Black hole quasinormal mode spectroscopy

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CASST 2021
The Canadian Astroparticle Summer Student Talk Competition

August 24, 2021

SNOLAB

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Overview

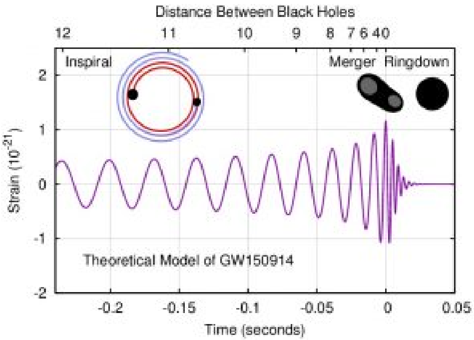
Motivation

- **Stability of black holes**: Given the initial perturbation, will the perturbation remain bounded at all times?

  [Regge, Tullio and Wheeler, John A. (*Phys. Rev* 1957)]

- **Gravitational wave Astronomy**: To extract information of the black hole like their mass, charge, and spin.
- **Tests of General Relativity**: Different modes should give us a stringent test for GR. Using different techniques within GR independently to test GR.
- **Confirmation of BHs**: Detection of these frequencies also confirm the existence of the BHs.

Inspiral-Merger-Ringdown



- Ringdown phase is well described by perturbing the background black hole spacetime by any test field.
- It is assumed that this model is valid only when black hole is very close to reaching its equilibrium.

The aim of the project

What will happen to the black hole when it is perturbed by an external field?

- A perturbed black hole will go through quasinormal modes ringdown. The aim of the project was to calculate these quasinormal modes.
- As a toy model I will treat scalar fields as an external test field.

Scalar Perturbation-Introduction

- Klein Gordan Equation for massless scalar field in spherically symmetric and static background

$$\Box\phi = 0$$

- Decomposing the function as

$$\phi(t, r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{\Psi_{\ell m}^{s=0}(r)}{r} P_{\ell m}(\theta) e^{-i\omega t} e^{im\phi}$$

- By using tortoise coordinate r_* defined as $\frac{dr}{dr_*} \equiv (fh)^{1/2}$, we get the Schrodinger equation:

$$\frac{d^2\Psi_l^{s=0}}{dr_*^2} + \left[\omega^2 - V_0\right] \Psi_l^{s=0} = 0$$

where effective potential reads as $V_0 \equiv f\frac{\ell(\ell+1)}{r^2} + \frac{(fh)'}{2r}$

Boundary Conditions

- Effective Potential $V_0^{\text{Schw}} = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$
- **Horizon**: Classically, nothing should leave horizon, therefore

$$\phi \sim e^{-i\omega(t+r_+)}$$

- **Infinity**: Discarding unphysical waves entering from infinity.

$$\phi \sim e^{-i\omega(t-r_+)}$$

- Now our aim is to solve for ω in the radial equation satisfying the above boundary conditions and show that the eigen functions are damped in nature.

WKB Method

- For any massless spin (s=0 scalar, s=1 Electromagnetic, s=2 Gravitational):

$$\frac{d^2\Psi_\ell^s}{dr_*^2} + Q(r_*)\Psi_\ell^s = 0$$

where $Q(r_*) = \left[\omega^2 - V_s\right]$, $V_s = f\left(\frac{\Lambda}{r^2} + \frac{2\beta}{r^3}\right)$, $\beta = 1 - s^2$, $\lambda = \ell(\ell+1)$

- **Validity**: Due to boundary conditions, the transmitted and reflected waves should have comparable amplitudes. Therefore, turning points degenerate or very close to each other.
- In middle region, expanding Q about maxima and using boundary conditions, we get

$$n + \frac{1}{2} = -\frac{\mathrm{i}Q_0}{\left(2Q_0''\right)^{1/2}}$$

- Using the above result to get

$$\omega_n^2 = i\left(n + \frac{1}{2}\right)\sqrt{2Q_0''} + \left(1 - \frac{2M}{r_0}\right)\left(\frac{\Lambda}{r_0^2} + \frac{2\beta M}{r_0^3}\right)$$

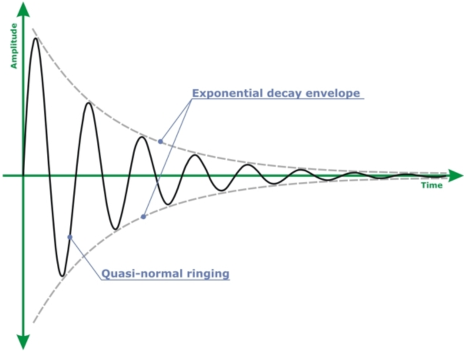
Application

$M = nM_{\odot}$
For gravitational field, s=2

l	n	$M\omega_0 + iM\omega_i$
2	0	$0.373162 - 0.0892174i$
	1	$0.346017 - 0.274915i$
	2	$0.302935 - 0.471064i$
	3	$0.247462 - 0.672898i$
3	0	$0.599265 - 0.0927284i$
	1	$0.582355 - 0.281406i$
	2	$0.5532 - 0.476684i$
	3	$0.515747 - 0.677429i$
4	0	$0.809098 - 0.0941711i$
	1	$0.796499 - 0.284366i$
	2	$0.773636 - 0.478974i$
	3	$0.743312 - 0.6783i$

For scalar field in RN spacetime assuming $q = 0.5$

l	n	Q	$M\omega_0 + iM\omega_i$
0	0	0.5	$0.247376 - 0.249018i$
	1	0.5	$0.887109 - 0.922351i$
	2	0.5	$1.37073 - 1.41889i$
	3	0.5	$0.142375 - 0.863119i$
1	0	0.5	$0.26604 - 0.145384i$
	1	0.5	$0.700671 - 0.66721i$
	2	0.5	$0.992813 - 0.97177i$
	3	0.5	$0.0604634 - 0.758635i$



Tests of GR

- Converting into Physical units

$$\nu = \frac{32.26}{n}\,(M\omega_0)\,\mathrm{kHz}$$

$$\tau = \frac{n \cdot 0.4937 \cdot 10^{-5}}{(M\omega_i)}\mathrm{s}$$

- Test no-hair theorem and stability of black holes
- In eikonal limit, we can relate QNM to lyapunov exponent

$$\omega_{QNM} = \Omega_c l + i\left(n + \frac{1}{2}\right)|\lambda|$$

where λ is the Lyapunov exponent (measures of the rate at which the trajectories diverge) of the null orbit.
Ω_c is the orbital angular velocity of the null orbit.

References

1. Emanuele Berti, Vitor Cardoso, and Andrei O Starinets. Quasinormal modes of black holes and black branes. Classical and Quantum Gravity, 26(16):163001, jul 2009.
2. E. S. C. Ching, P. T. Leung, A. Maassen van den Brink, W. M. Suen, S. S. Tong, and K. Young. Quasinormal-mode expansion for waves in open systems. Rev. Mod. Phys., 70:1545–1554, Oct 1998
3. Vitor Cardoso, Alex S Miranda, Emanuele Berti, Helvi Witek, and Vilson T Zanchin. Geodesic stability, lyapunov exponents, and quasinormal modes. Physical Review D, 79(6):064016, 2009.
4. MKostas D Kokkotas and Bernd G Schmidt. Quasi-normal modes of stars and black holes. Living Reviews in Relativity, 2(1):1–72, 1999.

Thank You

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