

# QCD corrections to the forward-backward asymmetry in $e^+e^-$ annihilation in the two-jet region

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# Outline

Motivation

Forward-backward asymmetry

Longitudinal structure function

Hagiwara, Kirilin, JHEP10 (2010) 093

Mistaged events

Hagiwara, Kirilin, in preparation

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## Motivation

Forward-backward asymmetry

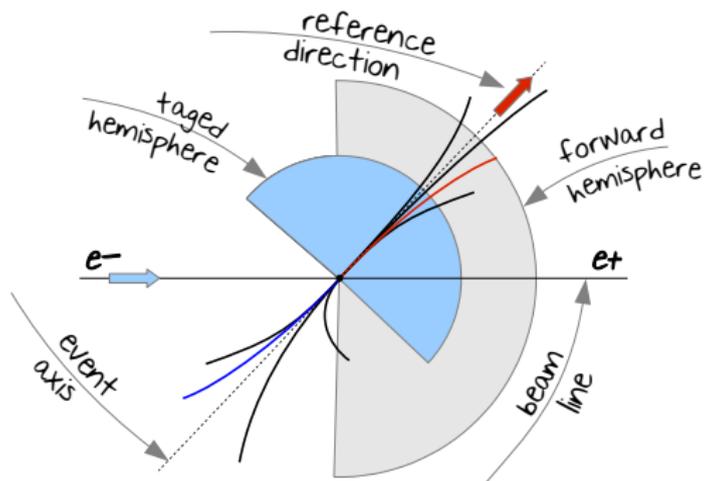
Longitudinal structure function

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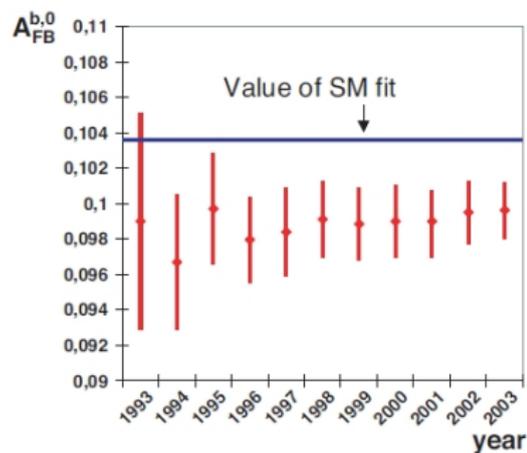
# $A_{FB}$ asymmetry



$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}$$

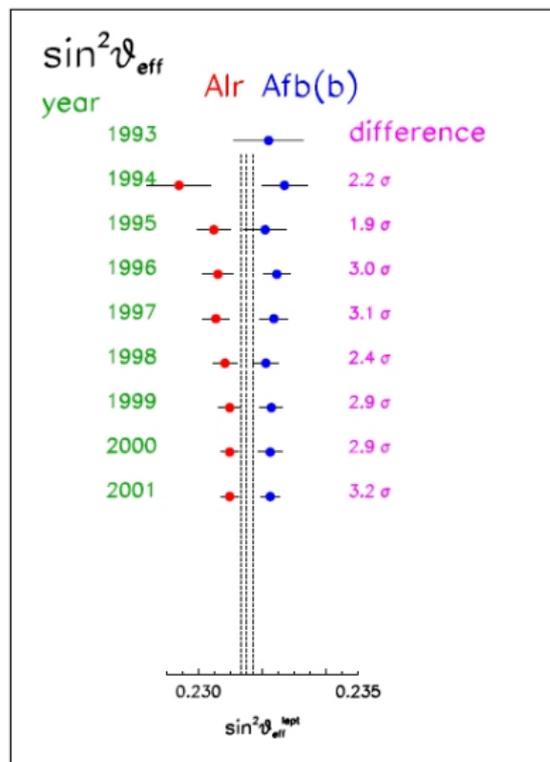
- Essential ingredients:
- ✿ event axis
  - ✿ “tag” procedure for a hemisphere

# Long-standing anomaly

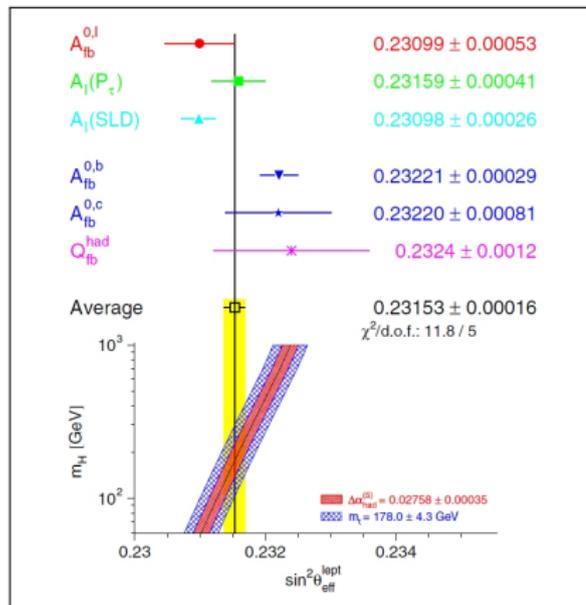


❖ M. Elsing, *Heavy Flavour Results from LEP I*, (HEP 2003) slides

❖ W. Venus, *A LEP Summary* (HEP 2001), hep2001/284



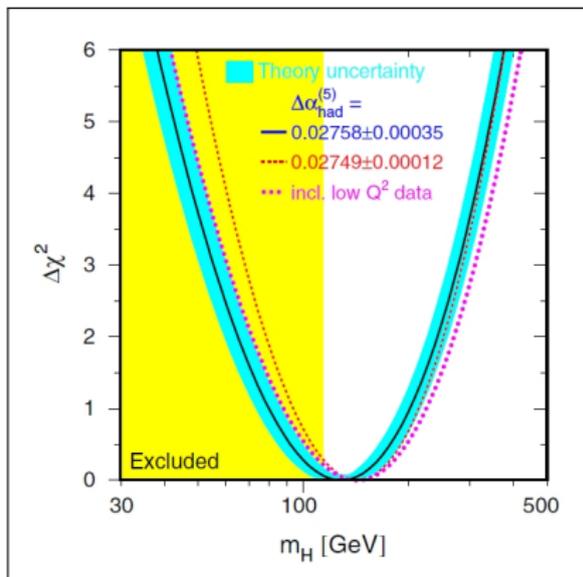
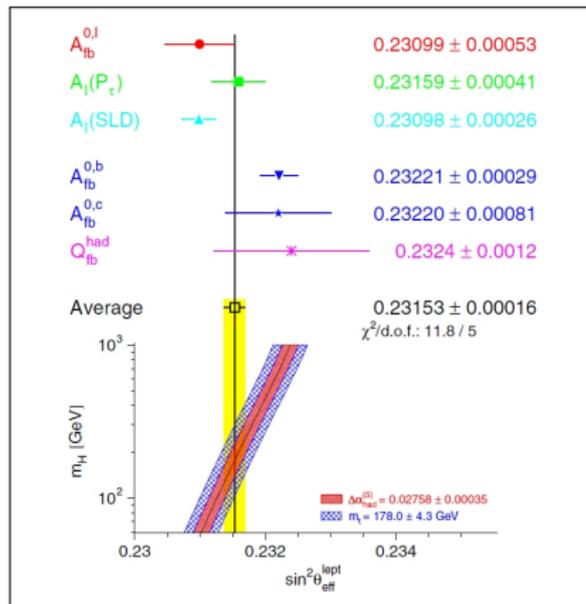
# SM parameters



❖ Precision electroweak measurements on the Z resonance, ALEPH, DELPHI, L3, OPAL, SLD, LEP EWWG, SLD EWHFG, Phys. Rep. 427 (2006) 257

	Measurement	Fit	$ \text{O}^{\text{meas}} - \text{O}^{\text{fit}}  / \sigma^{\text{meas}}$
$\Delta\alpha_{\text{had}}^{(5)}(m_Z)$	$0.02758 \pm 0.00035$	0.02767	0.1
$m_Z$ [GeV]	$91.1875 \pm 0.0021$	91.1874	0.001
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	2.4965	0.5
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	41.481	1.6
$R_l$	$20.767 \pm 0.025$	20.739	1.1
$A_{fb}^{0,l}$	$0.01714 \pm 0.00095$	0.01642	0.7
$A_1(P_z)$	$0.1465 \pm 0.0032$	0.1480	0.2
$R_b$	$0.21629 \pm 0.00066$	0.21562	0.1
$R_c$	$0.1721 \pm 0.0030$	0.1723	0.001
<b><math>A_{fb}^{0,b}</math></b>	<b><math>0.0992 \pm 0.0016</math></b>	<b>0.1037</b>	<b>2.8</b>
$A_{fb}^{0,c}$	$0.0707 \pm 0.0035$	0.0742	0.5
$A_b$	$0.923 \pm 0.020$	0.935	0.2
$A_c$	$0.670 \pm 0.027$	0.668	0.001
$A_1(\text{SLD})$	$0.1513 \pm 0.0021$	0.1480	1.5
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{fb})$	$0.2324 \pm 0.0012$	0.2314	0.8
$m_W$ [GeV]	$80.425 \pm 0.034$	80.389	1.1
$\Gamma_W$ [GeV]	$2.133 \pm 0.069$	2.093	0.7
$m_t$ [GeV]	$178.0 \pm 4.3$	178.5	0.1

# Higgs mass. Direct search and indirect constraints.



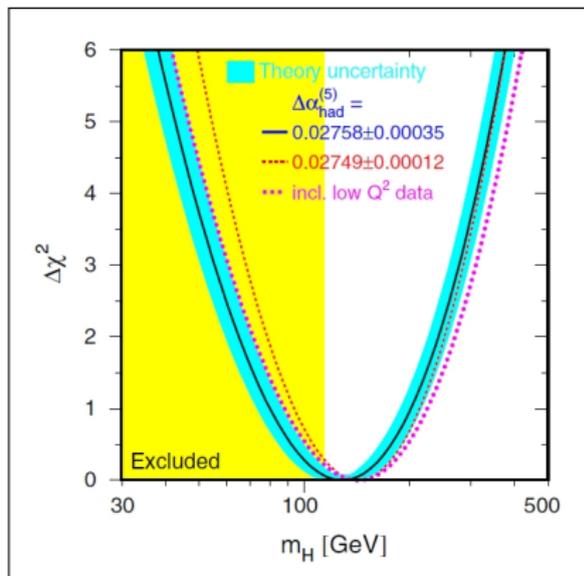
❖ Precision electroweak measurements on the Z resonance, ALEPH, DELPHI, L3, OPAL, SLD, LEP EWWG, SLD EWHFG, Phys. Rep. 427 (2006) 257

# Higgs mass. Direct search and indirect constraints.

$m_H > 114$  direct search

$m_H = 129 \pm_{49}^{74}$  fit 15%

$m_H = 76 \pm_{33}^{54}$  fit without  $A_{FB}^b$



❖ Precision electroweak measurements on the Z resonance, ALEPH, DELPHI, L3, OPAL, SLD, LEP EWWG, SLD EWHFG, Phys. Rep. 427 (2006) 257

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# Pole asymmetry

$$d\sigma \sim L^{\mu\nu} H_{\mu\nu}$$

Leptonic tensor:

$$L^{\mu\nu} = \langle 0 | j^\nu | e^+ e^- \rangle \langle e^+ e^- | j^\mu | 0 \rangle, \quad j^\mu = \bar{\psi}_e \gamma^\mu (g_{ve} - g_{ae} \gamma_5) \psi_e$$

Hadronic tensor:

$$H^{\mu\nu} = \sum_X \langle 0 | J^\nu | X \rangle \langle X | J^\mu | 0 \rangle, \quad J^\mu = \bar{\psi}_q \gamma^\mu (g_{vq} - g_{aq} \gamma_5) \psi_q$$

Asymmetry:

$$A_{FB}^0 = \frac{3}{4} \left[ \frac{2g_v g_a}{g_v^2 + g_a^2} \right]_e \left[ \frac{2g_v g_a}{g_v^2 + g_a^2} \right]_q$$

# QCD corrections

$$\left(A_{FB}^{(qq)}\right)_{\text{meas}} = \left[ 1 - \frac{\alpha_s(m_Z^2)}{\pi} c_1 - \left(\frac{\alpha_s(m_Z^2)}{\pi}\right)^2 c_2 \right] \left(A_{FB}^{(qq)}\right)_{\text{no QCD}}$$

For example, the second order QCD corrections:

- ❖ G. Altarelli and B. Lampe, Nucl. Phys. B391 (1993) 3
- ❖ V. Ravindran and W. L. van Neerven, Phys. Lett. (1998) 214
- ❖ S. Catani and M. H. Seymour, JHEP 07 (1999) 23.

# QCD corrections

$$\left(A_{FB}^{(qq)}\right)_{\text{meas}} = \left[ 1 - \frac{\alpha_s(m_Z^2)}{\pi} c_1 - \left(\frac{\alpha_s(m_Z^2)}{\pi}\right)^2 c_2 \right] \left(A_{FB}^{(qq)}\right)_{\text{no QCD}}$$

$A_{FB}$  is measured at the **per cent level** of accuracy:

$$A_{FB}^{0,b} = 0.0992 \pm 0.0016, \quad C_{\text{QCD}}^{\text{had,T}} = 0.0354 \pm 0.0063$$

The idea is to select events in the two-jet region

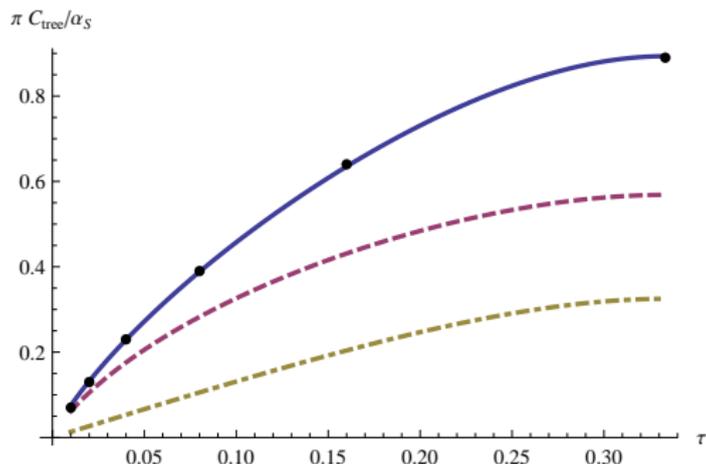
$$H^{\mu\nu}(y) = \sum_X \langle 0 | J^\nu | X \rangle \langle X | J^\mu | 0 \rangle \Theta[y - y(X)]$$

The analysis cuts lead to **the power suppression** of the QCD corrections.

# Bias factor

For example, thrust (hemisphere mass) as an experimental cut:

- ❖ A. Djouadi, B. Lampe, P.M. Zerwas, Z. Phys. C67 (1995) 123



*“Due to the strong interconnections between detector and QCD effects, a global correction is estimated usually by simulation”*

- ❖ B. Abbaneo et al., Euro. Phys. J. C4 (1998) 185

# Hadronic tensor structure

Structure functions:

$$H^{\mu\nu} = (g_{vq}^2 + g_{aq}^2) [-F(\tau) g_{\perp}^{\mu\nu} + G(\tau) g_{\parallel}^{\mu\nu}] - 2g_{vq}g_{aq}K(\tau)(ia^{\mu\nu})^*$$

Tensors:

$$g_{\perp} = g^{\mu\nu} - \frac{n^{\mu}n_{+}^{\nu} + n^{\nu}n_{+}^{\mu}}{n \cdot n_{+}}, \quad g_{\parallel} = \frac{(n^{\mu} - n_{+}^{\mu})(n^{\nu} - n_{+}^{\nu})}{4}, \quad a = \frac{\varepsilon^{\mu\nu\alpha\beta}n^{\alpha}n_{+}^{\beta}}{n \cdot n_{+}},$$

Light-cone directions:

$$\mathbf{n}_T^2 = 1, \quad n = (1, -\mathbf{n}_T), \quad n_{+} = (1, \mathbf{n}_T)$$

$$H^{\mu\nu}(n + n_{+})_{\nu} = 0$$

# Angular distribution

For simplicity, Z-boson in  $|1, -1\rangle$ . Angular distribution  $\cos \theta_T = \mathbf{n}_T \cdot \mathbf{n}_e$

$$\frac{d\sigma(T)}{d\cos\theta} \sim (g_v^2 + g_a^2) \left[ F(\tau) \left( 1 + \cos^2 \theta_T \right) + G(\tau) \sin^2 \theta_T \right] + 2g_v g_a K(\tau) 2\cos \theta_T,$$

Left and right quarks have different coupling constants

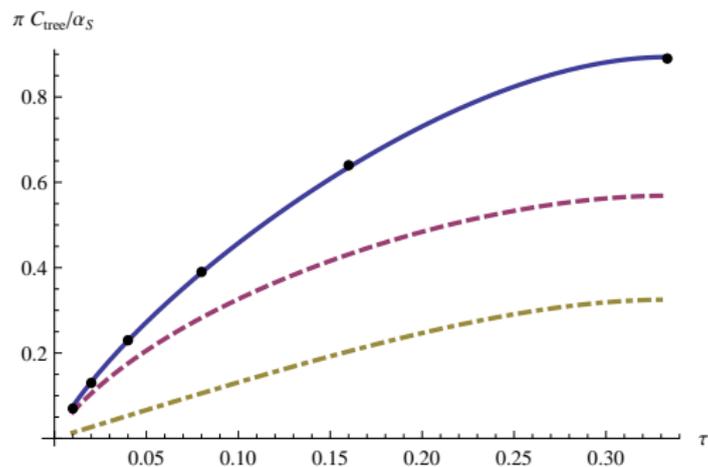
$$\begin{aligned} \frac{d\sigma}{d\cos\theta} &\sim (g_v + g_a)^2 \left| d_{-1,-1}^1 \right|^2 + (g_v - g_a)^2 \left| d_{1,-1}^1 \right|^2 \\ &= \frac{1}{2} \left[ (g_v^2 + g_a^2) \left( 1 + \cos^2 \theta_q \right) + (2g_v g_a) 2\cos \theta_q \right]. \end{aligned}$$

Relations in the two jet region:

$$F(\tau) = K(\tau), \quad G(\tau) = 0.$$

# Correction to the asymmetry

$$A_{FV}(\tau) = \frac{K}{F+G} A_{FV}^{(0)} = \left( 1 - \frac{F-K}{F+G} - \frac{G}{F+G} \right) A_{FV}^{(0)}$$



# Symmetries

## Operators

$$\hat{g} = g_{\perp}^{\mu\nu} = g^{\mu\nu} - \frac{n^{\mu} n_{+}^{\nu} + n^{\nu} n_{+}^{\mu}}{n \cdot n_{+}},$$
$$\hat{a} = a^{\mu\nu} = \frac{1}{n \cdot n_{+}} \varepsilon^{\mu\nu\alpha\beta} n_{\alpha} n_{+\beta}.$$

## Rotation $U(1)$

$$\hat{U} = \hat{g} \cos \phi + \hat{a} \sin \phi = \hat{g} \exp(\hat{a} \phi)$$
$$\hat{U}^{\dagger} = \hat{g} \cos \phi - \hat{a} \sin \phi = \hat{g} \exp(-\hat{a} \phi)$$

$$\hat{a} = -i \mathbf{n}_T \cdot \mathbf{J}$$

## Algebra

$$\hat{g}^2 = \hat{g},$$
$$\hat{a} \hat{g} = \hat{g} \hat{a} = \hat{a},$$
$$\hat{a}^2 = -\hat{g}.$$

## Hadronic tensor

$$\hat{H} = A \hat{g} + B \hat{a} = \hat{U}(\phi) \hat{H} \hat{U}^{\dagger}(\phi)$$
$$\hat{H}(n, n_{+}) = \hat{H}(\alpha n, \beta n_{+})$$

# Physical meaning

For simplicity,  $g_v = g_a$  – there are left quark and right antiquark only, thus the primary contribution:

$$\mathbf{n}_T \cdot \mathbf{J} |\Psi\rangle = - |\Psi\rangle$$

❁  $G$  function corresponds to the projection  $\mathbf{n}_T \cdot \mathbf{J} = 0$ :

$$a_{\mu\nu}(n^\nu - n_+^\nu) = 0$$

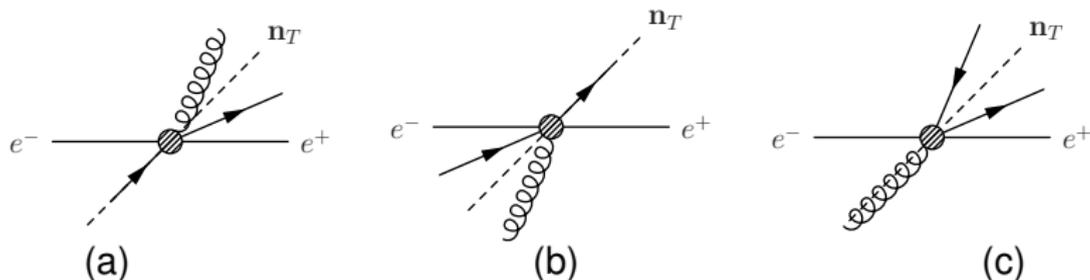
❁  $F - K$  combination corresponds to the eigenvalue  $\mathbf{n}_T \cdot \mathbf{J} = 1$ :

$$2(F\hat{g} + iK\hat{a}) = \frac{F+K}{2}(\hat{g} + i\hat{a})^2 + \frac{F-K}{2}(\hat{g} - i\hat{a})^2$$

# Gluon emissions

❁ Long distance contribution is negligible:  $G \sim \tau^2$ ,  $F - K \sim \tau^3$ .

❁ Short distance contribution is leading:  $G \sim \tau$ ,  $F - K \sim \tau \ln \tau$



What about large logarithmic corrections in the two-jet region?

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# Physically relevant degrees of freedom

❁ Our analysis is based on **the strategy of expanding by regions**

❖ M. Beneke and V. Smirnov, Nucl. Phys. B522 (1998) 321

❖ V. Smirnov and E. Rakhmetov, Theor. Math. Phys. 120 (1999) 870

❁ The small parameter is  $\lambda \ll 1$  and  $\tau \sim \lambda^2$ .

The moment of a particle in the right hemisphere ( $\mathbf{k} \cdot \mathbf{n} > 0$ )

$$k = u \frac{n_+}{2} + v \frac{n}{2} + k_{\perp}, \quad k^2 = uv - \mathbf{k}_{\perp}^2$$

Phase space restriction:  
 $\Theta(\tau - v)\Theta(u - v)\Theta(1 - u)$

$\Rightarrow$

**Right-collinear mode**

$u \sim 1, \quad v \sim \lambda^2, \quad \mathbf{k}_{\perp} \sim \lambda$

# Soft modes

$$k = u \frac{n_+}{2} + v \frac{n}{2} + k_{\perp}, \quad k^2 = uv - \mathbf{k}_{\perp}^2$$

For the collinear momentum, the condition  $\Theta(u - v)$  is not homogeneous, thus  $\Theta(u - v) = 1 - \Theta(v - u)$ :

Phase space restriction:  
 $-\Theta(\tau - v)\Theta(v - u)$



Soft mode

$$u \sim \lambda^2, v \sim \lambda^2, \mathbf{k}_{\perp} \sim \lambda^2$$

The integration of the standard two-parton antenna of the  $(n, n_+)$ -dipole:

$$\int_0^{\tau} dv v^{-\varepsilon} \left[ - \int_0^v du u^{-\varepsilon} \right] \frac{\alpha_s}{\pi UV} = \int_0^{\tau} dv v^{-\varepsilon} \left[ \int_v^{\infty} du u^{-\varepsilon} \right] \frac{\alpha_s}{\pi UV}$$

The same right-hemisphere condition:  $u > v$ .

# Bookkeeping of all modes

Region	Scale	Power counting $Q^{-1}(k \cdot n, k_{\perp}, k \cdot n_{+})$
Hard	$Q^2$	$(1, 1, 1)$
Right collinear	$\tau Q^2$	$(1, \lambda, \lambda^2)$
Left collinear	$\tau Q^2$	$(\lambda^2, \lambda, 1)$
Soft	$(\tau Q)^2$	$(\lambda^2, \lambda^2, \lambda^2)$

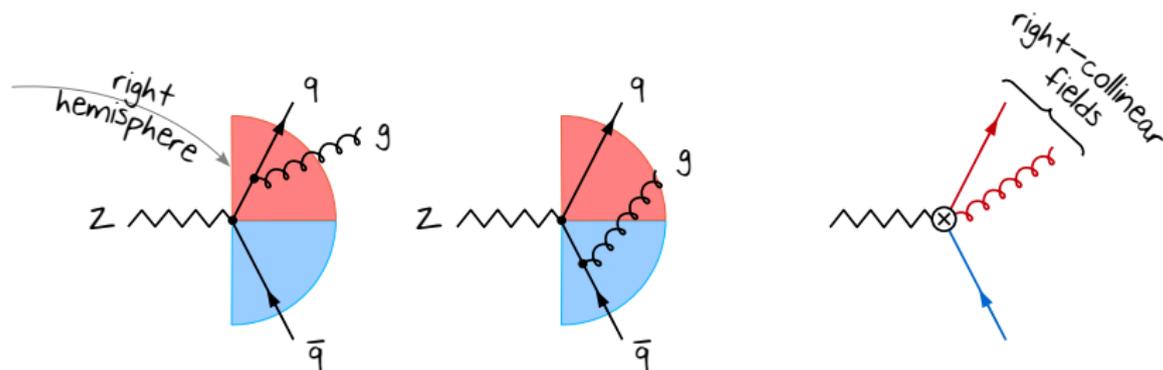
# Local operator

Leading operator:

$$\bar{\psi}\Gamma\psi \Rightarrow \hat{O}_2 = \bar{\xi}(n_+)\Gamma\xi(n)$$

Subleading operator:

$$\bar{\psi}\left(\frac{n}{2} - \frac{n_+}{2}\right)\psi \Rightarrow \hat{O}_3 = \bar{\xi}(n_+)A_{\perp}(n_+)\xi(n) - \bar{\xi}(n_+)A_{\perp}(n)\xi(n) \sim \lambda\hat{O}_2$$

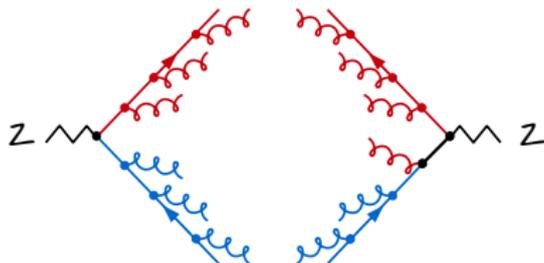


# Factorization

- ✿ Independent right and left collinear radiation.

Starting with planar gauge:

$$\langle A^\mu A^\nu \rangle = \frac{-i}{k^2} \left( g^{\mu\nu} - \frac{k^\mu n_0^\nu + k^\nu n_0^\mu}{k \cdot n_0} \right), \quad n_0 = \frac{n_+}{2} + \frac{n_-}{2}$$



Power suppression of the interference between hemispheres for collinear particles.

# Factorization

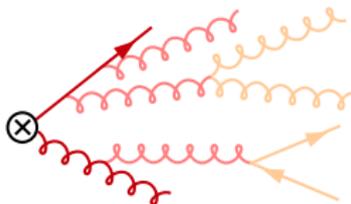
- ❁ Independent right and left collinear radiation.
- ❁ Independent gauge transformation. Independent QCD (SCET).  
copies

Light-cone gauges are extremely convenient:

$$n \cdot A(n_+) = 0, \quad n_+ \cdot A(n) = 0$$

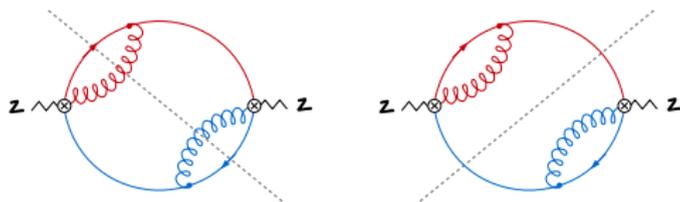
All terms of perturbative expansion has the same power counting in  $\lambda$ .

⇒ Automatic hierarchy from boost:



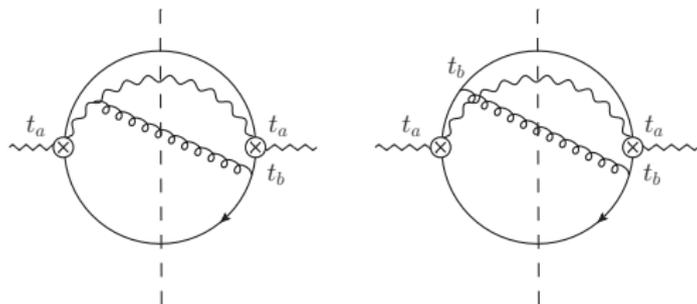
# Factorization

- ❁ Independent right and left collinear radiation.
- ❁ Independent gauge transformation. Independent QCD (SCET) copies
- ❁ No mixing with the leading operator. No interference between.  $\bar{\xi}(n_+)A_\perp(n_+)\xi(n)$  and  $\bar{\xi}(n_+)A_\perp(n)\xi(n)$



The reason is  $\hat{O}_3 \sim \lambda \hat{O}_2$ , the factor  $(\lambda^2)^{1/2}$  cannot appear.

# Soft modes



$$ifabc t^c \frac{e(k_{\text{soft}}) \cdot n_+}{k_{\text{soft}} \cdot n_+} + t^b t^a \frac{e(k_{\text{soft}}) \cdot n_+}{k_{\text{soft}} \cdot n_+} = t^a t^b \frac{e(k_{\text{soft}}) \cdot n_+}{k_{\text{soft}} \cdot n_+}$$

$$|M|^2 = |M_0|^2 \times \frac{\alpha_s C_F}{\pi} \left( \frac{n}{n \cdot k} - \frac{n_+}{n_+ \cdot k} \right)^2$$

# Local operator in SCET

$$\hat{O}_3 = \hat{O}_{3R} + \hat{O}_{3L}, \quad \hat{O}_{3R} = 2g_s \bar{\xi}_{n_+} \hat{A}_{\perp, n_+} \xi_{n_+}, \quad \hat{O}_{3L} = 2g_s \bar{\xi}_{n_+} \hat{A}_{\perp, n} \xi_n$$

Arbitrary gauge

❖ C. W. Bauer, D. Pirjol, I. W. Stewart, Phys. Rev. D65 (2002) 054022

❖ M. Beneke, T. Feldmann, Phys. Lett. B553 (2003) 267

$$\xi = Y W^\dagger \xi', \quad g_s A_\perp = Y \left( W^\dagger i D'_{\perp C} W - i \partial_\perp \right) Y^\dagger,$$

Wilson lines:

$$W_{n_+} = P \exp \left[ i g_s \int_0^\infty ds n \cdot A'_c(sn) \right], \quad Y_{n_+} = P \exp \left[ i g_s \int_0^\infty ds n \cdot A'_s(sn) \right]$$

SCET operator:

$$\hat{O}_3 = 2g_s \bar{\xi}'_{n_+} \tilde{A}_{\perp, n_+} W_{n_+} Y_{n_+}^\dagger Y_n W_n^\dagger \xi'_n + 2g_s \bar{\xi}'_{n_+} W_{n_+} Y_{n_+}^\dagger Y_n W_n \tilde{A}_{\perp, n} \xi'_n,$$

where

$$\tilde{A}_\perp = A'_\perp - \frac{i}{g_s} W \left[ \partial_\perp, W^\dagger \right].$$

# Factorization formula

We are under the conditions of:

- ❖ C.W. Bauer, S.P Fleming, C. Lee, and G. Sterman, Phys. Rev. D78 (2008) 034027

$$G(\tau) = 2 H_3(Q^2, \mu^2) \int dp_L^2 dp_R^2 dk \\ \times \Sigma_{\perp}(p_R^2, \mu^2) J(p_L^2, \mu^2) S_T(k, \mu^2) \Theta(Q^2 \tau - p_L^2 - p_R^2 - Qk)$$

- ❖ where  $S_T(k, \mu^2)$  is the same soft factor
- ❖  $J(p_L^2, \mu^2)$  is the jet function
- ❖  $H_3(Q^2, \mu^2)$  is the square of the hard matching coefficient of the QCD operator  $(n - n_+)^{\mu} j_{\mu}/2$  onto SCET operator  $\hat{O}_3$ .

# Factorization formula

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- ◆ C.W. Bauer, S.P Fleming, C. Lee, and G. Sterman, Phys. Rev. D78 (2008) 034027

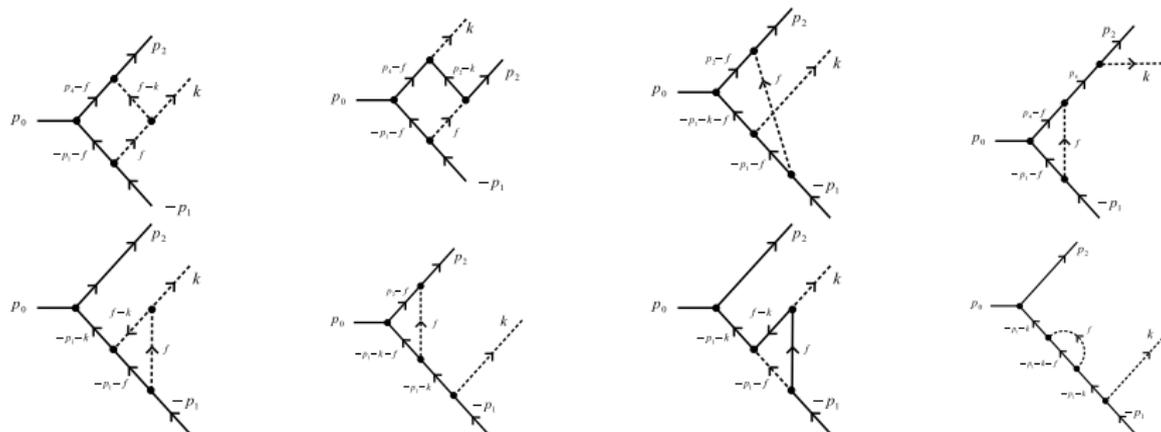
$$G(\tau) = 2 H_3(Q^2, \mu^2) \int dp_L^2 dp_R^2 dk \\ \times \Sigma_{\perp}(p_R^2, \mu^2) J(p_L^2, \mu^2) S_T(k, \mu^2) \Theta(Q^2 \tau - p_L^2 - p_R^2 - Qk)$$

New object:

$$\Sigma_{\perp}(p^2, \mu^2) = \frac{g_s^2}{(p \cdot n) Q^2 N_c} \\ \times \frac{1}{\pi} \text{Im} \left[ i \int d^{\mathcal{D}} x e^{-ipx} \left\langle 0 \left| T \left\{ \left( \bar{\xi}_{n_+} \tilde{A}_{\perp, n_+} W_{n_+} \right) (x) \frac{\hat{n}}{2} \left( W_{n_+}^{\dagger} \tilde{A}_{\perp, n_+} \xi'_{n_+} \right) (0) \right\} \right| 0 \right\rangle \right]$$

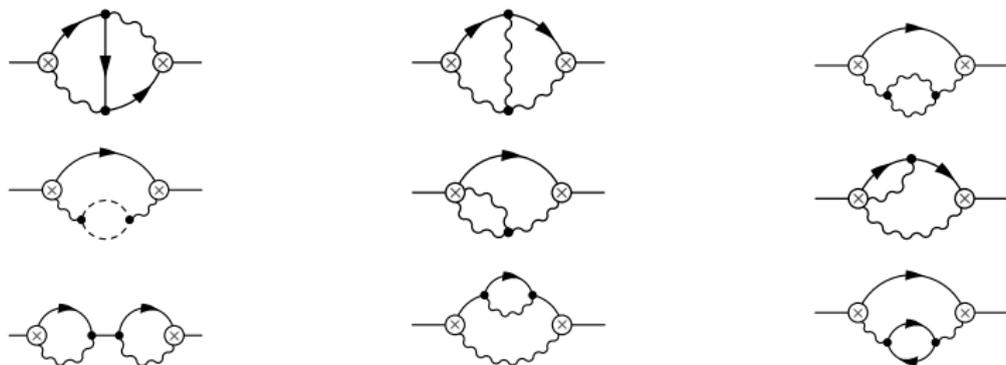
$$\text{If } \mathcal{D} = 4: \quad \Sigma_{\perp}^{(0)}(p^2, \mu^2) = \frac{\alpha_s C_F}{2\pi Q^2}, \quad G^{(0)} = \frac{\alpha_s \tau}{\pi}$$

# Matching coefficient



# Transverse self energy

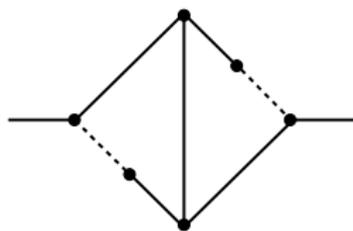
The set of two-loop diagrams:



Using IBP relation, any FI can be expressed through the master integrals:



# Master in main topology



$$\begin{aligned}
 J = & \frac{(2\pi) (\tilde{p}^2)^{d-3} (\rho \cdot n)^2}{\sin \pi d} \left[ \frac{2 \|2 - d/2\| \|d/2 - 1\|^3}{\|d - 1\|^2} \cos \frac{\pi d}{2} {}_3F_2 \left( \begin{matrix} 1, 1, d/2 - 1 \\ d - 1, d/2 \end{matrix} \middle| 1 \right) \right. \\
 & + \frac{2 \|d/2\|^2}{(d - 2)^4 \|2d - 4\|} {}_4F_3 \left( \begin{matrix} d - 2, d - 2, d - 2, d/2 \\ d - 1, d - 1, 2d - 4 \end{matrix} \middle| 1 \right) \\
 & \left. + \frac{2 \|d - 2\| \|d/2\|^2}{\|2d - 4\| (d - 2)^2} \sum_{n=0}^{\infty} \frac{\|n + 1\| \|d/2 + n - 1\|}{\|d/2 + n\| \|d + n - 1\|} {}_3F_2 \left( \begin{matrix} d - 2, d - 2, d/2 \\ 2d - 4, d + n - 1 \end{matrix} \middle| 1 \right) \right].
 \end{aligned}$$

where  $d = \mathcal{D} - 2$  and  $\|x\| = \Gamma(x)$ .

# Perturbative corrections

$$H_3^{(1)}(Q^2, \mu^2) = \left(\frac{Q^2}{\mu^2}\right)^{-\varepsilon} \left\{ 2C_F \left[ -\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left( 3 - \frac{2\pi^2}{3} \right) + \frac{\pi^2}{2} + 17 - 16\zeta(3) \right] \right. \\ \left. + C_A \left[ \frac{1}{\varepsilon} \left( \frac{2\pi^2}{3} - 4 \right) - 16 + \frac{2\pi^2}{3} + 16\zeta(3) \right] + O(\varepsilon) \right\}$$

$$\Sigma_{\perp}^{(1)}(p^2, \mu^2) = \left(\frac{p^2}{\mu^2}\right)^{-\varepsilon} \left\{ 2C_F \left[ \frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left( \frac{2\pi^2}{3} - \frac{9}{2} \right) - \frac{5\pi^2}{6} - \frac{85}{4} + 22\zeta(3) \right] \right. \\ \left. + C_A \left[ \frac{1}{\varepsilon} \left( \frac{23}{3} - \frac{2\pi^2}{3} \right) + \frac{503}{18} - 22\zeta(3) \right] - 2T_F N_f \left( \frac{2}{3\varepsilon} + \frac{19}{9} \right) + O(\varepsilon) \right\}$$

## Fixed order

$$G(\tau) = G^{(0)}(\tau) \left( 1 + \frac{\alpha_S}{4\pi} G^{(1)}(\tau) \right),$$

$$G^{(1)} = -4C_F \ln^2 \frac{1}{\tau} + \ln \frac{1}{\tau} \left[ C_F \left( \frac{4\pi^2}{3} - 14 \right) + C_A \left( \frac{23}{3} - \frac{2\pi^2}{3} \right) - \frac{4}{3} T_F N_f \right] \\ + C_F \left[ -\frac{31}{2} + 12\zeta(3) \right] + C_A \left[ \frac{353}{18} - 6\zeta(3) \right] - \frac{50}{9} T_F N_f,$$

Leading structure function:

$$F = 1 + \frac{\alpha_S}{4\pi} C_F \left( -4 \ln^2 \frac{1}{\tau} + 6 \ln \frac{1}{\tau} - 2 + \frac{2\pi^2}{3} \right).$$

# Resummation of large logarithms

$$G(\tau) = 2 H_3(Q^2, \mu^2) \frac{1}{2\pi i} \int_C \frac{dv}{v} \tilde{\Sigma}_\perp(sQ^2, \mu^2) j(sQ^2, \mu^2) s_T(sQ, \mu^2),$$

where  $s = (vQ^2 e^{\gamma_E})^{-1}$ . If we put  $\mu^2 = \tau Q^2$ :

$$\tilde{\Sigma}_\perp(sQ^2, \tau Q^2) \equiv \int_0^\infty dp^2 e^{-vp^2} \Sigma_\perp^{(0)}(p^2, \tau Q^2) \sim \frac{\alpha_S(\tau Q^2) C_F}{2\pi} \frac{1}{vQ^2},$$

then

$$G(\tau) \sim \frac{\alpha_S(\tau Q^2) C_F}{\pi} H_3(Q^2, \tau Q^2) \frac{1}{2\pi i} \int_C \frac{dv}{v^2 Q^2} s_T(sQ, \tau Q^2),$$

where “ $\sim$ ” implies a non-log factor  $1 + C\alpha_s(Q^2)$

# Ratio of the distributions

$$G(\tau) \sim \tau \frac{\exp[\mathcal{F}_{H_3}(L, \alpha_S) + \mathcal{F}_s(L, \alpha_S)]}{\Gamma[2 - \gamma(L, \alpha_S)]},$$

The same procedure for the leading structure function results in:

$$F(\tau) \sim \frac{\exp[\mathcal{F}_{H_2}(L, \alpha_S) + \mathcal{F}_s(L, \alpha_S)]}{\Gamma[1 - \gamma(L, \alpha_S)]}$$

the latter coincides with the classical NLL result

- ❖ S. Catani, L. Trentadue, G. Turnock, B.R. Webber, Nucl. Phys. B407 (1997) 3

$$\frac{G(\tau)}{F(\tau)} \sim \tau \frac{\exp[\mathcal{F}_{H_3}(L, \alpha_S) - \mathcal{F}_{H_2}(L, \alpha_S)]}{1 - \gamma(L, \alpha_S)}.$$

# Resummation factor

$$\frac{G(\tau)}{F(\tau)} = G^{(0)}(\tau) e^{\omega(\tau)}, \quad G^{(0)} = \frac{\alpha_S}{\pi} C_F \tau$$

where

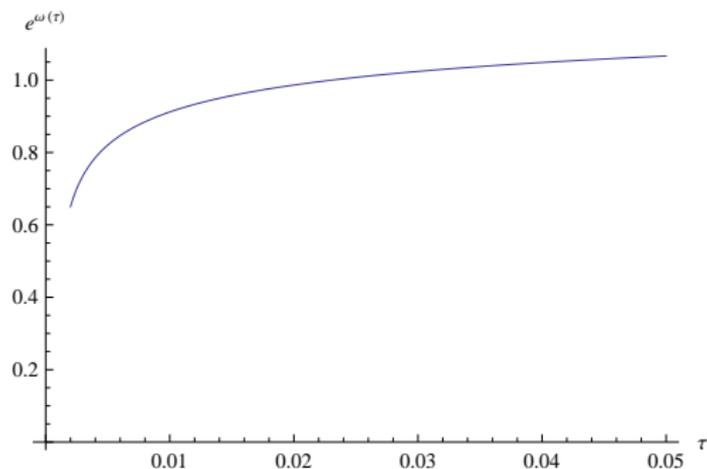
$$\omega(\tau) = \frac{\gamma_0^{H_3} - \gamma_0^{H_2} - \beta_0}{\beta_0} \ln(1 - \lambda) - \ln[1 - \gamma(\lambda)] + \alpha_S(Q^2) (\mathcal{C}_3 - \mathcal{C}_2),$$

$$\gamma(\lambda) = \frac{\Gamma_0}{\beta_0} [\ln(1 - 2\lambda) - \ln(1 - \lambda)], \quad \lambda = \frac{\beta_0 \alpha_S(Q^2)}{4\pi} \ln \frac{1}{\tau}$$

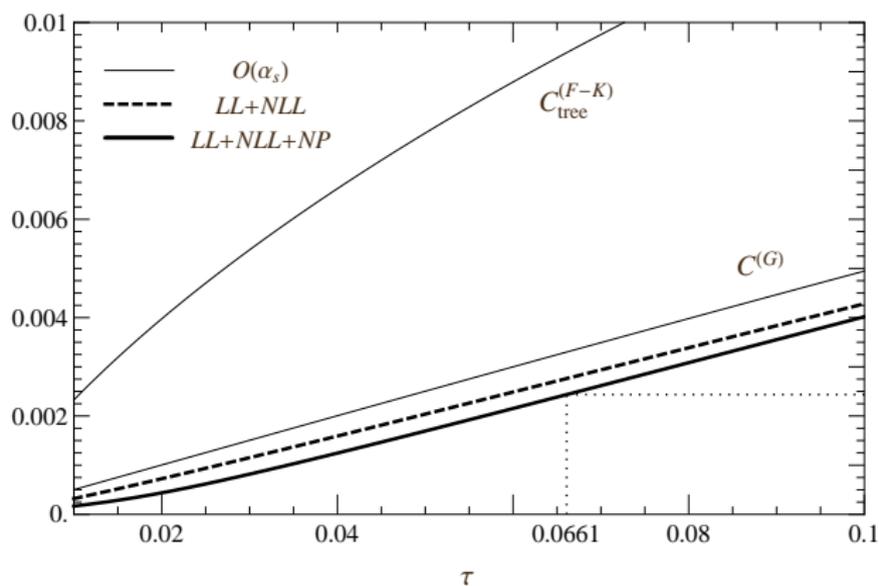
There are single-log terms only

# Resummation factor

$$\frac{G(\tau)}{F(\tau)} = G^{(0)}(\tau) e^{\omega(\tau)}, \quad G^{(0)} = \frac{\alpha_S}{\pi} C_F \tau$$

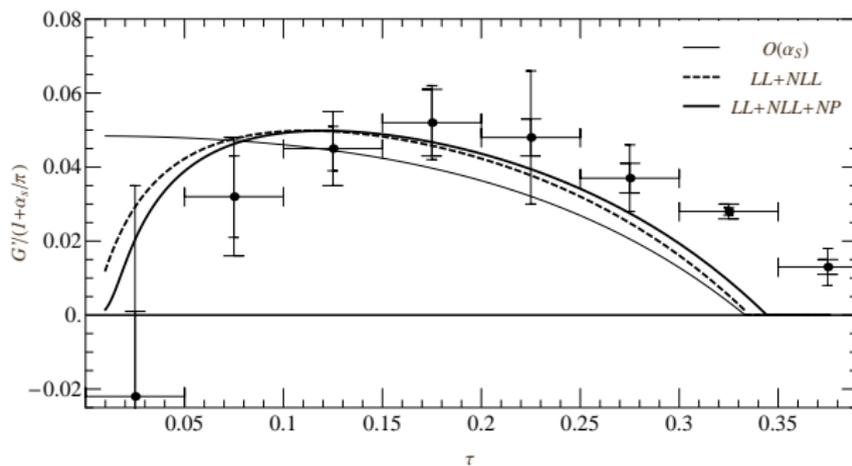


# Correction for the asymmetry



$$C^{(G)}(\tau) \Big|_{\tau=1-\langle T \rangle=0.066} = 0.0024 \pm 0.0002,$$

# Comparison with the data



❖ **OPAL** Collaboration, G. Abbiendi *et al.*, Phys. Lett. B440 (1998) 393

*Measurement of the longitudinal cross-section using the direction of the thrust axis in hadronic events at LEP*

# Outline

Motivation

Forward-backward asymmetry

Longitudinal structure function

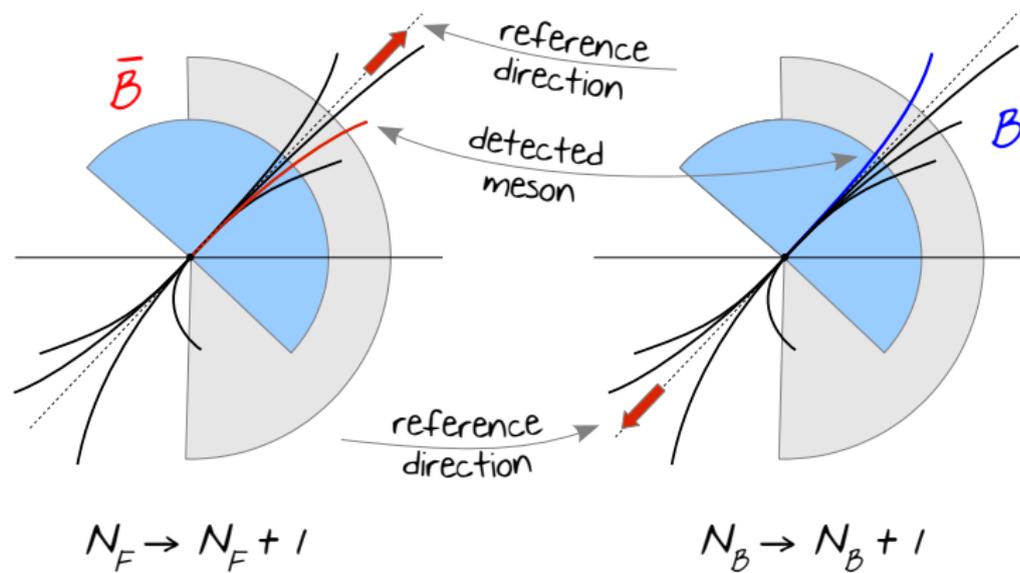
Hagiwara, Kirilin, JHEP10 (2010) 093

Mistaged events

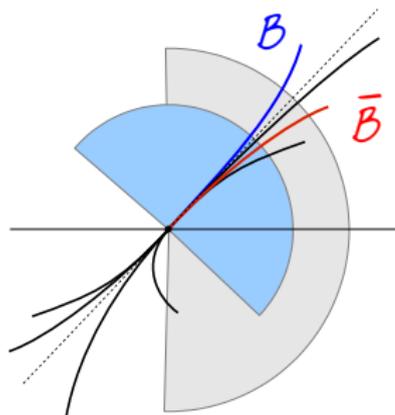
Hagiwara, Kirilin, in preparation

# Tag procedure

FB asymmetry is the correlation of a charge (flavour) and a momentum (reference direction)



# Neutral hemispheres



Irrelevant?

Detector mistake?

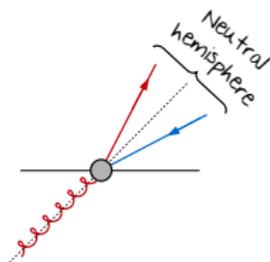
Analysis bug?

Reference direction?

$$N_F \rightarrow N_F + w_F$$

$$N_B \rightarrow N_B + w_B$$

$$w_F + w_B = 1$$



F-K leading contribution

e.g, for the inclusive charge reconstruction all events are sorted into five different categories:  $N_F$ ,  $N_B$ ,  $N_F^{(d)}$ ,  $N_B^{(d)}$ ,  $N_{same}$ .

# Hemisphere correlations

Almost all high-statistic b-physics analysis at LEP made control systematic uncertainties using data itself via the double hemisphere tagging.

## ✿ Flavour tag efficiency

$$F_S = \varepsilon_b R_b + \varepsilon_c R_c + \varepsilon_{uds}(1 - R_b - R_c)$$

$$F_d = \varepsilon_b^{(d)} R_b + \varepsilon_c^{(d)} R_c + \varepsilon_{uds}^{(d)}(1 - R_b - R_c)$$

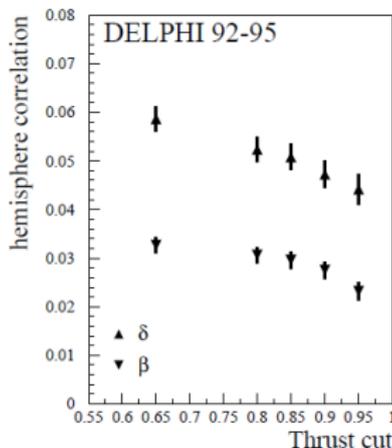
$$\varepsilon_f^2 \neq \varepsilon_f^{(d)} = (1 + \rho_f) \varepsilon_f^2$$

## ✿ Charge tag efficiency

$$A_{FB}^{meas} = (2\omega_q - 1) F_q A_{FB}^{(0)}$$

$$N_{opp} \sim \omega_b^2 + (1 - \omega_b)^2$$

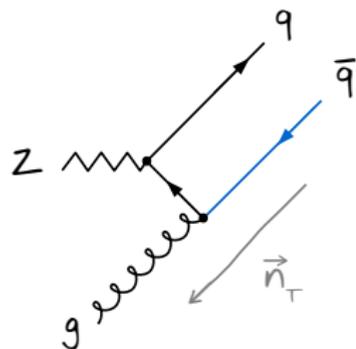
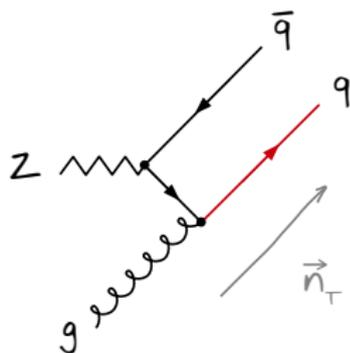
$$N_{same} \sim 2\omega_b(1 - \omega_b)$$



✿ W. Liebig, XXXVIII th Rencontres de Moriond (2003)

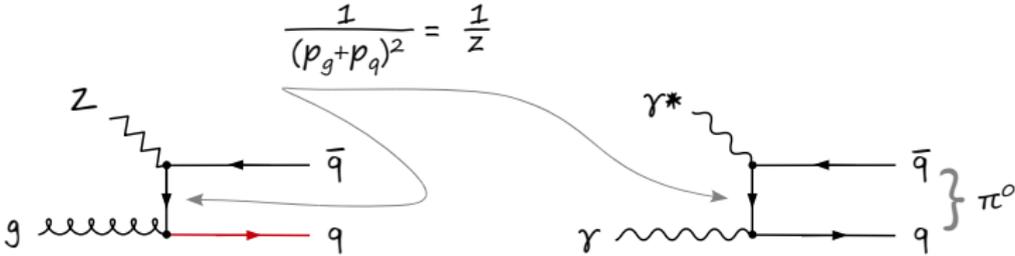
# Tree level

$$g_{\perp}^{\mu\nu}(n, n_+) \underbrace{F(x_q, x_{\bar{q}})}_{\text{symmetric}} + a^{\mu\nu}(n, n_+) \underbrace{K(x_q, x_{\bar{q}})}_{\text{antisymmetric}}, \text{ where } x_i = 2E_i/Q.$$



In  $F - K$  combination, only one diagram survives. Which one?  
Depends on the charge detected.

# Expansion by regions



$p_q^\mu = z \frac{n_+^\mu}{2} + \bar{z} p^2 \frac{n^\mu}{2} + p_\perp^\mu$ , the process is essentially nonlocal:



the contribution to the distribution is  $\sim \int_0^\tau dp^2 \int_0^1 dz \frac{\bar{z}}{z} \Theta(z - \bar{z}p^2)$

$$\int_0^\tau dp^2 \int_0^1 dz \frac{\bar{z} z^\epsilon}{z} - \int_0^\tau dp^2 \int_{p^2}^\infty dz \frac{z^\epsilon}{z} = \tau \ln \frac{1}{\tau}$$

# Region bookkeeping

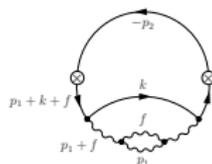
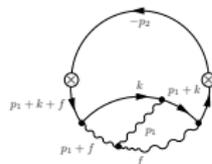
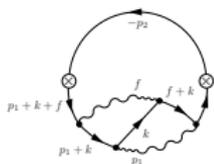
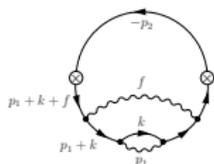
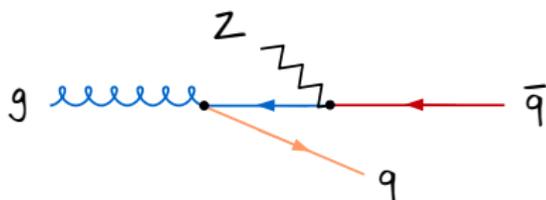
## ❁ Virtual

- ▶ hard gluon + r.collinear quark
- ▶ r.collinear gluon + r.collinear quark
  
- ▶ hard gluon + soft quark
- ▶ r.collinear gluon + soft quark
- ▶ soft gluon + soft quark

## ❁ Real

- ▶ r.collinear + r.collinear quark
- ▶ l.collinear gluon + r.collinear quark
- ▶ soft gluon + r.collinear quark
  
- ▶ r.collinear gluon + soft quark
- ▶ l.collinear gluon + soft quark
- ▶ soft gluon + soft quark

# I.collinear gluon + soft quark



$$\left(\frac{\tau Q^2}{\mu^2}\right)^{-\epsilon} \left(\frac{\tau^2 Q^2}{\mu^2}\right)^{-\epsilon} \left\{ C_A \left[ \frac{16}{\epsilon^3} + \frac{140}{3\epsilon^2} + \frac{1}{\epsilon} \left( \frac{1396}{9} - \frac{28\pi^2}{3} \right) \right. \right. \\ \left. \left. + \frac{13448}{27} - 26\pi^2 - \frac{344}{3} \zeta(3) \right] + C_F \left[ \frac{1}{\epsilon} \left( 8 - \frac{4\pi^2}{3} \right) + 40 - 4\pi^2 - 24\zeta(3) \right] \right\}.$$

# Perturbative result

Surprisingly simple:

$$\begin{aligned} & (F - K)_{(0)} \left( 1 - \frac{\beta_0}{\varepsilon} \frac{\alpha_S}{4\pi} \right) + (F - K)_{(1)}^{\log} \\ &= \frac{\alpha_S}{\pi} C_F \tau \ln \frac{1}{\tau} \left\{ 1 + \frac{\alpha_S}{4\pi} \left[ C_A \left( -2 \ln^2 \tau - \frac{43}{6} \ln \tau + \frac{5\pi^2}{3} + \frac{61}{9} \right) \right. \right. \\ & \quad \left. \left. + C_F \left( -2 \ln^2 \tau - \frac{3}{2} \ln \tau + \frac{\pi^2}{3} - 1 \right) \right] \right\} \end{aligned}$$

- ✿ Sudakov suppression as for one gluon and one quark jets!
- ✿ The ratio  $(F - K)/F$  has LL contribution

# Leading Log estimation

Resummation factor as the ratio of LL contributions for the gluon and quark jets:

$$\exp \left\{ -\ln \frac{1}{\tau} \times \frac{2(C_A - C_F)}{\beta_0 \lambda} [(1 - 2\lambda) \ln(1 - 2\lambda) - 2(1 - \lambda) \ln(1 - \lambda)] \right\},$$

where

$$\lambda = \frac{\beta_0}{4\pi} \alpha_s \ln \frac{1}{\tau}.$$

Numerically it is about the forth part of the correction:

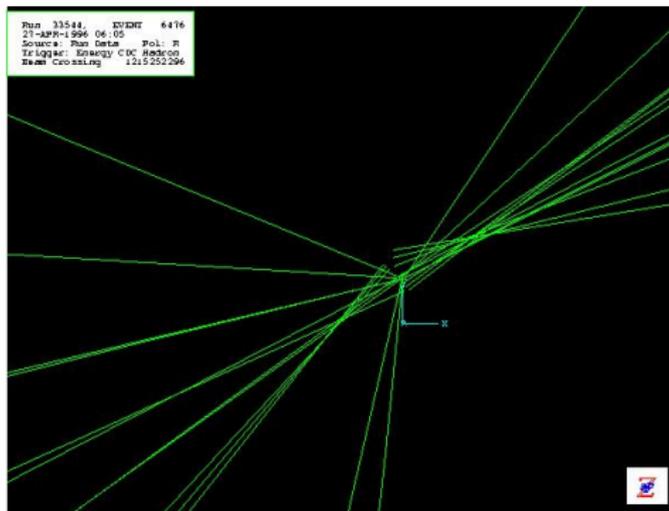
$$\exp \{ \dots \} \Big|_{\tau = \tau_{mean}} \approx 0.75$$

# Conclusion

- ✿ If the angular distributions  $1 + \cos^2 \theta$ ,  $\sin^2 \theta$  and  $\cos \theta$  are measured independently, one finds “jets” with different internal structure.
- ✿ SCET is the relevant framework to establish factorization formulae and perform resummation
- ✿ The corrections to the FB asymmetry are either negligible ( $G$ ) or can be **probably** correctly simulated by standard MC tools ( $F - K$ ). How well it was done?
- ✿ Probably only a future  $e^+e^-$  collider (ILC?) can find out the solution of the long-standing problem with the FB asymmetry.

# SLAC Event Display Collection

$Z \rightarrow b\bar{b}$  back-to-back event (x-y plane)



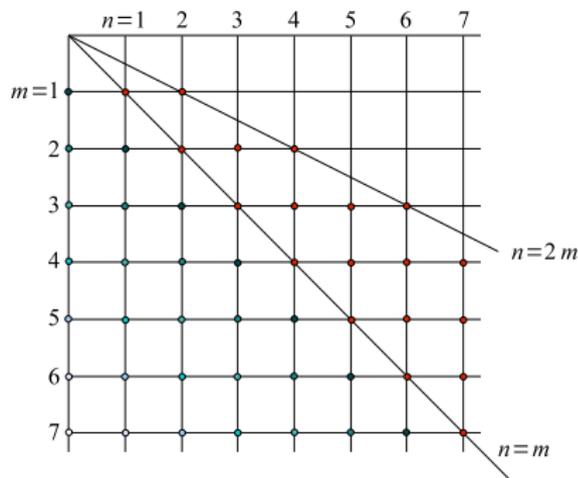
looks very simple but the physics is interesting.

Back up slides

# LL, NLL, NNLL...

$$R(y \ll 1) = \sum_{m,n} G_{mn} \alpha_s^m L^n,$$

$$\text{where } L = \ln \frac{1}{y}.$$



If  $\alpha_s L \sim 1$ , strictly speaking, we should sum over the whole red sector – not possible in real life.

$$R(y) = \mathcal{C}(\alpha_s) \exp [Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots] + D(y, \alpha_s),$$

# LL, NLL, NNLL...

However we can sum all logarithms in the sector which exponentiate:

$$R(y) = \left( 1 + \sum_{n=1}^{\infty} \mathcal{C}_n \alpha_s^n \right) \exp [Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots] + D(y, \alpha_s),$$

so that  $g_i(0) = 0$  and  $D(y, \alpha_s) = O(y)$ .

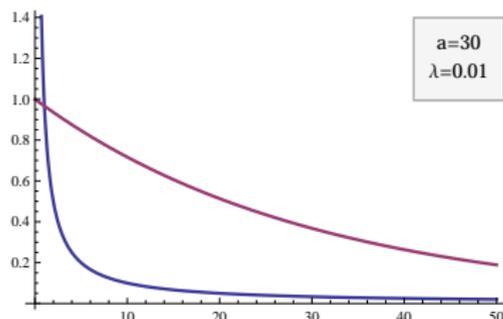
*LL*: exact  $g_1(\alpha_s L)$ ,

*NLL*: exact  $g_1(\alpha_s L)$ ,  $g_2(\alpha_s L)$ ,  $\mathcal{C}_1$ ,

*NNLL*: exact  $g_1(\alpha_s L)$ ,  $g_2(\alpha_s L)$ ,  $g_3(\alpha_s L)$ ,  $\mathcal{C}_1$ ,  $\mathcal{C}_2$ .

NLL level is the first reasonable approximation for a quantitative comparison to experiment.

# A very simple integral



The exponential integral function

$$\begin{aligned}\int_0^{\infty} \frac{e^{-x/a}}{x+\lambda} dx &= e^{\lambda/a} \int_{\lambda/a}^{\infty} \frac{e^{-x}}{x} dx \\ &= -e^{\lambda/a} \text{Ei}(-\lambda/a)\end{aligned}$$

Expansion in the  $\lambda \ll a$  limit:

$$\int_0^{\infty} \frac{e^{-x/a}}{x+\lambda} dx = \int_0^{\infty} \frac{e^{-x}}{x+\lambda/a} dx \neq \sum_{n=0}^{\infty} \left(-\frac{\lambda}{a}\right)^n \int_0^{\infty} \frac{e^{-x}}{x^{1+n}} dx$$

One can not ignore the region  $x \sim \lambda$ .

# Separation of the regions

- ▶ Dimensional regularization  $dx \rightarrow \left(\frac{x}{\mu}\right)^\varepsilon dx$

# Separation of the regions

- ▶ Dimensional regularization  $dx \rightarrow \left(\frac{x}{\mu}\right)^\varepsilon dx$
- ▶ Integrand expansion in the soft region:

$$I(x) = \frac{e^{-x/a}}{x+\lambda} \rightarrow I_{\text{soft}}(x) = \frac{1}{x+\lambda} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-x}{a}\right)^n,$$

Soft region

$x \sim \lambda$

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Soft region

$x \sim \lambda$

- ▶ Subtraction of the soft region:

$$\int_0^\infty I(x) \left(\frac{x}{\mu}\right)^\epsilon dx = \int_0^\infty [I(x) - I_{\text{soft}}(x)] \left(\frac{x}{\mu}\right)^\epsilon dx + \int_0^\infty I_{\text{soft}}(x) \left(\frac{x}{\mu}\right)^\epsilon dx$$

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- ▶ Integrand expansion in the hard region:

$$I_{\text{hard}}(x) = I(x) - I_{\text{soft}}(x) = \frac{e^{-x/a}}{x+\lambda} - \frac{1}{x+\lambda} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-x}{a}\right)^n$$
$$\rightarrow \sum_{n=0}^{\infty} \left(-\frac{\lambda}{x}\right)^n \frac{e^{-x/a}}{x}$$

Hard  
region

$$x \sim a$$

# Integration over regions

- ▶ In fact, the contributions are separated out

$$\int_0^\infty I(x) \left(\frac{x}{\mu}\right)^\varepsilon dx = \sum_{n=0}^\infty \left(-\frac{\lambda}{a}\right)^n \\ \times \left\{ \left(\frac{a}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{hard}}^{(n)}(x) x^\varepsilon dx + \left(\frac{\lambda}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{soft}}^{(n)}(x) x^\varepsilon dx \right\}$$

# Integration over regions

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$$\int_0^\infty I(x) \left(\frac{x}{\mu}\right)^\varepsilon dx = \sum_{n=0}^\infty \left(-\frac{\lambda}{a}\right)^n \\ \times \left\{ \left(\frac{a}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{hard}}^{(n)}(x) x^\varepsilon dx + \left(\frac{\lambda}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{soft}}^{(n)}(x) x^\varepsilon dx \right\}$$

- ▶ Each of which can be easily evaluated

$$\int_0^\infty I_{\text{hard}}^{(n)}(x) x^\varepsilon dx = \int_0^\infty \frac{e^{-x}}{x^{1+n}} x^\varepsilon dx = \Gamma(\varepsilon - n)$$

$$\int_0^\infty I_{\text{soft}}^{(n)}(x) x^\varepsilon dx = \frac{1}{n!} \int_0^\infty \frac{x^n}{x+1} x^\varepsilon dx = -\frac{\pi}{n! \sin \pi \varepsilon}$$

# Integration over regions

- ▶ In fact, the contributions are separated out

$$\int_0^\infty I(x) \left(\frac{x}{\mu}\right)^\varepsilon dx = \sum_{n=0}^{\infty} \left(-\frac{\lambda}{a}\right)^n \\ \times \left\{ \left(\frac{a}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{hard}}^{(n)}(x) x^\varepsilon dx + \left(\frac{\lambda}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{soft}}^{(n)}(x) x^\varepsilon dx \right\}$$

- ▶ The singularities with respect to  $\varepsilon$  and the  $\mu$ -dependence drop out of the sum of the all contributions

$$\int_0^\infty \frac{e^{-x/a}}{x+\lambda} dx = \sum_{n=0}^{\infty} \left(\frac{\lambda}{a}\right)^n \frac{\pi}{\sin(\pi\varepsilon)} \left[ \left(\frac{a}{\mu}\right)^\varepsilon \frac{1}{\Gamma(1-\varepsilon+n)} - \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^\varepsilon \right] \\ = \sum_{n=0}^{\infty} \left(\frac{\lambda}{a}\right)^n \frac{1}{n!} \left[ \ln \frac{a}{\lambda} + \psi^{(0)}(n+1) \right].$$