SCET analysis of inclusive top-quark pair production

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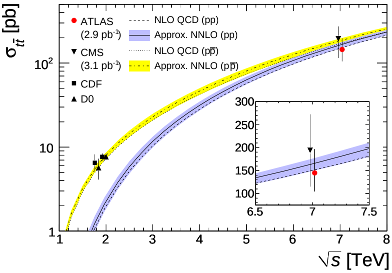
Why study top quark pair production?

Physics motivation

- large production rate at hadron colliders (signal or background)
- sensitive to new physics though bumps in $M_{t\bar{t}}$ or p_T distributions
- current discrepancy in A_{fb} at Tevatron
- already first measurements at the LHC



ATLAS: 37 top candidates in semi-leptonic/di-lepton channels

$$\sigma_{t\bar{t}} = 145 \pm 31^{+43}_{-27}\mathrm{pb}$$

CMS: 11 top candidates in di-lepton channel

$$\sigma_{t\bar{t}} = 194 \pm 72 \pm 24 \pm 21\mathrm{pb}$$

Figure from ATLAS, arXiv:1012.1792v2

SCET motivation

1) Practical: NLO calculations not enough, but NNLO slow in coming

- can improve on NLO through threshold resummation (in SCET)
  Kidonakis, Laenen, Sterman; Kidonakis, Laenen, Moch, Vogt '01; Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Kidonakis '10; Beneke et. al. '09; Ahrens, Ferroglia, Neubert, BP, Yang ('10, '11)

2) Conceptual: several interesting technical complications
(mainly due to presence of 4 colored partons)

- matching functions and RG eqns. are matrices in color space
- different soft limits for different observables ($M_{t\bar{t}}, p_T, \ldots$)

3) Earlier calculations (Kidonakis et. al.) found large discrepancy in total cross section from soft limits of "PIM" and "1PI" kinematics: true also in SCET?

Outline

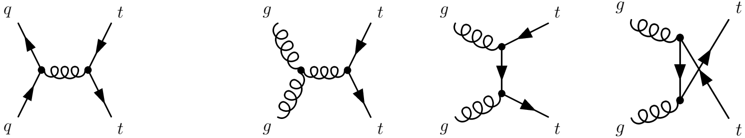
1) Threshold resummation in SCET

- different soft limits (PIM, 1PI, production threshold)
- threshold enhancement and power corrections

2) Phenomenology

- total cross section
- invariant mass and p_T distributions
- forward-backward asymmetry

Factorization and resummation in $h_1 h_2 \to t\bar{t}X$



$$d\sigma^{t\bar{t}X}_{h_1,h_2} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F) d\hat{\sigma}_{ij}(x_1,x_2,\ldots,\mu_F,\mu_R)$$

Threshold resummation in SCET (for given $d\sigma$):

- identify partonic expansion parameter λ which vanishes in limit of soft real emission (threshold limit)
- factorize $d\hat{\sigma} \to H \times S + \mathcal{O}(\lambda)$, resum logarithms with RG equations
- check that leading terms in partonic limit $\lambda \to 0$ are dominant for hadronic cross section (due to fall-off of PDFs = threshold enhancement)

Three soft limits (partonic thresholds)

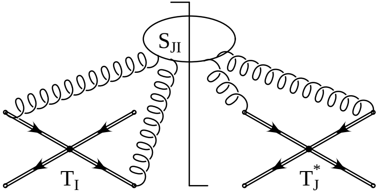
$$p_i(p_1) + p_j(p_2) \to t(p_3) + \bar{t}(p_4) + X(k) \qquad (p_i, p_j \in \{q, g\})$$

Name	Observable	Soft limit
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$ or σ	$s_4 = (p_4 + k)^2 - m_t^2 \to 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}} d\cos\theta$ or σ	$(1 - z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$

Note that $\beta \to 0$ is special case of PIM and 1PI, and $\langle\beta\rangle \sim 0.5$

$\Rightarrow$ will focus on PIM and 1PI kinematics

Hard-soft factorization in 1PI and PIM kinematics



(b)

$$T_I \sim C_I(\hat{s}, \hat{t}_1 \ldots) \times \bar{\chi}_{\bar{n}}^{a_2}\, \chi_n^{a_1}\, \bar{h}_{v_3}^{a_3} h_{v_4}^{a_4} \times c_I^{\{a\}}$$

$$d\hat{\sigma} \sim \sum_{I,J} C_I S_{IJ} C_J^* \times |\langle t\bar{t}|\bar{\chi}_{\bar{n}}\, \chi_n\, \bar{h}_{v_3} h_{v_4}|q\bar{q}\rangle|^2 \equiv \mathrm{Tr}[\mathbf{H}\,\mathbf{S}]$$

- **H** from virtual corrections is same in PIM and 1PI
- **S** from soft real emission (Wilson loop), different in PIM and 1PI (phase-space integrals depend on the observable)

As usual, need to derive and solve RG equations to resum logs

- same structure in both cases

Master formula and implementation

All-orders resummation formula:

$$d\hat{\sigma}(\lambda,\mu_f) \propto \exp\left[4a_{\gamma\phi}(\mu_s,\mu_f)\right] \mathrm{Tr}\Big[\mathbf{U}_\lambda(\mu_h,\mu_s)\,\mathbf{H}(\mu_h)$$
$$\mathbf{U}_\lambda^\dagger(\mu_h,\mu_s)\,\tilde{\mathbf{s}}_\lambda(\partial_\eta,\alpha_s(\mu_s))\Big]\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\left[\frac{1}{\lambda}\left(\frac{E_s(\lambda)}{\mu_s}\right)^{2\eta}\right]_+$$

Perturbative ingredients for NNLL

Γ_{cusp}	γ^h, γ^s_λ	$\mathbf{H}$, $\tilde{\mathbf{s}}_\lambda$
3-loop	2-loop	1-loop

Ahrens, Ferroglia, Neubert, BP, Yang ('10, '11)

Issues:

1) power corrections to threshold limit in PIM/1PI
2) How to choose μ_s and μ_h
3) Whether to use NLO+NNLL or approximate NNLO

$$d\hat{\sigma}^{\text{NNLO}} = \sum_{m=0}^{3} C_m\,[\ln^m(\lambda)]_+ + D_2\delta(\lambda) + R(\lambda)$$

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Choice of expansion parameter in SCET

To factorize and resum, must expand to leading order in soft limit

$$d\hat{\sigma}(\lambda, \mu_f) \propto \mathrm{Tr}\left[\mathbf{H}(\hat{s}, \hat{t}_1, m_t, \mu_f)\,\mathbf{S}\left(\frac{E_s(\lambda)}{\mu_f}, v_i \cdot v_j, \ldots, \alpha_s(\mu_f)\right)\right] + \mathcal{O}(\lambda)$$

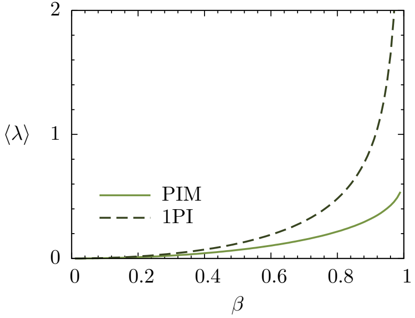
PIM kinematics: $E_s(\lambda) = M_{t\bar{t}}\lambda$	1PI kinematics: $E_s(\lambda) = m_t\lambda$
$\mathrm{PIM}_{\mathrm{SCET}}$: $\lambda = (1-z)/\sqrt{z}$	$1\mathrm{PI}_{\mathrm{SCET}}$: $\lambda = s_4/(m_t\sqrt{m_t^2 + s_4})$
PIM: $\lambda = (1-z)$	1PI: $\lambda = s_4/m_t^2$

- in $\mathrm{PIM}_{\mathrm{SCET}}$ ($1\mathrm{PI}_{\mathrm{SCET}}$) $E_s(\lambda)$ is the energy of the soft gluon radiation in the partonic center-of-mass frame ($\bar{t}$ rest frame)
- keeping the exact form of the energy re-organizes the threshold expansion, so that some formally subleading terms are kept compared to other calculations (Kidonakis et.al.), which use PIM and 1PI

The β-distribution and total cross section

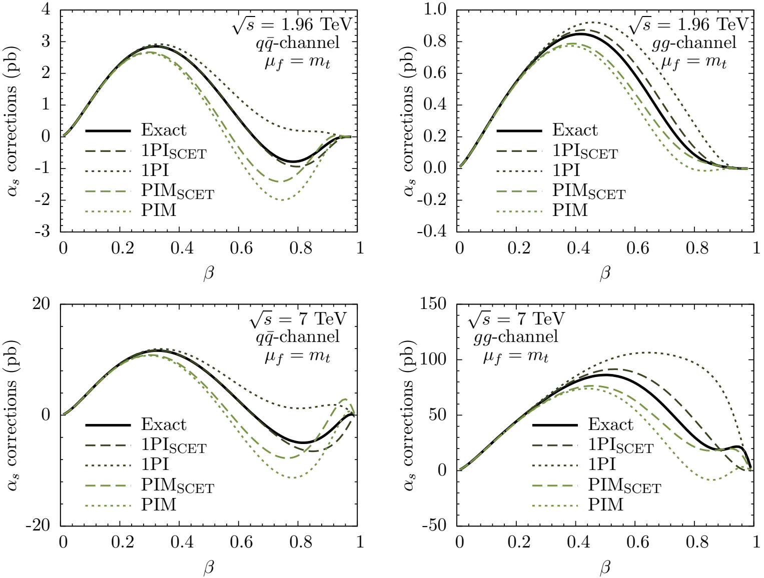
$$\frac{d\sigma}{d\beta} = \frac{1}{s}\frac{8\beta}{(1-\beta^2)^2}\sum_{ij} ff_{ij}\left(\frac{\hat{s}}{s},\mu_f\right)\alpha_s^2\, f_{ij}\left(\frac{4m_t^2}{\hat{s}},\mu_f\right); \quad \hat{s} = 4m_t^2/(1-\beta^2)$$

- ff_{ij} are parton luminosities
- $f_{ij} \sim \int_{\text{p.s.}}[\mathrm{Tr}[\mathbf{H}\,\mathbf{S}]_{\text{ij}} + \mathcal{O}(\lambda)]$ are partonic cross sections (known to NLO)



- for $\beta \to 0$, gluon emission is soft, 1PI, PIM, and exact QCD should agree
- for larger β, expect power corrections to be more important in 1PI kinematics than in PIM kinematics

PIM vs. 1PI at NLO



Results for the cross section in different schemes

$m_t/2 < \mu_f < 2m_t, m_t = 173.1$ GeV, MSTW2008

	Tevatron	LHC (7 TeV)
$\sigma_{\text{NLO}}, q\bar{q} + gg$	$6.80^{+0.27}_{-0.73}$	160^{+5}_{-15}
$\sigma_{\text{NLO leading}}$, 1PI_{SCET} (1PI)	$6.79^{+0.20}_{-0.70}$ $(7.23^{+0.45}_{-0.86})$	163^{+0}_{-11} (183^{+6}_{-18})
$\sigma_{\text{NLO leading}}$, PIM_{SCET} (PIM)	$6.42^{+0.42}_{-0.76}$ $(6.20^{+0.28}_{-0.69})$	152^{+7}_{-15} (143^{+1}_{-12})
$\sigma_{\text{NNLO approx}}$, 1PI_{SCET} (1PI)	$6.63^{+0.00}_{-0.27}$ $(7.06^{+0.00}_{-0.29})$	155^{+3}_{-2} (180^{+3}_{-8})
$\sigma_{\text{NNLO approx}}$, PIM_{SCET} (PIM)	$6.62^{+0.05}_{-0.40}$ $(6.46^{+0.18}_{-0.45})$	155^{+8}_{-8} (148^{+14}_{-11})
$\sigma_{\text{NLO+NNLL}}$, 1PI_{SCET}	$6.55^{+0.16}_{-0.14}$	150^{+7}_{-7}
$\sigma_{\text{NLO+NNLL}}$, PIM_{SCET}	$6.46^{+0.18}_{-0.19}$	147^{+7}_{-6}

- PIM and 1PI not really consistent with each other Kidonakis, Laenen, Moch, Vogt '01; Kidonakis '10
- PIM_{SCET} and 1PI_{SCET} are consistent with each other and exact result at NLO Ahrens, Ferroglia, Neubert, BP, Yang ('10, '11)
- NLO+NNLL and approximate NNLO consistent with each other

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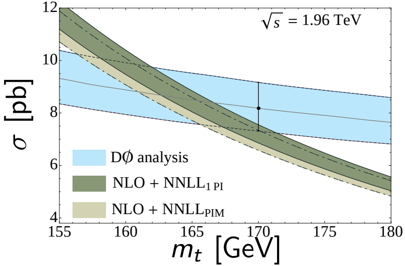
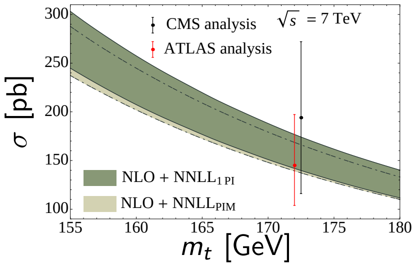
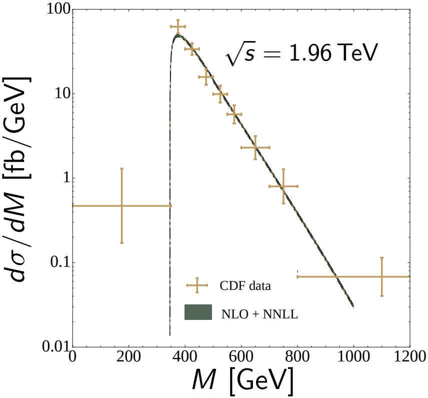
The total cross section at NLO+NNLL: comparison with experiment

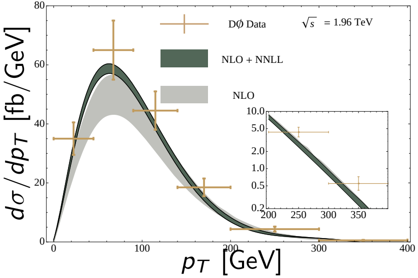




- PIM_{SCET} and 1PI_{SCET} are consistent
- can be used to extract m_t, although preferable to use $\overline{\text{MS}}$ mass (or other short-distance scheme)

Invariant mass and p_T distributions (Tevatron)





- normalization and shape of distributions consistent with data
- moments of distributions (such as $\langle M \rangle$) also sensitive to m_t

Inclusive forward-backward asymmetry at the Tevatron

$$A^c = \frac{N_t(y>0) - N_t(y<0)}{N_t(y>0) + N_t(y<0)}$$

DO, 4.3fb^{-1}

$$A_{fb}(\mathrm{t\bar{t}}) = 0.08 \pm 0.08 \text{ stat} \pm 0.01 \text{ syst}$$

CDF, 5.3fb^{-1}

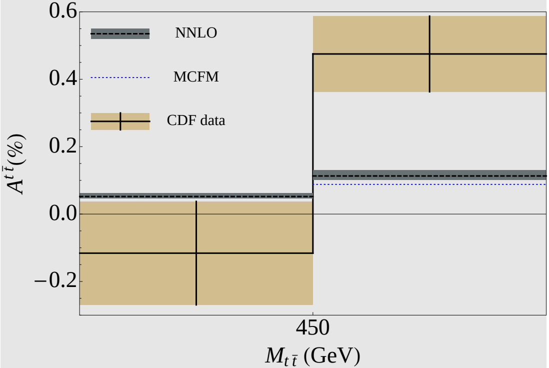
$$A_{fb}(\mathrm{t\bar{t}}) = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst}$$
$$A_{fb}(\mathrm{lab}) = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst}$$

Theory

$$A_{fb}(\mathrm{lab}) = 0.049 \pm 0.002 \ (\text{NLO+NNLL, 1PI}_{\text{SCET}})$$
$$A_{fb}(\mathrm{t\bar{t}}) = 0.073 + 0.011 - 0.007 \ (\text{NLO+NNLL, PIM}_{\text{SCET}})$$

Theory and experiment agree at about 2σ

Invariant mass dependent asymmetry at Tevatron (CDF arXiv:1101:0034)



- $M_{t\bar{t}} < 450$ GeV: compatible with NLO within 1σ
- $M_{t\bar{t}} > 450$ GeV: disagrees with NLO at 3.4σ
- approximate NNLO and NLO+NNLL doesn't change this much (preliminary)

Summary

Described calculation of top-quark pair production within SCET

Results for both PIM and 1PI kinematics were calculated at NLO+NNLL and approximate NNLO

- results for total cross section from the two kinematics are consistent with each other (in the SCET calculation)
- the NLO+NNLL $M_{t\bar{t}}$ and p_T distributions agree well with Tevatron data
- no large corrections to FB asymmetry at Tevatron, also at high $M_{t\bar{t}}$

backup slides

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Momentum-space resummation: NLO+NNLL

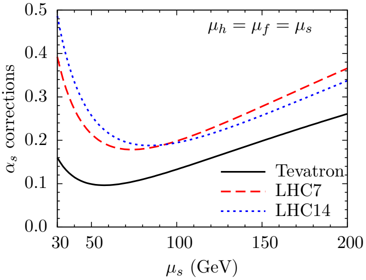
Philosophy: choose μ_s and μ_h such that there are no large logs in matching functions, run to μ_f using RG

Complication is with soft function (ignoring matrix structure)

$$S(\omega,\mu_f) = e^{-4S(\mu_s,\mu_f)+2a_{\gamma^s}(\mu_s,\mu_f)}\,\tilde{s}\left(\partial_\eta,\mu_s\right)\left[\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\right]_+ \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

- logs vanish for $\mu_s \sim \omega$, but ω is integrated down to $\omega = 0$, so hit Landau pole
- choose μ_s *after* integration over ω, such that correction from soft function is minimized
- keep parametric counting $\mu_s \sim \omega$, even though it is a *number*

Hard and soft scales for total cross section in 1PI kinematics



- left: NLO correction (%) from soft function $\Rightarrow \mu_s \sim 50 - 90$ GeV
- right: NLO correction (%) from hard function $\Rightarrow \mu_h \sim 400$ GeV

Approximate NNLO

Philosophy: plus distributions give dominant contributions at a given order, but corrections small enough for series to converge

In approximate formulas re-expand in limit $\mu_s \to \mu_f$ ($\eta \to 0$)

$$S(\omega,\mu_f) = \tilde{s}\,(\partial_\eta,\mu_s)\left[\frac{1}{\omega}\left(\frac{\omega}{\mu_f}\right)^{2\eta}\right]_+ \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\bigg|_{\eta\to 0}$$

$$= \delta(\omega) + \sum_{m=1}\left(\frac{\alpha_s}{4\pi}\right)^m\left[D^{(m)}\delta(1-z) + \sum_{k=0}^{2m-1} S^{(m,k)}P_m(\omega)\right]; \quad P_m(\omega) = \left[\frac{1}{\omega}\ln^m\frac{\omega}{\mu_f}\right]_+$$

- RG equations determine all two loop plus distributions in terms of one-loop matching function and two-loop anomalous dimensions
  - in form above, must derive two-loop logs in $\tilde{s}(L,\mu_s)$ separately and add on to NNLL solution, which only has one loop matching
- two-loop delta-function term undetermined

Resummation vs. approximate formulas

The two approaches are *different* in structure of plus distributions, at a given order in α_s.

Example: approximate NLO (use 1-loop anomalous dimensions and tree matching)

$$\tilde{s}(L, \alpha_s(\mu)) = 1 + \frac{\alpha_s(\mu)}{4\pi} \left[\frac{\Gamma_0}{2} L^2 + L\gamma_0^s + s^{(1,0)} \right]$$

$$\Rightarrow S_{\mathrm{NLO_{approx}}} = 1 + \frac{\alpha_s(\mu_f)}{4\pi} \left[4\Gamma_0 P_1(\omega) + 2\gamma_0^s P_0(\omega) - \frac{\pi^2}{3}\Gamma_0\, \delta(\omega) \right]$$

Compare with $\mathrm{NLL}|_{\mathrm{NLO}}$. To expand resummed formula, use $\alpha_s(\mu_s) = \alpha_s(\mu_f)[1 - \beta_0\alpha_s(\mu_f)/(4\pi)\ln(\mu_s^2/\mu_f^2) + \ldots]$

$$S_{\mathrm{NLL}|_{\mathrm{NLO}}} = 1 + \frac{\alpha_s(\mu_f)}{4\pi} \left[2\Gamma_0 L_s P_0(\omega) + \left(\frac{1}{2}\Gamma_0 L_s^2 + \gamma_0^s L_s \right) \delta(\omega) \right]; \quad L_s = \ln\left(\frac{\mu_s^2}{\mu_f^2} \right)$$

All orders exponentiation

In momentum space, can exponentiate the distributions using

$$S(\omega,\mu_f) = \left\{ \left[e^{-4S(\mu_s,\mu_f)+2a_{\gamma^s}(\mu_s,\mu_f)} \tilde{s}\left(0,\alpha_s(\mu_s)\right) \right] \bigg|_{\ln(\mu_s^2/\mu_f^2)\to\partial\eta} \right\} \frac{1}{\omega} \left(\frac{\omega}{\mu_f}\right)^{2\eta} \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \bigg|_{\eta\to 0}$$

where

$$\frac{\alpha_s(\mu_s)}{4\pi} = \frac{\alpha_s(\mu_f)}{4\pi}\frac{1}{X} - \frac{\alpha_s(\mu_f)^2}{(4\pi)^2}\frac{1}{X^2}\frac{\beta_1}{\beta_0}\ln X + \ldots\,; \qquad X = 1 + \beta_0\frac{\alpha_s(\mu_f)}{4\pi}\ln\frac{\mu_s^2}{\mu_f^2}$$

But this doesn't match the "philosophy" of RG-improved perturbation theory