Factorization in SCET II
Or
Finding Sudakov Logs in Real Calculations

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Logs and Scales: SCET I

SCET I simply matches scale dependence in all sectors:

$$-\text{Log}\left(\frac{\lambda^2 Q}{\mu}\right) + 2\text{Log}\left(\frac{\lambda Q}{\mu}\right) = \text{Log}\left(\frac{Q}{\mu}\right)$$

$$2\gamma_J + \gamma_{us} = -\gamma_H$$

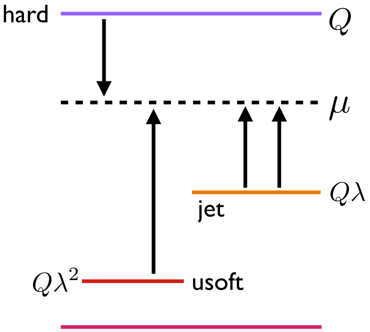


Figure: Scales for SCET I process

Logs and Scales: SCET II

SCET II must have different story:

$$C \text{Log}\left(\frac{\lambda Q}{\mu}\right) \neq \text{Log}\left(\frac{Q}{\mu}\right) \quad \forall C$$

$$2\gamma_J + \gamma_s =_? -\gamma_H$$

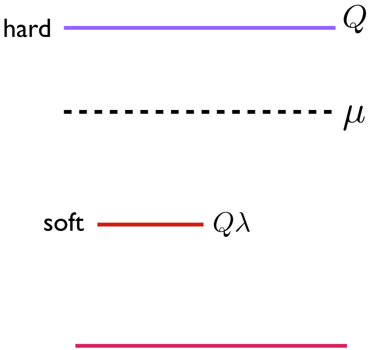


Figure: Scales for SCET II process

(Not Quite) Back to Back jets In SCET

$e^+e^- \to 2j$ with the event shape Jet Broadening.

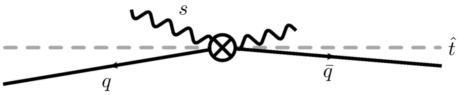


Figure: Jet Broadening Displacement From Thrust Axis

(Rakow,Webber, 1981; Catani, Turnock, Webber 1992; Dokshitzer, Lucenti, et al, 1998)

(Not Quite) Back to Back jets In SCET

- $e = \sum_i \frac{|\vec{k}_{ti}|}{Q}$ jet broadening event shape.
- Thrust axis $\hat{\mathbf{t}}$ of the event defines directions:

$$n = (1, \hat{\mathbf{t}}) \qquad \bar{n} = (1, -\hat{\mathbf{t}}) \qquad \vec{p}_t.\hat{\mathbf{t}} = 0$$

$$p = (\bar{n}.p, n.p, \vec{p}_t)$$

- Demand $e \ll 1$ for dijets.
- Relevant on-shell modes must have $\vec{p}_t \sim Q\lambda$.
- Softs $Q(\lambda, \lambda, \lambda)$ and collinears $Q(1, \lambda^2, \lambda)$ or $Q(\lambda^2, 1, \lambda)$

Factorization Theorem

$$\frac{d\sigma}{de} = N \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s)$$
$$\int \frac{d^2\vec{p}_{t1}}{(2\pi)^2} \frac{d^2\vec{p}_{t2}}{(2\pi)^2} J_n(e_n, \vec{p}_{t1}) J_{\bar{n}}(e_{\bar{n}}, \vec{p}_{t2}) S(e_s, \vec{p}_{1t}, \vec{p}_{2t})$$

Where the jet and soft functions are defined as:

$$J_{\bar{n}}(e_{\bar{n}}, \vec{p}_{t2}) = \frac{(2\pi)^3}{N_c} \mathrm{tr}\langle 0 | \bar{\chi}_{\bar{n}} \delta(n \cdot \hat{P} - Q) \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \delta(\hat{P}_{\perp} - p_{2\perp}) \frac{\not{n}}{2} \chi_{\bar{n}} | 0 \rangle$$
$$J_n(e_n, \vec{p}_{t1}) = \frac{(2\pi)^3}{N_c} \mathrm{tr}\langle 0 | \frac{\bar{\not{n}}}{2} \chi_n \delta(\bar{n} \cdot \hat{P} - Q) \delta(e_n - \hat{e}_{\hat{n}}) \delta(\hat{P}_{\perp} - p_{1\perp}) \bar{\chi}_n | 0 \rangle$$
$$S(e_s, \vec{p}_{1t}, \vec{p}_{2t}) = \frac{1}{N_c} \mathrm{tr}\langle 0 | S_{\bar{n}} S_n^{\dagger} \delta(\mathbb{P}_{n\perp} + p_{1\perp}) \delta(\bar{\mathbb{P}}_{n\perp} + p_{2\perp}) \delta(e_s - \hat{e}_s) S_n S_{\bar{n}}^{\dagger} | 0 \rangle$$

Where $\mathbb{P}_{n\perp}$ and $\bar{\mathbb{P}}_{n\perp}$ are operators picking out the transverse momenta being contributed to each hemisphere.

Blindly Calculate

- Bare jet function:

$$J_n(e_n,0) = \frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2}{Q^2 e_n^2}\right)^{\epsilon}(e_n)^{-1}\int_0^1 dz\,\frac{1+(1-z)^2}{z}$$

$$z = \frac{\bar{n}.l}{Q} \quad l \text{ momentum of gluon crossing cut}$$

- Integral ill-defined at $z = 0$, the soft region.
- Divergence multiplies non-zero e_n terms that virtuals cannot cancel.

The resolution of these divergences also solves problem of scales in SCETII

Lesson #1 of QFT Divergences: Proceed without fear!

We could...

- Choose favorite regulator for Wilson lines.
- Calculate all sectors.
- Sum and/or subtract.

...and (presummably) get a fine fixed order result.

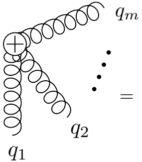
η Regulator

- Problem: All order structure?
- Solution:
  - Multiplicatively Renormalize
  - Regulator with dim reg like features.
- For non-singular gauges, we redefine wilson lines:

$$W_n(x) = \left[\sum_{\text{perm}} \exp\left(-g \frac{1}{\bar{n}\cdot\hat{P}} \left[\nu^{\eta} |\bar{n}\cdot\hat{P}|^{-\eta} \bar{n}\cdot A_{n,q}(x) \right] \right) \right]$$

$$S_n(x) = \left[\sum_{\text{perm}} \exp\left(-g \frac{1}{n\cdot\hat{P}} \left[\nu^{\frac{\eta}{2}} |\hat{P}^0|^{-\frac{\eta}{2}} n\cdot A_{s,q}(x) \right] \right) \right].$$

η Regulator



$$= \nu^{m\eta}\frac{g^m}{m!}\prod_{i=1}^{m}|\overline{n}.q_i|^{-\eta}\sum_{\text{perms}}\frac{(\overline{n}^{\mu_m}T^{A_m})...(\overline{n}^{\mu_1}T^{A_1})}{[\overline{n}.q_1][\overline{n}.(q_1+q_2)]...[\overline{n}.\sum_{j=1}^{m}q_j]}$$

Figure: Feynman Rule for Regulated Collinear Wilson Line

- Regulates the energy of each gluon coming off Wilson Line.
- Does not hide soft functions.
- Preserves Exponentiation Theorems.
- Sets zero-bins in collinear sectors to zero automatically.
- Preserves modes and their power counting.
- Dim-reg. style evolution equations.

η Regulator Comparison

- Analytic regulator:
  - Hides soft contributions
  - Does not exponentiation
- Δ regulator (Chui, Fuhrer, et al. 2009):
  - Introduces more scales into integrals
  - Exponentiates after zero bins
  - No known evolutions equation.
- Off the light cone (Collins, Soper 1981):
  - Introduces more scales into integrals
  - Proof of exponentiation straightforward
  - Introduces gauge modes that are not appropiate by strict power counting.
  - Understood evolution equation.

Jet Function Redux: ν Logs

Now with η regulator in place, we look at the laplace transformed jet function:

$$J_n(\tau,0) = \frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2\tau^2}{Q^2}\right)^{\epsilon}\Gamma(-2\epsilon)\int_0^1 dz\left(z+2\left(\frac{\nu}{Q}\right)^{\eta}\frac{1-z}{z^{1+\eta}}\right)$$

$$= -\frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2\tau^2}{Q^2}\right)^{\epsilon}\Gamma(-2\epsilon)\left(\frac{2}{\eta}\right) + \frac{\alpha_s C_f}{\pi}\frac{3}{4\epsilon} + \frac{\alpha_s C_f}{2\pi\epsilon}\mathrm{Log}\left(\frac{\nu^2}{Q^2}\right)$$

$$+ \frac{\alpha_s C_f}{2\pi}\mathrm{Log}\left(\frac{\nu^2}{Q^2}\right)\mathrm{Log}\left(\frac{\mu^2\tau^2}{Q^2}\right) + \frac{\alpha_s C_f}{\pi}\frac{3}{4}\mathrm{Log}\left(\frac{\mu^2\tau^2}{Q^2}\right)$$

$$\tau \sim \frac{Q}{\lambda}$$

Structure of η divergences

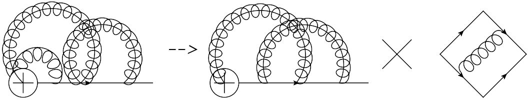


Figure: Factorization of η Divergences

- Combining jet and soft sectors, η divergences and ν dependence cancels.
- Within a sector η divergences exponentiate.
  - For jet functions, utilize factorization of soft limit.
  - Then invoke exponentiation theorems for eikonal processes.
  - Treat removal of η divergences with multiplicative renormalization.

ν Renormalization

Given exponentiation of η divergences, there are renormalization factors $Z_n, Z_{\bar{n}}, Z_s$ such

$$J_n^{\mathrm{B}}(\tau, b_1) J_{\bar{n}}^{\mathrm{B}}(\tau, b_2) S^{\mathrm{B}}(\tau, b_1, b_2) = \Big(Z_n(\nu,\mu) J_n^{\mathrm{R}}(\tau, b_1, \nu, \mu)\Big)\Big(Z_{\bar{n}}(\nu,\mu) J_{\bar{n}}^{\mathrm{R}}(\tau, b_2, \nu, \mu)\Big) \Big(Z_s(\nu,\mu) S^{\mathrm{R}}(\tau, b_1, b_2, \nu, \mu)\Big)$$

Where

$$Z_n(\nu,\mu) Z_{\bar{n}}(\nu,\mu) Z_s(\nu,\mu) = Z_H^{-1}(\mu)$$

ν RG

One can calculate the RG equations as:

$$\nu \frac{d}{d\nu} F^{\mathrm{R}}(\nu,\mu) = \gamma_F^{\nu} F^{\mathrm{R}}(\nu,\mu)$$
$$\mu \frac{d}{d\mu} F^{\mathrm{R}}(\nu,\mu) = \gamma_F^{\mu} F^{\mathrm{R}}(\nu,\mu)$$

For the case of the jet function at NLO:

$$\gamma_J^{\nu} = \frac{\alpha_s(\mu) C_f}{\pi} \mathrm{Log}\left(\frac{\mu^2 \tau^2}{Q^2}\right), \quad \gamma_J^{\mu} = \frac{\alpha_s(\mu) C_f}{\pi} \mathrm{Log}\left(\frac{\nu^2}{Q^2}\right) + \frac{3\alpha_s C_f}{2\pi}$$

NB: Running in ν and μ commute.

The Strategy of Running

- μ Run hard function down to scale eQ
- ν Run soft function up to scale Q

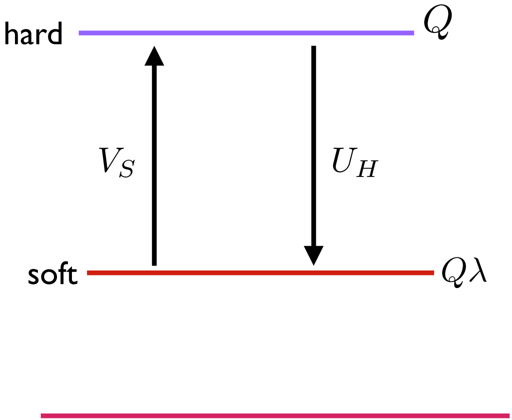


Figure: Running Strategy

Preliminary Results at NLL

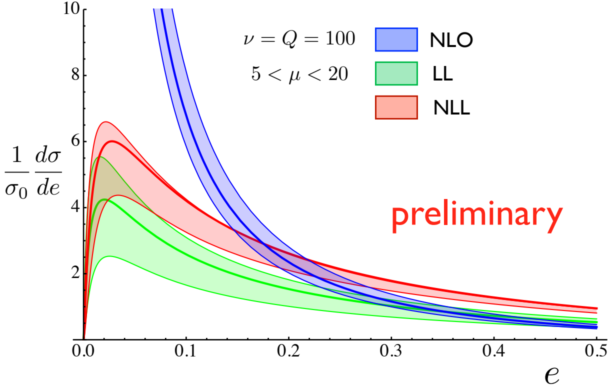


Figure: Jet Broadening Differential Cross-Section

Conclusion

As known since long ago (Collins, Soper 1981), observing transverse momentum dependent quantities gives...

- Uncanceled divergences in each sector.
- Important soft contributions.
- New logs to resum dependent on cusp angle (the high scale).

η regulator provides controllable form to divergences, and a way to resum.

η Regulator in Different Sectors

$$W_n(x) = \left[\sum_{\text{perm}} \exp\left(-g \frac{1}{\bar{n}\cdot\hat{P}} \left[\nu^{\eta} |\bar{n}\cdot\hat{P}|^{-\eta} \bar{n}\cdot A_{n,q}(x)\right]\right)\right]$$

$$S_n(x) = \left[\sum_{\text{perm}} \exp\left(-g \frac{1}{n\cdot\hat{P}} \left[\nu^{\frac{\eta}{2}} |\hat{P}^0|^{-\frac{\eta}{2}} n\cdot A_{s,q}(x)\right]\right)\right].$$

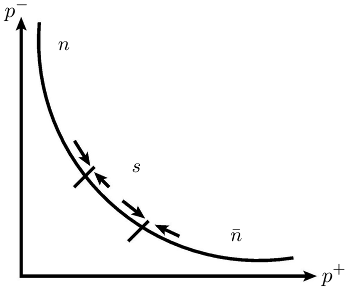


Figure: Mass-Shell Hyperbola in SCET II