Applications of heavy-to-light currents at NNLO in SCET

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Outline

- Introduction and Motivation
- Matching heavy-to-light currents from QCD onto SCET at NNLO
- Results and applications
  - Results for the matching coeffcients
  - Heavy-to-light form factor ratios
  - Exclusive radiative decays
  - Semi-inclusive $\bar{B} \to X_s\, \ell^+ \ell^-$

Introduction and motivation

- Heavy-to-light currents

$$\bar{q}\,\Gamma_i\,b \qquad \text{with} \qquad \Gamma_i = \{1,\, \gamma_5,\, \gamma^\mu,\, \gamma_5\gamma^\mu,\, i\sigma^{\mu\nu}\}$$

  govern many semi-leptonic and radiative B decays
  - $\bar{B} \to X_u\,\ell\,\nu$
  - Exclusive radiative decays
  - Semi-inclusive $\bar{B} \to X_s \ell^+ \ell^-$
  - Matrix elements of heavy-to-light currents (transition form factors) are inputs to factorization formulae in non-leptonic B decays *[Beneke,Buchalla,Neubert,Sachrajda'99,'00]*
- Experimental cuts (to eliminate backgrounds):
  Put us in kinematic region where the hadronic final state has large energy ($E \sim m_b$) but small invariant mass ($m_X \ll m_b$) $\quad\rightsquigarrow\quad$ SCET framework
- Many of these decays require precision beyond NLO
- Goal: Two-loop $\mathcal{O}(\alpha_s^2)$ matching coefficients for
  heavy-to-light currents from QCD onto SCET

Matching QCD onto SCET

- Generic heavy-to-light current $\bar{q}\,\Gamma_i\,b$ in SCET

$$[\bar{q}\,\Gamma_i\,b](0) = \sum_j \int ds\; \tilde{C}_i^j(s)\,\left[\bar{\xi}W_{hc}\right](sn_+)\,\Gamma_j'\,h_v(0) + \text{“three} - \text{body operators“} + \ldots$$

- Adopt momentum space representation for matching coefficients C_i^j

$$C_i^j(n_+p) = \int ds\; e^{isn_+p}\,\tilde{C}_i^j(s).$$

Γ_i	1	γ_5	γ^μ			$\gamma_5\gamma^\mu$			$i\sigma^{\mu\nu}$			
Γ_j'	1	γ_5	γ^μ	v^μ	n_-^μ	$\gamma_5\gamma^\mu$	$v^\mu\gamma_5$	$n_-^\mu\gamma_5$	$\gamma^{[\mu}\gamma^{\nu]}$	$v^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}v^{\nu]}$
C_i^j	C_S	C_P	C_V^1	C_V^2	C_V^3	C_A^1	C_A^2	C_A^3	C_T^1	C_T^2	C_T^3	C_T^4

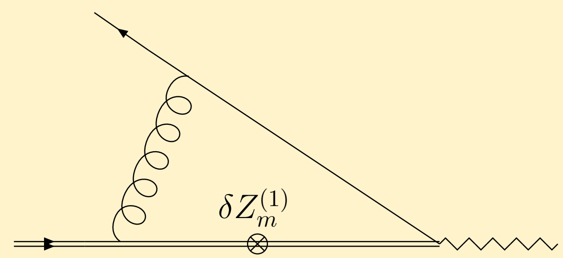
- Constraints: $C_P = C_S$, $C_A^i = C_V^i$ (in NDR scheme w/ anti-commuting γ_5)
- Moreover, $C_T^2 = C_T^4 = 0$ since pseudo-tensor current is reducible in four dimensions

Matching QCD onto SCET

- Perform matching with on-shell quarks. Use dim. reg. for UV and IR, $D = 4 - 2\epsilon$
- Intermediate step: parameterize QCD result in terms of 12 form factors F_i^j

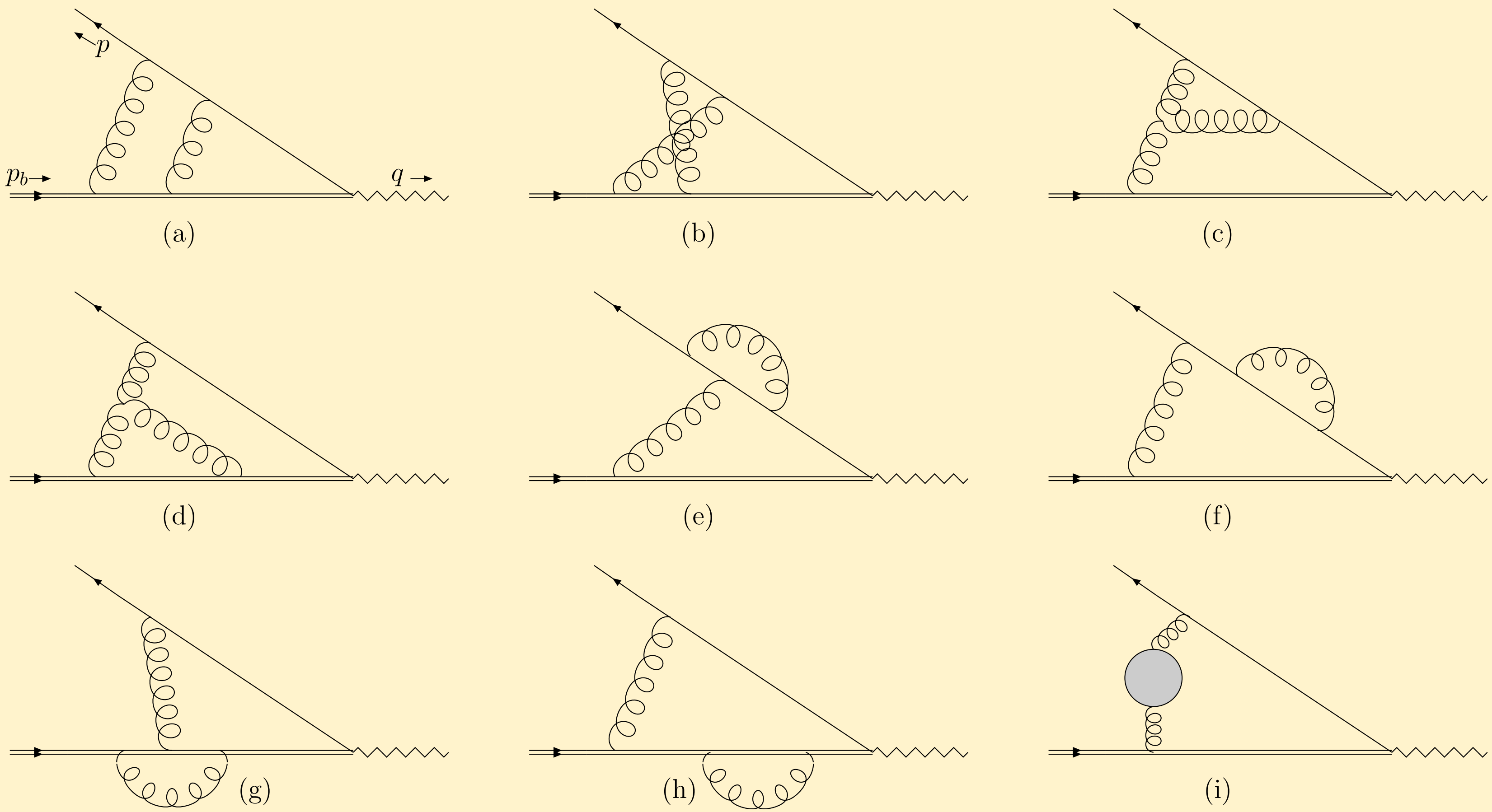
$$\langle q(p)|\,\bar{q}\,\Gamma_i\,b\,|b(p_b)\rangle = \sum_j F_i^j(q^2)\,\bar{u}(p)\,\Gamma'_j\,u(p_b)$$

- Kinematics: $q^2 = (p_b - p)^2 = (1 - u)\,m_b^2$
- The form factors F_i^j are UV finite, but IR divergent. Poles up to $1/\epsilon^{2L}$.
- UV renormalization:
  - Use on-shell scheme for m_b and for the quark fields, $\overline{\mathrm{MS}}$ scheme for α_s
  - Non-vanishing anomalous dimension of scalar and tensor current: Additional counterterms Z_S and Z_T *[Nanopoulos,Ross'79; Tarrach'81; Broadhurst,Grozin'94]*
- All UV renormalizations are simple multiplications except the one-loop mass counterterm



- F_i^j are (complicated) functions of u, up to transcendental weight 4.

Two-loop diagrams



- Work with $n_l = 4$ massless and one massive flavour (m_b)
- Charm mass can also be implemented (see plots later on)

Completing the matching

- Obtain C_i^j via $$C_i^j = Z_J^{-1}\, F_i^j$$
- Z_J is the renormalization factor of the SCET current $\left[\bar{\xi} W_{hc}\right] \Gamma'_j h_v$. It is universal, subtracts the IR divergences and yields finite C_i^j.
- Perturbative expansion

$$F_i^j = \sum_{k=0}^{\infty} \left(\frac{\alpha_s^{(5)}}{4\pi}\right)^k F_i^{j,(k)}, \qquad Z_J = 1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s^{(4)}}{4\pi}\right)^k Z_J^{(k)}, \qquad C_i^j = \sum_{k=0}^{\infty} \left(\frac{\alpha_s^{(4)}}{4\pi}\right)^k C_i^{j,(k)}$$

- Need D-dim. relation between four– and five – flavour α_s

$$\begin{aligned}
\alpha_s^{(5)} &= \alpha_s^{(4)} \left[1 + \frac{\alpha_s^{(4)}}{4\pi}\, \delta\alpha_s^{(1)} + \mathcal{O}(\alpha_s^2)\right] \\
\delta\alpha_s^{(1)} &= T_F \left[\frac{4}{3} \ln \frac{\mu^2}{m_b^2} + \left(\frac{2}{3} \ln^2 \frac{\mu^2}{m_b^2} + \frac{\pi^2}{9}\right) \epsilon + \left(\frac{2}{9} \ln^3 \frac{\mu^2}{m_b^2} + \frac{\pi^2}{9} \ln \frac{\mu^2}{m_b^2} - \frac{4}{9} \zeta_3\right) \epsilon^2 + \mathcal{O}(\epsilon^3)\right].
\end{aligned}$$

- Yields finite matching coefficients C_i^j

$$\begin{aligned}
C_i^{j,(0)} &= F_i^{j,(0)} \\
C_i^{j,(1)} &= F_i^{j,(1)} - Z_J^{(1)} F_i^{j,(0)} \\
C_i^{j,(2)} &= F_i^{j,(2)} + \delta\alpha_s^{(1)} F_i^{j,(1)} - Z_J^{(1)} \left(F_i^{j,(1)} - Z_J^{(1)} F_i^{j,(0)}\right) - Z_J^{(2)} F_i^{j,(0)}
\end{aligned}$$

[Bauer et. al.'00,'01; Beneke,Feldmann'00; Beneke,Kiyo,Yang'04]

Checks

- $C_T^2 = C_T^4 = 0$, although F_T^2, $F_T^4 \neq 0$ in $D \neq 4$
- Vector FFs done by several groups *[Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08; Bell'08]*
- Matching coefficients obey renormalization group equation

$$\frac{d}{d\ln\mu} C_i^j(u;\mu) \quad = \quad \left[\Gamma_{\text{cusp}}(\alpha_s^{(4)}) \ln\frac{u m_b}{\mu} + \gamma'(\alpha_s^{(4)}) + \gamma_i(\alpha_s^{(5)})\right] C_i^j(u;\mu)$$

  - Γ_{cusp} and γ': Universal, related to SCET current.
  - γ_i anomalous dimension of the QCD current

  $\Longrightarrow$ Distinguish two scales μ (from RGE in SCET) and ν (RGE in QCD)

$$C_i^j \quad \equiv \quad C_i^j(u;\,\mu,\,\nu)$$

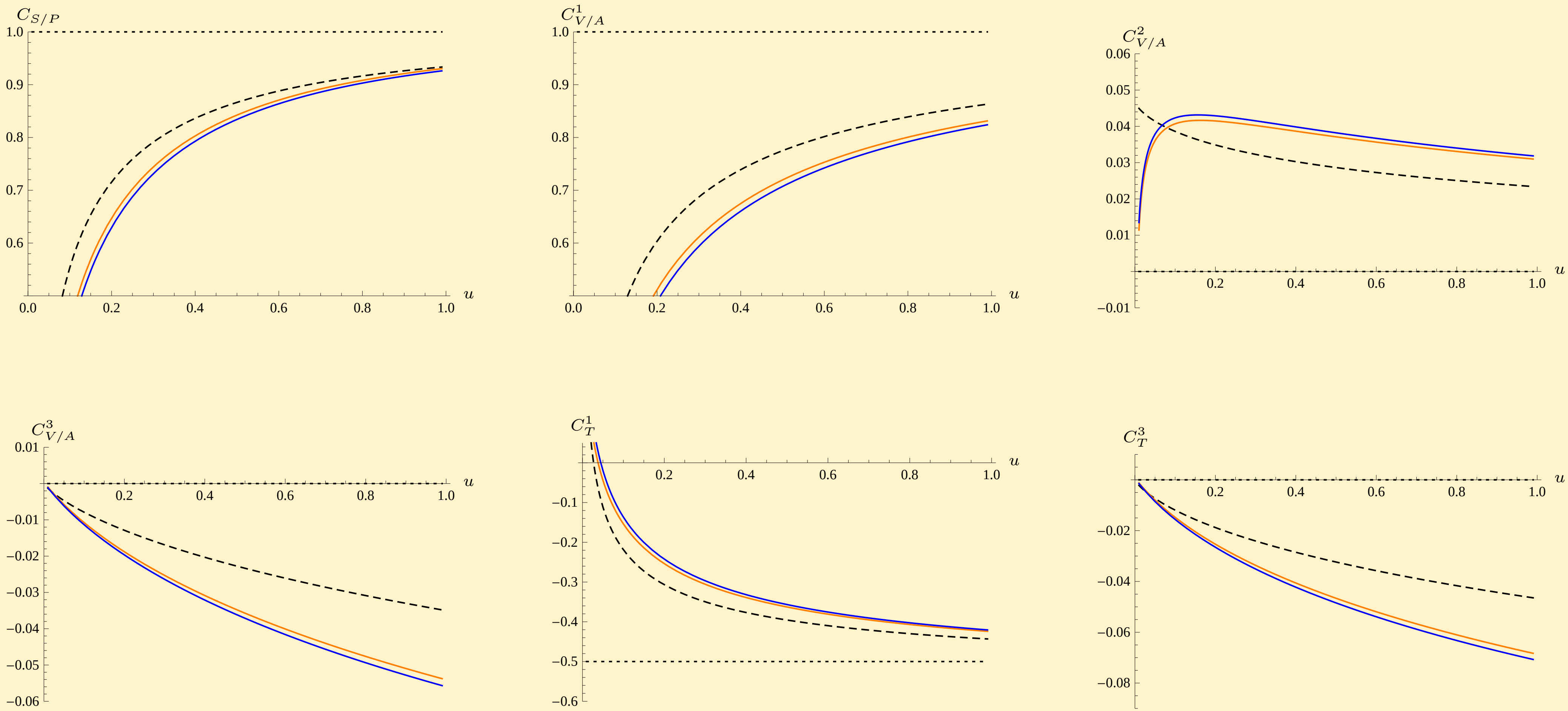
- Scalar coefficient C_S is not independent (EOM) *[Hill,Becher,Lee,Neubert'04; Bonciani,Ferroglia'08]*

$$C_V^1(u;\mu) + \left(1 - \frac{u}{2}\right) C_V^2(u;\mu) + C_V^3(u;\mu) \quad = \quad \frac{\overline{m}_b(\nu)}{m_b} C_S(u;\mu,\nu)$$

- Tensor coefficients in $u = 1$ (or $q^2 = 0$) enter the $\bar{B} \to X_s\gamma$ process. *[Ali,Greub,Pecjak'07]*

Matching coefficients

- Matching coefficients as a function of u for $\mu = \nu = m_b$
- Dotted: Tree-level. Dashed: NLO.
- Orange: NNLO, $m_c = 0$. Blue: NNLO, $m_c = 0.3 m_b$.



Heavy-to-light form factor ratios

- Factorization formula for heavy-to-light form factors at large recoil

$$F_i^{B\to M}(E) = C_i(E)\,\xi_a(E) + \underbrace{\int_0^\infty \frac{d\omega}{\omega} \int_0^1 dv\; T_i(E;\ln\omega,v)\,\phi_{B+}(\omega)\,\phi_M(v)}_{\text{spectator scattering}}$$

[Beneke,Feldmann'00,'03; Bauer,Pirjol,Stewart'02]

- For $B \to P$, have three FFs $\{f_+,\ f_0,\ f_T\}$ and a single ξ_P
- For $B \to V$, have seven FFs $\{V,\ A_{0,1,2},\ T_{1,2,3}\}$ and two $\xi_{\perp,\parallel}$
- Five independent A0-type matching coefficients, now at NNLO. $(u = 2E/m_B)$

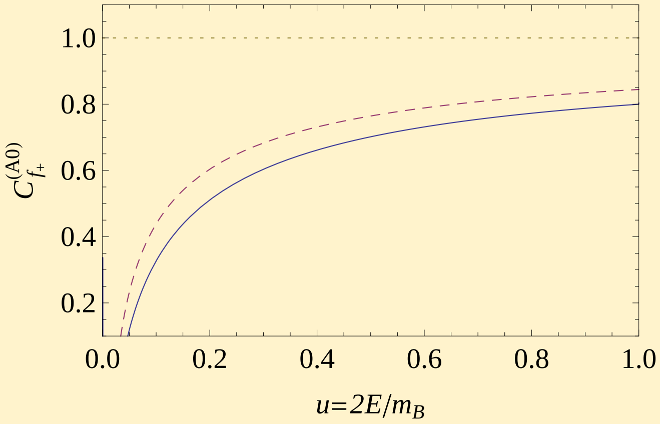
$$C_{f_+}^{(A0)} = C_V^1(u;\mu) + \frac{u}{2}C_V^2(u;\mu) + C_V^3(u;\mu)\,, \qquad C_V^{(A0)} = C_V^1(u;\mu)$$

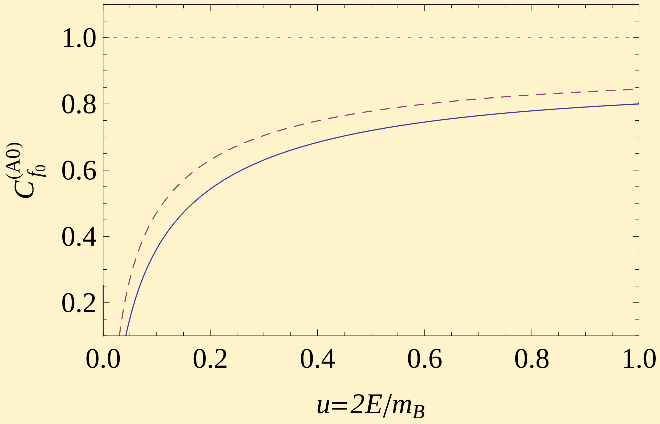
$$C_{f_0}^{(A0)} = C_V^1(u;\mu) + \left(1-\frac{u}{2}\right)C_V^2(u;\mu) + C_V^3(u;\mu)\,, \quad C_{T_1}^{(A0)} = -2C_T^1(u;\mu,\nu) + C_T^3(u;\mu,\nu)$$

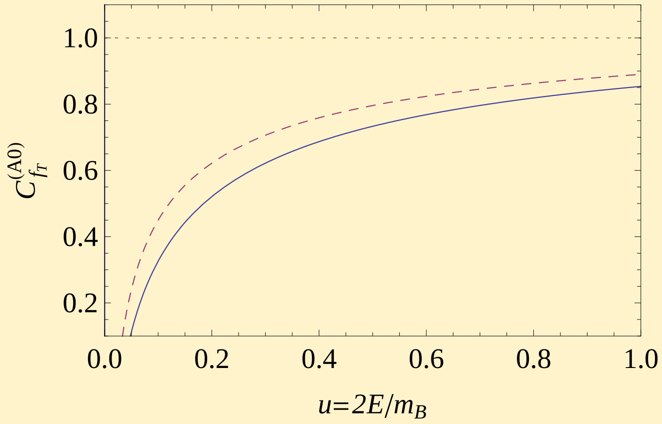
$$C_{f_T}^{(A0)} = -2C_T^1(u;\mu,\nu) - C_T^4(u;\mu,\nu)$$

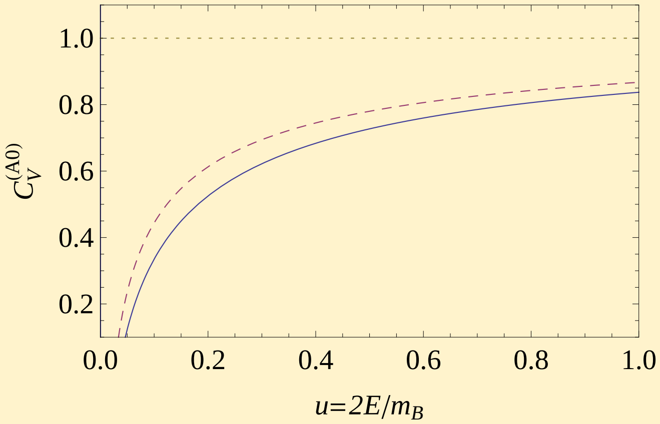
Heavy-to-light form factor ratios

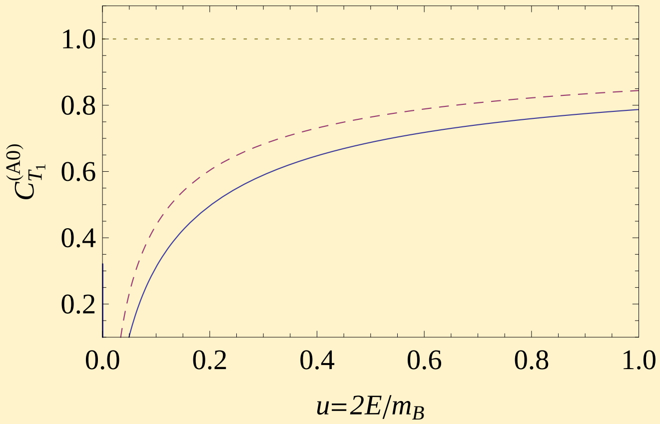
- A0-type matching coefficients as a function of $u = 2E/m_B$ for $\mu = \nu = m_b$
- Dotted: Tree-level (unity). Dashed: NLO. Solid: NNLO.
- NNLO corrections moderate. Add constructively to NLO result.









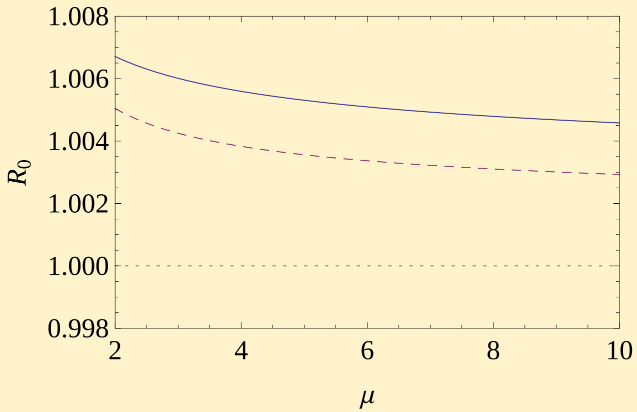


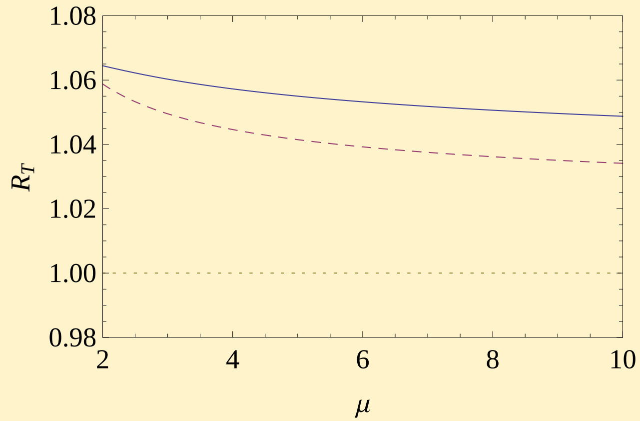
Heavy-to-light form factor ratios

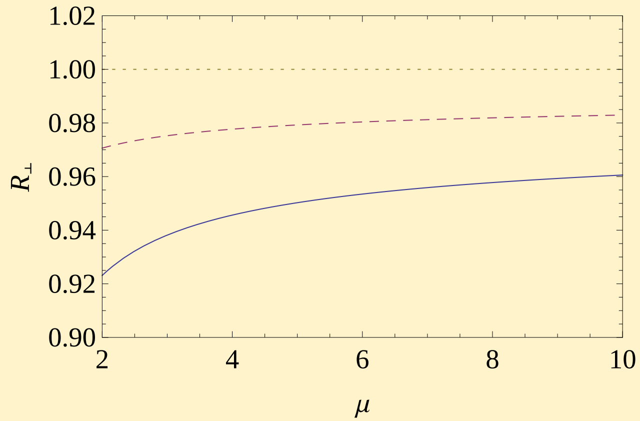
- In the physical form factor scheme, have only 3 independent ratios

$$R_0(u) \equiv C_{f_0}^{(A0)}/C_{f_+}^{(A0)}, \qquad R_T(u) \equiv C_{f_T}^{(A0)}/C_{f_+}^{(A0)}, \qquad R_\perp(u) \equiv C_{T_1}^{(A0)}/C_V^{(A0)}$$

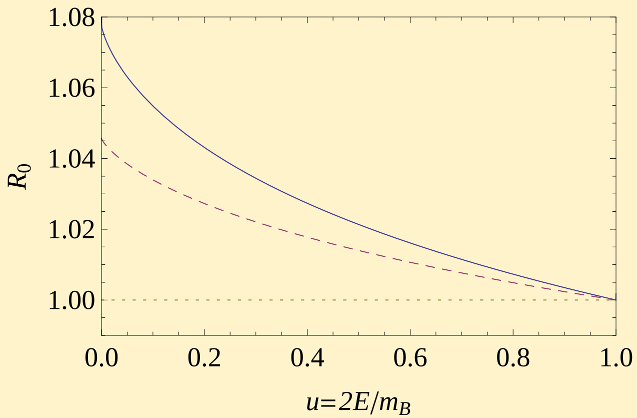
- $R_{0,T,\perp}$ as a function of μ for $u = 0.85$ and $\nu = m_b$

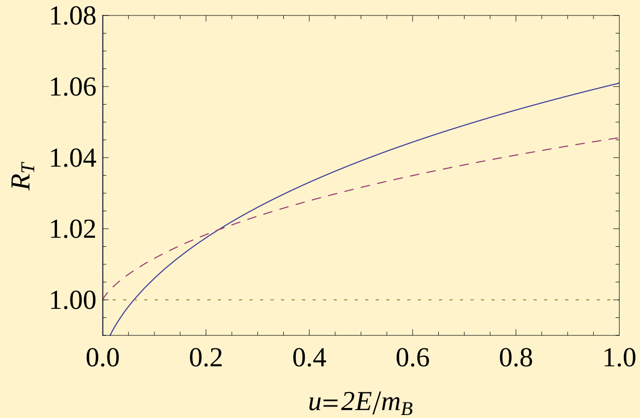


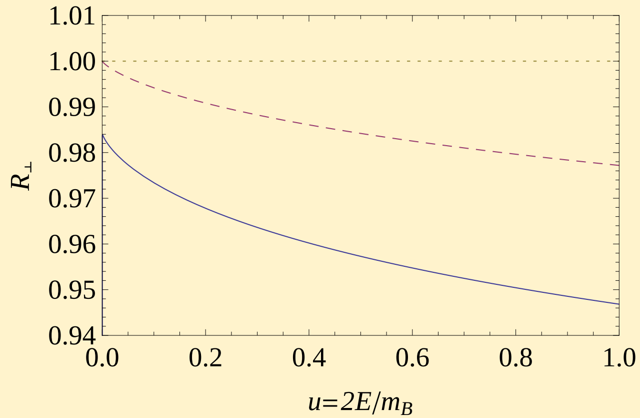




- $R_{0,T,\perp}$ as a function of u for $\mu = \nu = m_b$



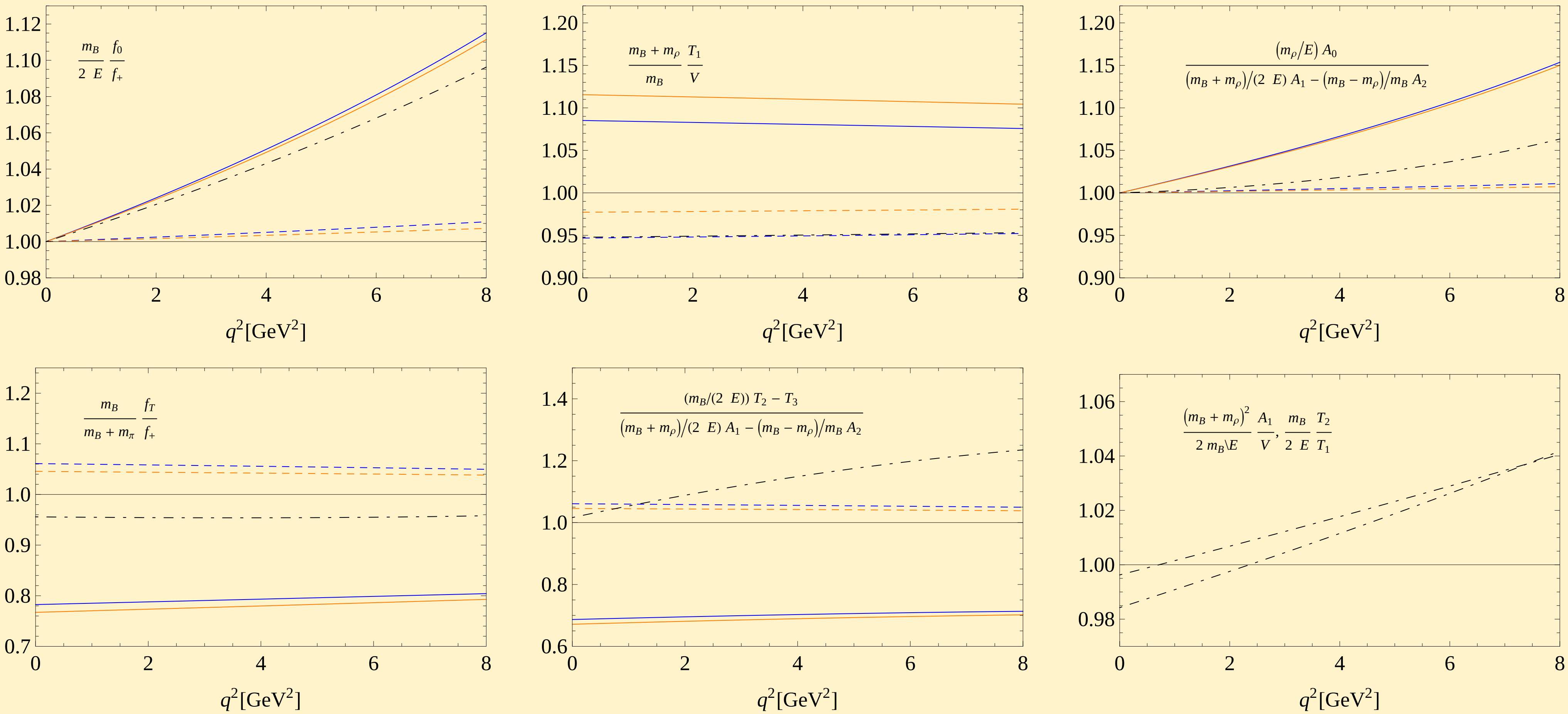




Heavy-to-light form factor ratios

- Corrections to $B \to \pi$ and $B \to \rho$ form factor ratios as a function of q^2
  - Solid: Fulls result with R_X at NNLO (blue), and at NLO (orange)
  - Dashed: Same as above, but without spectator scattering
  - Dash-dotted: Result from QCD sum rules

[Ball,Zwicky'04]



- Radiative corrections to A0-coefficients smaller than impact of spectator scattering

Exclusive radiative decays

- Exclusive radiative decays make use of form factors at maximum recoil, i.e. $u = 1$, or $E = m_B/2$, or $q^2 = 0$.
- Consider the two ratios in the physical form factor scheme

$$\mathcal{R}_1(E) \equiv \frac{m_B}{m_B + m_P}\frac{f_T(E)}{f_+(E)} = R_T(E) + \int_0^1 d\tau\, C^{(B1)}_{T+}(\tau, E)\,\frac{\Xi_P(\tau, E)}{f_+(E)},$$

$$\mathcal{R}_2(E) \equiv \frac{m_B + m_V}{m_B}\frac{T_1(E)}{V(E)} = R_\perp(E) + \frac{m_B + m_V}{m_B}\int_0^1 d\tau\, C^{(B1)}_{T_1 V}(\tau, E)\,\frac{\Xi_\perp(\tau, E)}{V(E)}$$

[Beneke,Yang'05; see also Beneke,Kiyo,Yang'04; Hill,Becher,Lee,Neubert'04]

- Specifying to the π ($\mathcal{R}_1$) meson and the ρ ($\mathcal{R}_2$) meson, numerically have

$$\mathcal{R}_1(E_{\rm max}) = 1 + \Big[0.046\,(\text{NLO}) + 0.015\,(\text{NNLO})\Big]\,(R_T) -0.160\,\Big\{1 + 0.524\,(\text{NLO spec.}) - 0.002\,(\delta^{\|}_{\rm log})\Big\} = 0.817,$$

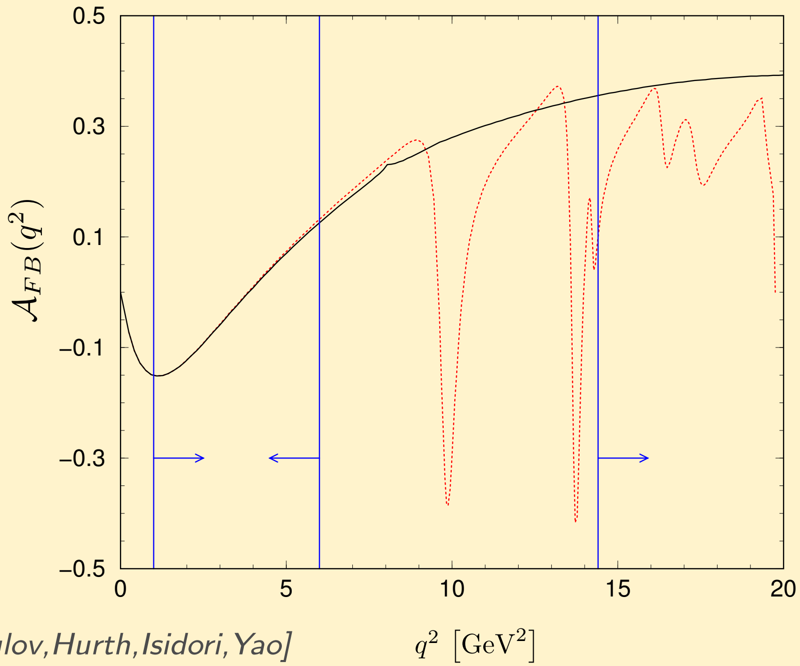
$$\mathcal{R}_2(E_{\rm max}) = 1 - \Big[0.023\,(\text{NLO}) + 0.030\,(\text{NNLO})\Big]\,(R_\perp) +0.084\,\Big\{1 + 0.406\,(\text{NLO spec.}) + 0.032\,(\delta^{\|}_{\rm log})\Big\} = 1.067.$$

[see also Bauer,Pirjol,Rothstein,Stewart'04; BBNS'04]

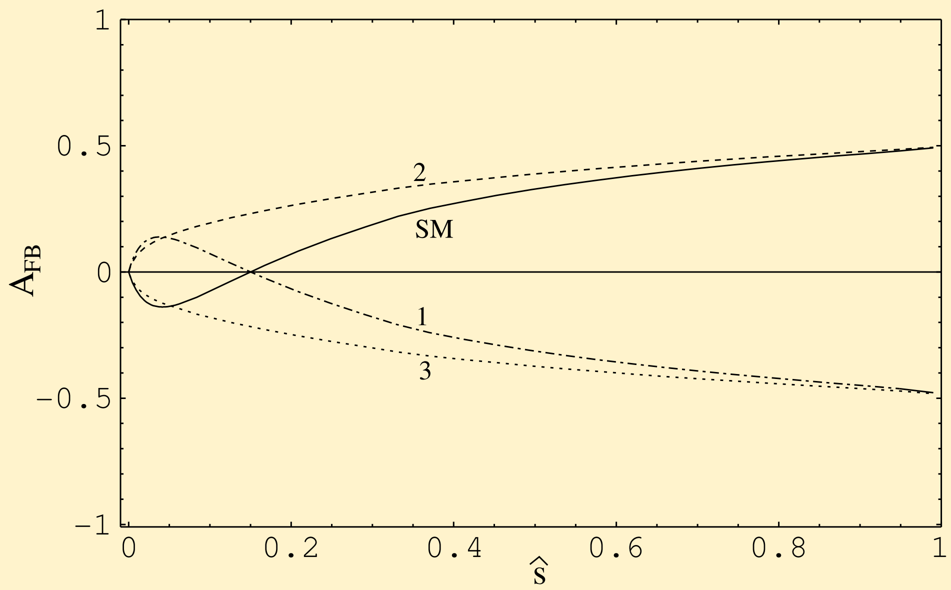
- A0-type and spectator scattering: Opposite sign, latter are larger
- Sum rule results: $\mathcal{R}_1 = 0.955$ and $\mathcal{R}_2 = 0.947$. *[Ball,Zwicky'04]*

Semi-inclusive $\bar{B} \to X_s \ell^+ \ell^-$

- $\bar{B} \to X_s \ell^+ \ell^-$ is a FCNC process, sensititve to NP, complementary to $\bar{B} \to X_s \gamma$
- Need cut on m_X to discriminate background from
  $b \to c \ell^- \bar{\nu}_\ell \to s \ell^+ \ell^- \bar{\nu}_\ell \nu_\ell \;=\; b \to s \ell^+ \ell^- + \not{E}$
- $m_X \le m_X^{\rm cut} = 1.8 \ldots 2.0\,{\rm GeV}$ and $1\,{\rm GeV}^2 \le q^2 \le 6\,{\rm GeV}^2 \quad\Rightarrow\quad$ "shape function region"
- Forward-backward asymmetry



[Ghinculov,Hurth,Isidori,Yao]



[Ali,Greub,Hiller,Lunghi, $\hat{s} \equiv q^2/m_b^2$]

Semi-inclusive $\bar{B} \to X_s\,\ell^+\ell^-$

- In the shape function region and at leading power in $\Lambda_{\rm QCD}/m_b$, have

$$\mathrm{d}\Gamma^{[0]} = h^{[0]} \times J \otimes S$$

[Lee,Stewart'05]

- $h^{[0]}$: process-dependent hard function. J, S: Universal jet- and shape-function
- For $h^{[0]}$, match first on two QCD currents with coefficients $C_{9/7}^{\rm incl}$

$$J_9^\mu = \bar{s}\,\gamma^\mu P_L b\,, \qquad J_7^\mu = \frac{2\,m_b}{q^2}\,\bar{s}\,iq_\rho\sigma^{\rho\mu}P_R b\Big|_{\nu=m_b}$$

- Then match QCD onto SCET ("split matching") [Lee,Stewart'05]

$$J_9^\mu = \sum_{i=1,2,3} c_i^9(u,\mu)\,[\bar{\xi}W_{hc}]\,\Gamma_{9,i}^\mu\,h_v\,, \qquad J_7^\mu = \frac{2m_b}{q^2}\sum_{i=1,2} c_i^7(u,\mu)\,[\bar{\xi}W_{hc}]\,\Gamma_{7,i}^\mu\,h_v$$

- Need here:

$$\begin{aligned} c_1^9(u,\mu) &= C_V^1(u;\mu) \\ c_1^7(u,\mu) &= -2\,C_T^1(u;\mu,\nu=m_b) + C_T^3(u;\mu,\nu=m_b) \end{aligned}$$

Semi-inclusive $\bar{B} \to X_s \ell^+ \ell^-$

- Differential decay rate

$$\frac{d^3\Gamma}{dq^2 dp_X^+ d\cos\theta} = \frac{3}{8}\left[(1+\cos^2\theta) H_T(q^2, p_X^+) + 2\,(1-\cos^2\theta) H_L(q^2, p_X^+) + 2\cos\theta\, H_A(q^2, p_X^+)\right]$$

- Position of the FBA zero occurs at q_0^2 with

$$0 = \int_0^{p_X^{+\mathrm{cut}}} dp_X^+\, H_A(q_0^2, p_X^+)$$

$$= \mathrm{const} \times \int_0^{p_X^{+\mathrm{cut}}} dp_X^+\, h_A^{[0]}(q_0^2, p_X^+)\, \frac{(q_{0+} - q_{0-})^2}{q_{0+}}\, q_0^2 \int d\omega\, p^- J(p^-\omega)\, S(p_X^+ - \omega)$$

$$h_A^{[0]}(q^2, p_X^+) = 2\mathcal{C}_{10}\, c_1^9(u)\, \mathrm{Re}\left[C_9^{\mathrm{incl}}(q^2) c_1^9(u) + \frac{2m_b}{q_-}\, C_7^{\mathrm{incl}}(q^2) c_1^7(u)\right]$$

- $h_A^{[0]}(q_0^2, p_X^+)$ hardly varies with p_X^+. Pull in front of integral.
  Condition for the zero becomes $h_A^{[0]}(q_0^2, \langle p_X^+\rangle) = 0$ or

$$\frac{q_0^2}{2m_b(m_B - \langle p_X^+\rangle)} = -\frac{\mathrm{Re}\,[C_7^{\mathrm{incl}}(q_0^2)]}{\mathrm{Re}\,[C_9^{\mathrm{incl}}(q_0^2)]}\, \underbrace{\frac{c_1^7(u_0)}{c_1^9(u_0)}}_{=R_\perp}$$

- Zero independent of J and S!

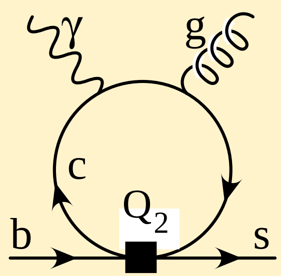
[Lee,Stewart'05; Lee,Ligeti,Stewart,Tackmann'05]

Semi-inclusive $\bar{B} \to X_s \ell^+\ell^-$

- For $m_X^{\rm cut} = (2.0 \ldots 1.8)$ GeV, have

$$
\begin{aligned}
q_0^2\big|_{R_\perp=1} &= (3.62 \ldots 3.69)\,\text{GeV}^2 \\
q_0^2\big|_{R_\perp\ \rm NLO} &= (3.55 \ldots 3.61)\,\text{GeV}^2 \\
q_0^2\big|_{R_\perp\ \rm NNLO} &= \left[(3.34 \ldots 3.40)\,{}^{+0.04}_{-0.13}{}_{\,\mu} \pm 0.08_{\,m_b}\,{}^{+0.05}_{-0.04}{}_{\,m_c} \pm 0.14_{\rm SF} \pm 0.14_{\,\langle p_X^+\rangle}\right]\text{GeV}^2 \\
&= \left[(3.34 \ldots 3.40)^{+0.22}_{-0.25}\right]\text{GeV}^2,
\end{aligned}
$$

- Perturbative NLO impact is -2.2%, NNLO another -3%
- Result includes -0.1 GeV^2 as estimate from subleading SF *[Lee,Tackmann'08]*
- Result also includes +0.07 GeV^2 from $1/m_c^2$ power corrections. However, it is not clear if these can be absorbed into $C_i^{\rm incl}$ in the presence of an invariant mass cut.
- With cut, the soft gluon matrix element is not a short-distance coefficient times a local matrix element
- Soft gluon attached to charm loop affects invariant mass of hadronic final state by an amount $\sqrt{m_b \Lambda_{\rm QCD}} \Longrightarrow$ subleading SF



Conclusion

- We computed the hard matching coefficients from QCD onto SCET at NNLO for all Dirac structures
- NNLO corrections are moderate and add contructively to NLO contributions
- We discussed Heavy-to-light form factor ratios and exclusive radiative decays. Only $R_\perp$ receives large NNLO corrections
- Perturbative NNLO shift on the FBA zero in semi-inclusive $\bar{B} \to X_s \, \ell^+ \ell^-$ amounts to -3%.
- Final result for the zero:

  $q_0^2 = \left[(3.34 \ldots 3.40)^{+0.22}_{-0.25} \right]$ GeV^2 for $m_X^{\rm cut} = (2.0 \ldots 1.8)$ GeV

Backup slides

Reduction methods

- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.

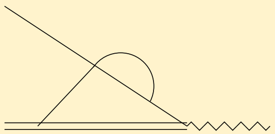
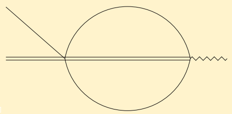
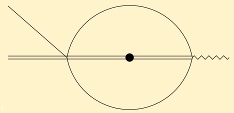
- Passarino-Veltman reduction to scalar integrals
  (in general with irreducible scalar products in the numerator) *[Passarino,Veltman'79]*

- Integration-by-parts (IBP) identities, 8 per diagram *[Tkachov'81; Chetyrkin,Tkachov'81]*

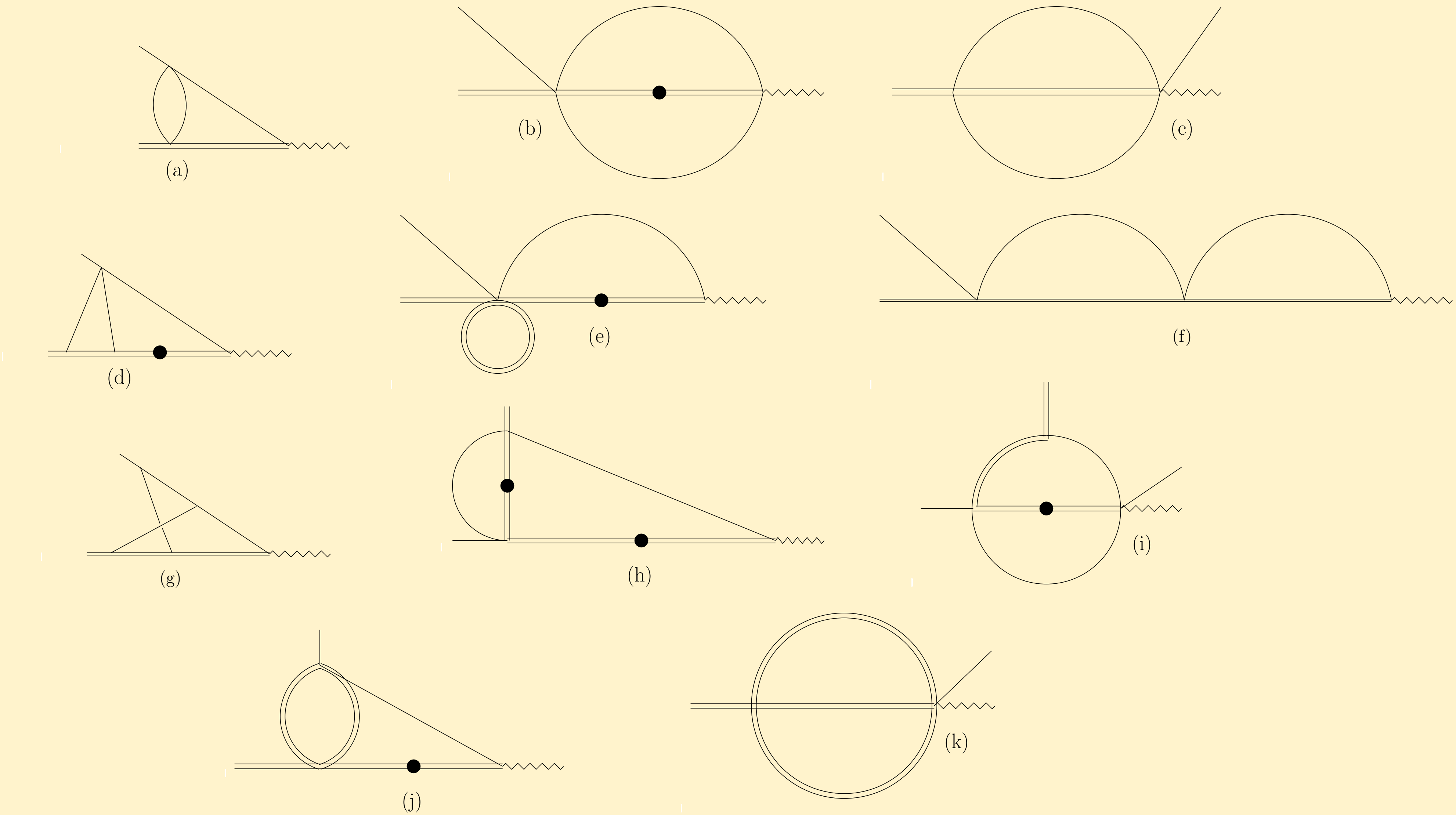
- Lorentz-Invarianz (LI) identities, 1 per diagram *[Gehrmann,Remiddi'99]*

- Solve system of equations with Laporta algorithm *[Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08]*

- Obtain scalar integrals as a linear combination of master integrals

 $= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$  $- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$ 

Master integrals



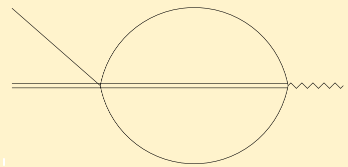
- Double lines are massive, single lines are massless
- Dots on lines denote squared propagators

Master Integrals

- Reduction yields 18 master integrals with poles up to $1/\epsilon^4$. Analytic calculation of coefficient functions yields harmonic polylogarithms up to weight 4 of argument u or $1-u$. *[Remiddi,Vermaseren'99]*

- Several calculations in agreement *[Bell'07; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08; Bell'08]*

- Applied techniques
  - Hypergeometric functions, use `HypExp` or XSummer for ϵ-expansion *[Moch,Uwer'05; Maitre,TH'05,'07]*



$$= \frac{(m_b^2)^{1-2\epsilon}}{(4\pi)^{4-2\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma(2\,\epsilon-1)}{\Gamma(2-\epsilon)} \, {}_2F_1(\epsilon, 2\,\epsilon-1\,; 2-\epsilon\,; 1-u)$$

  - Differential equations *[Kotikov'91; Remiddi'97]*

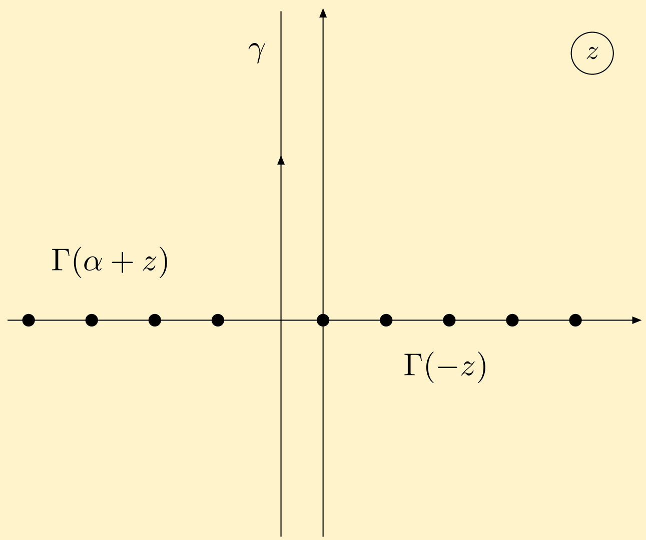
$$\frac{\partial}{\partial u}\mathrm{MI}_i(u) = f(u,\epsilon)\,\mathrm{MI}_i(u) + \sum_{j\neq i} g_j(u,\epsilon)\,\mathrm{MI}_j(u)$$

    * Requires result of Laporta reduction.
    * Boundary condition in $u=0$ or $u=1$ from Mellin-Barnes representation

Master Integrals (cont'd.)

- Applied techniques (cont'd.)
  - Mellin-Barnes representation *[Smirnov'99; Tausk'99]*

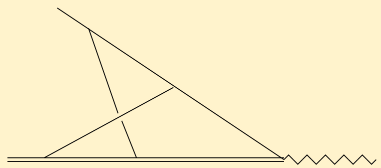
$$\frac{1}{(A_1+A_2)^\alpha} = \int_\gamma \frac{dz}{2\pi i}\, A_1^z\, A_2^{-\alpha-z}\, \frac{\Gamma(-z)\,\Gamma(\alpha+z)}{\Gamma(\alpha)}$$



  - ∗ partially automated
  - ∗ Numerical cross checks possible

[Czakon'05; Gluza,Kajda,Riemann'07]

- Most difficult master integral:



[TH'09]

  - Solved with differential equations technique
  - Possesses a three-fold Mellin-Barnes integral at $u = 1$

Some additional definitions and numbers

- In the physical form factor scheme, define to all orders in perturbation theory *[Beneke,Feldmann'00]*

$$\xi_P^{\rm FF} \equiv f_+, \qquad \xi_\perp^{\rm FF} \equiv \frac{m_B}{m_B+m_V}\,V, \qquad \xi_\parallel^{\rm FF} \equiv \frac{m_B+m_V}{2E}\,A_1 - \frac{m_B-m_V}{m_B}\,A_2$$

- Kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X| \qquad q_+ = m_B - p_X^+\,, \qquad q_- = q^2/q_+$

$$p_X^{+\rm cut} = \frac{1}{2m_B}\left[m_B^2 + (m_X^{\rm cut})^2 - q^2 - \sqrt{(m_B^2+(m_X^{\rm cut})^2-q^2)^2 - 4m_B^2(m_X^{\rm cut})^2}\right]$$

[Lee,Tackmann'08]

- Numerical inputs that we use in the phenomenological analysis of the forward-backward asymmetry zero

$\alpha_s(M_Z) = 0.1180$	$\lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12~\mathrm{GeV}^2$
$\sin^2\theta_W = 0.23122$	$m_t^{\rm pole} = 171.4~\mathrm{GeV}$
$M_W = 80.426~\mathrm{GeV}$	$m_c^{\rm pole} = (1.5\pm 0.1)~\mathrm{GeV}$
$M_Z = 91.1876~\mathrm{GeV}$	$m_b^{\rm PS}(2\,\mathrm{GeV}) = (4.6\pm 0.1)~\mathrm{GeV}$

Some additional plots

- Matching coefficients $c_i^9(u,\mu)$ and $c_i^7(u,\mu)$ relevant to semi-inclusive $\bar{B} \to X_s\,\ell^+\ell^-$
- Matching coefficients as a function of $u = 1 - q^2/m_b^2$
- Dotted: Tree-level. Dashed: NLO. Solid: NNLO.
- Orange: $\mu = 1.5\,\text{GeV}$. Blue: $\mu = m_b$.