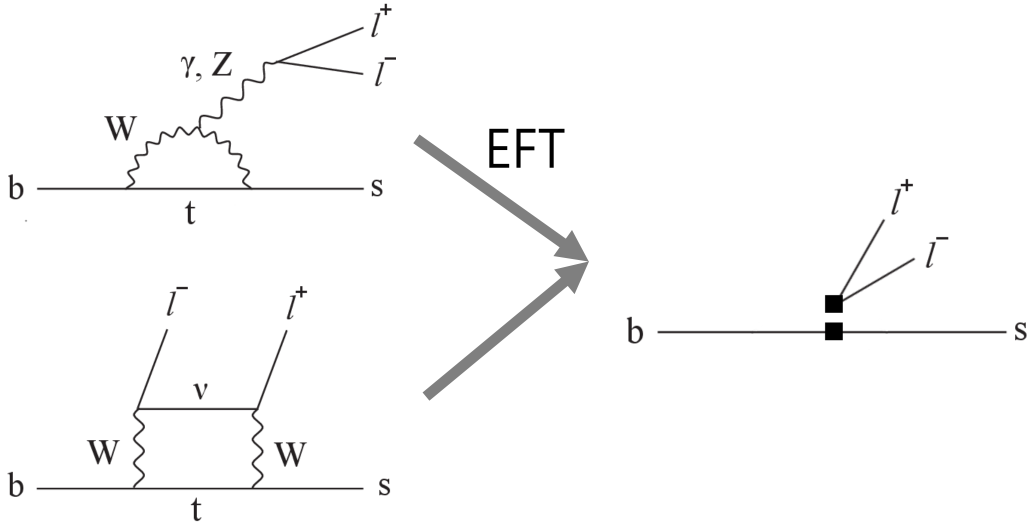
integrate out DOF heavier than the b
⇓
weak effective field theory



FCCC



FCNC

Hadronic matrix elements

study $\boldsymbol{B}$-meson decays to test the SM (neglect QED corrections)

FCCC $\langle D^{(*)}\ell\nu_\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu_\ell|\mathcal{O}_{lep}|0\rangle\langle D^{(*)}|\mathcal{O}_{had}|B\rangle$

FCNC $\langle K^{(*)}\ell\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\ell|\mathcal{O}_{lep}|0\rangle\langle K^{(*)}|\mathcal{O}_{had}|B\rangle$ + non−fact.

leptonic matrix elements: perturbative objects, high accuracy

hadronic matrix elements: non-perturbative QCD effects, usually large uncertainties

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leptonic matrix elements: perturbative objects, high accuracy

hadronic matrix elements: non-perturbative QCD effects, usually large uncertainties

decay amplitudes depend on:

- local hadronic matrix elements (form factors)
  $\langle K^{(*)}|\mathcal{O}(0)|B\rangle$
  $\langle D^{(*)}|\mathcal{O}(0)|B\rangle$
- nonlocal hadronic matrix elements (soft gluon contributions to the charm-loop)
  $\langle K^{(*)}|\mathcal{O}(0,x)|B\rangle$

Interesting observables

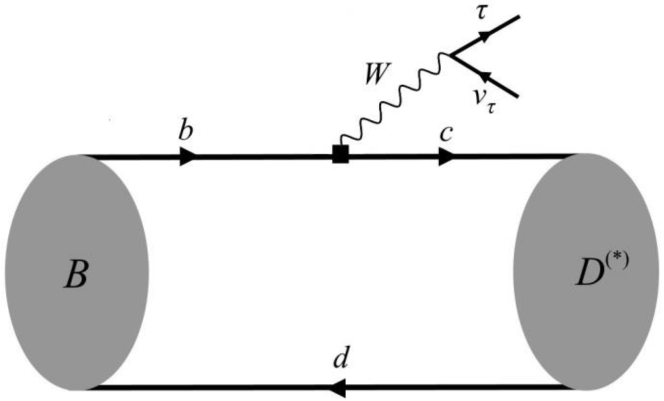
test the lepton flavour universality to test the SM

lepton flavour universality = the 3 lepton generations have the same couplings to the gauge bosons

violations of lepton flavour universality $\Longrightarrow$ new physics

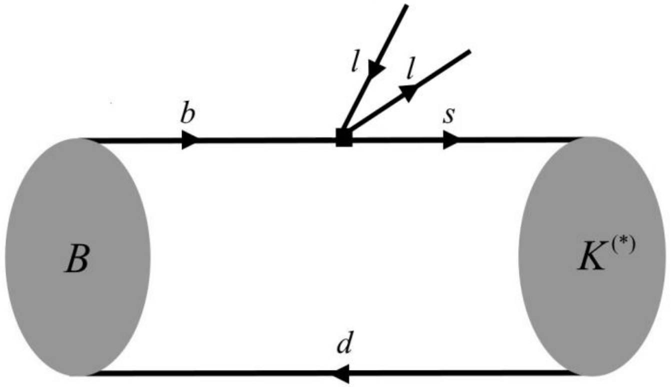
observables to test LFU

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)} \nu \tau)}{\Gamma(B \to D^{(*)} \nu \mu)} \qquad R_{K^{(*)}} = \frac{\Gamma(B \to K^{(*)} \mu^+ \mu^-)}{\Gamma(B \to K^{(*)} e^+ e^-)}$$

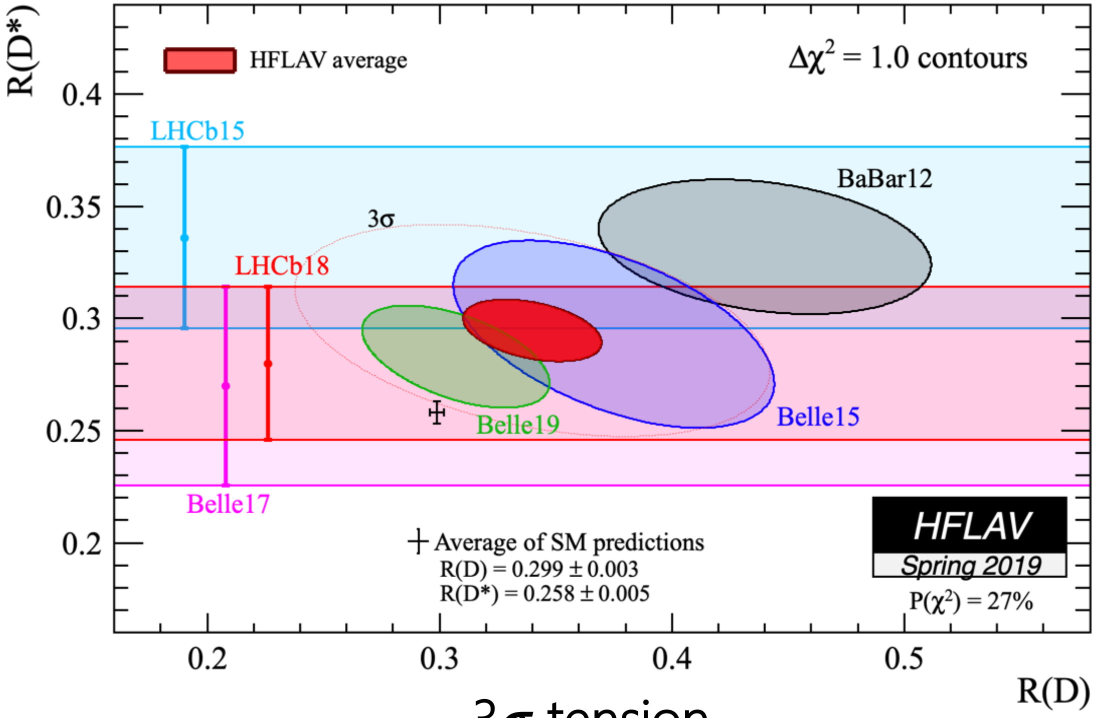


another test of the SM: angular observables in $B \to K^* \ell\ell$ (e.g. P_5')

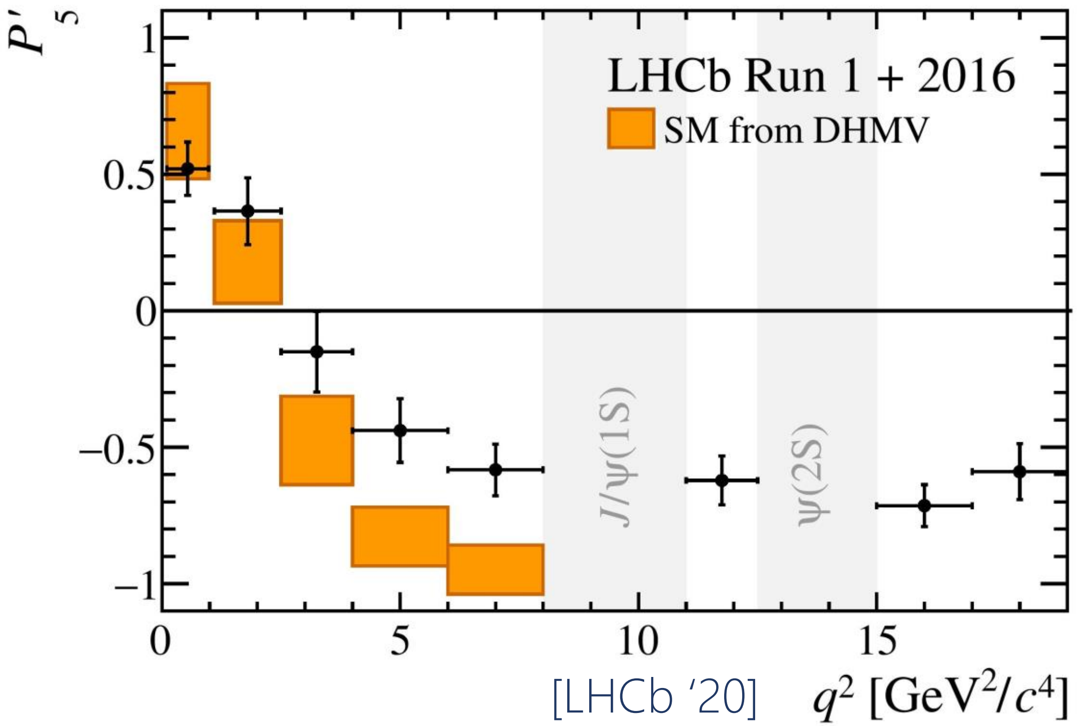
right choice of observables can reduce the hadronic uncertainties



B-anomalies



~3$\boldsymbol{\sigma}$ tension



[LHCb '20]

B-anomalies = tension between experimental measurements and theoretical predictions in B-meson decays involving different observables ($R_{D^{(*)}}, R_{K^{(*)}}, P'_5$ ...) and experiments

Standard Model predictions

Methods to compute hadronic matrix elements

non-perturbative techniques are needed
to compute hadronic matrix elements

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Lattice QCD

numerical evaluation of correlators in a
finite and discrete space-time

local matrix elements (usually at high q^2)

nonlocal matrix elements still
work in progress

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numerical evaluation of correlators in a finite and discrete space-time

local matrix elements (usually at high q^2)

nonlocal matrix elements still work in progress

Light-cone sum rules (LCSRs)

based on unitarity, analyticity, and quark-hadron duality approximation

need universal B-meson matrix elements

applicable for both local and nonlocal matrix elements (at low q^2)

Definition of the form factors

form factors (FFs) parametrize exclusive hadronic matrix elements

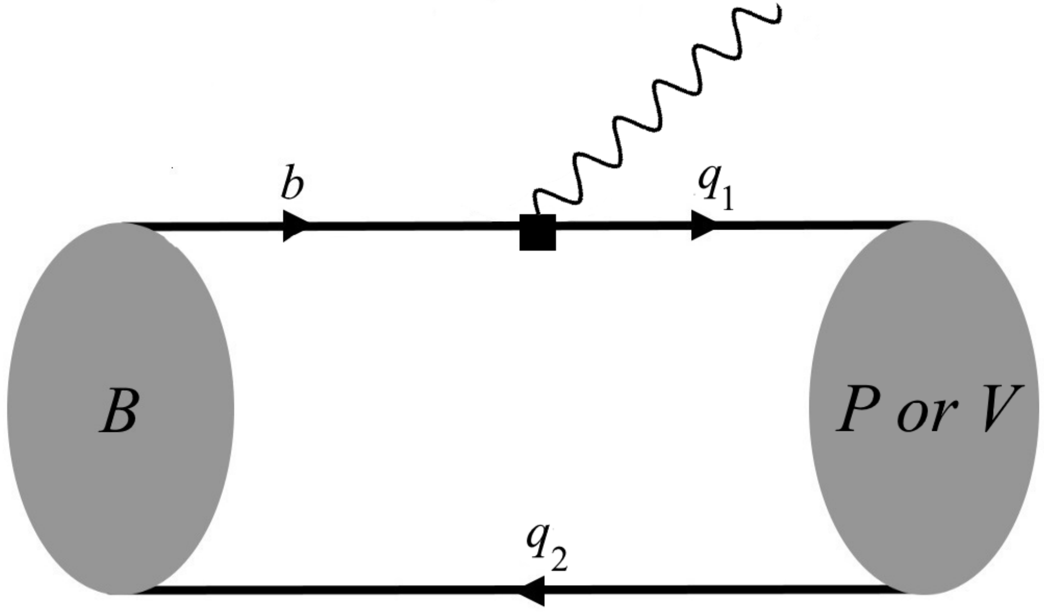
$$\langle P(k)|\bar{q}_1\gamma_\mu b|B(q+k)\rangle = 2\,k_\mu f_+(q^2) + q_\mu\big(f_+(q^2) + f_-(q^2)\big)$$

$$\langle P(k)|\bar{q}_1\sigma_{\mu\nu}q^\nu b|B(q+k)\rangle = \frac{i f_T(q^2)}{m_B + m_P}\left(q^2(2k+q)_\mu - (m_B^2 - m_P^2)q_\mu\right)$$

decomposition follows from Lorentz invariance

FFs are functions of the momentum transferred q^2
(q^2 is the dilepton mass squared)

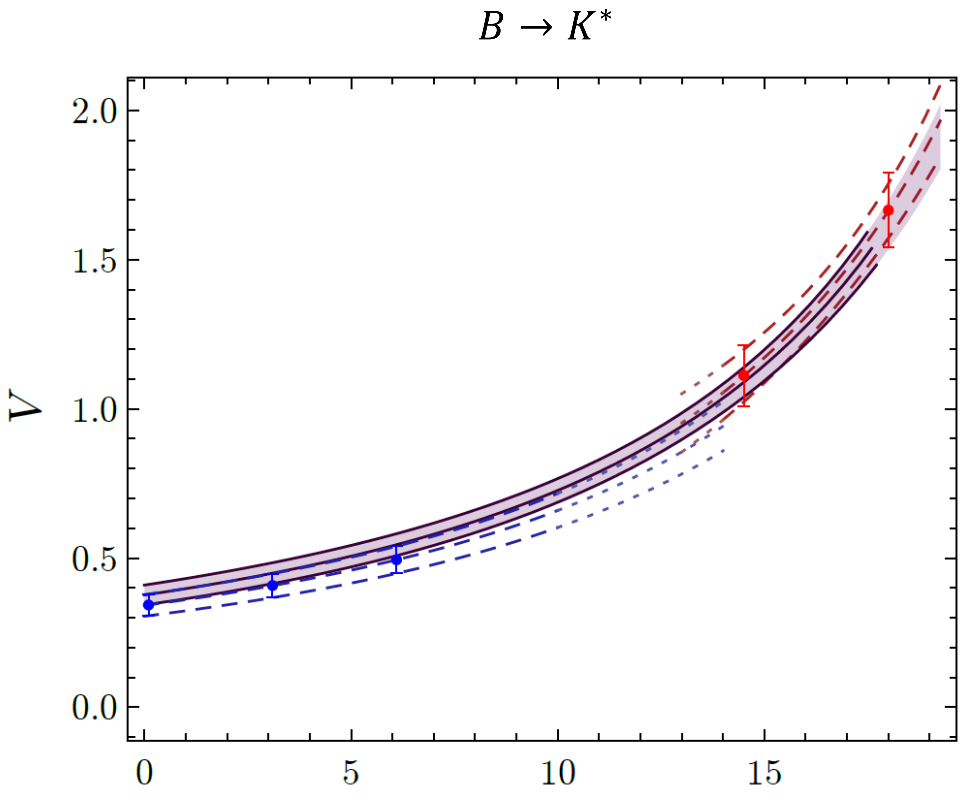
3 independent *B* to pseudoscalar meson (*P*) FFs
7 independent *B* to vector meson (*V*) FFs



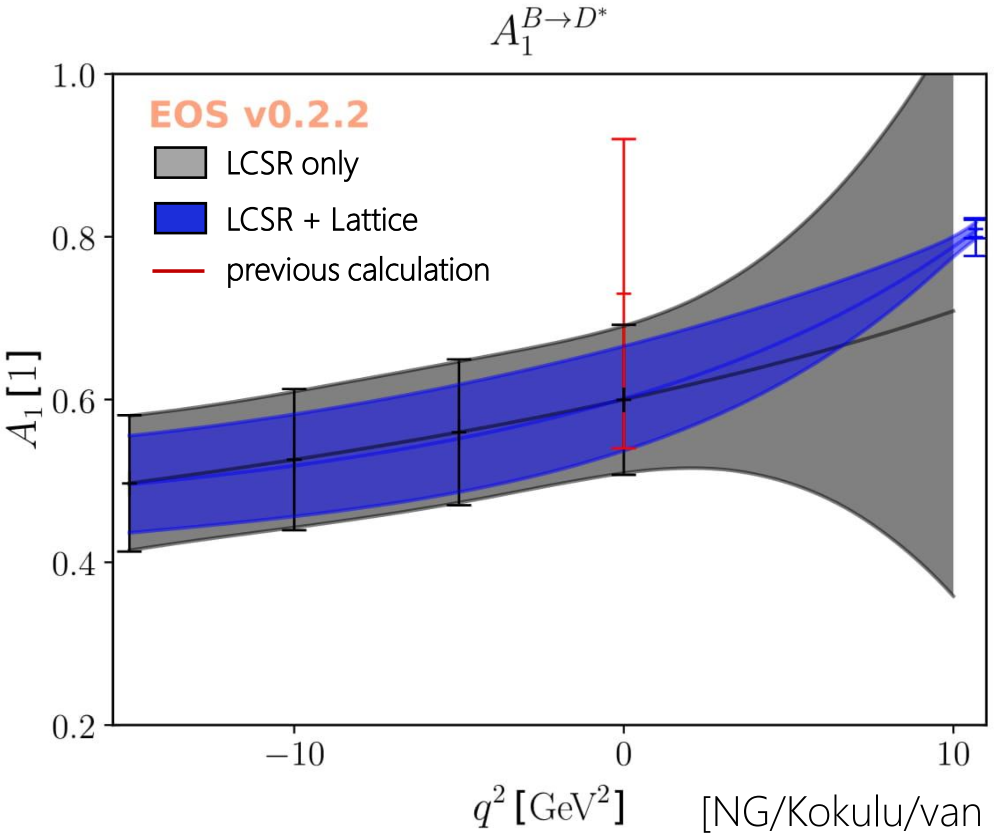
State of the art

	Transition	Lattice QCD	LCSR
$b \to s$	$B \to K$	high q^2	$q^2 < 12\ \mathrm{GeV}^2$
	$B \to K^*$	high q^2	$q^2 < 6\ \mathrm{GeV}^2$
	$B_s \to \phi$	high q^2	$q^2 < 6\ \mathrm{GeV}^2$
$b \to c$	$B \to D$	high q^2	$q^2 < 0\ \mathrm{GeV}^2$
	$B \to D^*$	high q^2	$q^2 < 0\ \mathrm{GeV}^2$
	$B_s \to D_s$	whole q^2 range	$q^2 < 0\ \mathrm{GeV}^2$
	$B_s \to D_s^*$	whole q^2 range	$q^2 < 0\ \mathrm{GeV}^2$

Combine lattice QCD and LCSRs for local FFs



[Bharucha/Straub/Zwicky '15]



[NG/Kokulu/van Dyk '18]

obtain the FF values to the whole spectrum (no additional assumptions required)

good agreement between lattice and LCSRs calculations

More on the $B_{(s)} \to D_{(s)}^{(*)}$ FFs

use heavy-quark limit ($m_{b,c} \to \infty$) to relate $B_{(s)} \to D_{(s)}$ FFs to $B_{(s)} \to D_{(s)}^*$ FFs

expand $B_{(s)} \to D_{(s)}^{(*)}$ FFs in the heavy-quark limit

$$FF^{B\to D^{(*)}}(q^2) = c_0 \xi(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i(q^2) + c_3 \frac{1}{m_c} L_i(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

$$FF^{B_s\to D_s^{(*)}}(q^2) = c_0 \xi^s(q^2) + c_1 \frac{\alpha_s}{\pi} C_i(q^2) + c_2 \frac{1}{m_b} L_i^s(q^2) + c_3 \frac{1}{m_c} L_i^s(q^2) + c_4 \frac{1}{m_c^2} l_i(q^2)$$

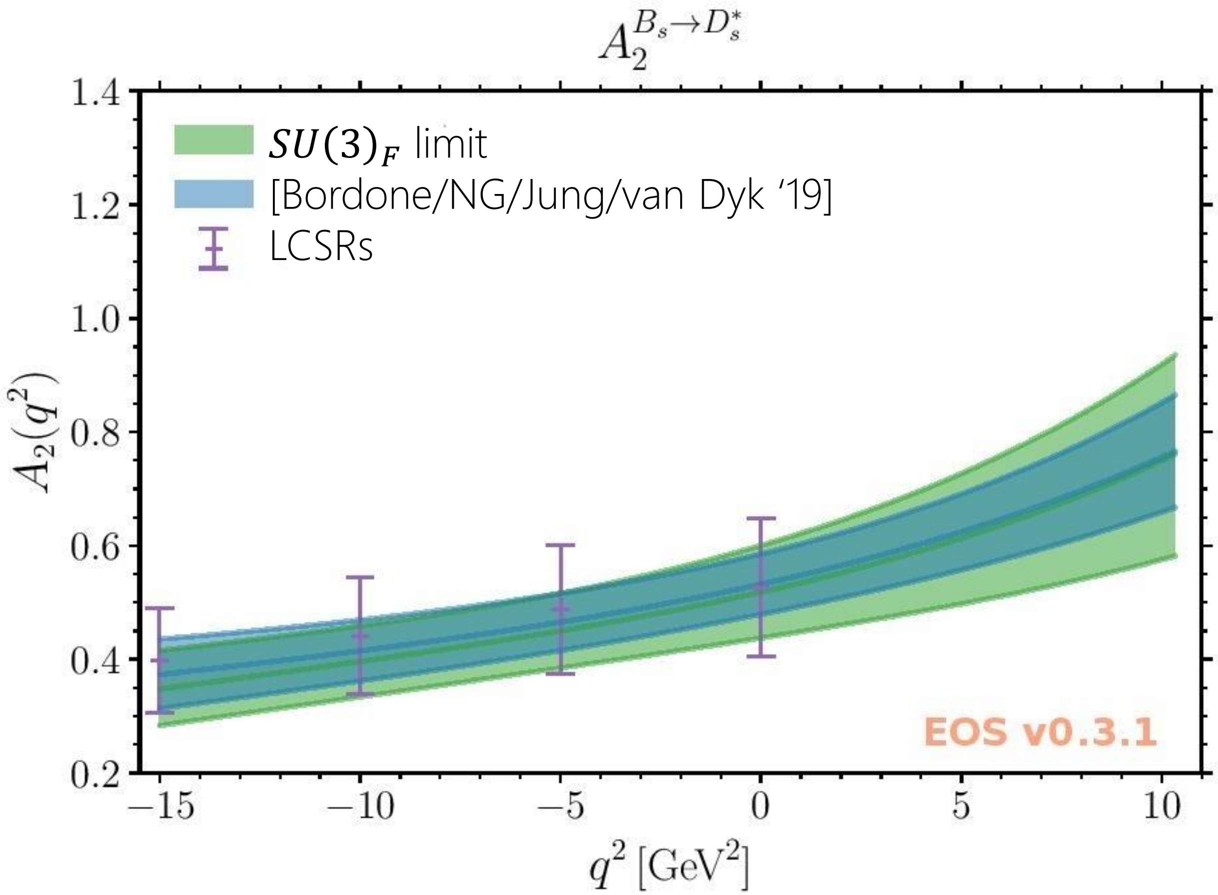
include $1/m_c^2$ corrections [Bordone/Jung/van Dyk '19]

all $B \to D^{(*)}$ and $B_s \to D_s^{(*)}$ FFs parametrized in terms of **14 Isgur-Wise (IW) functions**

More on the $B_{(s)} \to D_{(s)}^{(*)}$ FFs

constrain IW functions with

- lattice QCD (where available)
- light-cone sum rules for the FFs
- SVZ sum rules for Isgur-Wise functions
- with and w/o exp data
- dispersive bounds



results for all $B \to D^{(*)}$ FFs and $B_s \to D_s^{(*)}$ FFs in the whole physical phase space

improved precision going beyond the $SU(3)_F$ limit

The $B \to D^{**}$ FFs

why study $B \to D^{**}$ FFs? $D^{**} = \{D_0^*, D_1', D_1, D_2^*\}$

- alternative channel to study the (anomalous) $b \to c$ transitions
- important background for the $B \to D^{(*)}\ell\nu$ measurements
- improve the determination of $|V_{cb}|$ and $|V_{ub}|$

theoretical calculations of $B \to \{D_1', D_1\}$ FFs are very challenging
(both with LQCD and LCSRs)

- same quantum numbers ($J^P = 1^+$)
- almost the same mass

difficult to disentangle

extend the LCSR method to disentangle $\boldsymbol{D_1}$ and $\boldsymbol{D_1'}$

[NG/Khodjamirian/Mandal/Mannel w.i.p.]

More on the $b \to s$ transitions

rare decays amplitude written in term of (local) FFs and non-local FFs

$$\mathcal{A}(B \to K^{(*)}\ell\ell) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \boldsymbol{\mathcal{F}_\mu} - \frac{L_V^\mu}{q^2}\left(C_7\, \boldsymbol{\mathcal{F}_{T,\mu}} + \boldsymbol{\mathcal{H}_\mu}\right)\right]$$

(local) FFs:

- combine lattice QCD (high q^2) and LCSRs (low q^2) to get good precision $\sim 10\%$

non-local FFs (charm-loop effects):

- calculated using an Operator Product Expansion (OPE)
- large uncertainties → reduce uncertainties for a better understanding of rare B decays

Soft-gluon contribution to the charm loop

expand $\mathcal{H}_\lambda$ in a light-cone OPE for $q^2 \ll 4m_c^2$

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

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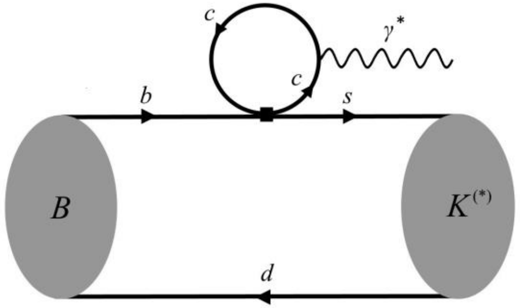
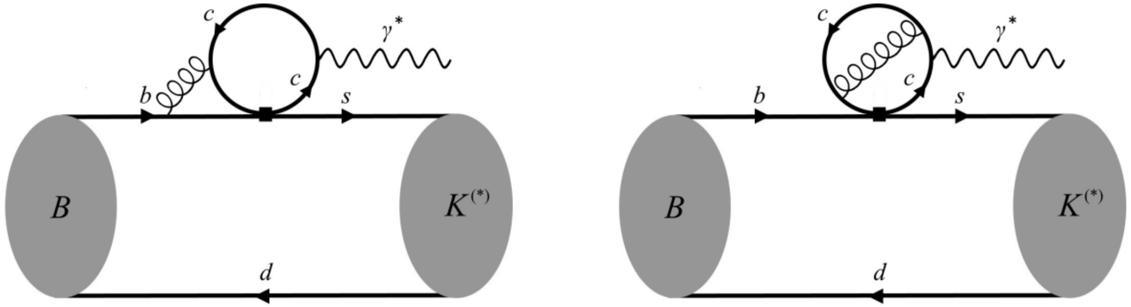
leading power (LO in α_s)



+ hard gluons (α_s) corrections



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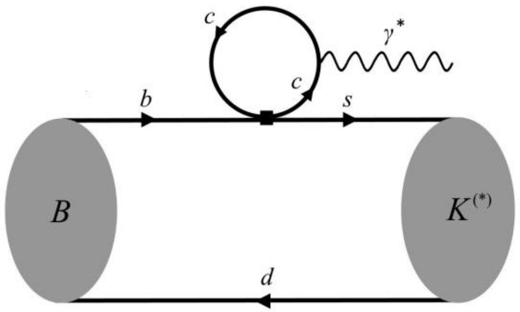

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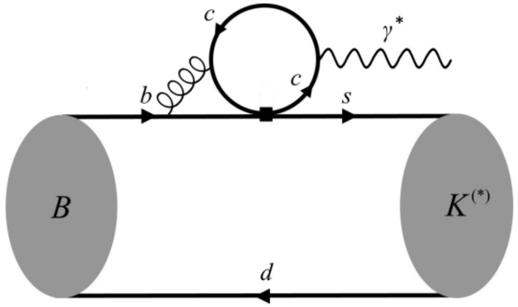
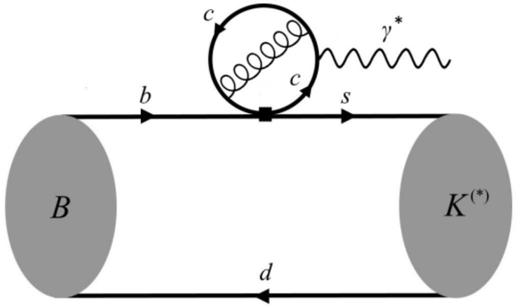
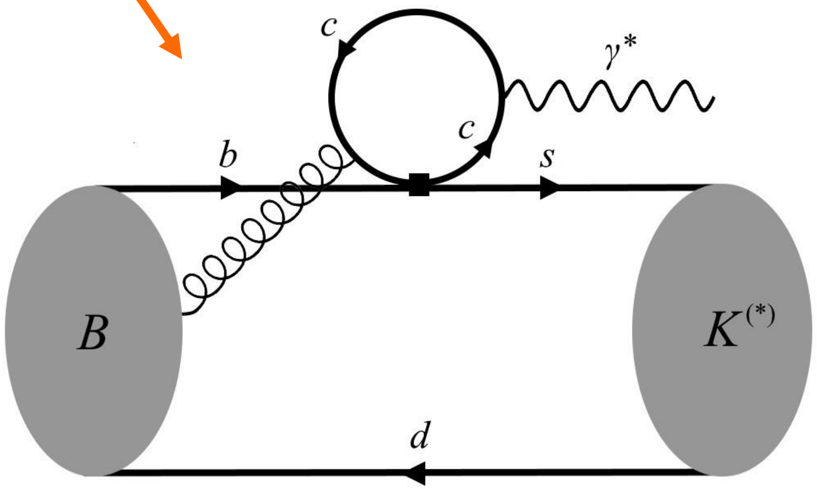




soft gluon correction
non-perturbative
$\Longrightarrow$ not α_s suppressed



Charm-loop results and comparison

$\Delta C_9(q^2 = 1\ \mathrm{GeV}^2)$		**KMPW2010**	**GvDV2019**
leading power (LO α_s)		0.27	0.27
$B \to K\ell\ell$	$\mathcal{V}_{\mathcal{A}}$	$-0.09^{+0.06}_{-0.07}$	$(1.9^{+0.6}_{-0.6}) \cdot 10^{-4}$
$B \to K^*\ell\ell$	$\mathcal{V}_1$	$0.6^{+0.7}_{-0.5}$	$(1.2^{+0.4}_{-0.4}) \cdot 10^{-3}$
	$\mathcal{V}_2$	$0.6^{+0.7}_{-0.5}$	$(2.1^{+0.7}_{-0.7}) \cdot 10^{-3}$
	$\mathcal{V}_3$	$1.0^{+1.6}_{-0.8}$	$(3.0^{+1.0}_{-1.0}) \cdot 10^{-3}$
$B_s \to \phi\ell\ell$	$\mathcal{V}_i$	—	see paper

[Khodjamirian/Mannel/Pivovarov/Wang 2010 (**KMPW2010**)]

- our results are **two orders of magnitude smaller** than in KMWP2010 ($\Longrightarrow$ **smaller unc.**)
- **we can reproduce the analytical results given in KMWP2010 and the differences are well understood**
- quick **convergence of the light-cone OPE**

Why such different results?

different inputs: LCDAs models depend on λ_H^2, λ_E^2 →

KMPW10:
$\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15\ \mathrm{GeV}^2$
$\Longrightarrow$ twist 3 does not contribute

we use $\lambda_E^2 = 0.03 \pm 0.02\ \mathrm{GeV}^2$
$\lambda_H^2 = 0.06 \pm 0.03\ \mathrm{GeV}^2$
$\Longrightarrow$ ~10 times smaller [Nishikawa/Tanaka 2014]

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three-particle LCDAs twist expansion →

KMPW10: the 3-pt LCDAs twist expansion was not known

we use Braun/Ji/Manashov 2017

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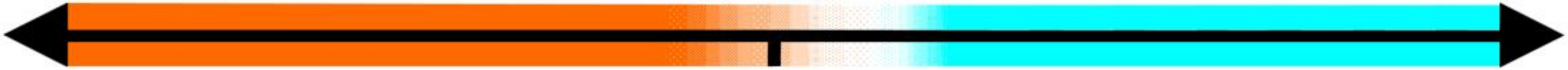
independent 3-particle LCDAs considered →

KMPW10: 4 Lorentz structures

all 8 independent Lorentz structures
$\Longrightarrow$partial cancelation (new structures come with an opposite sign)

Dispersive bounds for $\mathcal{H}_\lambda$

parametrize $\mathcal{H}_\lambda$ (z or q^2 expansion) → truncate the expansion **estimate truncation error** → obtain $\mathcal{H}_\lambda$ in the phenomenologically relevant region



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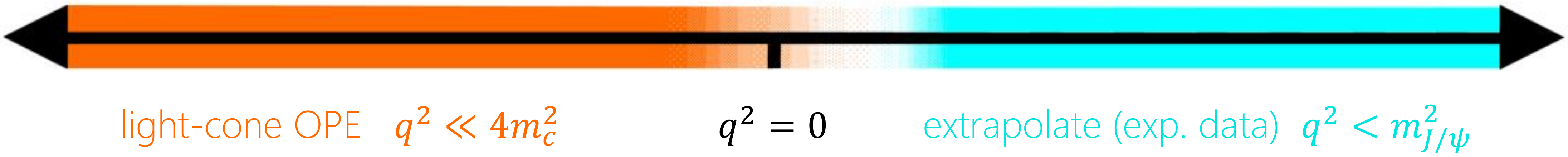

- **estimate truncation error using dispersive bounds**
- extend method already used for local form factors to non-local form factors $\mathcal{H}_\lambda$

  [BGL 1995] [CLN 1998]
- model independent constraints on $\mathcal{H}_\lambda$ → **control theoretical uncertainties**

Parametrizations for $\mathcal{H}_\lambda$

- q^2 parametrization [Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda{}'(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda''(0) + \cdots$$

- dispersion relation [KMPW2010]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi \mathcal{A}_\psi}{M_\psi^2\left(M_\psi^2 - q^2\right)} + \int_{4M_D^2}^{\infty} dt\, \frac{\rho(t)}{t(t-q^2)}$$

- z expansion [Bobeth/Chrzaszcz/van Dyk/Virto 2017]

$$\mathcal{H}_\lambda(z) = \sum_{n=0}^{\infty} c_n z^n$$

- **we propose a new parametrization** (z polynomials) [NG/van Dyk/Virto '20]

$$\hat{\mathcal{H}}_\lambda(z) = \sum_{n=0}^{\infty} a_n p_n(z)$$

The dispersive bound

expand $\hat{\mathcal{H}}_\lambda$ in orthogonal polynomials $p_n(z)$

$$\hat{\mathcal{H}}^{B\to K}(z) = \sum_{n=0}^{\infty} a_n^{B\to K}\, p_n^{B\to K}(z)$$

where

$$\hat{\mathcal{H}}^{B\to K}(z) = \mathcal{P}(z)\,\phi^{B\to K}(z)\;\mathcal{H}_\lambda^{B\to K}(z)$$

$$p_0^{B\to K}(z) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(z) = \left(z - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right)\sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}}$$

$$p_2^{B\to K}(z) = \left(z^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1} z + \frac{2\,\mathrm{si}\ldots}{}\right.$$

$$p_3^{B\to K}(z) = \cdots$$

the dispersive bound reads

$$1 > 2\sum_{n=0}^{\infty}|a_n^{B\to K}|^2 + \sum_\lambda\left(2\sum_{n=0}^{\infty}\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \sum_{n=0}^{\infty}\left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right)$$

the coefficients of the $\hat{\mathcal{H}}_\lambda$ are bounded! [NG/van Dyk/Virto '20]

Conclusions and outlook

Conclusion and outlook

$b \to c$ transitions:

- $B_{(s)} \to D_{(s)}^{(*)}$ FFs – lattice QCD (and LCSRs) calculations available
- use HQET and dispersive bounds for better precision
- non-local effects absent (neglect QED corrections)
- computation $B \to \{D_1', D_1\}$ FFs w.i.p.

$b \to s$ transitions:

- $B \to K^{(*)}$ and $B_s \to \phi$ FFs – lattice QCD (and LCSRs) calculations available
- non-local effects implies large uncertainties
- calculate non-local effects
- control these uncertainties (use dispersive bounds)

Thank you!