THE GLUON JET FUNCTION AT TWO-LOOP ORDER

[GUIDO BELL]

based on: Th. Becher, GB, Phys. Lett. B 695 (2011) 252-258

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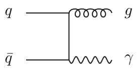
Introduction

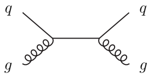
Inclusive gluon jet function:

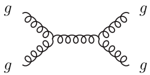
$$\int \mathrm{d}^4x\, e^{ipx}\, \langle 0|\, \mathcal{A}_\mu^a(x)\mathcal{A}_\nu^b(0)\,|0\rangle \;=\; \sum_X (2\pi)^4\, \delta^{(4)}(p-p_X)\, \langle 0|\, \mathcal{A}_\mu^a(0)\,|X\rangle\langle X|\, \mathcal{A}_\nu^b(0)|0\rangle$$

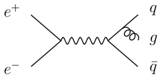
- gluon field in light-cone gauge $\mathcal{A}^\mu(x) = W^\dagger(x)\,[iD^\mu W(x)]$
- "probability" that gluon field produces a jet of particles with momentum p
- gauge invariant (under gauge transformations that vanish at $x \to \infty$)
- universal ingredient of many factorization theorems:

  threshold resummation, hadronic and e^+e^- event shapes, quarkonium decay, . . .









Gluon jet functions

Parametrization:

$$\int \mathrm{d}^4x\, e^{ipx} \langle 0|\, \mathcal{A}_\mu^a(x)\mathcal{A}_\nu^b(0)\,|0\rangle \;\sim\; \left[\left(-g_{\mu\nu} + \frac{n_\mu p_\nu + p_\mu n_\nu}{n\cdot p}\right) J_g(p^2) + \frac{n_\mu n_\nu}{(n\cdot p)^2} K_g(p^2)\right]$$

- respects $n \cdot \mathcal{A}(x) = 0$ and rescaling of light-cone vector n^μ
- in SCET we further decompose $\mathcal{A}^\mu(x) = \mathcal{A}_\perp^\mu(x) + \bar{n}\cdot\mathcal{A}(x)\,\frac{n^\mu}{2}$

$\langle 0|\, \mathcal{A}_\perp^\mu(x)\mathcal{A}_\perp^\nu(0)\,|0\rangle \quad \Rightarrow \quad$ leading jet function $J_g(p^2)$

$\langle 0|\, \bar{n}\cdot\mathcal{A}(x)\bar{n}\cdot\mathcal{A}(0)\,|0\rangle \quad \Rightarrow \quad$ subleading jet function $K_g(p^2)$

We computed both jet functions to two-loop accuracy

- $J_g(p^2) \quad \Rightarrow \quad$ will allow to perform NNNLL resummations in SCET
- $K_g(p^2) \quad \Rightarrow \quad$ renormalization of subleading operators

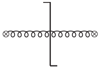
Leading jet function $J_g(p^2)$

Start from time-ordered product

$$\int \mathrm{d}^4x\, e^{ipx} \langle 0|\, T\{\mathcal{A}^a_\mu(x)\mathcal{A}^b_\nu(0)\}\, |0\rangle \;\sim\; \left[\left(-g_{\mu\nu} + \frac{n_\mu p_\nu + p_\mu n_\nu}{n\cdot p}\right) \mathcal{J}_g(p^2) + \ldots\right]$$

- tree level

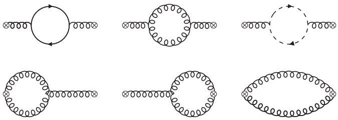
$$J_g(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\mathcal{J}_g(p^2)\right] = \delta(p^2)$$



- one-loop

[Fleming, Leibovich, Mehen 03, Becher, Schwartz 09]

$$i\mathcal{J}^{(1)}_g(p^2) = \frac{\alpha_s}{4\pi}\, \frac{(\mu^2 e^{\gamma_E})^\varepsilon}{(-p^2 - i0)^{1+\varepsilon}}\, \frac{8\Gamma(2-\varepsilon)^2\,\Gamma(\varepsilon)}{\Gamma(4-2\varepsilon)} \times \left[C_A\left(\frac{3}{\varepsilon} - \frac{9}{4}\right) - n_f T_F\right]$$



We performed the two-loop calculation in Feynman and in light-cone gauge

Feynman gauge

In Feynman gauge the calculation involves $\mathcal{O}(35)$ two-loop diagrams

- self energy diagrams

- wilson line diagrams

Reduction to master integrals

- integration by parts, Laporta algorithm, partial fractioning, symmetry relations

$\Rightarrow$ find same master integrals as in the two-loop quark jet calculation [Becher, Neubert 06]

Most complicated two-loop integral

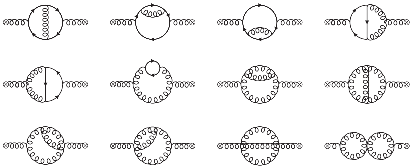
$$\int d^d k\, d^d l \; \frac{1}{k^2 (k+p)^2 (k+l+p)^2 (l+p)^2 l^2 \, (n\cdot k)\,(n\cdot l)} \sim \frac{(-p^2 - i0)}{(n\cdot p)^2}^{-1-2\varepsilon} \left[\frac{\pi^2}{3\varepsilon^2} - \frac{7\zeta_3}{\varepsilon} + \frac{23\pi^4}{360} \right]$$

$\Rightarrow$ single scale integrals (no zero-bins), dependence on $(n \cdot p)$ is analytic

Light-cone gauge

In LC gauge there are only 12 diagrams

- self energy diagrams (without ghosts)
- no wilson line diagrams



Gluon propagator in light-cone gauge

- $G_{\mu\nu}(k) = \frac{i}{k^2 + i0} \left[-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n \cdot k} \right]$
- we do not adopt the Mandelstam-Leibbrandt prescription $\frac{1}{n \cdot k} \to \frac{1}{n \cdot k + i\,(\bar{n} \cdot k)\,\eta}$

  (the prescrip. does not matter, but the IR-regulator η leads to unnecessary complications)

Overall the light-cone gauge calculation is much more efficient

$\Rightarrow$ both calculations agree on the level of the unrenormalized jet function

Two-loop result

Bare jet function

$$i\mathcal{J}_g(p^2) = \frac{1}{-p^2 - i0}\left\{1 + \frac{Z_\alpha \alpha_s}{(4\pi)}\left(\frac{\mu^2 e^{\gamma_E}}{-p^2 - i0}\right)^{\epsilon} \frac{8\Gamma(2-\varepsilon)^2\,\Gamma(\varepsilon)}{\Gamma(4-2\varepsilon)}\left[C_A\left(\frac{3}{\varepsilon} - \frac{9}{4}\right) - n_f T_F\right]\right.$$
$$\left. + \frac{Z_\alpha^2 \alpha_s^2}{(4\pi)^2}\left(\frac{\mu^2}{-p^2 - i0}\right)^{2\epsilon}\left[C_A^2 J_{AA}(\epsilon) + C_A n_f T_F J_{Af}(\epsilon) + C_F n_f T_F J_{Ff}(\epsilon) + n_f^2 T_F^2 J_{ff}(\epsilon)\right]\right\}$$

where

$$J_{AA}(\epsilon) = \frac{8}{\epsilon^4} + \frac{55}{3\epsilon^3} + \frac{1}{\epsilon^2}\left(-\frac{\pi^2}{3} + \frac{152}{3}\right) + \frac{1}{\epsilon}\left(-\frac{184\zeta_3}{3} - \frac{11\pi^2}{6} + \frac{3638}{27}\right)$$
$$- \frac{23\pi^4}{180} - \frac{1496\zeta_3}{9} - \frac{161\pi^2}{27} + \frac{57415}{162}$$

$$J_{Af}(\epsilon) = -\frac{20}{3\epsilon^3} - \frac{188}{9\epsilon^2} + \frac{1}{\epsilon}\left(\frac{2\pi^2}{3} - \frac{536}{9}\right) + \frac{400\zeta_3}{9} + \frac{74\pi^2}{27} - \frac{12880}{81}$$

$$J_{Ff}(\epsilon) = -\frac{2}{\epsilon} + 16\zeta_3 - \frac{55}{3}$$

$$J_{ff}(\epsilon) = \frac{16}{9\epsilon^2} + \frac{160}{27\epsilon} - \frac{8\pi^2}{27} + 16$$

Renormalization

Renormalization is most conveniently discussed in Laplace space

$$\widetilde{j}_g\left(\ln\frac{Q^2}{\mu^2},\mu\right)=\int_0^\infty dp^2\,e^{-\nu p^2}\,J_g(p^2,\mu)\qquad\text{with}\qquad \nu=\frac{1}{Q^2\,e^{\gamma_E}}$$

- multiplicative renormalization $\widetilde{j}_g = Z_{\tilde{j}_g}\,\widetilde{j}_g^{\text{bare}}$
- Z-factor fulfills renormalization group equation

$$\frac{\mathrm{d}Z_{\tilde{j}_g}}{\mathrm{d}\ln\mu}=\left[-2\Gamma^A_{\text{cusp}}\ln\frac{Q^2}{\mu^2}-2\gamma^{J_g}\right]Z_{\tilde{j}_g}$$

- γ^{J_g} known to three-loop from direct photon production analysis [Becher, Schwartz 09]

$\Rightarrow$ we can explicitly check the cancellation of $1/\varepsilon^4\ldots 1/\varepsilon$ divergences

Laplace transform

The renormalized two-loop result reads

$$\widetilde{j}_g(L,\mu) \;=\; 1+\left(\frac{\alpha_s}{4\pi}\right)\left[\Gamma_0^A\,\frac{L^2}{2}+\gamma_0^{J_g}\,L+c_1^{J_g}\right]+\left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(\Gamma_0^A\right)^2\frac{L^4}{8}+\left(-\beta_0+3\gamma_0^{J_g}\right)\Gamma_0^A\,\frac{L^3}{6}\right.$$
$$\left.+\left(\Gamma_1^{J_g}+(\gamma_0^{J_g})^2-\beta_0\gamma_0^{J_g}+c_1^{J_g}\Gamma_0^A\right)\frac{L^2}{2}+(\gamma_1^{J_g}+\gamma_0^{J_g}c_1^{J_g}-\beta_0 c_1^{J_g})L+c_2^{J_g}\right]$$

Our calculation yields the two-loop constant

$$c_2^{J_g} \;=\; C_A^2\left(\frac{20215}{162}-\frac{362\pi^2}{27}-\frac{88\zeta_3}{3}+\frac{17\pi^4}{36}\right)+C_A n_f T_F\left(-\frac{1520}{27}+\frac{134\pi^2}{27}-\frac{16\zeta_3}{3}\right)$$
$$+\,C_F n_f T_F\left(-\frac{55}{3}+16\zeta_3\right)+n_f^2 T_F^2\left(\frac{400}{81}-\frac{8\pi^2}{27}\right)$$

Numerically for $n_f=5$ the gluon jet function becomes

$$\widetilde{j}_g(L,\mu)=1+\left(\frac{\alpha_s}{4\pi}\right)\left(6L^2-7.667\,L-2.961\right)+\left(\frac{\alpha_s}{4\pi}\right)^2\left(18\,L^4-61.33\,L^3+82.46\,L^2+44.39\,L-58.58\right)$$

Subleading jet function $K_g(p^2)$

We can simply change the projectors to obtain the bare subleading jet function

$$
\begin{aligned}
i\mathcal{K}_g(p^2) &= -\frac{Z_\alpha \alpha_s C_A}{(4\pi)} \left(\frac{\mu^2 e^{\gamma_E}}{-p^2 - i0} \right)^\epsilon \frac{4\Gamma(2-\epsilon)\Gamma(-\epsilon)\Gamma(\epsilon)}{\Gamma(2-2\epsilon)} \\
&\quad + \frac{Z_\alpha^2 \alpha_s^2}{(4\pi)^2} \left(\frac{\mu^2}{-p^2 - i0} \right)^{2\epsilon} \left[C_A^2 K_{AA}(\epsilon) + C_A n_f T_F K_{Af}(\epsilon) \right]
\end{aligned}
$$

where

$$
\begin{aligned}
K_{AA}(\epsilon) &= \frac{8}{\epsilon^4} + \frac{19}{\epsilon^3} + \frac{148}{3\epsilon^2} + \frac{1}{\epsilon} \left(-\frac{151\zeta_3}{3} - \frac{7\pi^2}{6} + \frac{1082}{9} \right) \\
&\quad + \frac{17\pi^4}{72} - \frac{428\zeta_3}{3} - \frac{41\pi^2}{9} + \frac{7672}{27} \\
K_{Af}(\epsilon) &= -\frac{4}{\epsilon^3} - \frac{38}{3\epsilon^2} - \frac{298}{9\epsilon} + \frac{44\zeta_3}{3} + \frac{13\pi^2}{9} - \frac{2198}{27}
\end{aligned}
$$

$K_g(p^2)$ vanishes at tree level, but is divergent at one-loop $\Rightarrow$ renormalization?

Operator mixing

Consider the scalar operators

$$O_{\mathcal{J}}(x) = \frac{-g_{\mu\nu}}{d-2} T\Big\{\mathcal{A}^a_\mu(x)\,\mathcal{A}^b_\nu(0)\Big\} \qquad \langle 0|\,O_{\mathcal{J}}(x)\,|0\rangle \to J_g(p^2)$$

$$O_{\mathcal{K}}(x) = T\Big\{\partial\cdot\mathcal{A}^a(x)\,\partial\cdot\mathcal{A}^b(0)\Big\} + \Box\,O_{\mathcal{J}}(x) \qquad \langle 0|\,O_{\mathcal{K}}(x)\,|0\rangle \to K_g(p^2)$$

Starting from $O_{\mathcal{J}}$ one can construct infinitely many operators of dimension 4, e.g.

$$O_1(x) = -\Box\,O_{\mathcal{J}}(x) \quad \to \quad J_1(p^2) = p^2 J_g(p^2)$$

$$O_2(x) = \frac{4}{-x^2+i0}\,O_{\mathcal{J}}(x) \quad \to \quad J_2(p^2) = \frac{2}{d-2}\left\{J_3(p^2) + \int_{p^2}^\infty \mathrm{d}p'^2 \left(\frac{p'^2}{p^2}\right)^{\frac{d-2}{2}} J_g(p'^2)\right\}$$

$$O_3(x) = 4\,\frac{\partial}{\partial x^2}\,O_{\mathcal{J}}(x) \quad \to \quad J_3(p^2) = \int_0^{p^2} \mathrm{d}p'^2\, J_g(p'^2)$$

How to construct a minimal basis that closes under renormalization? [cf. also Paz 09]

Operator basis

At tree level: $J_1(p^2) = 0 \qquad J_2(p^2) = \frac{1}{1-\varepsilon} \qquad J_3(p^2) = 1$

- $\{O_1, O_{\mathcal{K}}\}$ does not close under renormalization
- $J_2(p^2) - J_3(p^2)$ vanishes in four dimensions

Operators formed out of $O_{\mathcal{J}}$ have same field content and spin structure

$\Rightarrow$ their matrix elements cannot be distinguished

$\Rightarrow$ sufficient to include one arbitrary operator with non-vanishing tree level matrix element

If several such operators are included

$\Rightarrow$ can choose a scheme where only one operator has non-vanishing ren. matrix element

$\Rightarrow$ additional operators are "evanescent", not needed to form a (complete) basis

Back to $K_g(p^2)$

We choose $\{O_3, O_{\mathcal{K}}\}$ and obtain a matrix equation in Laplace space

$$\begin{pmatrix} \widetilde{j}_3 \\ \widetilde{k}_g \end{pmatrix} = \begin{pmatrix} Z_{\tilde{j}g} & 0 \\ Z_{kj} & Z_{kk} \end{pmatrix} \begin{pmatrix} \widetilde{j}_3^{\,\rm bare} \\ \widetilde{k}_g^{\,\rm bare} \end{pmatrix}$$

- one-loop $\Rightarrow$ determines $Z_{kj} = -\frac{C_A \alpha_s}{\pi} \frac{1}{\epsilon}$
- two-loop $\Rightarrow$ to single out Z_{kj} and Z_{kk} contributions one would have to consider a matrix element that is sensitive to the gluon spin

Final remark:

- power corrections to $B \to X_s \gamma$ reveal an UV-divergent convolution integral [Benzke, Lee, Neubert, Paz 10]
- formally, we find a similar situation in $J_2(p^2) = \frac{2}{d-2} \left\{ J_3(p^2) + \int_{p^2}^{\infty} \mathrm{d}p'^2 \left(\frac{p'^2}{p^2} \right)^{\frac{d-2}{2}} J_g(p'^2) \right\}$
- maybe a different choice of the operator basis may help to circumvent the problem?

Conclusion

We computed the inclusive gluon jet function to two-loop order

$\Rightarrow$ universal ingredient to perform NNNLL resummations in SCET

SCET calculations simplify considerably in light-cone gauge

$\Rightarrow$ "spurious" LC singularities are regularized in DR and natural in SCET – use it!

We also computed the subleading gluon jet function

$\Rightarrow$ showed how to construct an operator basis that closes under renormalization

Backup slides

Toy integral with different $i\varepsilon$-prescriptions

Consider the integral ($n^2 = 0$): $$I = \tilde{\mu}^{2\varepsilon} \int \frac{d^dk}{(2\pi)^d} \, \frac{n\cdot p}{[(k-p)^2 + i\eta] \, [k^2 + i\eta] \, n\cdot k}$$

$+i\eta$:
$$\begin{aligned} I_+ &= \tilde{\mu}^{2\varepsilon} \int \frac{d^dk}{(2\pi)^d} \, \frac{n\cdot p}{[(k-p)^2 + i\eta] \, [k^2 + i\eta] \, [n\cdot k + i\eta]} \\ &= \frac{-i}{(4\pi)^2} \left(\mu^2 e^{\gamma_E}\right)^{\varepsilon} \Gamma(1+\varepsilon) \int_0^1 dx \int_0^\infty dy \; n\cdot p \left[-x(\bar{x}p^2 + y\, n\cdot p) - i\eta\right]^{-1-\varepsilon} \\ &= \frac{i}{(4\pi)^2} \left(\frac{\mu^2 e^{\gamma_E}}{-p^2 - i\eta}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)\Gamma(\varepsilon)\Gamma(-\varepsilon)}{\Gamma(1-2\varepsilon)} \end{aligned}$$

$-i\eta$:
$$\begin{aligned} I_- &= \tilde{\mu}^{2\varepsilon} \int \frac{d^dk}{(2\pi)^d} \, \frac{n\cdot p}{[(k-p)^2 + i\eta] \, [k^2 + i\eta] \, [n\cdot k - i\eta]} \\ &= \tilde{\mu}^{2\varepsilon} \int \frac{d^dk}{(2\pi)^d} \, \frac{-n\cdot p}{[(k-p)^2 + i\eta] \, [k^2 + i\eta] \, [-n\cdot k + i\eta]} \\ &= I_+\big|_{n\to -n} = I_+ \end{aligned}$$

PV:
$$I_{PV} = \frac{I_+ + I_-}{2} = I_+$$

Mandelstam-Leibbrandt prescription

We now work in LC coordinates and choose $p_\perp = 0$ and $p_- > 0$

$$I_{ML} = \tilde{\mu}^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p_-}{[k_+k_- + k_\perp^2 - p_+k_- - p_-k_+ + p_+p_- + i\eta]\,[k_+k_- + k_\perp^2 + i\eta]} \left[\frac{\theta(k_+)}{k_- + i\eta} + \frac{\theta(-k_+)}{k_- - i\eta}\right]$$

$$= \frac{-i}{4\pi}\,\tilde{\mu}^{2\varepsilon} \int_0^{p_-} dk_- \int \frac{d^{d-2}k_\perp}{(2\pi)^{d-2}} \frac{1}{[k_\perp^2 + \frac{p_+}{p_-}(p_- - k_-)k_- + i\eta]} \left[\frac{\theta(-k_\perp^2/k_-)}{k_- + i\eta} + \frac{\theta(k_\perp^2/k_-)}{k_- - i\eta}\right]$$

$$= \frac{i}{(4\pi)^2} \frac{(\mu^2 e^{\gamma_E})^\varepsilon}{\Gamma(1-\varepsilon)} \int_0^{p_-} dk_- \int_0^\infty dk^2 \frac{(k^2)^{-\varepsilon}}{[k^2 - \frac{p_+}{p_-}(p_- - k_-)k_- - i\eta]} \frac{1}{k_- + i\eta}$$

$$= \frac{i}{(4\pi)^2} \left(\frac{\mu^2 e^{\gamma_E}}{-p^2 - i\eta}\right)^\varepsilon \Gamma(\varepsilon)\, p_-^{2\varepsilon} \int_0^{p_-} dk_- \frac{k_-^{-\varepsilon}}{k_- + i\eta} (p_- - k_-)^{-\varepsilon}$$

$$= \frac{i}{(4\pi)^2} \left(\frac{\mu^2 e^{\gamma_E}}{-p^2 - i\eta}\right)^\varepsilon \frac{\Gamma(1-\varepsilon)\Gamma(\varepsilon)\Gamma(-\varepsilon)}{\Gamma(1-2\varepsilon)} = I_+$$

$\Rightarrow$ the $i\eta$-prescription of the light-cone propagators does not matter!

(as long as one does not interchange the order the limits $\eta \to 0$ and $\varepsilon \to 0$ are taken ...)