# Subtractions for SCET

*in collaboration with:*

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# Why subtractions?

- future in SCET: jet algorithms
- jet functions and soft functions with algs are hard
- only handful of (relatively simple!) examples:
  - N-jets in e+e- with the shape of N measured — Ellis, Hornig, Lee, Walsh, Vermilion
  - N-jet threshold with N > 0 (or 1) — Bauer, Dunn, Hornig
  - now, N-jettiness (see Teppo's talk) — Tackmann, Stewart, Waalewijn, Jouttenus
- alg dependence on measured jets fncs power suppressed

⟹ focus on soft functions

# Original Subtractions

- Add & subtract to get finite Real and Virtual:

Ellis, Kunzst, Soper

Catani, Seymour

$$\sigma^{NLO} = \int_{m+1} \left[ \left( d\sigma^R \right)_{\epsilon=0} - \left( d\sigma^A \right)_{\epsilon=0} \right] + \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

- subtractions are universal and analytically calculable, same singularities as Virtual and Real (point-by-point!)
- difference here Soft/Jet Functions: depend on observable/algorithm
- however, will see that all obs/algs belong in one of 2 universality classes

# Outline

- UV study: all alg/obs combos 2 types of subtractions
- Angularities as basis for all UV limits for φ-symm obs
- Examples:
  - cross-check theta'-phi from theta-phi jets
  - jet shapes for eta-phi jets @ pp
  - 1-jettiness
  - others...
- Compare/contrast with related work (see Teppo's talk on Tues)

# Soft Functions: $\mathcal{S}^{(1)}(\mathcal{A},\mathcal{M};\{\sigma_m\}) = \sum_{\langle i,j\rangle}\int\frac{\mathrm{d}^d k}{(2\pi)^d}N_{ij}(k)\sum_{k=0}^{N}\Theta_{\mathcal{A}}^{k}\Delta_{\mathcal{M}}^{k}$

- integrand:

$$N_{ij} = -g^2\mu^{2\epsilon}\,\mathbf{T}_i\cdot\mathbf{T}_j\,\frac{n_i\cdot n_j}{n_i\cdot k\, n_j\cdot k}2\pi\,\delta(k^2)\theta(k^0)$$

- measurement:

$$\Delta_{\mathcal{M}}^{k}(k) = \delta\left(\sigma_k - \sigma_k(k)\right)\prod_{l\neq k}\delta\left(\sigma_l\right)$$

- algorithm: $\Theta_{\mathcal{A}}^{k}$
  - e.g., $\Theta_{\text{cone}}^{k}(k) = \theta\left(\frac{n_k\cdot k}{\bar{n}_k\cdot k} < \tan^2\frac{R}{2}\right)$

$$\Theta_{\text{cone}}^{0}(k) = \left(1 - \sum_{k=1}^{N}\Theta_{\text{cone}}^{k}(k)\right)\theta(k^0<\Lambda)$$

# Divergences in Soft Functions

$$N_{ij} = -g^2\mu^{2\epsilon}\,\mathbf{T}_i\cdot\mathbf{T}_j\;\frac{n_i\cdot n_j}{n_i\cdot k\,n_j\cdot k}2\pi\,\delta(k^2)\theta(k^0)$$

- IR: $k^0 \to 0$
- Collinear: $n_i \cdot k \to 0$
- UV: $k^0 \to \infty$

$$\left.\begin{array}{l} k^0 \to 0 \\ n_i\cdot k \to 0 \end{array}\right\} 1/\epsilon^2 \qquad \left.\begin{array}{l} n_i\cdot k \to 0 \\ k^0 \to \infty \end{array}\right\} 1/\epsilon^2$$

# Divergences in *Differences of* Soft Functions

- take moments for easier comparison:

$$\langle \sigma_k^n \rangle = \prod_l \int_0^1 \mathrm{d}\sigma_l \, \sigma_k^n \, \mathcal{S}^{(1)}\left(\mathcal{A}, \mathcal{M}; \{\sigma_m\}\right) \equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d} I_k^n(k)$$

- IR & Coll. trivial due to IR safety:
  - n > 0:

$$\lim_{k^0 \to 0} \sigma_i(k) = 0 \qquad \lim_{k^0 \to 0} \left(I_k^n(k) - \tilde{I}_k^n(k)\right) = 0$$

$$\Rightarrow$$

$$\lim_{n_i \cdot k \to 0} \sigma_i(k) = 0 \qquad \lim_{n_i \cdot k \to 0} \left(I_k^n(k) - \tilde{I}_k^n(k)\right) = 0$$

  - n = 0:

$$\lim_{k^0 \to 0} \left(I^0(k) - \tilde{I}^0(k)\right) \propto \sum_{k=0}^{N} \Big[\Theta_{\mathcal{A}}^{k}(k) - \Theta_{\widetilde{\mathcal{A}}}^{p}(k)\Big] = 0$$

$$\lim_{n_i \cdot k \to 0} \left(I^0(k) - \tilde{I}^0(k)\right) \propto \sum_{k=0}^{N} \Big[\Theta_{\mathcal{A}}^{k}(k) - \Theta_{\widetilde{\mathcal{A}}}^{k}(k)\Big] = 0$$

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- UV safety:
  - n > 0:

$$\lim_{k^0\to\infty}\left(I^0(k) - \tilde{I}^0(k)\right) \propto \sum_{k\in\text{meas}} \left[\Theta_{\mathcal{A}}^k(k)\theta(\sigma_k(k)<1) - \Theta_{\widetilde{\mathcal{A}}}^k(k)\theta(\widetilde{\sigma}_k(k)<1)\right] + \sum_{k\notin\text{meas}} \left[\Theta_{\mathcal{A}}^k(k) - \Theta_{\widetilde{\mathcal{A}}}^k(k)\right]$$

what matters depends on whether alg. or obs. “wins”

only alg. matters

  - n = 0:

$$\lim_{k^0\to\infty}\left(I_k^n(k) - \tilde{I}_k^n(k)\right) \propto \sigma_k^n(k)\Theta_{\mathcal{A}}^l(k)\theta(\sigma_k(k)<1) - \widetilde{\sigma}_k^n(k)\Theta_{\widetilde{\mathcal{A}}}^k(k)\theta(\widetilde{\sigma}_k(k)<1)$$

⇒ 2 types of soft functions

# Type I vs. Type II

- Type I: $\lim_{k^0\to\infty} \Theta^{\,l}_{\mathcal{A}}(k)\theta(\sigma_l(k)<1) = \lim_{k^0\to\infty} \theta(\sigma_l(k)<1)$ (obs. “wins”)
  - incl. kt/cone with any angularity a < 2 (or obs. with same UV)
  - excl. kt with 1 < a < 2
  - N-jettiness
  - N-jet threshold at e+e- and pp colliders
- Type II: $\lim_{k^0\to\infty} \Theta^{\,l}_{\mathcal{A}}(k)\theta(\sigma_l(k)<1) = \lim_{k^0\to\infty} \Theta^{\,l}_{\mathcal{A}}(k)$ (alg. “wins”)
  - “unmeas.” jets (no shape is probed)
  - excl. kt with a < 1

# Angularities as Basis of Obs. in UV

- def'n (Q arbitrary normalization):

Berger, Kucs, Sterman

$$\tau_a = \frac{1}{Q}\sum_i E_i (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a} = \frac{1}{Q}\sum_i |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$

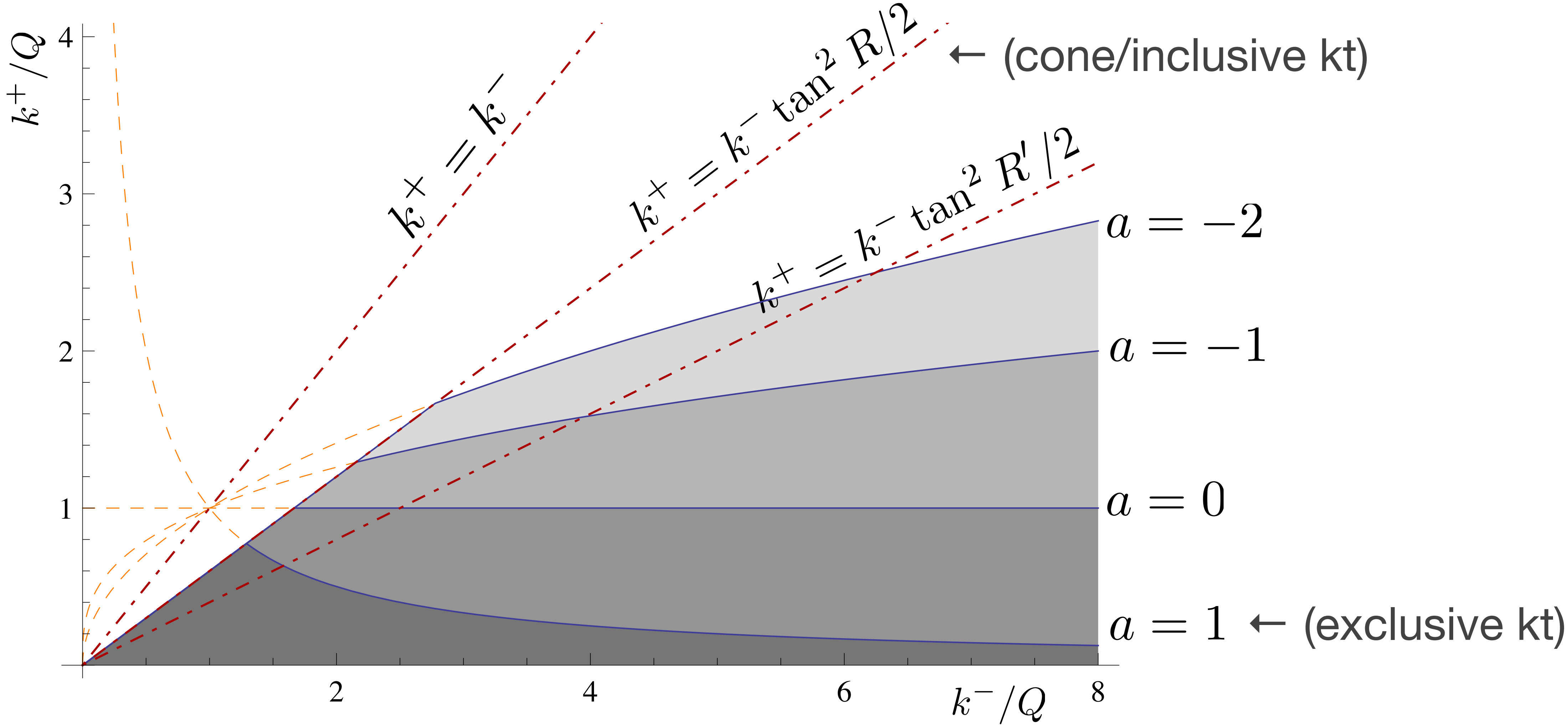
- for fixed τ, as E → ∞ (so θ → 0):

$$\tau_a \to E_i\, \theta^{2-a}$$

- this is the most general, IR safe (for a < 2) result in θ → 0 limit
- results exist for jet shapes for all a < 1 (will use as "subtractions")...

Ellis, Hornig, Lee, Walsh, Vermilion

# Demonstration of Type I vs. II in $k^+k^-$-plane




- Note: any measurement (w/ a < 1) does not change divergences of Type II

# Final Result

$$\mathcal{S}^{(1)}(\mathcal{A},\mathcal{M};\{\sigma_m\}) = \widetilde{\mathcal{S}}^{(1)}(\widetilde{\mathcal{A}},\widetilde{\mathcal{M}};\{\widetilde{\sigma}_m \to \sigma_m\}) + \mathcal{D}^0 \prod_i \delta(\sigma_i) + \sum_i \mathcal{D}_i^1 \prod_{j\neq i} \delta(\sigma_j) \left(\frac{1}{\sigma_i}\right)_+$$

$$\mathcal{D}^0 = \langle \sigma^0 \rangle - \langle \tilde{\sigma}^0 \rangle \qquad \mathcal{D}_k^n = \langle \sigma_k^n \rangle - \langle \widetilde{\sigma}_k^n \rangle$$

($0^{th}$ moment w.r.t. all obs) ($n^{th}$ moment w.r.t. $k^{th}$ obs)

- we use jet shape calc. as subtraction (variation needed for threshold)
- moments only involve ∫dΩ:

$$\mathcal{D}^0 = -\frac{\alpha_s}{\pi} \sum_{\langle i,j \rangle} \mathbf{T}_i \cdot \mathbf{T}_j \int \frac{d\Omega}{4\pi} \frac{n_i \cdot n_j}{(1-\hat{k}\cdot\hat{n}_i)(1-\hat{k}\cdot\hat{n}_j)}$$

$$\times \Bigg[ \underbrace{\sum_{k=1}^N (\Theta_{\mathcal{A}}^k - \Theta_{\tilde{\mathcal{A}}}^k) \log \frac{\tilde{k}_{\max}^k(\Omega)}{\Lambda}}_{\text{diff. from alg.}} + \underbrace{\left(1 - \sum_{k=1}^N \Theta_{\mathcal{A}}^k\right) \log \frac{E_{\rm cut}(\Omega)}{\Lambda}}_{\text{diff. from veto}} + \underbrace{\sum_{k=1}^N \Theta_{\mathcal{A}}^k \log \frac{k_{\max}^k(\Omega)}{\tilde{k}_{\max}^k(\Omega)}}_{\text{diff. from obs.}} \Bigg]$$

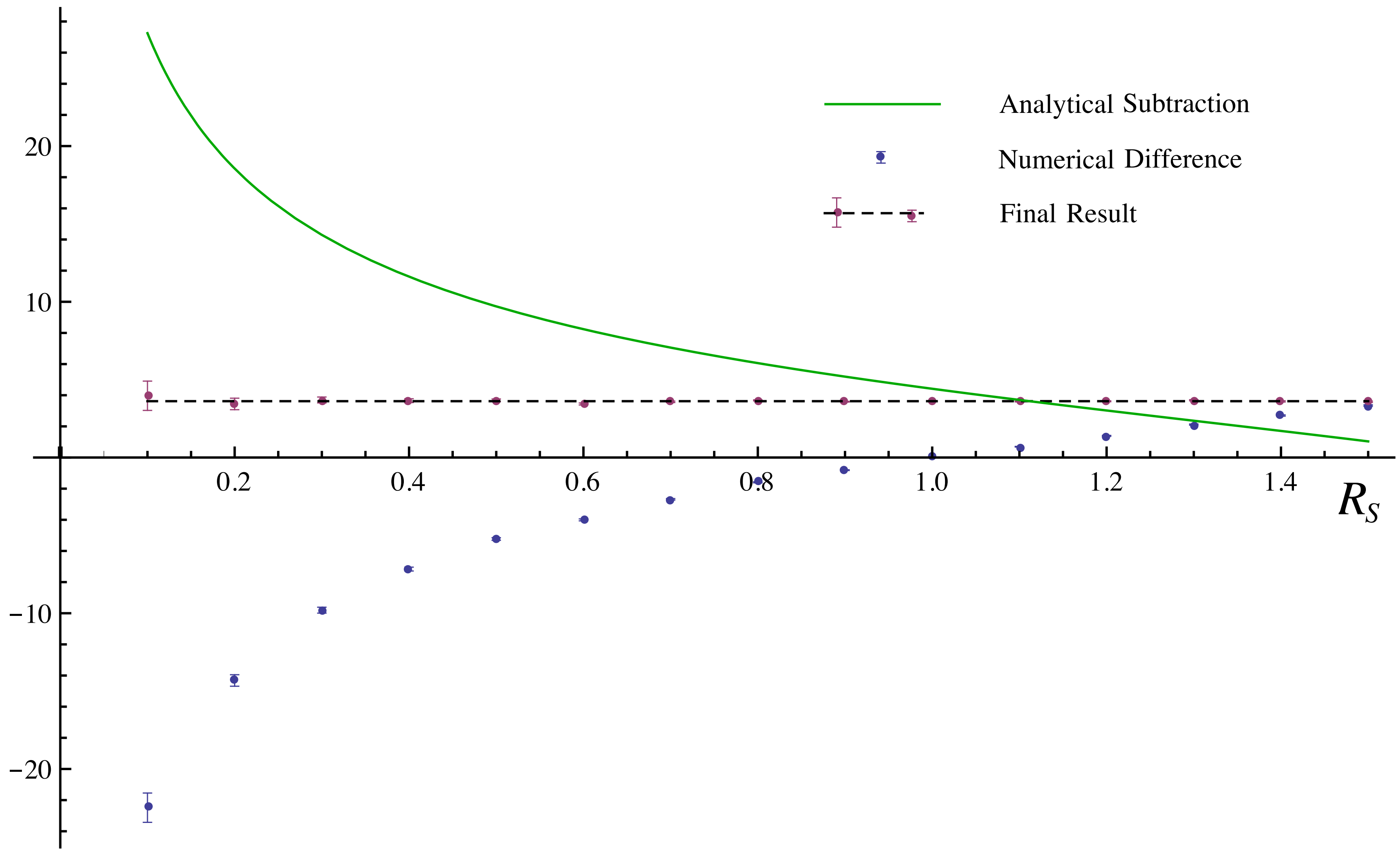
$$k_{\max}^k(\Omega) = k^0/\sigma_k(k)$$

$$\tilde{k}_{\max}^k(\Omega) = Q_i\,(1+\hat{k}\cdot n_k)^{-a/2}(1-\hat{k}\cdot n_k)^{-1+a/2}$$

$$E_{\rm cut}(\Omega) = \begin{cases} E_{\rm cut} \\ p_{\rm cut}^T/\sin\theta \\ \vdots \end{cases}$$

# θφ jets of $R = R_t$ from $R = R_s$ jets (cross-check)

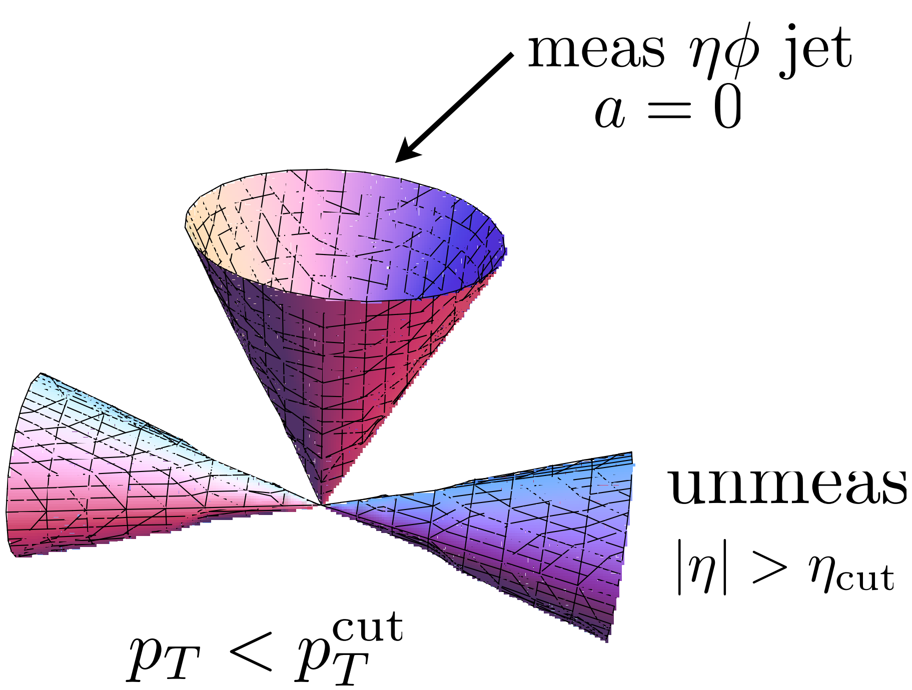
- Changing subtraction $R_s$ in mercedes benz (target $R_t$ fixed):



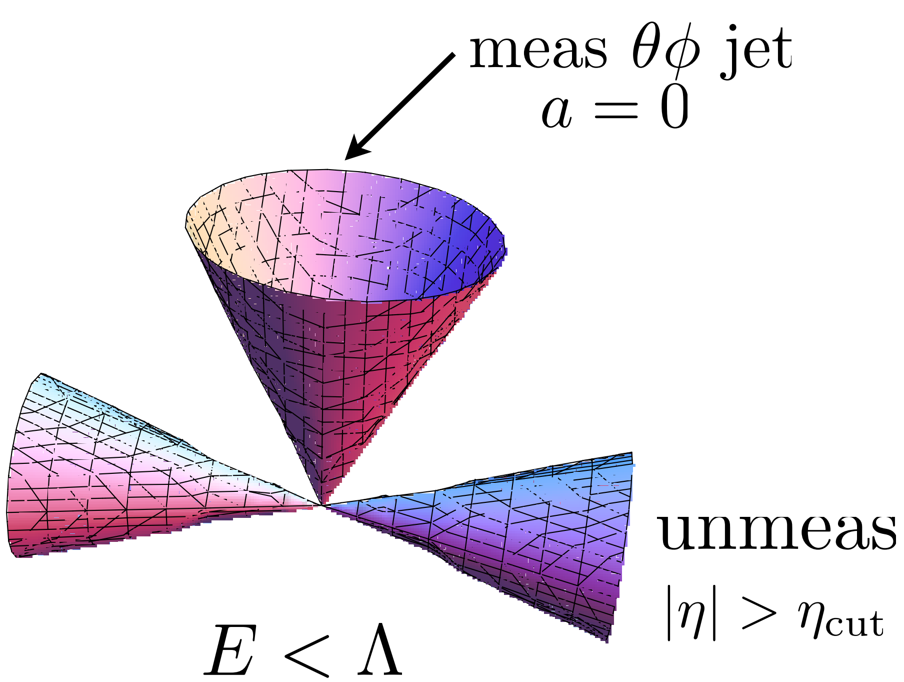
# jet shapes of ηφ jets (w/ $pT_{cut}$ & $\eta_{cut}$)

- one jet w/ angularity measured, two beams with $|\eta| > \eta_{cut}$ cut-out
  - needs Type I for meas. jet, Type II for beams

Target:



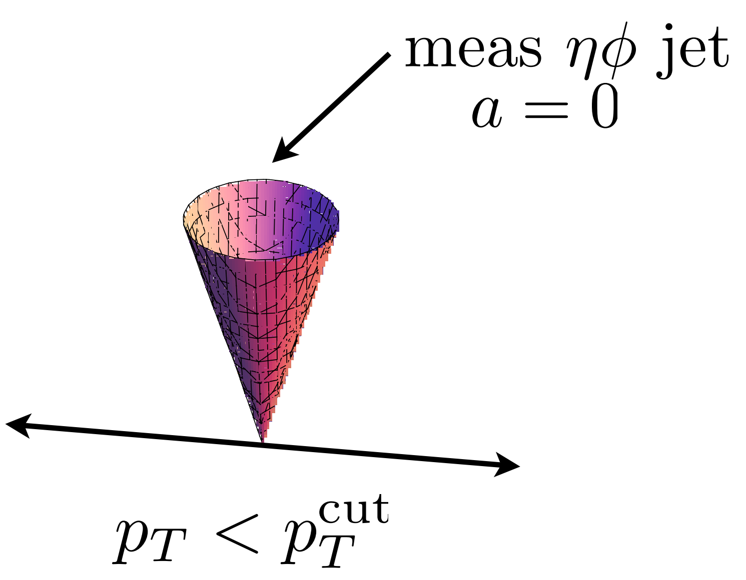

Subtraction:



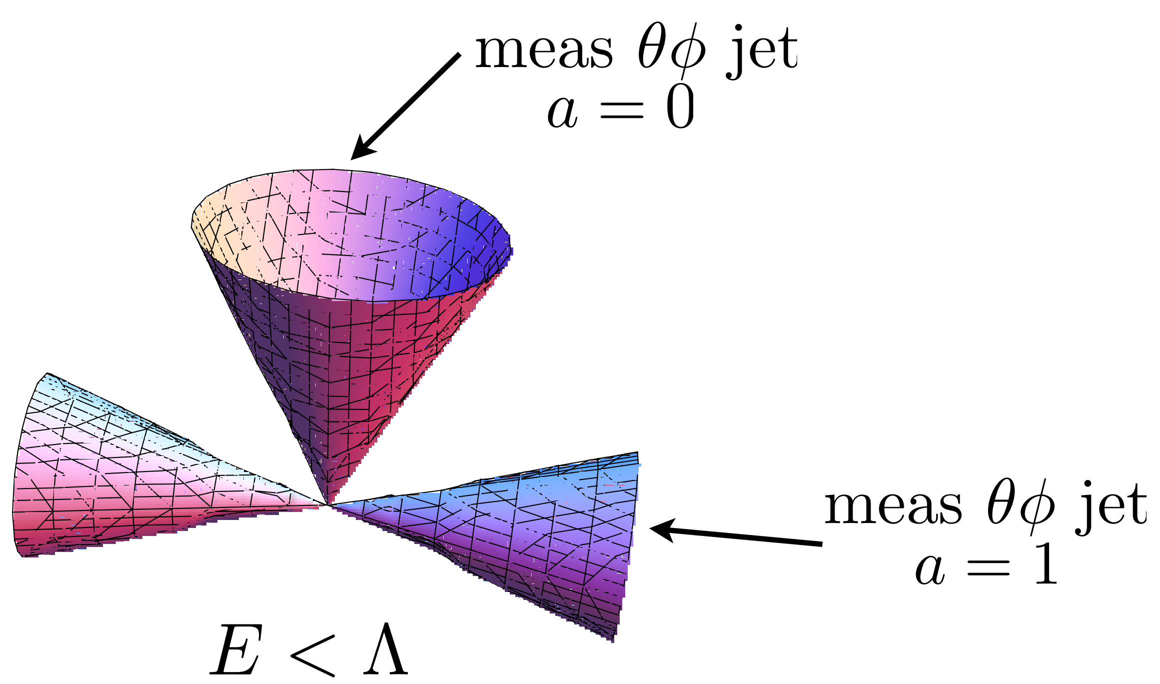
# jet shapes of **ηφ** jets (w/ $pT_{cut}$ & $\eta_{cut}$)

- note: could also do IF we had a=1 (no logs of $\eta_{cut}$)

Target:



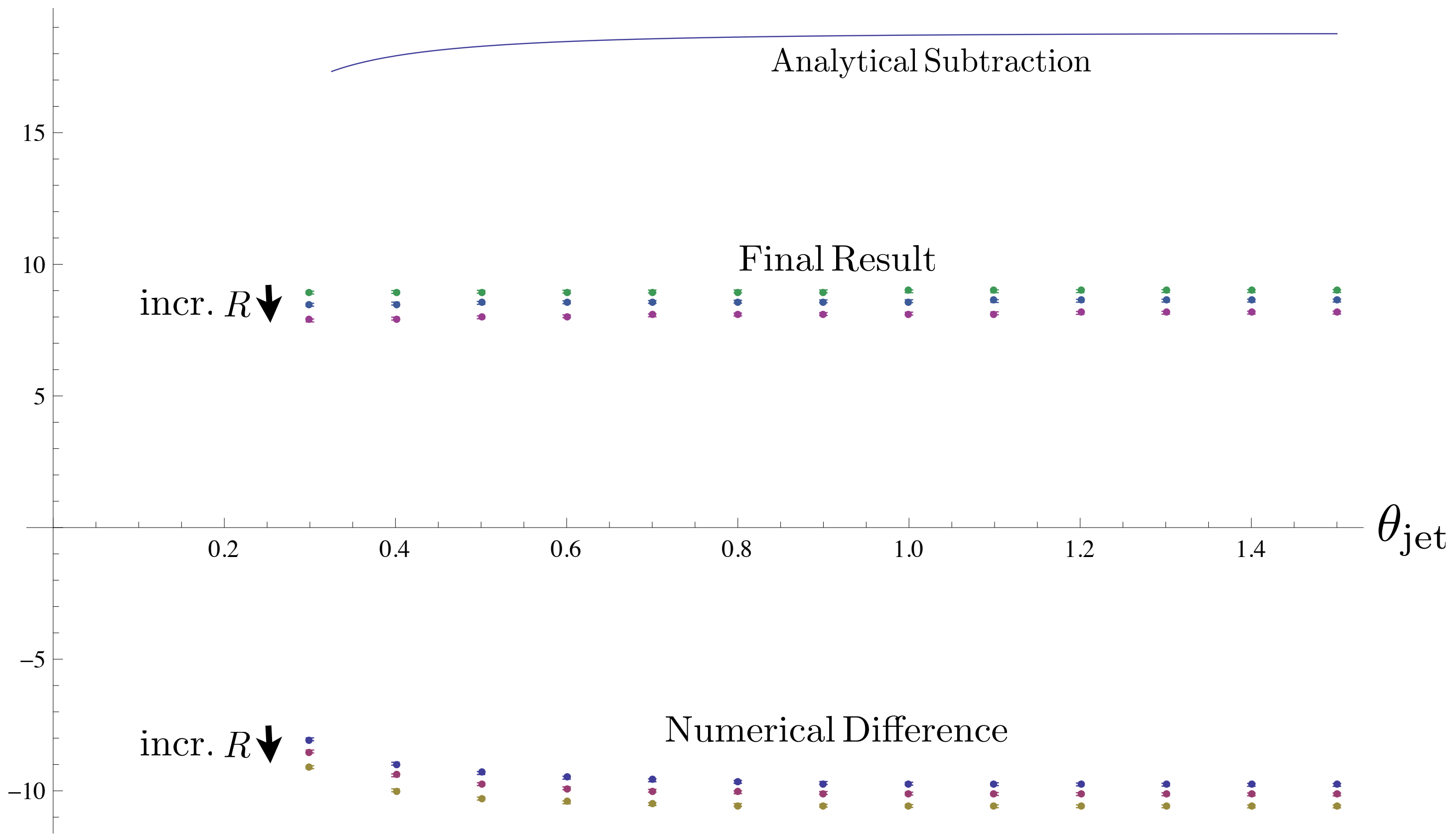

Subtraction:




- question: ok that a=1 weights near beam more than a=0 ($\theta^1$ vs $\theta^2$)

# jet shapes of $\eta\varphi$ jets (w/ $pT_{cut}$ & $\eta_{cut}$)

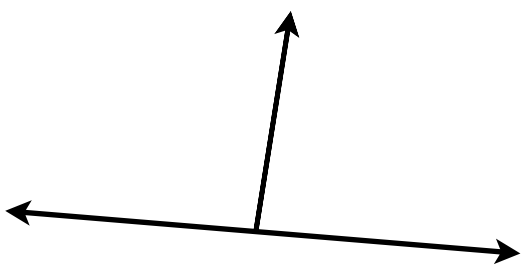
- one jet w/ angularity measured, two beams with $|\eta| > \eta_{cut}$ cut-out
  - needs Type I for meas. jet, Type II for beams
- beam-beam color-connection contribution as a function of $R_{jet}$:



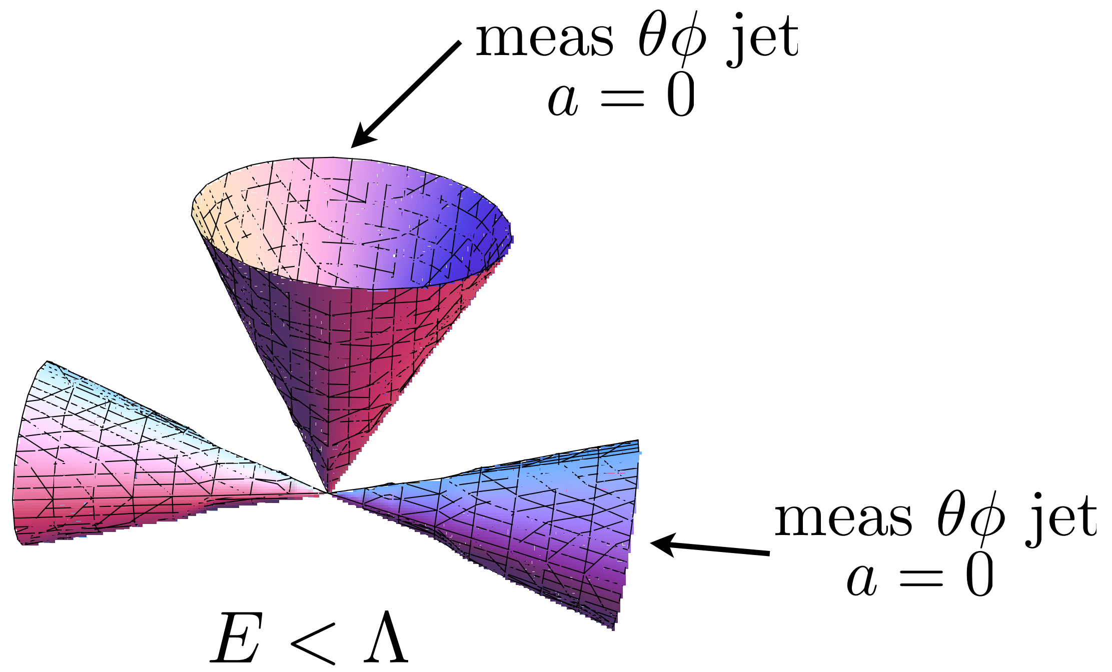
# 1-Jettiness from θφ jets w/ $E_{cut}$

- 1-jettiness has measurement everywhere:

Target:

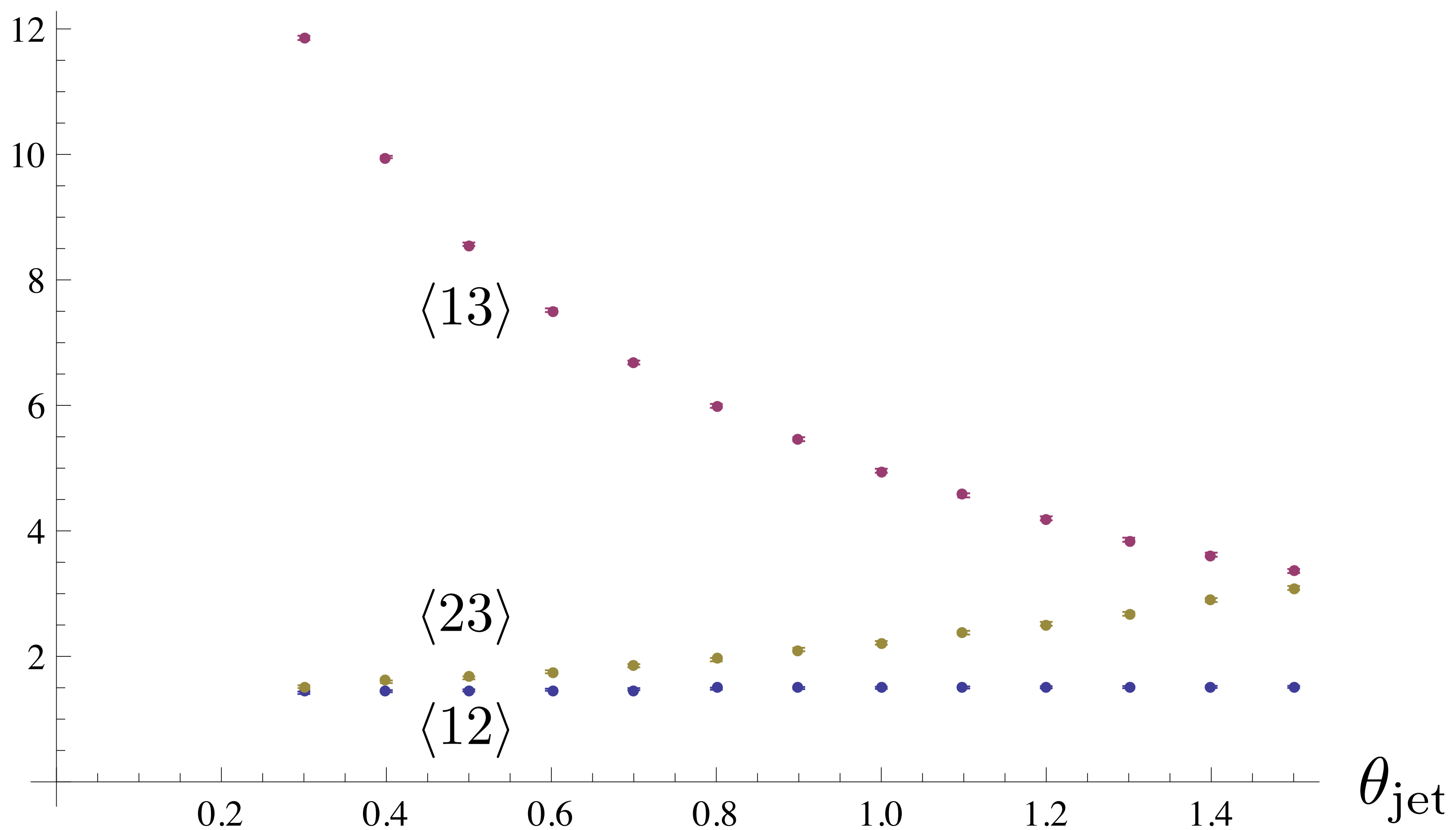


Subtraction:



# 1-Jettiness from θφ jets w/ $E_{cut}$

- 1,2 = beams; 3 = jet



# More examples

- jet shapes that only asymptote to an angularity
- N-jet threshold at pp (subtraction = N-jet threshold at e+e-)
- $pT_{cut}$ everywhere (no large logs of $\eta_{cut}$)
- azimuthally asymmetric observables (e.g., planarity)

# Comparison to the “Hemisphere Decomposition” (MIT) (see Teppo’s talk)

$$F(\{k_i\},p) = F_{ij,\mathrm{hemi}}(\{k_i\},p) + F_{ji,\mathrm{hemi}}(\{k_i\},p) + \sum_{m\neq i} F_{ij,m}(\{k_i\},p) + \sum_{m\neq j} F_{ji,m}(\{k_i\},p)$$

Analytical, Divergent:

$$F_{ij,\mathrm{hemi}}(\{k_i\},p) = \theta(p^j - p^i)\,\delta[k_i - f_i(p)] \prod_{l\neq i}\delta(k_l)$$

No R-Dependence

Numerical, Finite:

$$F_{ij,m}(\{k_i\},p) = \theta(p^j - p^i)\,\Theta_m(p) \prod_{l\neq i,m}\delta(k_l)\Big\{\delta(k_i)\,\delta[k_m - f_m(p)] - \delta[k_i - f_i(p)]\,\delta(k_m)\Big\}$$

- $F_{ij,\mathrm{hemi}}$ easier than $S_{ij}^{\mathrm{meas}}(\tau^k)$, but not known for a != 0
- Scope: only works for Type I (UPDATE: maybe Type II ??)

# Comparison to the "Hemisphere Decomposition" (MIT) (see Teppo's talk)

- Numerics: unbounded y integral
  ⇒ analytical evaluation needed (just for N-jettiness)

$$I_0(\alpha,\beta,\{\alpha_l,\beta_l,\phi_l\}) = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi \int \frac{\mathrm{d}y}{y}\, \theta\big(y-\sqrt{\beta/\alpha}\big)\, \theta\big(1/\alpha - 1 - y^2 + 2y\cos\phi\big) \times \prod_l \theta\Big[\alpha_l - 1 + (\beta_l - 1)y^2 - 2y\big[\sqrt{\alpha_l\beta_l}\,\cos(\phi+\phi_l) - \cos\phi\big]\Big]\,,$$

$$I_1(\alpha,\beta,\{\alpha_l,\beta_l,\phi_l\}) = \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi \int \frac{\mathrm{d}y}{y}\, \ln(1+y^2-2y\cos\phi)\, \theta\big(y-\sqrt{\beta/\alpha}\big)\, \theta\big(1/\alpha - 1 - y^2 + 2y\,\cos\phi\big) \times \prod_l \theta\Big[\alpha_l - 1 + (\beta_l - 1)y^2 - 2y\big[\sqrt{\alpha_l\beta_l}\,\cos(\phi+\phi_l) - \cos\phi\big]\Big]\,.$$

Unbounded dy/y

Quadratic Eq. in y:

$$y_\pm(\phi,\alpha_l,\beta_l,\phi_l) = \frac{1}{1-\beta_l}\left\{\cos\phi - \sqrt{\alpha_l\beta_l}\cos(\phi+\phi_l) \pm \sqrt{\big[\cos\phi - \sqrt{\alpha_l\beta_l}\cos(\phi+\phi_l)\big]^2 - (1-\alpha_l)(1-\beta_l)}\right\}$$

1. $\beta_l < 1$:
   - (a) $\alpha_l \geq 1$: $y \leq y_+(\phi,\alpha_l,\beta_l,\phi_l)$
   - (b) $\alpha_l < 1$: $y_-(\phi,\alpha_l,\beta_l,\phi_l) \leq y \leq y_+(\phi,\alpha_l,\beta_l,\phi_l)$
     $\cos\phi - \sqrt{\alpha_l\beta_l}\cos(\phi+\phi_l) \geq \sqrt{(1-\alpha_l)(1-\beta_l)}$
2. $\beta_l > 1$:
   - (a) $\alpha_l \leq 1$: $y \geq y_+(\phi,\alpha_l,\beta_l,\phi_l)$
   - (b) $\alpha_l > 1$: $y \leq y_-(\phi,\alpha_l,\beta_l,\phi_l)$ or $y \geq y_+(\phi,\alpha_l,\beta_l,\phi_l)$
     $\cos\phi - \sqrt{\alpha_l\beta_l}\cos(\phi+\phi_l) \geq \sqrt{(1-\alpha_l)(1-\beta_l)}$
3. $\beta_l = 1$:
   - (a) $\alpha_l \leq 1$: $y \geq \dfrac{1-\alpha_l}{2\cos\phi - 2\sqrt{\alpha_l}\,\cos(\phi+\phi_l)}$
     $\cos\phi \geq \sqrt{\alpha_l}\,\cos(\phi+\phi_l)$
   - (b) $\alpha_l > 1$: $y \leq \dfrac{\alpha_l - 1}{2\sqrt{\alpha_l}\,\cos(\phi+\phi_l) - 2\cos\phi}$
     $\cos\phi \leq \sqrt{\alpha_l}\,\cos(\phi+\phi_l)$

# Conclusions

- subtractions can easily be used to get LHC-ready soft functions
- ingredients for all azimuthally symmetric obs (a<1) with Type I algorithms
- need for future:
    - a >= 1
    - $S_{ij}(\tau^k)$, $S_{ij}^k$ for azimuthally-asymmetric classes
    - 2-loop anom dims of jet/soft fncs for a != 0 for NNLL
    - understanding resummation for Type II