Jet Quenching Parameter via SCET

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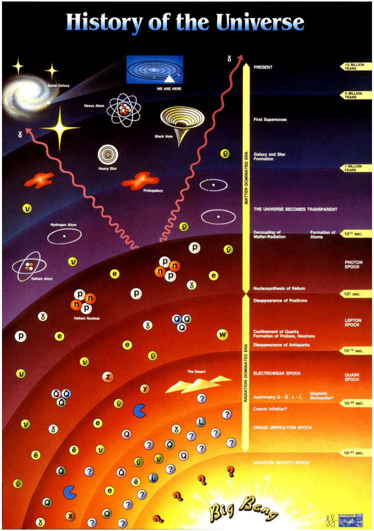
in collaboration with H. Liu and K. Rajagopal, arxiv:1006.1367 [hep-ph]

Outline

1. Introduction and motivations
2. EFT approach to jet propagation in a dense medium
3. Rate and jet quenching from Wilson lines
4. Summary and future directions

Outline

Thermal History of the Universe and QGP



Our universe originated in a "Big Bang"

Matter as we know it has not existed forever

First few microseconds after the Big Bang:
hadrons could not form, quark, antiquarks and gluons deconfined in the quark-gluon plasma (QGP);
first hadrons formed only when $T_{\rm cr} \simeq 170\,\mathrm{MeV}$.

In particular QGP hidden behind the CMB.
To study it we have to recreate on the Earth.

Relativistic Heavy Ion Collisions

The quark-gluon plasma can be recreated today by
colliding large nuclei at high energies

RHIC

RHIC began operation in 2000

p+p and Au+Au collisions at $\sqrt{s} \simeq 200\,\mathrm{GeV} \times A$

LHC

LHC began operation in 2009

p+p and Pb+Pb collisions at $\sqrt{s} \simeq 5.5\,\mathrm{TeV} \times A$

We can probe the QGP only indirectly

Hard Probes for the Quark Gluon Plasma

We will focus on the rare high p_T final state particles.

High p_T particles as probes

From fragmentation of even higher p_T partons, produced in the early stage of the collision (for $p_T \simeq 2\,\mathrm{GeV}$, $\tau_{\mathrm{form}} \simeq 1/\sqrt{p_T^2} \simeq 0.1\,\mathrm{fm}/c$).

Interacting with the medium (as much as $5-10\,\mathrm{fm}$) and probing its properties.

Exploring properties of the medium by detecting high p_T hadrons outside it: Jet Emission Tomography (JET).

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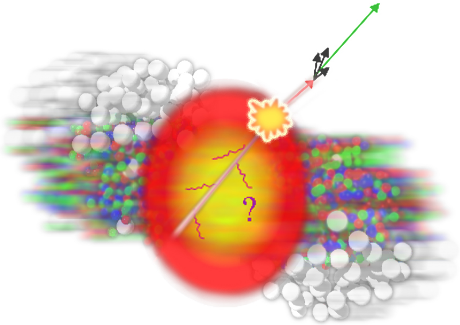
Jets propagation in dense media



Hard Probes for the QGP

Jet Quenching observables make it possible to study how parton fragmentation is affected by the medium.

At RHIC: $p_T \simeq$ tens of GeV.

At LHC: $p_T \simeq$ hundreds of GeV.

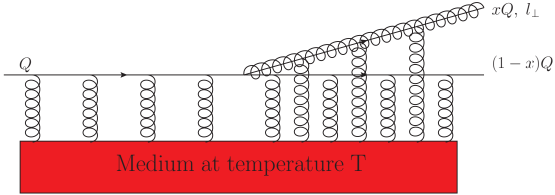
The medium has two main effects on the propagating hard parton:

- changing direction of its momentum (transverse momentum broadening);
- inducing parton energy loss.

Energy loss in the high energy limit

Radiative energy loss

High energy limit: QCD analogue of bremsstrahlung dominates.



Hard partons constantly kicked by the medium: all subjects to transverse momentum broadening.

The jet quenching parameter $\hat{q}$

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L}$$

$\hat{q}$ plays central role in the energy loss calculation, but defined via transverse momentum broadening only.

Hope for a factorized description

Separation of scales

Energy loss and $k_\perp$ broadening involve widely separated scales

$$Q \gg l_\perp \gg T$$

We can ultimately hope for a factorized description

- physics at each scale cleanly separated at leading power;
- correction to factorization systematically calculable, order by order in the small ratio between the scales.

Current theoretical formulations of jet quenching are not systematically improvable in this sense

First step

Formulation of the momentum broadening in SCET

Our focus

Non-radiative $k_\perp$ broadening in the high energy limit:
easiest case to handle, natural context in which $\hat{q}$ arises.

”Semi-controlled“ calculation:
radiation artificially turned off, other than that controlled in the $Q \gg T$ limit.

Our language: SCET

In the $T \ll Q$ limit:

- natural separation of scales;
- natural organization of the modes into kinematic regimes.

Set-up of the problem

Energy scales

Hard parton with initial four momentum: $q_0 \equiv (q_0^+, q_0^-, q_{0\perp}) = (0, Q, 0)$

propagating through some form of QCD matter.

Example: QGP in equilibrium at temperature T
(our analysis would apply to other forms of matter).

We assume $Q \gg T$, we have a small dimensionless ratio $\lambda \equiv \frac{T}{Q} \ll 1$.

Goal: compute $P(k_\perp)$ and $\hat{q}$

$P(k_\perp)$: probability distribution for the hard parton to acquire transverse momentum $k_\perp$ after traversing the medium.

From $P(k_\perp)$ it is straightforward to obtain $\hat{q}$

$$\hat{q} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} \, k_\perp^2 \, P(k_\perp), \qquad \int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$$

$k_\perp$ broadening and relevance of Glauber gluons

Momentum broadening for $Q \to \infty$ limit dominated by interactions between the hard collinear parton and Glauber gluons from the medium.

We need an effective Lagrangian describing the interaction between:

- Collinear partons: $q = Q(\lambda^2, 1, \lambda)Q$;
- Glauber gluons: $p = (\lambda^2, \lambda^2, \lambda)Q$.

SCET + Glauber Lagrangian

Start from the QCD Lagrangian and keep only the relevant degrees of freedom.

Power counting in λ at the level of the Lagrangian. At the leading order in λ

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp} \, \bar{\xi}_{\bar{n},q'_\perp} \left[i\bar{n} \cdot D + \frac{q_\perp^2}{2Q} \right] \not{n} \xi_{\bar{n},q_\perp}$$

Idilbi, Majumder, Phys.Rev.D80:054022,2009 [arXiv:0808.1087]

IM:
momentum broadening for a DIS process in the Breit frame.

DLR:
momentum broadening in the target frame.

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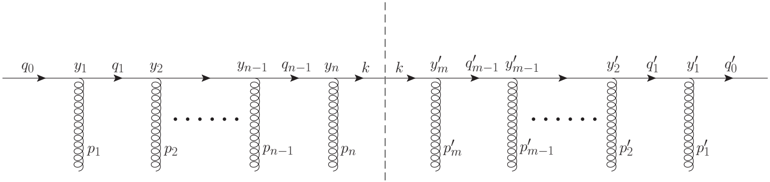
Relating $P(k_\perp)$ to the S-matrix

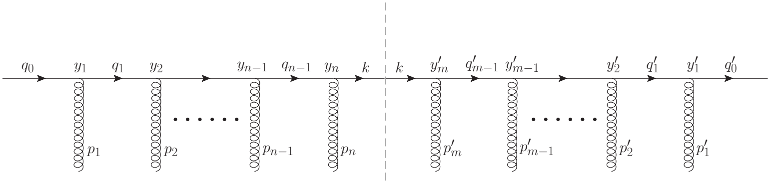
Strategy

- Probability amplitude for $\alpha \to \beta$ is $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$;
- S-matrix is unitary: $\sum_\beta |S_{\beta\alpha}|^2 = 1 \Rightarrow 2\,\mathrm{Im}\ M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$;
- Compute $2\,\mathrm{Im}\ M_{\alpha\alpha}$ by cutting the appropriate diagrams, use the unitarity relation to identify $|M_{\beta\alpha}|^2$ and then evaluate $P(k_\perp)$.
  (We identify: $\sum_\beta = L^2 \int \frac{d^2k_\perp}{(2\pi)^2}$.)



$$2\,\mathrm{Im}\, M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$$

Forward scattering amplitude evaluation I



SCET Lagrangian Feynman rules give:

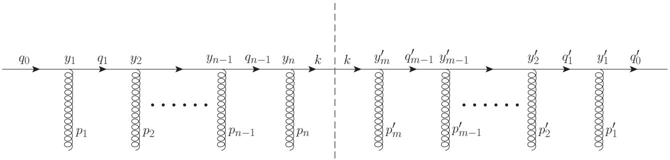
$$\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = \frac{1}{\sqrt{2}QL^3}\int \frac{dk^+dk^-}{(2\pi)^2}\prod_{i=1}^{n-1}\frac{d^4q_i}{(2\pi)^4}\prod_{j=1}^{m-1}\frac{d^4q'_j}{(2\pi)^4}$$

$$\times\, \bar{\xi}_{\bar{n}}(q'_0)\prod_{j=m-1}^{1}\left[(-ig)A^+(-p'_j)\,\not{n}\,\frac{-iQ}{2Qq'^{+}_j - q'^{\,2}_{j\perp} - i\epsilon}\,\not{\bar{n}}\right](-ig)A^+(-p'_m)\not{n}$$

$$\times\, 2\pi Q\delta\left(2k^+Q - k_\perp^2\right)\not{\bar{n}}\; igA^+(p_n)\not{n}\prod_{i=1}^{n-1}\left[\frac{iQ}{2Qq_i^+ - q_{i\perp}^2 + i\epsilon}\not{\bar{n}}\; igA^+(p_i)\not{n}\right]\xi_n(q_0)$$

(Analysis is analogous if the hard parton is a collinear gluon.)

Gluons from the medium



Gluon operators $A^+ = A^{a+}\, t_F^a$ ordering

$\prod_{j=m-1}^{1} A^+(-p'_j) \equiv A^+(-p'_1) \cdots A^+(-p'_{m-1})$; $\qquad \prod_{i=1}^{n-1} A^+(p_i) \equiv A^+(p_{n-1}) \cdots A^+(p_1)$

A_μ as a background field

We descrive the hard parton propagation in a specific field configuration $A_\mu(p)$.

Calculation valid no matter whether the medium is strongly coupled or weakly coupled.

This question only arise at the end, when we average over the field configurations.

Forward scattering amplitude evaluation II

Three simple steps

- average over the color indices;
- some Dirac algebra;
- gluon fields in the coordinate space $A^+(p_i) = \int d^4 y_i e^{i p_i y_i} A^+(y_i)$.

$$\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = \frac{2^{n+m}}{\sqrt{2}L^3 N_c} \int \prod_{i=1}^{n} d^4 y_i \prod_{j=1}^{m} d^4 y'_j \, e^{-iq_0\cdot(y_1 - y'_1)} \, \mathrm{Tr}\left[\prod_{j=m}^{1}(-ig)A^+(y'_j) \prod_{i=1}^{n} igA^+(y_i)\right]$$

$$\times\, g(y_n - y'_m, k_\perp) \prod_{j=1}^{m-1} f^*(y'_j - y'_{j+1}) \prod_{i=1}^{n-1} f(y_i - y_{i+1})$$

$$f(z) \equiv \int \frac{d^4q}{(2\pi)^4} \frac{iQ}{2Qq^+ - q_\perp^2 + i\epsilon} e^{iq\cdot z} = \delta(z^+)\theta(-z^-)\frac{iQ}{4\pi z^-} e^{-i\frac{Q}{2z^-}z_\perp^2},$$

$$g(z, k_\perp) \equiv \int \frac{dk^+dk^-}{(2\pi)^2} 2\pi Q\delta\left(2k^+Q - k_\perp^2\right) e^{ik\cdot z} = \frac{1}{2}\delta(z^+)e^{-ik_\perp\cdot z_\perp + i\frac{k_\perp^2}{2Q}z^-} .$$

The $Q \to \infty$ limit and its physical significance

$Q \to \infty$ for $f(z)$ and $g(z, k_\perp)$

So far not used the $Q \to \infty$ limit (although used in setting up the problem). In this limit both $f(z)$ and $g(z, k_\perp)$ simplify.

- $Q \gg p_\perp^2 z^- \quad \Rightarrow \quad f(z) \approx \frac{1}{2}\delta\left(z^+\right)\theta\left(-z^-\right)\delta^2\left(z_\perp\right)$
- $Q \gg k_\perp^2 z^- \quad \Rightarrow \quad g(z, k_\perp) \approx \frac{1}{2}\delta\left(z^+\right) e^{-ik_\perp \cdot z_\perp}$

Criterion for $g(z, k_\perp)$ stronger, we require: $Q \gg k_\perp^2 L \sim \hat{q} L^2$.

Physical significance of the $Q \to \infty$ limit: $Q \gg k_\perp^2 L \sim \hat{q} L^2$

The propagators of the internal quarks are $f(z) \propto \delta^2\left(z_\perp\right)$.

It requires L is short enough that the hard parton trajectory in position space remains well-approximated as a straight line, even though it picks up transverse momentum.

Forward scattering amplitude evaluation III

Final form for the forward scattering amplitude

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{1}{N_c} \int d^2 x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr} \left[\left(W_F^\dagger[0, x_\perp] - 1 \right) \left(W_F[0,0] - 1 \right) \right] \right\rangle$$

$|M_{\beta\alpha}|^2$ from the unitarity relation

The unitarity relation $\int \frac{d^2 k_\perp}{(2\pi)^2} \sum_{n=1,m=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = 2 \, \mathrm{Im} \; M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$

allows us to identify

$$|M_{\beta\alpha}|^2 = \frac{1}{L^2 \, N_c} \int d^2 x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr} \left[\left(W_F^\dagger[0, x_\perp] - 1 \right) \left(W_F[0,0] - 1 \right) \right] \right\rangle$$

Probability distribution $P(k_\perp)$

Expression for $P(k_\perp)$

$|M_{\beta\alpha}|^2$ and $2\,\mathrm{Im}\;M_{\alpha\alpha}$ are all we need to find $P(k_\perp)$. Thus

$$P(k_\perp) = \int d^2x_\perp\, e^{-ik_\perp\cdot x_\perp}\,\mathcal{W}_\mathcal{R}(x_\perp), \qquad \mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d\,(\mathcal{R})}\left\langle \mathrm{Tr}\left[W^\dagger_\mathcal{R}[0,x_\perp]\,W_\mathcal{R}[0,0]\right]\right\rangle$$

for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d\,(\mathcal{R})$.

Casalderrey-Solana and Salgado, Acta Phys.Polon. B 38, 3731 (2007) [arXiv:0712.3443 [hep-ph]]
Liang, Wang and Zhou, Phys. Rev. D 77, 125010 (2008) [arXiv:0801.0434 [hep-ph]]

Properties of $P(k_\perp)$

- $P(k_\perp)$ depends only on the medium property (thus also $\hat{q}$ does).
- Transverse momentum broadening without radiation: field theoretically well-defined property of the medium.
- This is the kind of factorization we hope to find once radiation is included.

Operator ordering for the $\hat{q}$ evaluation

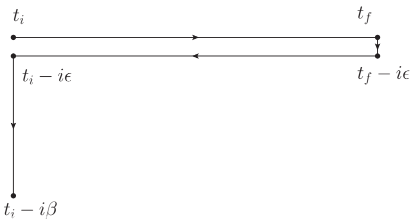
$\hat{q}$ from light-like Wilson lines

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{\sqrt{2}}{L^-} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \int d^2 x_\perp \, e^{-ik_\perp \cdot x_\perp} \mathcal{W}_\mathcal{R}(x_\perp)$$

$\mathcal{W}_\mathcal{R}(x_\perp)$: different operator ordering than a standard Wilson loop ($A^+ = (A^+)^a t^a$).

Standard Wilson loop

$(A^+)^a$ time ordered, t^a path ordered.



Wilson lines in $\mathcal{W}_\mathcal{R}(x_\perp)$

$(A^+)^a$ path ordered, t^a path ordered.

$\mathcal{W}_\mathcal{R}(x_\perp)$ should be described using the Schwinger-Keldysh contour

- one of the light-like Wilson line on the $\mathrm{Im}\, t = 0$ segment
- the other light-like Wilson line on the $\mathrm{Im}\, t = -i\epsilon$ segment

Evaluating the thermal average

$\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM

At RHIC physics of the QGP at scales $\sim T$ is not weakly coupled.

Insights by calculating $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM, by using AdS/CFT duality.
Liu, Rajagopal, Wiedemann, PRL97,2006 [hep-ph/0605178]

LRW evaluated $\mathcal{W}_{\mathcal{R}}(x_\perp)$ with the standard, i.e. wrong, operator ordering.

Our procedure to take the order into account: specific example of the more general Lorentzian AdS/CFT.
Skenderis, van Rees, JHEP 0905:085,2009. [arXiv:0812.2909]

Subtlety resolved, result unchanged: $\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{g^2 N_c}T^3$.

$\hat{q}$ in weakly coupled QCD plasma

$\hat{q}$ for the QCD plasma such that physics at scales $\sim T$ is weakly coupled.

Arnold, Xiao, Phys.Rev.D78:125008,2008, arXiv:0810.1026 [hep-ph];
Caron-Huot, Phys.Rev.D79:065039,2009, arXiv:0811.1603 [hep-ph].

Mindaugas Lekaveckas talk!

Outline

Summary

Probability distribution $P(k_\perp)$ for momentum broadening evaluated within the SCET formalism.

Upon turning radiation off by hand we have a nice factorization: $P(k_\perp)$ and $\hat{q}$ by a field theoretically well-defined property of the medium.

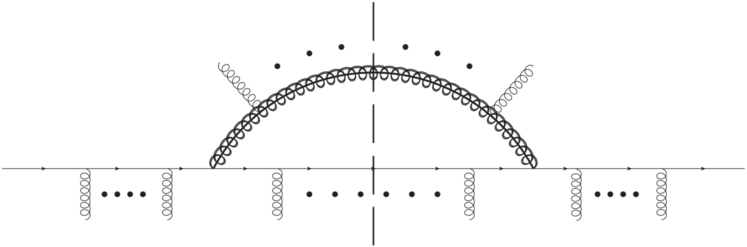
$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp), \qquad \mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr} \left[W^\dagger_\mathcal{R}[0, x_\perp] \, W_\mathcal{R}[0,0] \right] \right\rangle ,$$

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{\sqrt{2}}{L^-} \int \frac{d^2k_\perp}{(2\pi)^2} \, k_\perp^2 \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

Current calculation

Medium induced collinear gluon radiation (in progress)

Differential probability distribution for emitting one gluon collinear to the incoming hard parton and with arbitrary energy.



- See whether and how factorization arises for the collinear radiation;
- See how $\hat{q}$ enters in the spectrum of the radiated gluons.

More future directions

- Allow collinear radiation for a arbitrary direction;
- Allow soft radiation;
- Consider corrections to factorization (i.e. non-infinite Q), use the SCET formalism to compute the corrections suppressed by powers of T/Q.

A related work

Momentum broadening and collinear gluon emission from SCET at weak coupling.

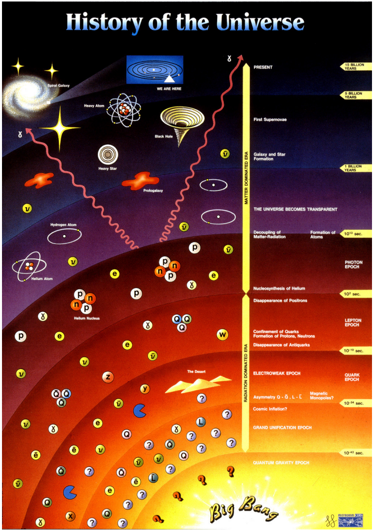
Ovanesyan, Vitev, arXiv:1103.1074 [hep-ph].

Grigory Ovanesyan talk!

Backup slides

BACKUP
SLIDES

Thermal History of the Universe



Our universe originated in a "Big Bang"

Matter as we know it has not existed forever

First few microseconds after the Big Bang:
hadrons could not form, quark, antiquarks and gluons deconfined in the quark-gluon plasma (QGP);
first hadrons formed only when $T_{\rm cr} \simeq 170\,\mathrm{MeV}$.

3 minutes after the Big Bang:
small atomic nuclei could form (chemical freeze-out).

300000 years after the Big Bang:
electrons and nuclei combined into neutral atoms;
universe transparent to electromagnetic radiation;
radiation decoupled (thermal freeze-out).

Impossible to directly see anything happened before 300000 years after the Big Bang (due to the opacity of the Early Universe).

In particular QGP hidden behind the CMB.
To study it we have to recreate on the Earth.

Phases of Quantum ChromoDynamics (QCD)

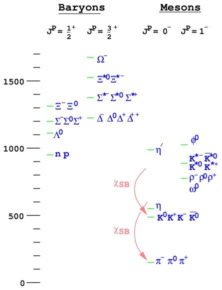
QCD: $SU(3)_c$ gauge theory of quarks of gluons.

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + i\overline{\psi}\gamma^\mu\left(\partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu\right)\psi - m\overline{\psi}\psi$$

Quarks and gluons have color charge,
and in the $m \to 0$ limit chiral symmetry.

QCD spectrum: hadrons, colorless (confinement)
and heavy (chiral symmetry breaking) objects.

QCD is more than its vacuum....

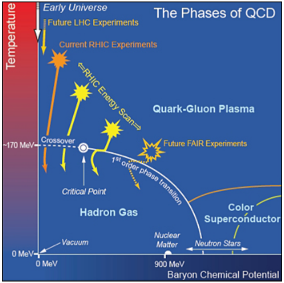


QCD phase diagram (μ_B, T)

Nuclear matter (hadrons): $(\mu_B, T) \simeq (900\,\mathrm{MeV}, 0)$.

QGP: $T \to \infty$ phase of QCD.

In the QGP phase quarks and gluons are
de-confined, and chiral symmetry is restored.



How do we know that QGP exists?

Lattice QCD

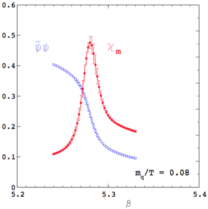
Method for calculating numerically equilibrium properties of QCD.

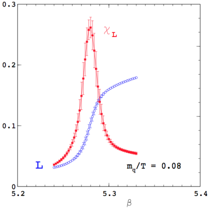
Order parameters

quark condensate: $\langle\overline{\psi}(x)\psi(x)\rangle$ (chiral symmetry breaking);

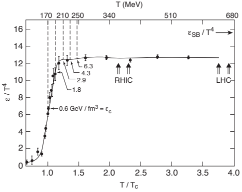
Polyakov loop: $\frac{1}{3}\mathrm{Tr}(\mathcal{P}e^{ig\int_0^\beta A_4(x,\tau)d\tau})$ (confinement).

Both susceptibilities peak for $T_c \simeq 170\,\mathrm{MeV}$.

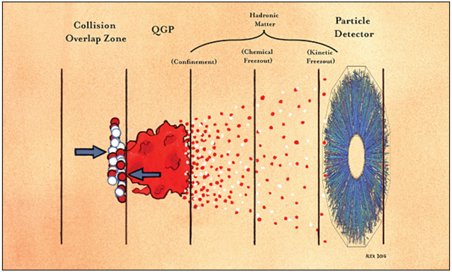


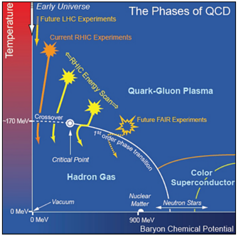


Near $\mu_B \sim 0$ the transition between the two phases is a rapid crossover: the medium properties change quickly and dramatically.



The Little Bang





Incoming nucleons cannot escape right away into the surronding vacuum.
They rescatter each other, and form a dense and strongly interacting matter.

If quick thermalization and sufficiently large energy density ($e > e_{\rm cr}$) this is a quark-gluon plasma.

Thermal pressure leads to collective (hydrodynamic) expansion of the collision fireball.

The fireball cools, and when the energy density reaches $e_{\rm cr}$ the partons convert to hadrons.

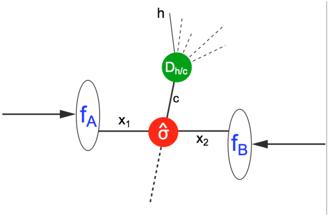
We can probe the QGP only indirectly

Hard processes in heavy ion collisions

Factorization assumed between the perturbative hard part and the non-perturbative fragmentation (FF) and parton distribution functions (PDF).

The PDF are assumed to be universal (known from DIS).

Hard scattering cross sections computed in pQCD.



$$\sigma^{AB\to h} = f_A(x_1, Q^2) \otimes f_B(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{c\to h}(z, Q^2)$$

Medium modified fragmentation function

FF modified when the fragmentation takes place in the medium.

Measuring these modifications: characterization of the medium properties.

Jet Quenching: experimental results I

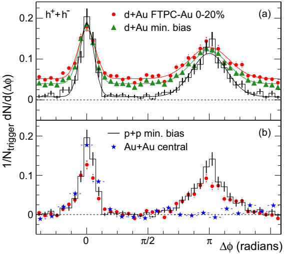
Azimuthal angular correlations

Trigger on fast particle (say $p_T > 4\,\mathrm{GeV}$), then look for correlations with other not too soft hadrons (say $p_T > 2\,\mathrm{GeV}$, to remove background from uncorrelated hadrons)

Fast partons fragment into a narrow angular cone, positive angular correlations at small angles, peak at $\Delta\phi = 0$.

In pQCD hard partons produced back to back in their c.o.m. frame, this leads to a peak at $\Delta\phi = \pi$ (in p+p).

In Au+Au the away-side peak at $\Delta\phi = \pi$ disappears.



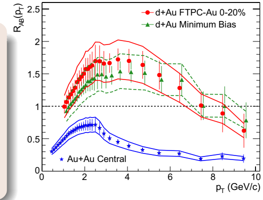
This is a very pictorial way to convince that jets are quenched. However it is hard to be quantitative about that...

Suppression of high p_T partons in central Au+Au collisions

Nuclear modification factor

$$R_{AA} = \frac{1}{N_{\rm coll}^{AA}} \frac{dN^{AA}/dydp_T}{dN^{pp}/dydp_T}$$

No quenching $\rightarrow$ $R_{AA} = 1$



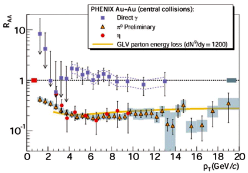


FIG. 1: R_{AA} for π^0 and η mesons, as well as direct photons, as measured by the PHENIX collaboration.

R^{AA} same for all hadrons, must be parton energy loss.

The parton loses energy before hadronization.

Transverse momentum broadening

Change in momentum direction: “transverse momentum broadening”.

tranverse: perpendicular to the original direction of motion

broadening: many hard partons within a jet are kicked from the medium, no change in the mean momentum but the spread of the momenta of the individual partons broadens

The jet quenching parameter $\hat{q}$

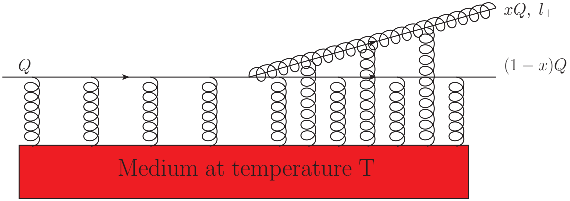
Effect quantified by the jet quenching parameter $\hat{q}$, defined as the mean transverse momentum picked up by the hard parton per unit distance travelled (or, in the high energy limit, per unit time).

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L}$$

Energy loss in the high energy limit

Radiative energy loss

High energy limit: QCD analogue of bremsstrahlung dominates.

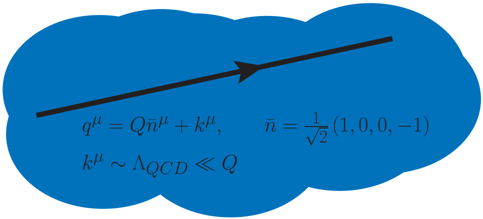


Hard partons constantly kicked by the medium: all subjects to transverse momentum broadening.

The jet quenching parameter enters in the energy loss calculation, thought of as the QGP property constrained by radiative parton energy loss.

However $\hat{q}$ defined via transverse momentum broadening only, i.e. by looking at just one hard parton in the absence of radiation.

Soft Collinear Effective Theory (SCET)



SCET

Effective theory of highly energetic, approximately massless particles interacting with a soft background.

C. Bauer et all., Phys. Rev. D 63: 014006, 2001
C. Bauer et all., Phys. Rev. D63: 114020, 2001
C. Bauer et all., Phys. Lett. B516: 134, 2001
C.Bauer et all., Phys. Rev. D65: 054022, 2002

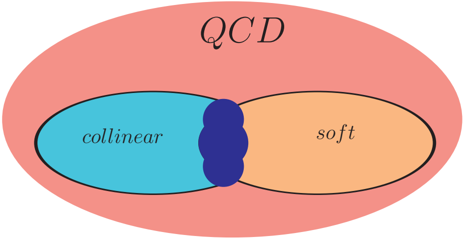
SCET degrees of freedom

Introduce fields for infrared degrees of freedom (in operators)

modes	$q^\mu = (q^+, q^-, q_\perp)$	q^2	fields
collinear	$Q\,(\lambda^2, 1, \lambda)$	$\lambda^2 Q^2$	$\xi_{\bar{n}},\ A^\mu_{\bar{n}}$
soft	$Q\,(\lambda, \lambda, \lambda)$	$\lambda^2 Q^2$	$\xi_s,\ A^\mu_s$
ultra-soft	$Q\,(\lambda^2, \lambda^2, \lambda^2)$	$\lambda^4 Q^2$	$\xi_{us},\ A^\mu_{us}$

Offshell modes with $q^2 \gg \lambda^2 Q^2$ are integrated out (in coefficients).

Soft Collinear Effective Theory (SCET) II



SCET Lagrangian

$$\mathcal{L}_{\rm SCET} = \mathcal{L}_{\rm c} + \mathcal{L}_{\rm s} + \mathcal{L}_{\rm c,s}$$

Collinear sector (QCD in boosted frame) and soft sector (QCD) coupled through a single term.

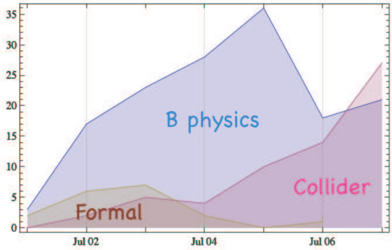
What is SCET good for?

Factorization: obtained from field redefinition and simple algebraic manipulations (decouple soft from collinear in the Lagrangian).

Summation of logarithms at the edges of phase space: obtained from Renormalization Group Equations (RGEs).

Systematically incorporate power corrections in λ.

SCET applications



B physics

Understand many new processes

Power corrections for better precisions

Improve perturbative results by proper resummation of logarithms

Collider physics

Factorization easier to understand

Perturbative calculation by standard EFT steps: sequences of matching and running

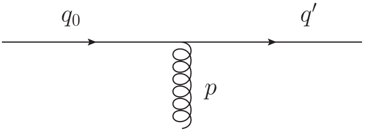
Processes with several scales easily understood

SCET gives operator definitions of all non-perturbative quantities

SCET is a systematic expansion order by order.

from C. Bauer, talk at SCET08

$k_\perp$ broadening in the high energy limit



$$q_0 = (0, Q, 0)$$

$$q' = q_0 + p$$

Soft gluon: $p = (\lambda, \lambda, \lambda)Q$

Final state $Q(\lambda, 1, \lambda)$ not collinear.
Kicked off-shell by $q'^2 \sim \lambda Q^2$.
Process suppressed by $\alpha_s(\sqrt{TQ})$.
Subsequent radiation induced.

Glauber gluon: $p = (\lambda^2, \lambda^2, \lambda)Q$

Final state $Q(\lambda^2, 1, \lambda)$ is collinear
Further Glaubers keep the parton collinear.
Not induced radiation.
Interaction vertex: $\alpha_s(T)$

Relevance of Glauber gluons

Both processes yield $k_\perp$ broadening of order $\lambda Q \sim T$, soft suppressed by $\alpha_s(\sqrt{TQ})$.
Glauber gluons responsible for momentum broadening in the absence of radiation.
All processes (including radiation) must be included before comparing to data.

Glauber gluons

Transverse momentum in excess of their longitudinal momentum.
They cannot be be thought of as being on the mass-shell.

Glauber gluons in SCET

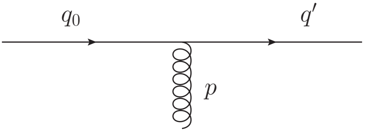
Attempt to prove Glauber factorization in Drell-Yan (flaws in the argument...).
Liu and Ma, arxiv:0802.2973 [hep-ph].

Explicit shown that Glaubers need to be included for a certain class of processes.
Bauer et all., arxiv:1010.1027 [hep-ph].

They might be integrated out of the effective theory, leading to a potential between pairs of collinear fields in opposite directions.
Stewart and Rothstein, in progress, talk at SCET2010.

Used to describe jet broadening in dense QCD medium.
Idilbi and Majumder, PRD80(2009); D'Eramo, Liu and K. Rajagopal, arxiv:1006.1367 [hep-ph].

$k_\perp$ broadening in the high energy limit



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SCET + Glauber Effective Lagrangian I

Goal

Derive an effective Lagrangian to describe the interaction between:

- Collinear partons (quarks or gluons): $q = Q(\lambda^2, 1, \lambda)Q$;
- Glauber gluons: $p = (\lambda^2, \lambda^2, \lambda)Q$.

EFT fields

Light-cone unit vectors: $\bar{n} \equiv \frac{1}{\sqrt{2}}(1, 0, 0, -1)\,, \qquad n \equiv \frac{1}{\sqrt{2}}(1, 0, 0, 1).$

Quark field decomposition:

$\xi(x) = \xi_{\bar{n}}(x) + \xi_n(x), \qquad \xi_{\bar{n}}(x) \equiv \frac{\not{\bar{n}}\not{n}}{2}\xi(x), \qquad \xi_n(x) \equiv \frac{\not{n}\not{\bar{n}}}{2}\xi(x).$

Collinear quark field: "large" component $\xi_{\bar{n}}(x)$,
the "small" component $\xi_n(x)$ is integrated out.

Collinear gluon field: $A^{\mu}_{\bar{n}}(x)$.

Glauber gluon field: $A^{\mu}_{G}(x)$ (background field).

SCET + Glauber Effective Lagrangian II

Effective Lagrangian derivation for collinear quarks

Start from the QCD Lagrangian and keep only the relevant d.o.f.

Integrate out $\xi_n(x)$ by using its equations of motion

$$\mathcal{L}_{QCD} = \bar{\xi} i \not{D} \xi \quad \Rightarrow \quad \mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\,(\bar{n}\cdot D)\,\xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, i\not{D}_\perp \frac{1}{2\, in\cdot D}\, i\not{D}_\perp\, \not{n}\,\xi_{\bar{n}}$$

Restrict to interactions with Glauber gluons only in $D_\mu \equiv \partial_\mu - igA_\mu$, which can only change the perpendicular momentum $q_\perp$ of the collinear quark field.

Remove "large" phases from $\xi_{\bar{n}}(x)$: $\xi_{\bar{n}}(x) = e^{-iQx^+} \sum_{q_\perp} e^{iq_\perp \cdot x_\perp} \xi_{\bar{n},q_\perp}(x)$

Power counting in λ: $\xi_{\bar{n}}(x) \sim \lambda, \quad i\partial_\mu \xi_{\bar{n},q_\perp}(x) \sim \lambda^2 \xi_{\bar{n},q_\perp}(x), \quad A^+ \sim \lambda^2$.

At the leading order in λ:

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp}\, \bar{\xi}_{\bar{n},q'_\perp} \left[i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n}\xi_{\bar{n},q_\perp}$$

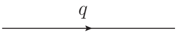
Idilbi, Majumder, Phys.Rev.D80:054022,2009. [arXiv:0808.1087]

SCET + Glauber Effective Lagrangian III

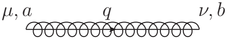
Collinear gluon case

Derivation analogous, Glauber fields in the adjoint.

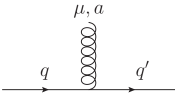
Fenyman rules



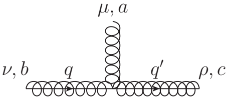
$= i\, \not{\bar{n}} \frac{Q}{2q^+Q-q_\perp^2+i\epsilon}$



$= -i g_{\mu\nu} \frac{1}{2q^+Q-q_\perp^2+i\epsilon} \delta^{ab}$



$= i g t^a n_\mu \not{\bar{n}}$



$= -2ig\, q^- \left(t^a_G\right)_{bc} \bar{n}_\mu g_{\nu\rho}$

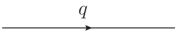
We are ready to compute Feynman diagrams!

SCET + Glauber Feynman rules

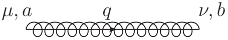
Collinear gluon case

Derivation analogous, Glauber fields in the adjoint.

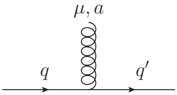
Fenyman rules



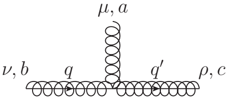
$= i\,\not{\bar n}\,\frac{Q}{2q^+Q-q_\perp^2+i\epsilon}$



$= -i g_{\mu\nu}\frac{1}{2q^+Q-q_\perp^2+i\epsilon}\,\delta^{ab}$



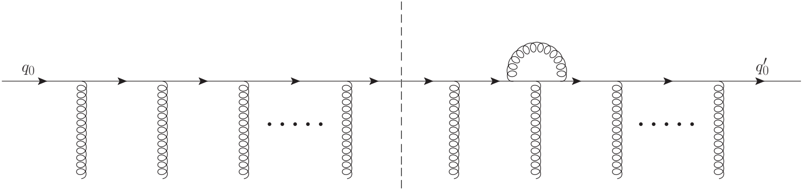
$= i g t^a n_\mu \not{\bar n}$



$= -2ig\, q^- \left(t_G^a\right)_{bc} \bar{n}_\mu g_{\nu\rho}$

We are ready to compute Feynman diagrams!

About loop diagrams....



- Glauber gluon in the loop: Feyman gauge $\propto \eta_{\mu\nu} n^\mu n^\nu = 0$.
- Collinear gluon in the loop: nonvanishing diagram. Cutting across the loop describes a radiative process, which we do not consider.

Optical theorem

Field theory tools

Use the optical theorem to relate $P(k_\perp)$ to a matrix element that we can calculate using the Feynman rules we have just derived.

Unitarity of the S-matrix

Probability amplitude for the process $\alpha \to \beta$: $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$.

The S-matrix is unitary: $\sum_\beta |S_{\beta\alpha}|^2 = 1 \Rightarrow 2\,\mathrm{Im}\ M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$.

Cubic box of sides L. Periodic BC $\Rightarrow \mathbf{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$.

With radiation turned off β differs from α only on $k_\perp$: $\sum_\beta = L^2 \int \frac{d^2 k_\perp}{(2\pi)^2}$.

Probability distribution $P(k_\perp)$

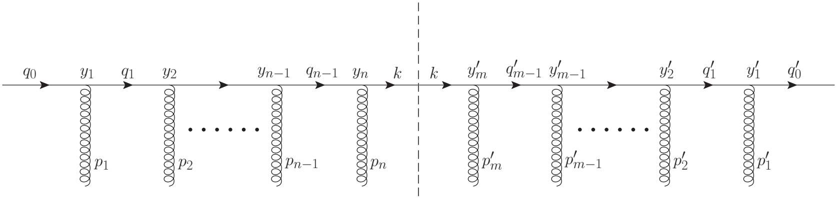
We identify: $P(k_\perp) = L^2 \begin{cases} |M_{\beta\alpha}|^2 & \beta \neq \alpha \\ 1 - 2\mathrm{Im}\, M_{\alpha\alpha} + |M_{\alpha\alpha}|^2 & \beta = \alpha \end{cases}$

Unitarity of S-matrix $\quad \leftrightarrow \quad$ $P(k_\perp)$ is normalized.

Forward scattering amplitude

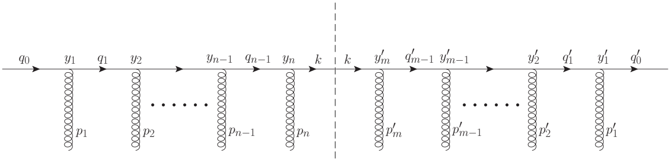
Strategy

- Compute $2\,\mathrm{Im}\ M_{\alpha\alpha}$ by cutting the appropriate diagrams;
- Use the unitarity relation to identify $\sum_\beta |M_{\beta\alpha}|^2$;
- Read off $|M_{\beta\alpha}|^2$, and evaluate $P(k_\perp)$ for $k_\perp \neq 0$;
- The normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$ fixes $P(0)$.



$$2\,\mathrm{Im}\,M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$$

A few comments on $\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$



Gluon momenta

Gluon momenta p_i and p'_j fixed by four-momentum convervation at each vertex
$p_i = q_i - q_{i-1}$ $(i = 1, \ldots, n-1)$; $p_n = k - q_{n-1}$; $p'_j = q'_j - q'_{j-1}$ $(j = 1, \cdots, m-1)$.

$n + m$ gluon field insertions, but only $n + m - 1$ independent momentum integrations

The cut momentum: $k_\perp$ not integrated over

The cut momentum k is the four-momentum of the hard parton in the final state.

For forward scattering amplitude: $q_0 = q'_0 \Rightarrow k_\perp = \sum_{i=1}^{n} p_{i\perp} = \sum_{i=1}^{m} p'_{i\perp}$

$p_{i\perp}$'s and $p'_{i\perp}$'s are of order $\lambda Q = T$, $k_\perp$ may turn out to be larger.

Typical value of $k_\perp^2$ is $\hat{q}L$, in particular $k_\perp^2$ grows with L.

Forward scattering amplitude evaluation IIIold

Summing over all the diagrams

Summing over m and n and taking the $\langle \ldots \rangle$ at the end of the calculation

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3\, N_c} \int dy^+ dy_\perp dy'_\perp\, e^{-ik_\perp \cdot (y_\perp - y'_\perp)} \left\langle \mathrm{Tr} \left[\left(W_F^\dagger [y^+, y'_\perp] - 1 \right) \left(W_F [y^+, y_\perp] - 1 \right) \right] \right\rangle$$

where we have introduce the fundamental Wilson line along the lightcone

$$W_F \left[y^+, y_\perp \right] \equiv P \left\{ \exp \left[ig \int_0^{L^-} dy^- \, A^+ (y^+, y^-, y_\perp) \right] \right\}$$

Cleaning up the result

The medium is translation invariant:
result independent on y^+ and depends only on $x_\perp = y_\perp - y'_\perp$.

The incident flux is $1/L^3$, so t/L particles going through the box in time t.
Divide the result by t/L to obtain $P(k_\perp)$ for a single particle.

$\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM

At RHIC physics of the QGP at scales $\sim T$ is not weakly coupled.

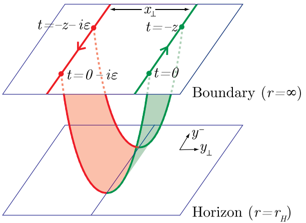
Insights can be obtained by calculating $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM, by using gauge/gravity duality. Liu, Rajagopal, Wiedemann, PRL97,2006 [hep-ph/0605178]

LRW evaluated $\mathcal{W}_{\mathcal{R}}(x_\perp)$ with the standard, i.e. wrong, operator ordering.

Standard AdS/CFT evaluation

- $\mathcal{N} = 4$ $SU(N_c)$ gauge theory, large N_c and $g^2_{YM}N_c$ limit;
- Gravity dual: 4+1dimensional AdS Schwarzschild black hole with Hawking temperature T;
- $\langle W(\mathcal{C})\rangle = \exp\left[i\left\{S(\mathcal{C}) - S_0\right\}\right]$.

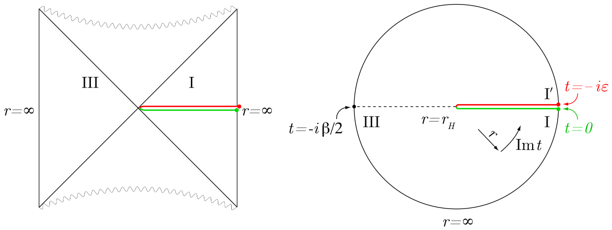
($S(\mathcal{C})$ is the action of an extremized string worldsheet, bounded by the Wilson lines along the contour $\mathcal{C}$ located at the 3+1 dimensional boundary, "hanging" into the AdS black hole spacetime. S_0 is twice the action of a disconnected world sheet hanging straight down from one Wilson line to the horizon.)



Taking the order into account

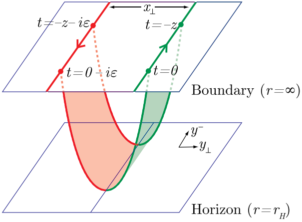
Our procedure to take the order into account: specific example of the more general Lorentzian AdS/CFT. (Skenderis, van Rees, JHEP 0905:085,2009. [arXiv:0812.2909].)

Construct the bulk geometry for the $\mathrm{Im}\, t = -i\epsilon$ segment of the Schwinger-Keldysh contour.



Any string world sheet connecting the Wilson lines at $\mathrm{Im}\, t = 0$ and $\mathrm{Im}\, t = -i\epsilon$, as in our case, must touch the horizon.

$\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM revisited



The old result is unchanged!

The LRW world sheet is the only one which touches the horizon.
Subtlety resolved, result unchanged.

For a hard parton in the adjoint, we find

$$\mathcal{W}_{\mathcal{A}}(x_\perp) = \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L^- x_\perp^2\right], \qquad P(k_\perp) = \frac{4\sqrt{2}\pi}{\hat{q}L^-}\exp\left[-\frac{\sqrt{2}k_\perp^2}{\hat{q}L^-}\right],$$

where $\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{g^2 N_c}T^3$.

At strong coupling $P(k_\perp)$ is a Gaussian and describes diffusion in $k_\perp$ space.

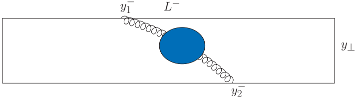
$\hat{q}$ in the same ballpark as the values of $\hat{q}$ inferred from RHIC data.

$\hat{q}$ not proportional to s (i.e. N_c^2), so does not count scattering centers.

$\hat{q}$ in weakly coupled QCD plasma

$\hat{q}$ can be computed for the QCD plasma at high enough temperatures that physics at scales $\sim T$ is weakly coupled.

Arnold, Xiao, Phys.Rev.D78:125008,2008, arXiv:0810.1026 [hep-ph];
Caron-Huot, Phys.Rev.D79:065039,2009, arXiv:0811.1603 [hep-ph].



We find:

$P(k_\perp) = \sqrt{2} g^2 C_{\mathcal{R}} L \int \frac{dk^-}{2\pi} \, k_\perp^2 \, G^{>}_{\mu\nu}(0, k^-, k_\perp) \, \bar{n}^\mu \bar{n}^\nu$;

$P(k_\perp)$ very different than a strong coupling;

$P(k_\perp)$ falls off slowly $\sim k_\perp^{-4}$ at large $k_\perp$;

$\hat{q}_{\mathcal{R}} \propto g^4 N_c^2$, and UV log divergent.

Work in progress..... (in collaboration with M. Lekaveckas)

RHIC data and strong coupling result

High p_T suppression entirely due to parton energy loss.

$$d\sigma_{med}^{AA\to h\,rest} = \sum_f d\sigma_{vac}^{AA\to f\,X} \otimes P_f(\Delta E, L, \hat{q}) \otimes D_{f\to h}^{vac}(z)$$

High p_T limit: properties of the medium enter P_f only through $\hat{q}$.

RHIC data fit

Introduce: $\hat{q} = 2\,K\,e^{3/4}$

More stable on K rather than $\hat{q}$.

Fitting RHIC data: $K = 4.1 \pm 0.6$.

At RHIC temperature regime:
$e \sim (9 - 11)\,T^4$.

Therefore we get: $\hat{q} \sim 4.5\,\mathrm{GeV}^2/\mathrm{fm}$

Strong coupling result

We rewrite the result:
$\hat{q} = 57\sqrt{\alpha_{SYM}\frac{N_c}{3}}\;T^3$

By comparing with the energy density we get a good match for $\alpha_{SYM} \sim 0.66$ and $N_c = 3$.

Extraction of $\hat{q}$ from LHC data should be under better control, since the separation of scale will be more quantitatively reliable.

The jet quenching parameter $\hat{q}$

$\hat{q}$ as a diffusion parameter

$\mathcal{W}(x_\perp) = \exp\left[-\frac{1}{4\sqrt{2}}\,\hat{q}\,L^-\,x_\perp^2\right]$ $\Rightarrow$ expression self-consistent ($\hat{q}$).

The probability distribution results: $P(k_\perp) = \frac{4\sqrt{2}\pi}{\hat{q}\,L^-}\,\exp\left[-\frac{\sqrt{2}k_\perp^2}{\hat{q}\,L^-}\right]$

Kicks from Glauber gluons, the parton performs Brownian motion in momentum space even though it stays on a light-like trajectory in coordinate space.

The diffusion constant is $D = \hat{q}L$.

Future directions

- Compute soft and collinear radiation, and see how $\hat{q}$ enters in the spectrum of the radiated gluons;
- See whether and how factorization arises for the radiation;
- Compute corrections to factorization (i.e. non-infinite Q), use the SCET formalism to compute the corrections suppressed by powers of T/Q.

(in progress, in collaboration with C. Lee)

- weak-coupling $\hat{q}$ evaluation for QCD plasma at high enough T (in progress, in collaboration with M. Lekaveckas);
- compare our $P(k_\perp)$ with the corresponding quantity in $N = 4$ SYM.