NON-GLOBAL LOGS IN SCET

work in progress w/ S. Zuberi, J. Walsh, I. Stewart, & A. Hornig

I. What are NGLs?
II. NGLs in hemisphere $S(k_L, k_R)$
III. Phase Space Factorization and NGLS

I. What are NGLs?

No one knows!
(or, has not given precise definition)

Generically, logs that are not resummed by "naive" methods
due to sharp divisions in phase space causing
sensitivity to disparate (soft) scales.
(or soft becoming hard)

Facetiously, people seem to call any log they don't know how to
resum an NGL! $\Rightarrow$ "Non-Local Globs" — Zoltan Ligeti

1^{st} known example in e^+e^-, $\sigma(m_{hemi}^2)$:

(Dasgupta, Salam 2001)

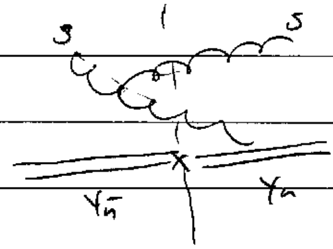
"Naive": $\sigma(m_{hemi}^2) = H_2(Q)\, J_n(m_{hemi}^2) \otimes S_{n\bar{n}}\left(\frac{m_{hemi}^2}{Q}\right) - \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{\pi^2}{3} \ln^2 \frac{m_{hemi}^2}{Q^2}$

using usual, known $\Gamma_{cusp}, \gamma_{J,S}$

"missing" NGL

2 possible sources :

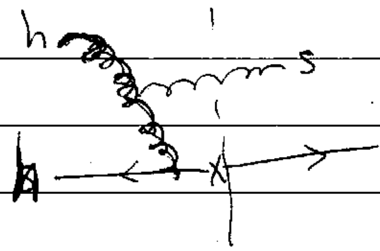
"soft NGLs"



→ considered by Dasgupta-Salam

applicable to $m_1^2 \ll m_2^2 \ll Q^2$

"hard NGls"

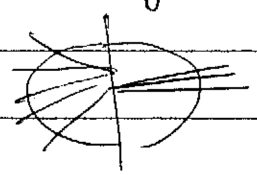


→ never considered before

applicable to $m_1^2 \ll m_2^2 \sim Q^2$

→ requires hard OPE

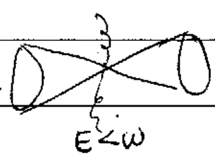
I will discuss only <u>soft NGLs</u> today.

So consider $\sigma(m_1^2, m_2^2)$ m_1^2  m_2^2

$m_2^2 \ll m_1^2 \ll Q^2$

Naively $\sigma(m_1^2, m_2^2) = H_2(Q)\, J_1(m_1)\, J_2(m_2) \otimes S_{n\bar{n}}\left(\frac{m_1^2}{Q_1}\, \frac{m_2^2}{Q}\right)$

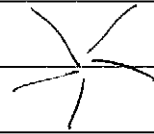
$+$ NGLs of $\frac{m_1}{m_2}$

Another soft example : k_L  k_R $S(k_L, k_R, \omega)$ → Kelley Schwartz Zhu (2011)

$E < \omega$

↓

<u>has NGLs</u> of $\ln^2 \frac{\omega}{k_{L,R}}$. cf. Banfi et al (2010)

or "N-multijettiness"  $\frac{d\sigma}{d\tau_1 \cdots d\tau_N}$ with $\tau_i \ll \tau_j$

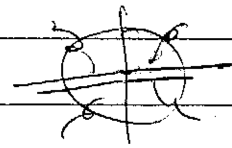
Jouttenus, Stewart, Tackmann, Waalewijn

One strategy is to construct only "global" observables with no NGLs.
But experiments / theorists at LHC are using NGL observables,
so cannot avoid dealing with them.

We are Effective Field Theorists. We sum logs.

So let's sum NGLs.

II. NGLs in $S(k_L, k_R)$



$$S(k_L,k_R) = \frac{1}{N_c} \mathrm{Tr} \sum_{X_S} |\langle X_S | Y_n \bar{Y}_{\bar{n}} | 0 \rangle|^2 \, \delta(k_L - \sum_{i\in L} p_i^-) \, \delta(k_R - \sum_{i \in R} p_i^+)$$

$\big\downarrow$ go to position space

$$S(x_1, x_2) = \int dk_L \, dk_R \, e^{-ix_1 k_L} e^{-ix_2 k_R} S(k_L, k_R)$$

Known properties:

RG inv $\Rightarrow$ $S(x_1, x_2) = U_S(x_1, \mu, \mu_0) \, U_S(x_2, \mu, \mu_0) \, S(x_1, x_2, \mu_0)$ Hoang Stewart (07)

$$= U_S(x_1, \mu, (ix_1)^{-1}) \, U_S(x_2, \mu, (ix_2)^{-1}) \, \tilde{S}(x_1, x_2)$$

$\downarrow$ $e^{\tilde{K} + i\tilde{\omega}}$ $\swarrow$ with usual anom. dims. Γ_S, γ_S $\qquad$ $\downarrow$ indep. of μ

Non-Abelian exp $\Rightarrow$ $\tilde{S}(x_1, x_2) = e^{T(x_1, x_2)}$

Symmetry $\Rightarrow$ $T(x_1, x_2) = T(x_2, x_1)$

known ↓ ↓ ansatz

Ansatz: (Hoang-Kluth 08)

$$T(x_1,x_2) = \frac{\alpha_s((x_1)^{-1})}{4\pi} + \frac{\alpha_s((x_2)^{-1})}{4\pi} + \underbrace{2\left(\frac{\alpha_s}{4\pi}\right)^2\left[s_1 + s_2 \ln^2\frac{x_1}{x_2}\right]}_{\equiv\, t_2(x_1,x_2)}$$

?'s:
- any other funcs of $\frac{x_1}{x_2}$ appear?
- calculate $s_{1,2}$?

Assuming H-K ansatz, extract s_2 from heavy jet mass

$$\Rightarrow \quad s_2^{C_F C_A} = -14 \qquad \text{(cf. V. Mateu)}$$

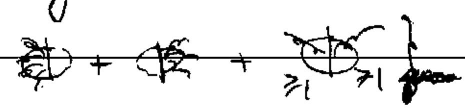
$$s_2^{C_F N_F} = -3 \rightarrow \text{note D-S imply } 0$$

Caveat: ρ_{heavy} only extracts an integral over t_2 — cf. Chien, Schwartz

so other non-log fns of $x_{1,2}$ can contaminate this extraction.

III. Phase Space Factorization — HLSWZ

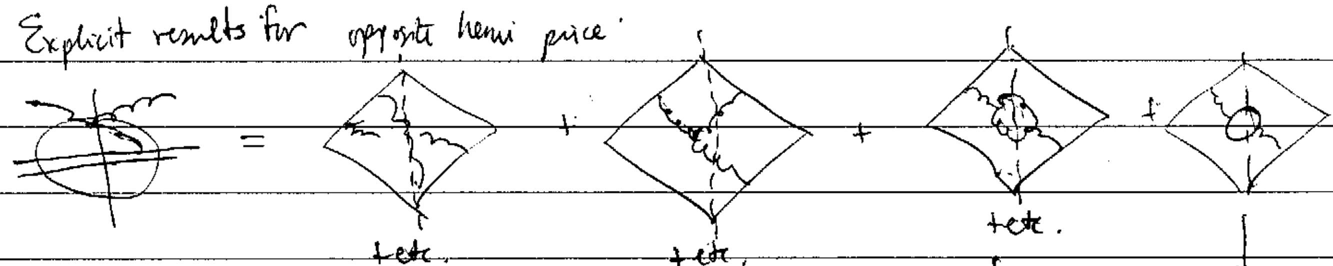
$S(x_1,x_2) = e^{\{\ \ldots\ \}}$



For now, view as split up of fixed-order calcs,

but we have operator defs. of each piece in an EFT.

Explicit results for opposite hemi piece:



Showed power-suppressed for $k_L \ll k_R$, $k_R \gg k_L$

suspect also power suppressed

$\Rightarrow$ no $C_F N_F$ NGL

remaining diags give

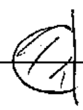
$$S_{\circ}(x_1,x_2) = \frac{2\alpha_s^2 C_F C_A}{\pi^2}\left(\mu^2 i x_1 i x_2\right)^{2\epsilon}\Gamma(-2\epsilon)^2$$

$$\times \underbrace{\int_{-1}^{0} d\cos\theta_1 \int_0^1 d\cos\theta_2 \frac{1}{1-\cos\theta_1}\frac{1}{1+\cos\theta_2}\frac{1}{\cos\theta_2-\cos\theta_1}}_{=\frac{\pi^2}{12}} \qquad (*)$$

$$\times\left\{1 + f\left(\theta_{1,2}, \frac{x_1}{x_2}\right)\right\}$$

we know this func. analytically but it is power suppressed for $\frac{x_1}{x_2}\to 0$ or $\frac{x_2}{x_1}\to 0$

$\Rightarrow$ contains double logs $\frac{\alpha_s^2 C_F C_A}{4\pi^2}\frac{\pi^2}{3}\ln^2(\mu^2 i x_1 i x_2)$

Since anom. dim. of full $S(k_+,k_-,\mu)$ already known we cannot have any new contribution like this. So μ dependence must be cancelled by a new term in  +  pieces

$$\Rightarrow \quad t_2(x_1,x_2) = t_2^{gl}(x_1,\mu) + t_2^{gl}(x_2,\mu) + t_2^{ng}(x_1,x_2,\mu)$$

$t_2^{gl}(x_1,\mu)$: $=$ usual logs $-\frac{\alpha_s^2 C_F C_A}{4\pi^2}\frac{2\pi^2}{3}\ln^2(\mu i x_1)$

$t_2^{gl}(x_2,\mu)$: usual logs $-\frac{\alpha_s^2 C_F C_A}{4\pi^2}\frac{2\pi^2}{3}\ln^2(\mu i x_2)$

$t_2^{ng}(x_1,x_2,\mu)$: $\frac{\alpha_s^2 C_F C_A}{\pi^2}\frac{\pi^2}{3}\ln^2(\mu^2 i x_1 i x_2)$

$$-2\ln^2(\mu i x_1) - 2\ln^2(\mu i x_2) + \ln^2(\mu^2 i x_1 i x_2)$$

$$= -\ln^2\frac{x_1}{x_2} \quad \checkmark$$

Two conclusions:

1) $t_2(x_1,x_2) = S_1 - \frac{\alpha_s^2}{8\pi^2} \underbrace{C_F C_A \frac{2\pi^2}{3}}_{=-S_2} \ln^2 \frac{x_1}{x_2}$

$$\boxed{S_2 = -C_F C_A \frac{2\pi^2}{3} = -26.3}$$

S_2 now known analytically!
(HJM extractions must be picking up non-log terms too)

2) Evolution of $S(\cancel{x_1,x_2})$ is

$$S(x_1,x_2,\mu) = U_S(x_1,\mu,\mu_0)\, S_{ge}(x_1,\mu_0)\, U_S(x_2,\mu,\mu_0)\, S_{ge}(x_2,\mu_0)\, U_{ng}(x_1x_2,\mu,\mu_0) \times S_{ng}(x_1x_2,\mu_0)$$

gain new contrib to 2-loop cusp anom dim

$$\Gamma_1^{\text{cusp}} \to \Gamma_1^{\text{cusp}} - \frac{\alpha_s^2 C_F C_A}{\pi^2} \frac{\pi^2}{3} \ln(\mu x_{1,2})$$

one conjecture:

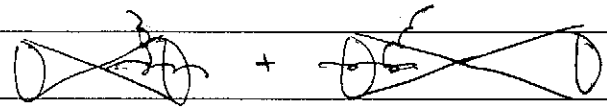
3) ~~powers~~ terms in $f(\theta_{12}, x_1/x_2)$ in integral $(*)$ on p. 5 generate non-log fns. e.g. $\mathrm{Li}_2\left(1-\frac{x_1}{x_2}\right)$, $\frac{x_1}{x_2} \ln \frac{x_1}{x_1+x_2}$, ... that would violate H-K ansatz. (away from $x_1 \gg x_2$, $x_2 \gg x_1$ limits)

Comment on $S(k_L, k_R, \omega)$ of Kelley, Schwartz, Zhu:

KSZ conjecture $S(k_L, k_R, \omega) \overset{\text{factors}}{=} S_{in}(k_L, k_R)\, S_{out}(\omega)$

~~and claim no logs of $\frac{\omega}{k_L}, \frac{\omega}{k_R}$,~~

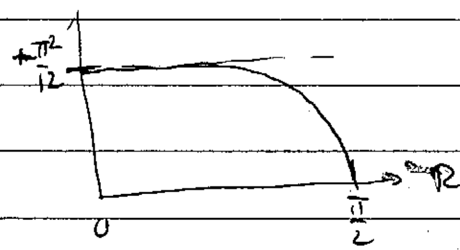
We have calculated



$C_F C_A$ pieces

$\Rightarrow$ give NGL $\quad -\frac{\alpha_s^2 C_F C_A}{\pi^2}\left(\ln^2\frac{\omega}{k_L} + \ln^2\frac{\omega}{k_R}\right) F(R)$

where $f(R) =$



(know analytically)

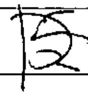
$$f(R) = \int_{\theta_1 = R}^{\pi - R} d\cos\theta_1 \int_{\theta_2 = 0}^{R} d\cos\theta_2 \, \frac{1}{1-\cos\theta_1}\,\frac{1}{1+\cos\theta_2}\,\frac{1}{\cos\theta_2 - \cos\theta_1}$$

There are also terms dependent on $\frac{\omega}{k_{L,R}}$ not logs and are power-suppressed for $\frac{\omega}{k_{L,R}} \ll 1$ or $\gg 1$, but if they survive in complete power-suppressed result, in general violate KSZ conjecture.

In $\frac{\omega}{k_{L,R}} \ll 1$ or $\gg 1$ limits, KSZ formula is consistent w/ presence of NGLs, but anom dim of each factor must be modified

$S_{in}(k_L)\, S_{in}(k_R)\, S_{out}(\omega)$

have to add $\sim \frac{\alpha_s^2 C_F C_A}{\pi^2} F(R) \ln(\mu/\chi_{L,R})$ $\quad \frac{\alpha_s^2 C_F C_A}{\pi^2} f(R) \ln(\mu/\omega)$ to anom dims.

Checks remaining: 1) finish explicit calc. of  +  pieces to find new contribs to Γ_1^{amp} explicitly.

2) interpret relative factor of ~2 in EVENT2 extractions of s_2.

→ Next steps (in progress): Renormalize and run EFT operators giving factorized contribs. above.

⇒ EFT resummation of NGLs!

Conclusion: There are Globs of NGLs at LHC.

Go forth and sum them!

SCET12: many NGL-resummed predictions for LHC!