Modified SCET Lagrangian And Its Applications

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A. Idilbi, I. Scimemi, Phys. Lett. B695 (2011) 463-468

M. G. Echevarria, A. Idilbi, I. Scimemi, In Prepapration

Outline

- Why to Modify? Cross-Sections With P_T Dependence
- Formulation Of SCET In Covariant Gauges And Light-Cone Gauge : SCET Lagrangian Valid in Covariant and LC Gauges
- Gauge Invariant TMDPDFs And Beam Functions In SCET
- One-Loop Calculation for Quark Jet In Light-Cone Gauge vs. Feynman Gauge [Clarify Subtle Issues of LCG In SCET]
- Conclusion

Factorization Theorems In SCET

- Identify The Relevant Scales. Ones Which Are Perturbative vs. Non-Perturbative Ones.
- At Each Scale Write The Most General Gauge Invariant Operator That Captures The Physics At That Scale
- Impose Some Assumptions On The Hilbert Space Of The Final States:

$$|X\rangle = |X_n\rangle \otimes |X_{\bar{n}} \otimes |X_s\rangle \otimes |A\rangle$$

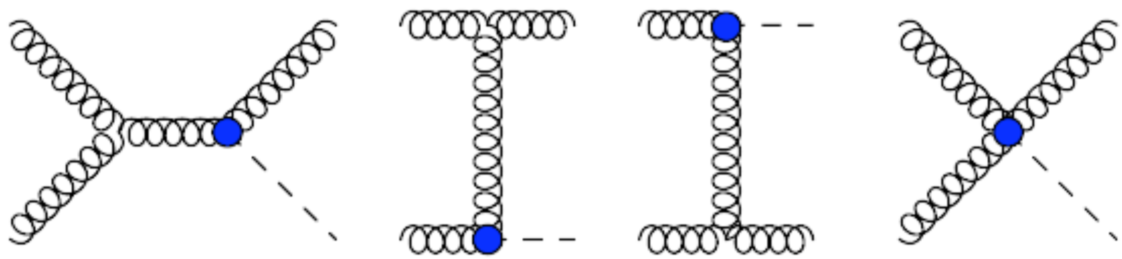
Factorization Theorem With Well-Defined Quantities At The Operator Level

[Soft, Jet Functions, PDFs, TMDPDFs, Beam Functions, etc.]

The Non-Perturbative Matrix Elements Are Inherently Gauge Invariant

Higgs At Low-P_T In SCET

[Mantry and Petriello 10]



The Most General SCET Operator Mediating The Hard Reaction:

$$\mathcal{O}(\omega_1,\omega_2) = g_{\mu\nu} h\, T\{\mathrm{Tr}\Big[S_n (gB^{\mu}_{n\perp})_{\omega_1} S^{\dagger}_n S_{\bar{n}} (gB^{\nu}_{\bar{n}\perp})_{\omega_2} S^{\dagger}_{\bar{n}}\Big]\},$$

$$S_n(x^\mu) = P\exp\left(ig_s \int_{\infty}^{0} ds\, n\cdot A^a_s(x^\mu + sn^\mu)\right),$$

$$\begin{aligned}\frac{d^2\sigma}{du\,dt} &= \frac{1}{2Q^2}\Big[\frac{1}{4}\Big] \int \frac{d^2 p_{h_\perp}}{(2\pi)^2} \int \frac{dn\cdot p_h d\bar{n}\cdot p_h}{2(2\pi)^2} (2\pi)\theta(n\cdot p_h + \bar{n}\cdot p_h)\delta(n\cdot p_h \bar{n}\cdot p_h - \vec{p}^{\,2}_{h_\perp} - m_h^2) \\ &\times\ \delta(u-(p_2-p_h)^2)\delta(t-(p_1-p_h)^2) \sum_{\text{initial pols.}} \sum_{X} \Big|C(\omega_1,\omega_2)\otimes \langle hX_n X_{\bar{n}} X_s|\mathcal{O}(\omega_1,\omega_2)|pp\rangle\Big|^2 \\ &\times\ (2\pi)^4 \delta^{(4)}(p_1+p_2-P_{X_n}-P_{X_{\bar{n}}}-P_{X_s}-p_h),\end{aligned}$$

The Factorized Double Differential Cross-Section

$$\frac{d^2\sigma}{dudt} = H \otimes J \otimes J \otimes S$$

[Mantry and Petriello 10]

$$J_n^{\alpha\beta}(\omega_1, x^-, x_\perp, \mu) = \sum_{\text{initial pols.}} \langle p_1 | \left[g B^A_{1n\perp\beta}(x^-, x_\perp) \delta(\bar{\mathcal{P}} - \omega_1) g B^A_{1n\perp\alpha}(0) \right] | p_1 \rangle$$

$$J_{\bar{n}}^{\alpha\beta}(\omega_1, y^+, y_\perp, \mu) = \sum_{\text{initial pols.}} \langle p_2 | \left[g B^A_{1n\perp\beta}(y^+, y_\perp) \delta(\bar{\mathcal{P}} - \omega_2) g B^A_{1n\perp\alpha}(0) \right] | p_2 \rangle$$

$$S(z, \mu) = \langle 0 | \bar{T} \left[\mathrm{Tr}\left(S_{\bar{n}} T^D S^\dagger_{\bar{n}} S_n T^C S^\dagger_n \right)(z) \right] T \left[\mathrm{Tr}\left(S_n T^C S^\dagger_n S_{\bar{n}} T^D S^\dagger_{\bar{n}} \right)(0) \right] |0\rangle.$$

$$g\mathcal{B}^\mu_{n\perp} \equiv \left[\frac{1}{\bar{\mathcal{P}}_n} [i\bar{n}\cdot\mathcal{D}_n, i\mathcal{D}^{\perp\mu}_n] \right], \qquad \mathcal{D}^\mu_n \equiv W^\dagger D^\mu_n W_n$$

$$W_{n,y} = \mathrm{P}\ \exp\left(ig \int_{-\infty}^{y} \mathrm{d}s\ \bar{n}\cdot A_n(s\bar{n}) \right)$$

The Gluon Jet Functions Are Genuine Non-Perturbative Physical Quantities, However They Are Not Gauge Invariant: No Gauge Link In The Transverse Space.

Drell-Yan at Small q_T In SCET

[Becher and Neubert 01]

Factorization Theorem Includes Transverse-Momentum PDF (TMDPDF) Of Quark In A Hadron

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p)|\, \bar{\chi}(t\bar{n} + x_\perp)\, \frac{\not{\bar{n}}}{2}\, \chi(0)\, |N(p)\rangle$$

$$\chi_{\overline{hc}} = W^\dagger_{\overline{hc}}\, \xi_{\overline{hc}}$$

Not Gauge Invariant

In SCET We Start From The Most General ``Gauge Invariant" Operators Mediating The Hard Reaction. We End-Up With Non-Gauge Invariant Physical Quantities.

Where Are The Transverse Gauge Links In SCET?

In pQCD The TMDPDF:

$$q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \times \left\langle P \left| \overline{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp) \gamma^+ \mathcal{L}_v(\infty; 0) \psi_q(0) \right| P \right\rangle ,$$

$$\mathcal{L}_v(\infty;\xi) = \exp\left(-ig\int_0^\infty d\lambda v\cdot A(\lambda v + \xi)\right) .$$

[Collins and Soper 82]

Consider The TMDPDF In Light-Cone Gauge With: $A^+ = 0,$

[Ji and Yuan 02]
[Belitsky, Ji and Yuan 03]

$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{[k^+]}\right)$$

Classically, In LCG: $A^+ = A^- = 0, \quad A_\perp = -\frac{e}{2\pi}\theta(\xi^-)\nabla \ln \mu r_\perp$

A Gauge Transformation At Infinity Can Be Performed And Has To Be Accounted For By A Gauge Link.

$$\longrightarrow \quad L_0(\infty, 0) \to \Delta L = P\exp\left(-ig\int_0^\infty d\xi_\perp\cdot A_\perp(\xi^- = \infty, \xi_\perp)\right)$$

We Need To Distinguish Gauges With Vanishing Gluon Fields At Infinity (Like Covariant Gauges) And Those Ones With Non-Vanishing Gluon Also At Infinity Fields (Like LCG)

The T-Wilson Line

$$A^{(\infty)}(x^+, x_\perp) \stackrel{def}{=} A(x^+, \infty^-, x_\perp)$$

$$\tilde{A}(x^+, x^-, x_\perp) \stackrel{def}{=} A(x^+, x^-, x_\perp) - A^{(\infty)}(x^+, x_\perp)$$

$$i\not{D}_\perp = i\not{\partial}_\perp + g\not{A}_\perp = i\not{\partial}_\perp + g\tilde{\not{A}}_\perp + g\not{A}_\perp^{(\infty)} \stackrel{def}{=} i\tilde{\not{D}}_\perp + g\not{A}_\perp^{(\infty)}$$

In Covariant Gauges Both Covariant Derivatives Are Identical.

Introduce: $T^\dagger = P\exp\left[-ig\int_0^\infty d\tau l_\perp \cdot A_\perp(x^+, \infty^-; x_\perp - l_\perp\tau)\right]$

One Can Show That For Arbitrary Gauge (Regular Or Singular)

$$i\not{D}_\perp = Ti\tilde{\not{D}}_\perp T^\dagger$$

$$i\not{D}_\perp = T i\tilde{\not{D}}_\perp T^\dagger$$

$$\begin{aligned} i l_\perp \partial_\perp T^\dagger &= i T^\dagger l_\perp \partial_\perp \left[-ig \int_0^\infty d\tau l_\perp A_\perp^{(\infty)}(x^+, x_\perp - \tau l_\perp) \right] \\ &= g T^\dagger \int_0^\infty d\tau \int \frac{d^4 k}{(2\pi)^4} (i l_\perp k_\perp)\, l_\perp A_\perp^{(\infty)}(k) e^{ik(x - l_\perp \tau)} \\ &= g T^\dagger l_\perp A_\perp^{(\infty)}(x) \end{aligned}$$

One Also Needs To Show:

$$[A_\perp(x^+, \infty^-, x_\perp - \tau l_\perp), \tilde{A}_\perp(x^+, x^-, x_\perp)] = 0$$

Space-Time Separation

$$[T(x^+, \infty^-, x_\perp - \tau l_\perp), \tilde{A}_\perp(x^+, x^-, x_\perp)] = 0$$

The RHS:

$$T[g T^\dagger A_\perp^{(\infty)} + g \tilde{A}_\perp T^\dagger + T^\dagger \partial_\perp] = D_\perp$$

The T-Wilson and the ``Collinear'' Lagrangian

The Lagrangian For Collinear Fermions Interacting With The Full Gluon Field:

$$\mathcal{L} = \bar{\xi}_n \left(inD + i\not{D}_\perp \frac{1}{i\bar{n}D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \xi_n$$

In LCG: $\bar{n}A = 0,$

Invoke: $i\not{D}_\perp = T i\tilde{\not{D}}_\perp T^\dagger$

$$\mathcal{L} = \bar{\xi}_n \left(inD + T i\tilde{\not{D}}_\perp \frac{1}{i\bar{n}\partial} i\tilde{\not{D}}_\perp T^\dagger \right) \frac{\not{\bar{n}}}{2} \xi_n$$

SCET_I Lagrangian Valid Also In LCG

$$\mathcal{L}_I = \bar{\xi}_n \left(i n D_n + g n A_s(x^+) + T_n i \tilde{\not{D}}_{n\perp} \frac{1}{i\bar{n}\partial} i \tilde{\not{D}}_{n\perp} T_n^\dagger \right) \frac{\not{\bar{n}}}{2} \xi_n$$

$$iD_n^\mu = i\partial^\mu + gA_n^\mu$$

Transverse Collinear Of Collinear Gluon

$$T_n = \bar{P} \exp \left[ig \int_0^\infty d\tau l_\perp \cdot A_{n\perp}(x^+, \infty^-; x_\perp - l_\perp \tau) \right]$$

The SCET_I Lagrangian Valid In Both Gauges:

$$\mathcal{L}_I = \bar{\xi}_n \left(i n D_n + g n A_s(x^+) + T_n i \tilde{\not{D}}_{n\perp} W_n \frac{1}{i\bar{n}\partial} W_n^\dagger i \tilde{\not{D}}_{n\perp} T_n^\dagger \right) \frac{\not{\bar{n}}}{2} \xi_n$$

Gauge Invariant Operators In SCET

$$T_n = \bar{P} \exp\left[ig \int_0^\infty d\tau l_\perp \cdot A_{n\perp}(x^+, \infty^-; x_\perp - l_\perp \tau)\right]$$

Gluon Jet:

$$g\mathcal{B}^\mu_{n\perp} = [T_n^\dagger W_n^\dagger i D^\mu_{n\perp} W_n T_n],$$

Higgs Production Revisited:

$$\mathcal{O}(\omega_1, \omega_2) = g_{\mu\nu} h\, T\{\mathrm{Tr}\left[S_n (g B^\mu_{n\perp})_{\omega_1} S_n^\dagger S_{\bar{n}} (g B^\nu_{\bar{n}\perp})_{\omega_2} S_{\bar{n}}^\dagger\right]\},$$

[Mantry and Petriello 10]

For Low-P_T

$$J_n^{\alpha\beta}(\omega, x^+, x_\perp) = \sum_{pols} \langle P | [g\mathcal{B}^\alpha_{n\perp}(x^+, x_\perp) \delta(\overline{\mathcal{P}}_n - \omega) g\mathcal{B}^\beta_{n\perp}(0)] | P \rangle$$

The Two Gluon Fields Are Connected In LC direction And The Transverse One
So Complete Gauge Invariance Is Etablished

Remark: For The Inclusive Cross-Section, The T-Wilson Line Cancels

Quark Jet:

$$\chi(x) = T_n^\dagger W_n^\dagger \xi_n(x)$$

Drell-Yan Revisited:

$$J^\mu \to C_V(-q^2 - i\varepsilon, \mu) \sum_q \left(g_L^q \, \bar{\chi}_{\overline{hc}} \, S_{\bar{n}}^\dagger \, \gamma^\mu \, \frac{1-\gamma_5}{2} \, S_n \, \chi_{hc} + g_R^q \, \bar{\chi}_{\overline{hc}} \, S_{\bar{n}}^\dagger \, \gamma^\mu \, \frac{1+\gamma_5}{2} \, S_n \, \chi_{hc} \right)$$

[Becher and Neubert 01]

$$\underline{\chi}_{\bar{n}}(y) \equiv T_{\bar{n}}^\dagger(y^+, \mathbf{y}_\perp) W_{\bar{n}}^\dagger(y) \xi_{\bar{n}}(y),$$

$$\phi_{q/P} = \langle P_{\bar{n}} | \underline{\overline{\chi}}_{\bar{n}}(y) \delta\left(x - \frac{n\mathcal{P}}{np}\right) \delta^{(2)}(p_\perp - \mathcal{P}_\perp) \frac{\not{n}}{\sqrt{2}} \underline{\chi}_{\bar{n}}(0) | P_{\bar{n}} \rangle$$

The Two Quark Fields Are Connected In LC Direction And The Transverse One

SCET_II And Transverse Wilson Line

Collinear: $(\bar{n}k, nk, k_\perp) \sim Q(1, \eta^2, \eta)$

(u)Soft: $(\bar{n}k, nk, k_\perp) \sim Q(\eta, \eta, \eta)$

The Transverse Components Of The Collinear And Soft Gluon Fields Have The Same Scaling

$$\mathcal{L} = \bar{\xi}_n \left(inD + i\not{D}_\perp \frac{1}{i\bar{n}D} i\not{D}_\perp \right) \frac{\not{\bar{n}}}{2} \xi_n$$

The Transverse Covariant Derivative Contains Both Gluon Fields

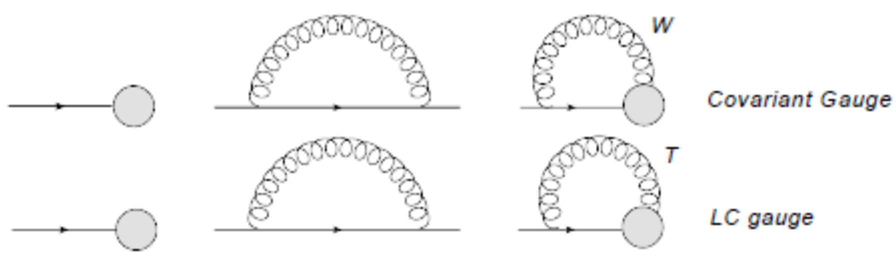
$$\mathcal{L}_{II} = \bar{\xi}_n \left(inD_n + gnA_s + Ti\tilde{\not{D}}_\perp W_n \frac{1}{i\bar{n}\partial} W_n^\dagger i\tilde{\not{D}}_\perp T^\dagger \right) \frac{\not{\bar{n}}}{2} \xi_n$$

$$T = \bar{P} \exp \left[ig \int_0^\infty d\tau \right.$$
$$\left. \times\, l_\perp \cdot (A_{n\perp} + A_{s\perp})(x^+, \infty^-; x_\perp - l_\perp \tau) \right]$$

Quark Jet In Feynman Gauge And LCG

$$\chi(x) = T_n^\dagger W_n^\dagger \xi_n(x)$$

[Idilbi and Scimemi 10]



$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2+i0}\left(g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{[k^+]}\right) \qquad \frac{1}{[k^+]} = \frac{\theta(k^-)}{k^+ + ip^+\eta} + \frac{\theta(-k^-)}{k^+ - ip^+\eta}.$$

Attains Different Values In Feynman Gauge And In LCG When Mandelstam-Leibrandt Prescription To Regularize The Spurious Singularity. Only With The Contribution from T We Get Gauge Invariance.

Note: Zero-Bin Does NOT Help In ML: $\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+i0)(k^+ + ip^+\eta Sgn(k^-))} \frac{p^+}{p^2 + k^-p^+ + i0} = 0.$

Conclusion

- We Treated SCET In LCG Which Led To The Emergence Of Transverse Gauge Links Both In SCET_I And SCET_II.

- Gauge Invariant SCET Building-Blocks (In Covariant Gauges And In LCG) Have Been Obtained With Which We Can Properly Define Non-Perurbative Matrix Elements With P_T dependence.