πK AMPLITUDES IN τ DECAYS

October 8, 2021 | Leon von Detten | IAS-4/IKP-3   LHCB Jahrestreffen 2021

based on arXiv:2103.01966   in collaboration with F. Noël, C. Hanhart, M. Hoferichter and B. Kubis

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MOTIVATION

- Consistent description of πK needed for:
  - ▶ heavy meson decays, eg. B $\to J/\Psi K\pi$
  - ▶ description of crossed channels in search of exotic states
  - ▶ investigation of CP asymmetry in eg. $\tau \to K\pi\nu_\tau$

- High accuracy dispersive analysis of low energy phase shifts already exist
  - ▶ based on experimental data and constraints from chiral symmetry

- Problematic when extending to higher energies
  - ▶ inelastic channels become relevant
  - ▶ higher energetic resonances need to be included

$\Rightarrow$ Formalism consistent with unitarity and analyticity which maps to low energy amplitudes and incorporates resonances determining high energy dynamics
  - ▶ firstly introduced as parametrization of the pion vector form-factor [Hanhart, 2012] and applied to $\pi\pi$-scattering [Ropertz et al., 2018]

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FORMALISM

- Implement unitarity and analitcity with Bethe Salpeter equation

$$T_{if} = V_{if} + V_{im}G_{mm}T_{mf}$$

- Separate scattering potential into two parts and thus $T = T_0 + T_R$

- T_0 fixes low energy behaviour
  - ▶ elastic and unitary, can be calculated from scattering phase shift δ_0
  - ▶ used results from [Pelaez and Rodas, 2016]
  - ▶ implicitly incorporates effects of $K_0^*(700)/\kappa$

- T_R determines high energy dynamics
  - ▶ explicitly includes $K_0^*(1430)$ and $K_0^*(1950)$

- Use 2-channel formalism with πK and $\eta' K$ (ηK decouples)
  - ▶ 6 free parameters: 4 couplings constants and 2 resonance masses

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PARAMETRIZATION OF THE FORM FACTOR

- Calculate form factor Γ with correct analytic structure, containing information of both phase and modulus of the underlying T-matrix


$$\Gamma_i = \Omega_{im} \left[\mathbb{1} - V_R \Sigma\right]^{-1}_{mn} M_n, \qquad M_i = c_i - \sum_r g_i^r \frac{s - s_0}{(s - m_r^2)(s_0 - m_r^2)} \alpha_r$$

$$\mathrm{disc}\,\Gamma = 2i\sigma T^* \Gamma, \quad \Omega_{\pi K} = \exp\left[\frac{s}{\pi} \int_{s_{\mathrm{th}}}^{\infty} \mathrm{d}z \frac{\delta_0(z)}{z(z-s)}\right], \quad \mathrm{disc}\,\Sigma = 2i\Omega^\dagger \sigma \Omega$$

- g_i^r and m_r fixed by fit to scattering data
- Normalization c_i and resonance coupling to source term α_r are free parameters

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πK SPECTRUM IN $\tau \to K_S \pi^- \nu_\tau$

- $\tau \to K_S \pi^- \nu_\tau$ decay data by Belle [Epifanov et al., 2007]
  - ▶ scalar form factor parameterised by new formalism
  - ▶ vector form factor parameterised by RChPT
- Investigate 4 Fit variants
  - ▶ source term coupling $\alpha^{(2)}$ to $K_0^*(1950)$ free or set to 0
  - ▶ additional term linear in s in normalisation $c^{(0)} + c^{(1)}s$
- Difficulty: overlapping of $K^*(1410)$ and $K_0^*(1430)$
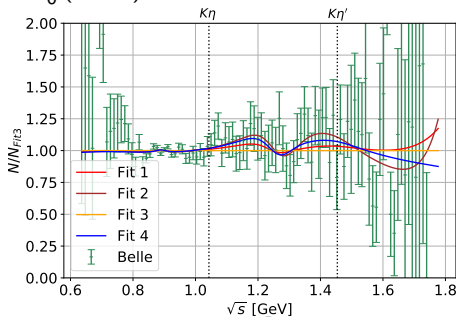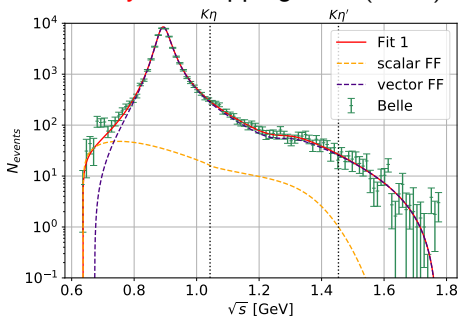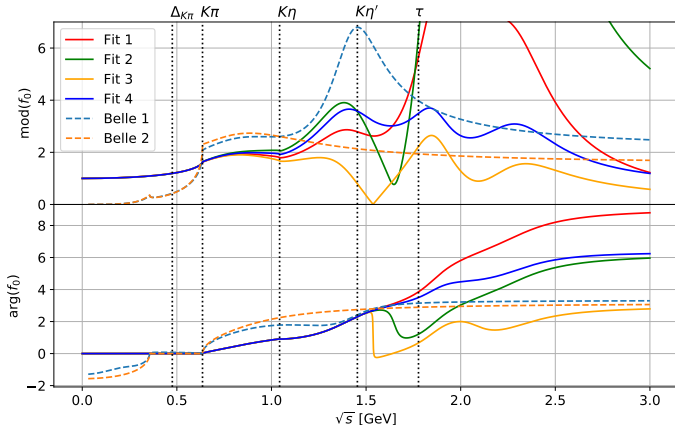

- for all fits $\chi^2/\text{d.o.f.} \in [0.98 - 1.07]$
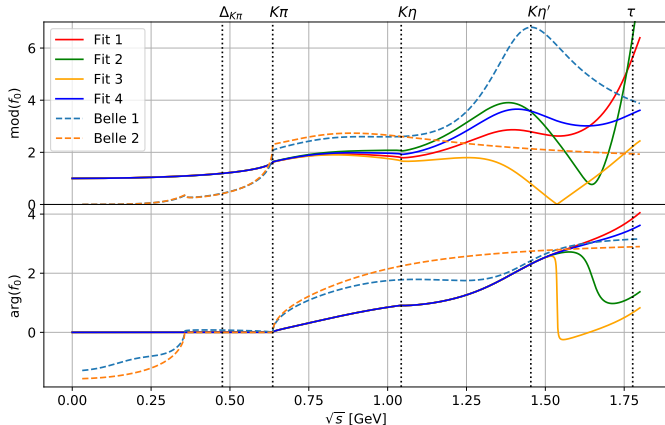
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SCALAR FORM FACTOR

- ► correct phase motion and normalisation by construction
- ► Callan-Treiman low energy theorem fulfilled (up to at least 0.5%)

→ all 4 fits are equally well and indistinguishable with present data

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SCALAR FORM FACTOR

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RESULTS

- *CP* assymetry in τ decays $A_{CP}^{\tau} = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}$
  - ▶ 2.8σ tension between SM prediction and [BaBar, 2012] measurement
  - ▶ only BSM option left: vector tensor interference
  - ▶ constraints on tensor form factor form lattice QCD and large N_C
  - ▶ estimate on BSM CP asymmetry agrees with [Cirigliano et al., 2018]

    $$A_{CP}^{\tau,\text{BSM}} = -0.034(14)\,\text{Im}(c_T) \quad \text{(Tensor Wilson coefficient } c_T)$$

- Pole extraction using Padé approximants
  $$\sqrt{s_{K_0^*(1430)}} = [1408(48) - i\,180(48)]\text{MeV}, \quad \text{BR}_{K_0^*(1430) \to \pi K} = 0.87(12)$$
  $$\sqrt{s_{K_0^*(1950)}} = [1863(12) - i\,136(20)]\text{MeV}, \quad \text{BR}_{K_0^*(1950) \to \pi K} = 0.70(8)$$

- Averaged over all fits: $\text{BR}_{\tau \to K_S \pi \nu_\tau} = 4.35(10) \times 10^{-3}$
  - ▶ agrees at 1.5σ with PDG [Zyla et al., 2020] and [Ryu et al., 2014]

- Improved estimate for $\text{BR}_{\tau \to K_0^*(1430)\nu_\tau} < 1.6 \times 10^{-4}$ (at 95% confidence level)

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CONCLUSION

- Applied new formalism consistent with unitarity and analyticity for πK scattering and production
  - low energy regime fixed by input phase
  - high energy dynamics determined by resonances

- Model able to reproduce scattering data up to $2.3\,\mathrm{GeV}$ including the $K_0^*(700), K_0^*(1430)$ and $K_0^*(1950)$ resonances

- Successful application to scalar form factor in $\tau \to K\pi\nu_\tau$

Outlook:
  - extending formalism to other partial waves (eg. P-wave)
  - forward-backward asymmetry in τ decays
    $\to$ separation of S and P wave
  - exotica in crossed B and D decays

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[1] C. Hanhart. "A New Parameterization for the Pion Vector Form Factor".
    In: Phys. Lett. B 715 (2012), pp. 170–177.
    DOI: 10.1016/j.physletb.2012.07.038. arXiv: 1203.6839 [hep-ph].

[2] S. Ropertz, C. Hanhart, and B. Kubis.
    "A new parametrization for the scalar pion form factors".
    In: Eur. Phys. J. C 78.12 (2018), p. 1000.
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[3] J. Pelaez and A. Rodas. "Pion-kaon scattering amplitude constrained
    with forward dispersion relations up to 1.6 GeV".
    In: Phys. Rev. D 93.7 (2016), p. 074025.
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    In: Phys. Lett. B 654 (2007), pp. 65–73.
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LITERATUR II

[5]   V. Cirigliano, A. Crivellin, and M. Hoferichter. "No-go theorem for
      nonstandard explanations of the $\tau \to K_S\pi\nu_\tau$ CP asymmetry".
      In: Phys. Rev. Lett. 120.14 (2018), p. 141803.
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      pi+ n at 11-GeV/c". In: Nucl. Phys. B 296 (1988), pp. 493–526.
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[7]   P. Estabrooks et al. "Study of K pi Scattering Using the Reactions K+- p
      —¿ K+- pi+ n and K+- p —¿ K+- pi- Delta++ at 13-GeV/c".
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OMNÉS PROBLEM

- Dispersion theory: express function $f(s)$ obeying unitarity and causality by its imaginary part

$$\text{disc } f(s) = 2i \operatorname{Im} f(s) \qquad f(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} dz \frac{\operatorname{Im} f(z)}{z - s}$$

- Consider f(s) to be a two particle amplitude with definite isospin and angular momentum
  - f(s) can be expressed by Watson's theorem in elastic regime with on-shell T-matrix $T(s)$ and scattering phase shift $\delta(s)$

$$\operatorname{Im} f = T^* \sigma f \quad \rightarrow \quad f(s) = |f(s)| e^{i\delta(s)}$$

  - Solution $f(s) = P(s)\Omega(s)$ with polynomial $P(s) \in \mathbb{R}$ and Omnes-function $\Omega(s)$

$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{s_{th}}^{\infty} dz \frac{\delta(z)}{z(z - s)} \right)$$

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FORMALISM

- Implement unitarity and analyticity via Bethe-Salpeter equation

$$T_{if} = V_{if} + V_{im} G_{mm} T_{mf}$$

  - V_{if} : scattering potential between initial channel i and final channel f
  - G_{mm} : loop function describing free propagation of particles in channel m
    $\rightarrow V_{if} \in \mathbb{R} \wedge \text{disc} G_{mm} = 2i\sigma_m$

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FORMALISM

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   $\rightarrow V_{if} \in \mathbb{R} \wedge \mathrm{disc} G_{mm} = 2i\sigma_m$

• Separate scattering potential into two parts $V = V_0 + V_R$ and thus $T = T_0 + T_R$

▶ T_0 calculated from elastic input phase δ_0
▶ V_0 not explicitly needed $\rightarrow$ absorbed into T_0

$$T_0 = V_0 + V_0 G T_0 \qquad T_0(s) = \begin{pmatrix} [\sigma(s)(\cot\delta_0(s) - i)]^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

▶ 2 channel setup including πK and $\eta' K$ as ηK decouples

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FORMALISM

- Define vertex function $\Omega = \mathbb{1} + T_0 G$
  - ▶ disc $\Omega = 2i T_0^* \sigma \Omega$
  - ▶ Coincides with discontinuity of Omnes function calculated from T_0

$$\Omega_{\pi K} = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\mathrm{d}z\frac{\delta_0(z)}{z(z-s)}\right] \qquad \Omega(s) = \begin{pmatrix} \Omega_{\pi K}(s) & 0 \\ 0 & 1 \end{pmatrix}$$

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- Use $T_R = \Omega t_R \Omega^T$ to obtain Bethe-Salpeter like equation for t_R

$$t_R = V_R + V_R (G\Omega) t_R$$
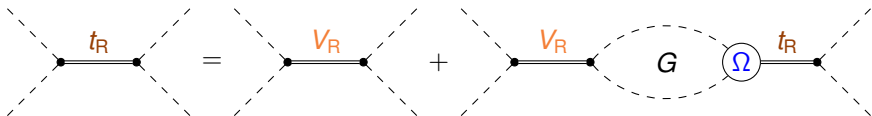
  - Selfenergy $\Sigma = G\Omega$ incorporates effects of T_0 in order to conserve unitarity

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FORMALISM

- Define self energy $\Sigma = G\Omega$
  - disc $\Sigma = 2i\Omega^\dagger \sigma \Omega$
  - Express Σ as a once-subtracted dispersion integral

$$\Sigma(s) = \frac{s}{2\pi i} \int_{s_{th}}^{\infty} dz \frac{\text{disc}\,\Sigma(z)}{z(z-s)} \qquad \Sigma(s) = \begin{pmatrix} \Sigma_{\pi K}(s) & 0 \\ 0 & \Sigma_{\eta' K}(s) \end{pmatrix}$$

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- General potential with coupling constants g_i, g_j given by

$$\bar{V}_R(s)_{ij} = \sum_r g_i^{(r)} \frac{1}{s - m_r^2} g_j^{(r)}$$

  - Subtract potential at some point s_0 to reduce its impact at low energies

$$V_R(s)_{if} = \bar{V}_R(s)_{ij} - \bar{V}_R(s_0)_{ij} = g_i \frac{s - s_0}{(s - m_r^2)(s_0 - m_r^2)} g_j$$

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FORMALISM

- Expression for T-matrix

$$T = T_0 + \Omega \left[\mathbb{1} - V_R \Sigma \right]^{-1} V_R \Omega^{\mathsf{T}}$$

- ► T_0 covers effects of $K_0^*(700)$
- ► T_R explicitly incorporates $K_0^*(1430), K_0^*(1950)$
- ► 6 free parameters: 4 couplings constants and 2 resonance masses
- $L = 0$ partial wave measured in combination of $I = \tfrac{1}{2}$ and $I = \tfrac{3}{2}$

$$T = T^{\frac{1}{2}} + T^{\frac{3}{2}}/2$$

- Results fixing low energy behaviour taken from [Pelaez and Rodas, 2016]

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FIT TO PHASE AND MODULUS
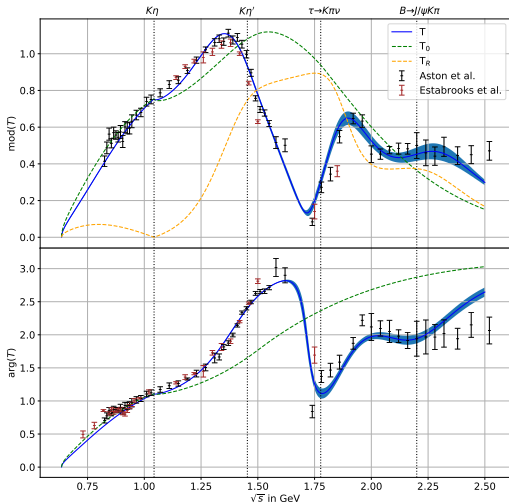
- Model reproduces data up to $2.3\,\mathrm{GeV}$

- Fixed $s_0 = (m_K + m_\eta)^2$
  - minimizes T_R at low energies

- Fit to Aston et al.
  $\chi^2/\mathrm{d.o.f.} = 370/112 \approx 3.5$

- Large incompatibilities between the two data sets
  - Underestimated systematic uncertainties

- Cover πK phase space in
  - $\tau \to K\pi\nu_\tau$
  - $B \to J/\Psi K\pi$

data: [Aston et al., 1988],[Estabrooks et al., 1978]

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POLE EXTRACTION

- Breit-Wigner parameterization model- and reaction-dependent
  - ▶ violate unitarity for overlapping resonances
  - ▶ problematic for broad or near-threshold resonances

- Information of a resonance encoded in its pole
  - ▶ mass M_R and width Γ_R from pole position $\sqrt{s_R} = M_R - i\frac{\Gamma_R}{2}$
  - ▶ coupling constant $\tilde{g}_j^R$ from residue $\mathcal{R}_{ij} = -\lim_{s \to s_R} T_{ij} \to \tilde{g}_j^R = \mathcal{R}_{ij}/\sqrt{\mathcal{R}_{ii}}$
  - ▶ $BR_{R \to i} = \Gamma_{R \to i}/\Gamma_{tot.}$ from partial width $\Gamma_{R \to i} = |\tilde{g}_i^R|^2 \rho(M_R{}^2)/M_R$
  - ▶ coupling C_{us}^R to $\bar{s}\gamma^\mu u$ current from residue $\lim_{s \to s_R} f_0 \propto \tilde{g}_i^R C_{us}^R$
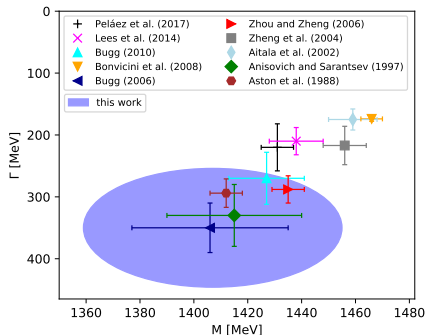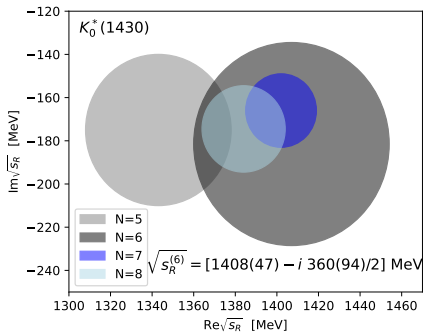
    Convention from PDG resonance review [P.A. Zyla et al., 2020]

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  - ▶ coupling C_{us}^R to $\bar{s}\gamma^\mu u$ current from residue $\lim_{s \to s_R} f_0 \propto \tilde{g}_i^R C_{us}^R$

    Convention from PDG resonance review [P.A. Zyla et al., 2020]

- T_0 has complicated analytic structure due to left-hand cuts
  - ▶ Use Padé approximants to extract resonance properties

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PADÉ APPROXIMANTS

- $K_0^*(1430)$ located in close proximity to the $K\eta'$ threshold
  - use conformal variable $\omega(s)$ to improve convergence of $P_1^N(s, s_0)$

$$\omega(s) = \frac{\sqrt{s - s_1^{\text{th}}} - \sqrt{s_2^{\text{th}} - s}}{\sqrt{s - s_1^{\text{th}}} + \sqrt{s_2^{\text{th}} - s}}$$
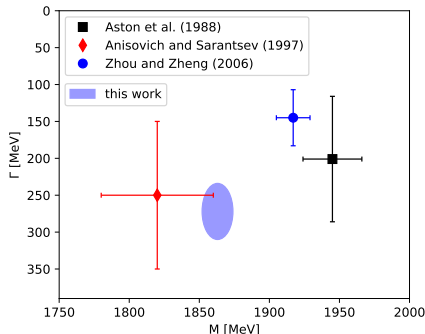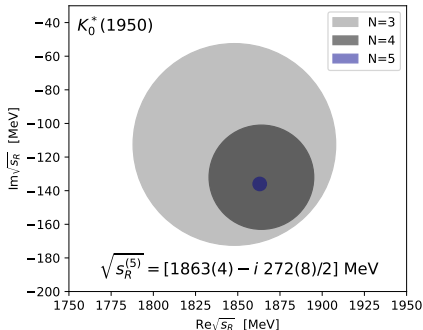
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PADÉ APPROXIMANTS

$$P_1^N(s, s_0) = \frac{\sum_{n=0}^{N} a_n^{(N)}(s - s_0^{(N)})^n}{1 + b^{(N)}(s - s_0^{(N)})}, \qquad \Delta_{sys}^{(N)} = \left| \sqrt{s_R^{(N)}} - \sqrt{s_R^{(N-1)}} \right|$$

- fit $P_1^N(s, s0)$ to T-matrix and form factor
- choose $s_0^{(N)}$ that minimizes $\Delta_{sys}^{(N)}$ for each N

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PADÉ APPROXIMANTS

$$P_1^N(s, s_0) = \frac{\sum_{n=0}^{N} a_n^{(N)}(s - s_0^{(N)})^n}{1 + b^{(N)}(s - s_0^{(N)})}, \qquad \Delta_{\text{sys}}^{(N)} = \left| \sqrt{s_R^{(N)}} - \sqrt{s_R^{(N-1)}} \right|$$
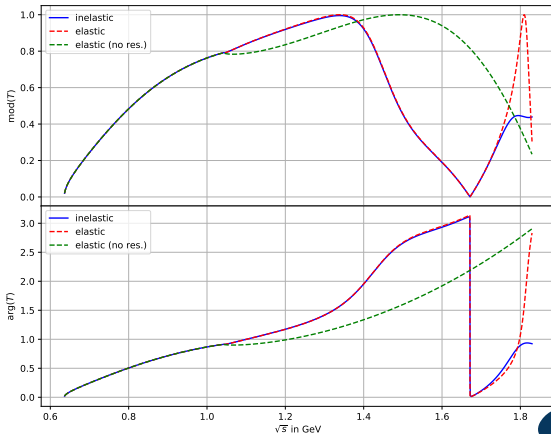
- ▶ fit $P_1^N(s, s0)$ to T-matrix and form factor
- ▶ choose $s_0^{(N)}$ that minimizes $\Delta_{\text{sys}}^{(N)}$ for each N
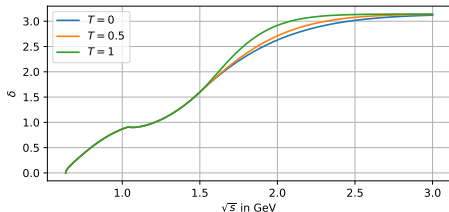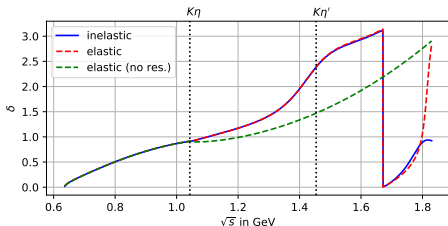
PELAEZ AND RODAS PARAMETERIZATION

$$t_0^{1/2} = \frac{S_0^b S_1^r S_2^r}{2i\sigma}$$

$$S_0^b = \exp\left(2i p_{\eta K}(\phi_0 + \phi_1 p_{\eta K}^2)\right)$$

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INPUT PHASE

- Results fixing low energy behaviour taken from [Pelaez et al., 2016]
  - $I = \frac{1}{2}$: reduced formalism with pure elastic phase without resonance contributions

- Phase must converge to multiple of π to ensure proper high energy behaviour of Omnes function
- Continuation matched at $\sqrt{s_m} = 1.5\,\mathrm{GeV}$ with additional tuning parameter T

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INPUT PHASE

- $I = \frac{3}{2}$: purly elastic up to $1.8\,\mathrm{GeV}$
  - $T_0^{\frac{3}{2}}$ directly calculated from phase
  - no resonances present in this channel (exotic quantum numbers)
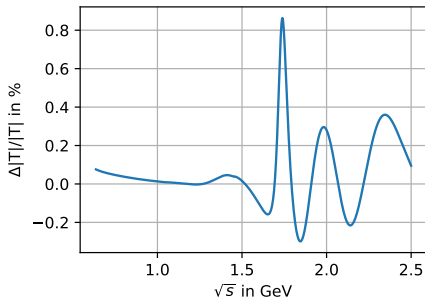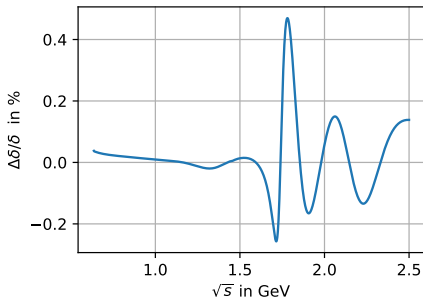 $\rightarrow$ phase continued to 0 and $T_R = 0$
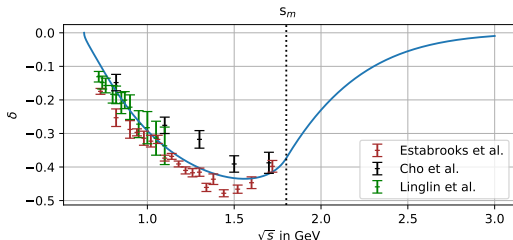
- original phase taken up to $\sqrt{s_m} = 1.8\,\mathrm{GeV}$ to conserve its prominent structure

INCLUSION OF ηK-CHANNEL

- ▶ Explicit inclusion of ηK-channel statistically insignificant
- ▶ Check relative difference between 2-channel formalism ($\pi K, \eta' K$) and 3-channel formalism ($\pi K, \eta K, \eta' K$)

$$\frac{\Delta \delta}{\delta} = \frac{\arg(T_{2c}) - \arg(T_{3c})}{\arg(T_{2c})} \qquad \frac{\Delta |T|}{|T|} = \frac{\mathrm{mod}(T_{2c}) - \mathrm{mod}(T_{3c})}{\mathrm{mod}(T_{2c})}$$
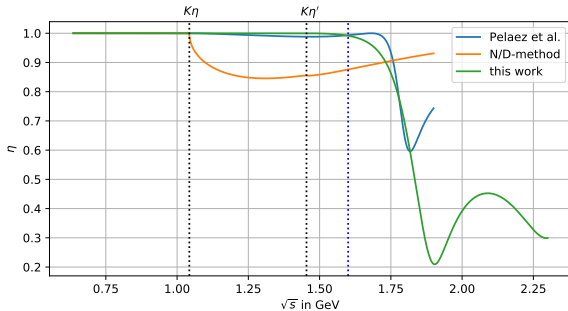

- ▶ Fit indicates vanishing couplings of resonances to the ηK-channel
  πK-channel mostly elastic up to $1.6\,\mathrm{GeV} \rightarrow \eta K$-channel decouples

ELASTICITY

$$\eta_{\pi K} = \text{mod}(1 + 2i\sigma_{\pi K} T_{\pi K})$$

▶ Elasticity of new model compatible with results from [Pelaez et al., 2016]

FIT TO τ-DECAY SPECTRUM

▶ $\tau \to K_S \pi^- \nu_\tau$ differential decay rate

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{c_\Gamma}{s} \left(1 - \frac{s}{m_\tau^2} \right)^2 \left(1 + 2\frac{s}{m_\tau^2} \right) q_{\pi K} \left(q_{\pi K}^2 |\bar{f}_+|^2 + \frac{3\Delta_{\pi K}^2}{4s(1 + 2\frac{s}{m_\tau^2})} |\bar{f}_0|^2 \right)$$

▶ defining matrix elements

$$\langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = (p_K - p_\pi)^\mu f_+(s) + (p_K + p_\pi)^\mu f_-(s) \quad (1)$$

$$\langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}u|0\rangle = \frac{\Delta_{\pi K}}{m_s - m_u} f_0(s), \quad (2)$$

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τ-DECAY SPECTRUM

- πK invariant mass spectrum extracted from $\tau \to K_s \pi^- \nu_\tau$ decays by Belle [Epifanov et al., 2007]

  ▶ Parameterized in terms of scalar F_S and vector F_V form factor

  $$\frac{d\Gamma}{d\sqrt{s}} \propto \frac{1}{s} \left(1 - \frac{s}{m_\tau^2}\right) \left(1 + 2\frac{s}{m_\tau^2}\right) P \left[P^2 |F_V|^2 + \frac{3(m_K^2 - m_\pi^2)^2}{4s(1 + 2\frac{s}{m_\tau^2})} |F_S|^2\right]$$

  ▶ Form factors of Belle based on addition of Breit-Wigners, thus violating unitarity

  $$F_S^{\text{Belle}} = \kappa \frac{s}{M_{K_0^*(700)}^2} \text{BW}_{K_0^*(700)} + \gamma \frac{s}{M_{K_0^*(1430)}^2} \text{BW}_{K_0^*(1430)}$$

  $$F_V^{\text{Belle}} = \frac{1}{1 + \beta + \chi} \left[\text{BW}_{K^*(892)} + \beta \text{BW}_{K^*(1410)} + \chi \text{BW}_{K^*(1680)}\right]$$

  ▶ $\kappa \in \mathbb{R}$ $\quad \gamma, \beta, \chi \in \mathbb{C}$ free parameters

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