New definition of TMD parton densities

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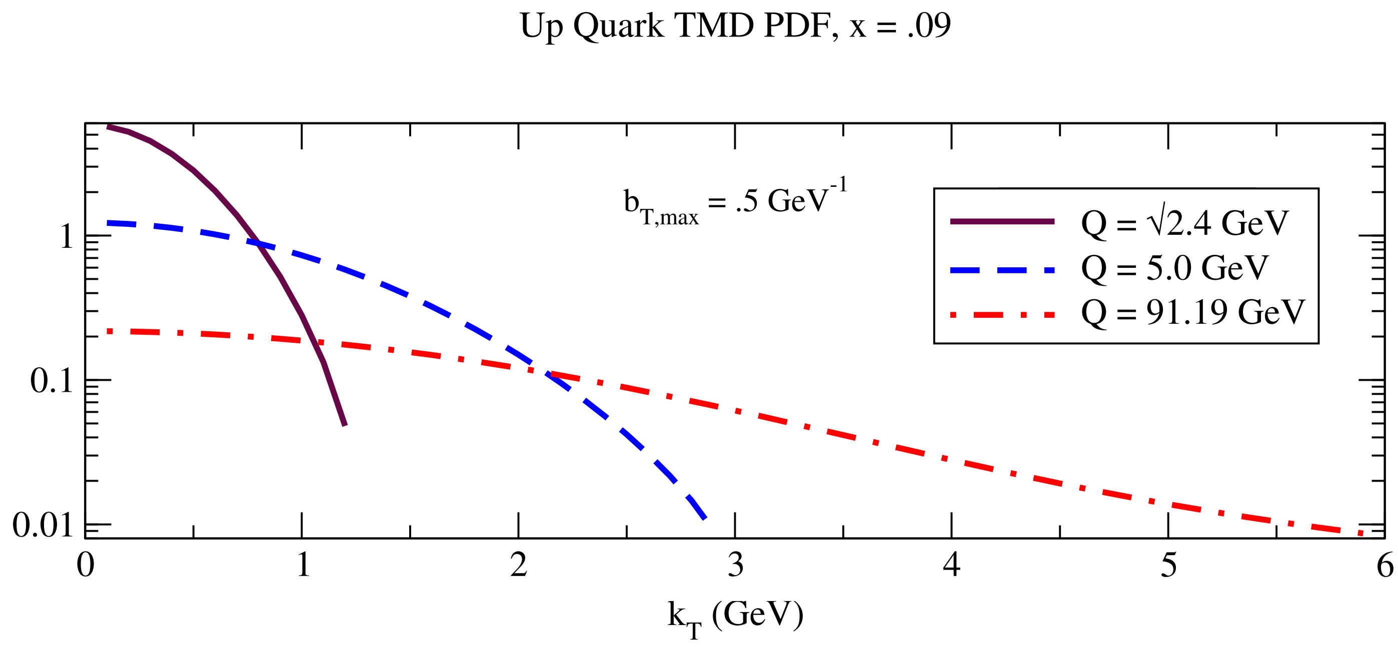


$f_{u/p}(x = 0.09, \boldsymbol{k}_{\rm T})$ (GeV^{-2}), from Aybat & Rogers arXiv:1101.5057

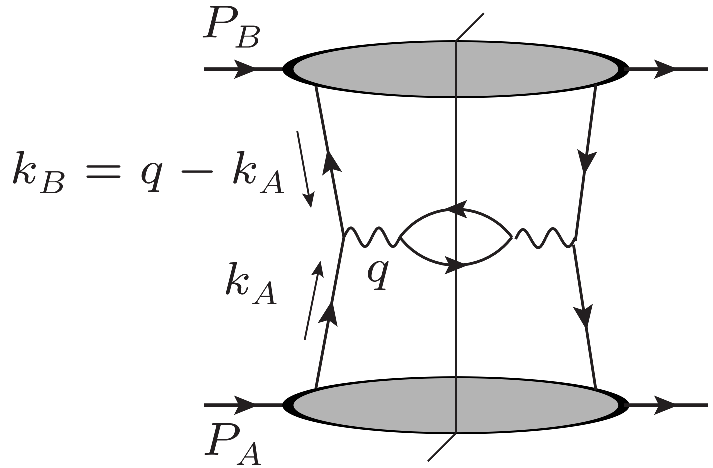
Overview

- Genesis of TMD pdfs à la parton model
- Difficulties
- New definition
- Implications

References:

JCC, "Foundations of Perturbative QCD" (Cambridge University Press, April 2011)
M. Aybat & T.C. Rogers arXiv:1101.5057

Whence TMD pdfs? Parton model for Drell-Yan



- Use cancellation of spectator-spectator interactions
- Assume other topologies unimportant
- Assume limited k_T and virtuality

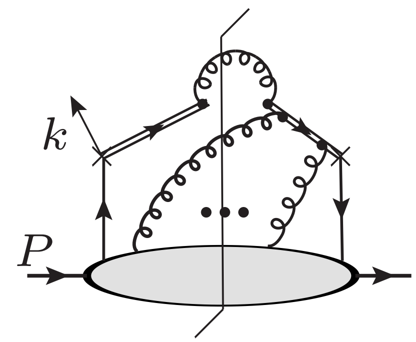
Get TMD factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} \simeq \sum_j \int \mathrm{d}^2\boldsymbol{k}_{A\,\mathsf{T}}\; f_{j/h_A}(x_A, \boldsymbol{k}_{A\,\mathsf{T}})\; f_{\bar{j}/h_B}(x_B, \boldsymbol{q}_{\mathsf{T}} - \boldsymbol{k}_{A\,\mathsf{T}})\; \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}}{\mathrm{d}\Omega}$$

Explicit definition of TMD pdf: complications in QCD

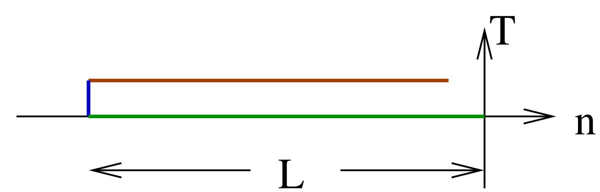
First attempt: $A^+ = 0$ gauge or use Wilson line in direction $n = (0, 1, \mathbf{0}_\mathsf{T})$:

$$f_{j/h}(\xi, \boldsymbol{k}_\mathsf{T}) \stackrel{?}{=} \int \frac{\mathrm{d}k^-}{(2\pi)^4} \, \mathrm{Tr} \, \frac{\gamma^+}{2}$$



$$= \mathsf{F.T.} \, \langle P | \, \overline{\psi}_j(0, w^-, \boldsymbol{w}_\mathsf{T}) \, W[w, n]^\dagger \, \frac{\gamma^+}{2} \, [\text{tr. link}] \, W[0, n] \psi_j(0) \, | P \rangle_{\mathsf{c}}$$

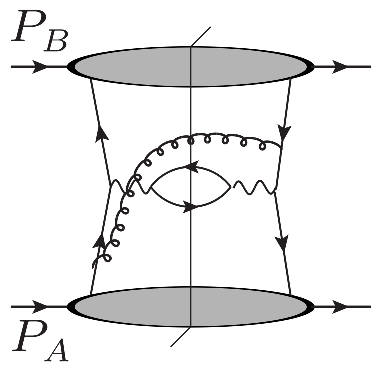
Wilson line from side: $\lim\limits_{L \to \infty}$

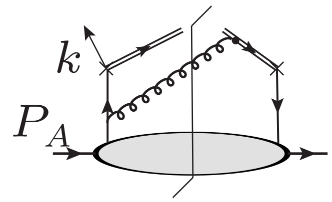


Complications:

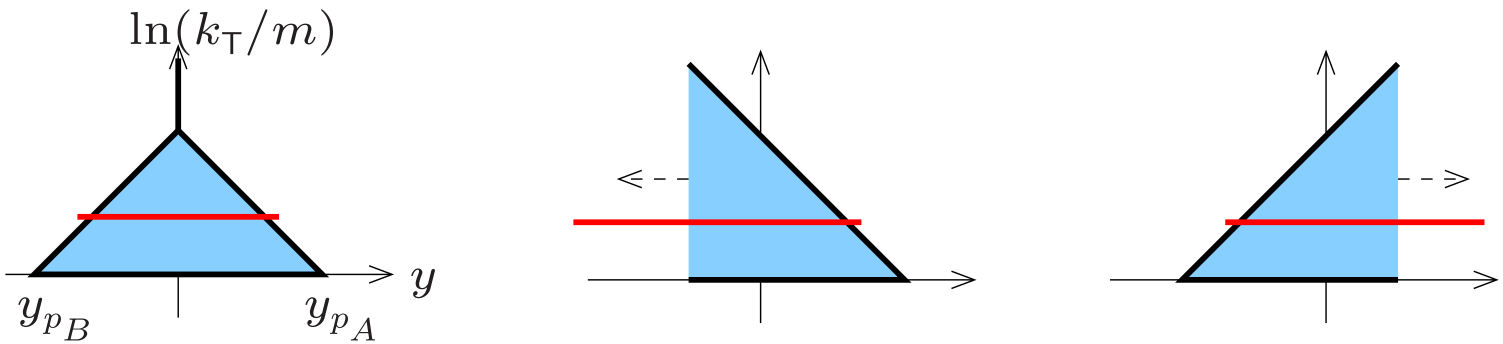
- (UV divergences)
- Rapidity divergences
- Wilson line self energies, . . .

Example: Regions for one gluon



A-pdf: 

B-pdf



Results:

- TMD pdf correctly gives its own collinear region (by Ward identity)
- Fails badly in opposite region
- Double counting of gluon contributions

TMD factorization in QCD

- Basic form:

  $\mathrm{d}\sigma = H \times$ convolution of ABS + high-k_T correction (Y) + power-suppressed

  - Double-counting subtractions in definitions of factors;
  - Suitable cut-offs in definitions of TMD pdfs;
  - (CSS) evolution equations for Q-dependence of A, B and S.

  But soft factor S has no independent experimental access.

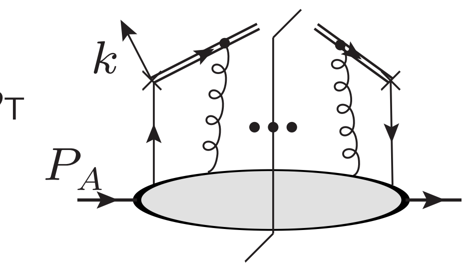
  N.B.

$$\text{Errors in} \left\{ \begin{array}{c} \text{collinear factorization} \\ \text{TMD factorization} \\ \text{TMD factorization} + Y \end{array} \right\} \text{are a power of} \left\{ \begin{array}{c} \dfrac{\Lambda}{q_\mathsf{T}} \\ \dfrac{\Lambda}{Q}, \dfrac{q_\mathsf{T}}{Q} \\ \dfrac{\Lambda}{Q} \end{array} \right.$$

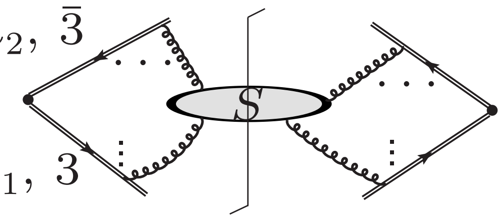
New definition of TMD pdf

Use non-light-like Wilson lines in basic unsubtracted definitions

$$\tilde{f}^{\text{unsub}}_{f/H_A}(x,\boldsymbol{b}_\mathsf{T};y_{P_A}-y_2) \overset{\text{def}}{=} \operatorname*{Tr}_{\text{color}} \operatorname*{Tr}_{\text{Dirac}} \frac{\gamma^+}{2} \int \frac{dk^-\, d^{2-2\epsilon}\boldsymbol{k}_\mathsf{T}}{(2\pi)^n}\, e^{-i\boldsymbol{k}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}}$$



$$\tilde{S}(\boldsymbol{b}_\mathsf{T}) \overset{\text{def}}{=} \frac{1}{N_c}\int \frac{d^{4-2\epsilon}k_S}{(2\pi)^{4-2\epsilon}} e^{-i\boldsymbol{k}_{S\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}}$$

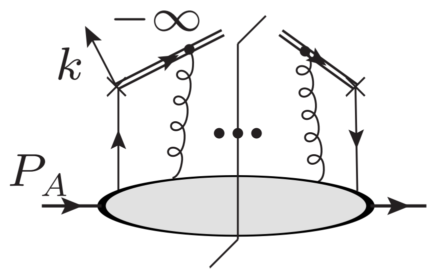


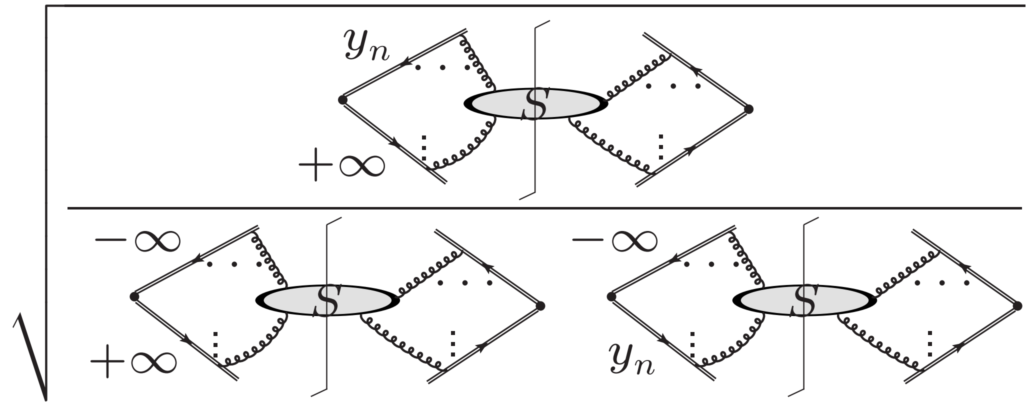
Define TMD pdf (UV renormalization implicit)

$$\tilde{f}_{f/H_A}(x,\boldsymbol{b}_\mathsf{T};\zeta_A;\mu) \overset{\text{def}}{=} \lim_{\substack{y_1\to+\infty\\ y_2\to-\infty}} \tilde{f}^{\text{unsub}}_{f/H_A}(x,\boldsymbol{b}_\mathsf{T};y_{P_A}-y_2)\sqrt{\frac{\tilde{S}(b_\mathsf{T},y_1,y_n)}{\tilde{S}(b_\mathsf{T},y_1,y_2)\ \tilde{S}_{(0)}(b_\mathsf{T},y_n,y_2)}}$$

$$= \tilde{f}^{\text{unsub}}_{f/H_A}\big(x,\boldsymbol{b}_\mathsf{T};y_{p_A}-(-\infty)\big)\sqrt{\frac{\tilde{S}(b_\mathsf{T};+\infty,y_n)}{\tilde{S}(b_\mathsf{T};+\infty,-\infty)\ \tilde{S}_{(0)}(b_\mathsf{T};y_n,-\infty)}}$$

Why this strange definition?



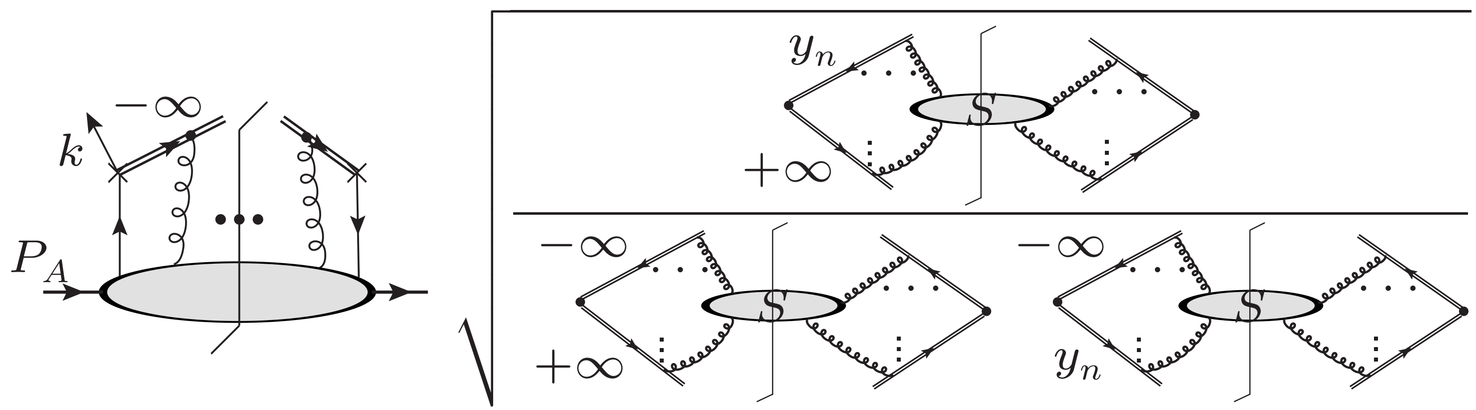


Unique (up to UV prescription) given:

- Product of basic pdf and powers of soft factor (in $\boldsymbol{b}_\mathrm{T}$ space).
- Light-like Wilson lines if possible:
  - Basic pdf has only light-like Wilson line
  - Soft factors have at most one non-light-like line
- No soft factor in factorization formula ($\Longrightarrow$ square roots natural).
- Rapidity and $L \to \infty$ divergences cancel

N.B. Only factorization-compatible definitions should be considered.

Consequences of the strange definition



- Factorization.
- Rapidity divergences canceled
- Link-at-infinity contributions cancel in Feynman gauge.
- Wilson-line self-energy divergences cancel.
- Energy-dependence $\Longrightarrow$ y_n-dependence $\Longrightarrow$ CSS evolution, etc
- CSS evolution equations are simpler.
- They are without power-law corrections.

Comparisons

- CSS-style:
  - CSS originally used non-light-like $n \cdot A = 0$ gauge
    Ignored error terms in evolution, etc
    Implicit Wilson-line self-energy problems
    Lack of actual proof for TMD factorization for Drell-Yan
  - Ji et al.: Multiple non-light-like Wilson lines

- Becher & Neubert
  - Use Smirnov subtraction method in SCET style
  - Define product AB but not individual TMD pdfs
  - Rest of structure is same/close

- Mantry & Petriello
  - More complicated formula with more variables:
  - "Fully unintegrated pdf" = "impact parameter beam function" (iBF) instead of "TMD pdf"
  - iBFs have zero bin subtractions: something like my soft factors
  - What is relation to operator definitions in QCD?

Implications of new definitions and associated derivations

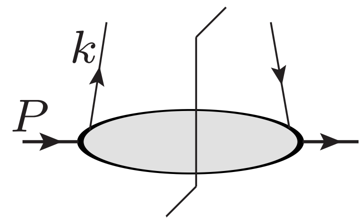
- Precise definition that can be taken literally, in Feynman gauge
- Much better subtraction methods
- Full proof of TMD factorization for Drell-Yan

  (CSS gave up! . . .)
- Clean formalism
- Fully specified relation between exact and approximated parton kinematics.
- Need to relate to other definitions, including those in SCET
- . . .

APPENDIX

Parton model definition of TMD pdf

$$f_{j/h}(\xi, \boldsymbol{k}_{\mathsf{T}}) \overset{\text{p.m.}}{=} \int \frac{\mathrm{d}k^-}{(2\pi)^4} \, \mathrm{Tr} \, \frac{\gamma^+}{2}$$



$$= \int \frac{\mathrm{d}w^- \, \mathrm{d}^2\boldsymbol{w}_{\mathsf{T}}}{(2\pi)^3} \, e^{-i\xi P^+ w^- + i\boldsymbol{k}_{\mathsf{T}}\cdot\boldsymbol{w}_{\mathsf{T}}} \, \langle P | \, \overline{\psi}_j(0, w^-, \boldsymbol{w}_{\mathsf{T}}) \, \frac{\gamma^+}{2} \, \psi_j(0) \, | P \rangle_{\mathsf{c}}$$

- Need for TMD factorization:

$$\text{Errors in} \left\{ \begin{array}{c} \text{collinear} \\ \text{TMD} \end{array} \right\} \text{factorization are a power of} \left\{ \begin{array}{c} \Lambda/q_{\mathsf{T}} \\ \Lambda/Q \end{array} \right.$$

- Importance of a definition as an operator matrix element:
  - Knowing what we're talking about
  - Unambiguous prescription for use in perturbative calculations
  - Unambiguous specification for non-perturbative calculations/analysis

Three views of factorization

1. Main TMD factorization formula
2. After evolution of TMD functions to fixed scales
3. Maximum perturbative content

Main factorization formula

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f H_f^{\mu\nu}(\hat{k}_A, \hat{k}_B) \int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} \tilde{f}_{f/H_A}(x_A, \boldsymbol{b}_\mathsf{T}; \zeta_A; \mu)\, \tilde{f}_{\bar{f}/H_B}(x_B, \boldsymbol{b}_\mathsf{T}; \zeta_B; \mu)$$

$+$ polarized terms $+$ large $q_{h\,\mathsf{T}}$ correction, Y.

$+$ power suppressed

where

$$x_A = \frac{Qe^y}{\sqrt{s}}, \quad x_B = \frac{Qe^{-y}}{\sqrt{s}},$$

$$\zeta_A = M_A^2 x_A^2 e^{2(y_{P_A} - y_n)}, \quad \zeta_B = M_B^2 x_B^2 e^{2(y_n - y_{P_B})},$$

Implement Y by

$W = F(q_\mathsf{T})$low-q_T term $+$ Collinear factorization on $(W - F(q_\mathsf{T})$low-q_T term$)$

$+$ power suppressed

Evolution, etc for TMD pdfs

CSS:

$$\frac{\partial \ln \tilde{f}_{f/H_A}(x, b_\mathsf{T}; \zeta; \mu)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\mathsf{T}; \mu).$$

RG:

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\ln \mu} = -\gamma_K(g(\mu))\,.$$

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu)}{\mathrm{d}\ln \mu} = \gamma_f\big(g(\mu); \zeta/\mu^2\big).$$

$$\gamma\big(g(\mu); \zeta/\mu^2\big) = \gamma(g(\mu); 1) - \frac{1}{2}\gamma_K(g(\mu)) \ln \frac{\zeta}{\mu^2}.$$

Small-b_T:

$$\tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu) = \sum_j \int_{x-}^{1+} \frac{\mathrm{d}\hat{x}}{\hat{x}}\, \tilde{C}_{f/j}(x/\hat{x}, b_\mathsf{T}; \zeta, \mu, g(\mu))\ f_{j/H}(\hat{x}; \mu) + O[(m b_\mathsf{T})^p]$$

Factorization with fixed TMD pdfs

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f H_f^{\mu\nu}(\hat{k}_A, \hat{k}_B) \int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} e^{-S(b_\mathsf{T};Q;\mu_Q,\mu_0)} \times$$

$$\times\, \tilde{f}_{f/H_A}(x_A, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0)\, \tilde{f}_{\bar{f}/H_B}(x_B, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0)$$

$$+ \text{ polarized terms} + \text{large } q_{h\,\mathsf{T}} \text{ correction, } Y + \text{p.s.c.}$$

$$e^{-S(b_\mathsf{T};Q;\mu_Q,\mu_0)} = \exp\left\{ \ln\frac{Q^2}{m^2} \tilde{K}(b_\mathsf{T};\mu_0) \right\} \times$$

$$\times \exp\left\{ \int_{\mu_0}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma(g(\mu');1) - \ln\frac{Q^2}{(\mu')^2} \gamma_K(g(\mu')) \right] \right\}$$

Factorization with maximum perturbative content

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_{f,j_A,j_B} H_f^{\mu\nu}(Q; g(\mu_Q), \mu_Q) \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathsf{T}}{(2\pi)^2} e^{-i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}}$$
$$\times \int_{x_A}^1 \frac{\mathrm{d}\hat{x}_A}{\hat{x}_A} f_{j_A/H_A}(\hat{x}_A; \mu_b)\ \tilde{C}_{f/j_A}\left(\frac{x_A}{\hat{x}_A}, b_*; \mu_b^2, \mu_b, g(\mu_b)\right)$$
$$\times \int_{x_B}^1 \frac{\mathrm{d}\hat{x}_B}{\hat{x}_B} f_{j_B/H_B}(\hat{x}_B; \mu_b)\ \tilde{C}_{\bar{f}/j_B}\left(\frac{x_B}{\hat{x}_B}, b_*; \mu_b^2, \mu_b, g(\mu_b)\right)$$
$$\times \exp\left[-g_{f/H_A}(x_A, b_\mathsf{T}) - g_{\bar{f}/H_B}(x_B, b_\mathsf{T})\right]$$
$$\times \exp\left[-\ln\frac{Q^2}{m^2} g_K(b_\mathsf{T}) + \ln\frac{Q^2}{\mu_b^2}\tilde{K}(b_*; \mu_b)\right]$$
$$\times \exp\left\{\int_{\mu_b}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(g(\mu'); 1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(g(\mu'))\right]\right\}$$

+ polarized terms + large-$q_{h\,\mathsf{T}}$ correction, Y + p.s.c.

where: $\mathbf{b}_* = \boldsymbol{b}_\mathsf{T}/\sqrt{1 + b_\mathsf{T}^2/b_{\max}^2}$, $\mu_b = C_1/b_*(b_\mathsf{T})$, $\mu_Q = C_2 Q$