Effect of the laser field on the branching ratios of the negatively charged pion π^-

S. Mouslih[2,1], M. Jakha[1], S. Taj[1], B. Manaut[1] and E. Siher[2]
[1] Sultan Moulay Slimane University, Polydisciplinary Faculty.
[2] Sultan Moulay Slimane University, Faculty of Sciences and Techniques.

Abstract
This work revives the controversial debate that has arisen over the last two decades about the possibility that the electromagnetic field affects the lifetime or the decay rate of an unstable particle. In this research, we show, by performing analytical calculations and extracting numerical results, that the pion lifetime can be changed notably by inserting the decaying pion into an electromagnetic field only if the number of photons transferred between the decaying system and the laser field does not reach the well-known sum-rule. The influence of the laser parameters on the decay rate and branching ratio is also discussed. The surprising result obtained for the pion lifetime is referred to the well-known quantum Zeno effect.

1 Introduction

The study of particle behavior and their properties when they are inserted into an electromagnetic field has received much attention in recent times [1], due to the progress made by laser technology in terms of both its intensity and its sources [2]. As one of these properties, we find the lifetime or the decay rate of a particle. In the meantime, the study of decay in the presence of an electromagnetic field and the size of the effect that the latter may have on the lifetime were and are still the subject of many scientific researches [3]. 13 years ago, in a controversial scientific research [4], Liu, Li, and Berakdar studied the effect of a strong laser field on the decay rate of the muon and found a dramatic change in its lifetime, as much as an order of magnitude. This result is criticized by Narozhny and Fedotov in their comment [5] and they consider it to be invalid and contradictory to physical intuition. Two other authors have done their own calculation with different laser polarization and also reach a very different conclusion [6]. They found the effect of the laser on the muon lifetime to be very small. The weak decay processes in the presence of strong laser fields can be divided into two categories: first, laser-assisted processes which also exist in the absence of the field but may be modified due to its presence. Second, field-induced processes which can occurred only when a background field is present, providing an additional energy reservoir [7]. Our aim in this work is to study the effect of a circularly polarized laser field on the decay rate and the lifetime of the negatively charged pions, which are unstable and decay into two leptons $\pi^- \to e^-\bar{\nu}_e$ or $\mu^-\bar{\nu}_\mu$ by virtue of weak interaction with a lifetime $\tau_{\pi^-} = (2.6033 \pm 0.0005) \times 10^{-8}$ sec [8]. Note that, in this work, natural units $c = \hbar = 1$ and the space-time metric $g = diag(1, -1, -1, -1)$ are employed throughout. In many equations of this paper, the Feynman slash notation is used. For any 4-vector, A, $\not{A} = A^\mu \gamma_\mu$ where the matrices γ are the well-known Dirac matrices. This paper is structured as follows. A detailed note on theoretical model is presented in section II. The results are discussed in section III. Finally, the conclusion of this study is given in section IV.

2 Theory

We consider the decay of a charged pion into two leptons,

$$\pi^-(p_1) \longrightarrow l^-(p_2) + \bar{\nu}_l(k'), \qquad (l = e, \mu) \tag{1}$$

We assume that this decay occurs in the presence of a circularly polarized monochromatic laser field , which can be expressed by:

$$A^\mu(\phi) = a_1^\mu \cos(\phi) + a_2^\mu \sin(\phi), \quad \phi = (k.x), \tag{2}$$

where $k = (\omega, \mathbf{k})$ is the wave 4-vector ($k^2 = 0$), ϕ is the phase of the laser field and ω its frequency. The polarization 4-vectors a_1^μ and a_2^μ are equal in magnitude and orthogonal:

$$a_1^\mu = |\mathbf{a}|(0,1,0,0), \quad a_2^\mu = |\mathbf{a}|(0,0,1,0), \tag{3}$$

which implies $(a_1.a_2) = 0$ and $a_1^2 = a_2^2 = a^2 = -|\mathbf{a}|^2 = -(\mathcal{E}_0/\omega)^2$ where $\mathcal{E}_0$ is the amplitude of the laser's electric field. Therefore, in the first Born approximation, the S-matrix element for the laser-assisted π^- decay can be written as [9]:

$$S_{fi}(\pi^- \to l^-\bar{\nu}_l) = \frac{-iG}{\sqrt{2}} \int d^4x J_\mu^{(\pi)\dagger}(x) J_{(l^-)}^\mu(x). \tag{4}$$

where $J_{(l^-)}^\mu(x)$ and $J_\mu^{(\pi)}(x)$ are, respectively, the leptonic and hadronic currents in the laser field, which can be expressed by:

$$J_{(l^-)}^\mu(x) = \bar{\psi}_l(x,t)\gamma^\mu(1-\gamma_5)\psi_{\bar{\nu}_l}(x,t), \tag{5}$$

and

$$J_\mu^{(\pi)} = i\sqrt{2} f_\pi p_{1\mu} \frac{1}{\sqrt{2Q_1 V}} \times e^{-iS(q_1,x)}, \tag{6}$$

the wave functions of outgoing particles:

$$\psi_l(x) = \left[1 + \frac{e\not{k}\not{A}}{2(k.p_2)}\right] \frac{u(p_2,s_2)}{\sqrt{2Q_2 V}} \times e^{iS(q_2,x)}, \qquad \psi_{\bar{\nu}_l}(x) = \frac{v(k',t')}{\sqrt{2E_2 V}} e^{ik'.x}, \tag{7}$$

with

$$S(q_2,x) = -q_2.x - \frac{e(a_1.p_2)}{k.p_2}\sin(\phi) + \frac{e(a_2.p_2)}{k.p_2}\cos(\phi). \tag{8}$$

After some manipulation, we find

$$S_{fi} = \frac{-Gf_\pi}{2\sqrt{2Q_1Q_2E_2V^3}} \sum_{s=-\infty}^{\infty} M_{fi}^s (2\pi)^4 \delta^4(k' + q_2 - q_1 - sk) \tag{9}$$

The decay rate of the pion is defined by:

$$W(\pi^- \to l^-\bar{\nu}_l) = \sum_{s=-\infty}^{+\infty} W_s(\pi^- \to l^-\bar{\nu}_l), \tag{10}$$

where

$$W_s(\pi^- \longrightarrow l^- + \bar{\nu}_l) = \frac{G^2 f_\pi^2}{8Q_1} \int \frac{d^3q_2}{(2\pi)^3 Q_2} \int \frac{d^3k'}{(2\pi)^3 E_2} \times (2\pi)^4 \delta^4(k' + q_2 - q_1 - sk) |\overline{M_{fi}^s}|^2, \tag{11}$$

The term $|\overline{M_{fi}^s}|^2$ can be calculated as follows:

$$|\overline{M_{fi}^s}|^2 = \mathrm{Tr}\left[(\not{p}_2 + m_l)\Gamma^s \not{k}' \bar{\Gamma}^s\right], \tag{12}$$

The total decay rate of the charged pion π^- in the laser field

$$W_{total} = W(\pi^- \to \mu^-\bar{\nu}_\mu) + W(\pi^- \to e^-\bar{\nu}_e) \tag{13}$$

In our case, we define the branching ratios (Br) of the muonic and electronic decay channels as follows:

$$\mathrm{Br}(\pi^- \to \mu^-\bar{\nu}_\mu) = \frac{W(\pi^- \to \mu^-\bar{\nu}_\mu)}{W_{total}}, \tag{14}$$

$$\mathrm{Br}(\pi^- \to e^-\bar{\nu}_e) = \frac{W(\pi^- \to e^-\bar{\nu}_e)}{W_{total}}. \tag{15}$$

3 Results and Discussion

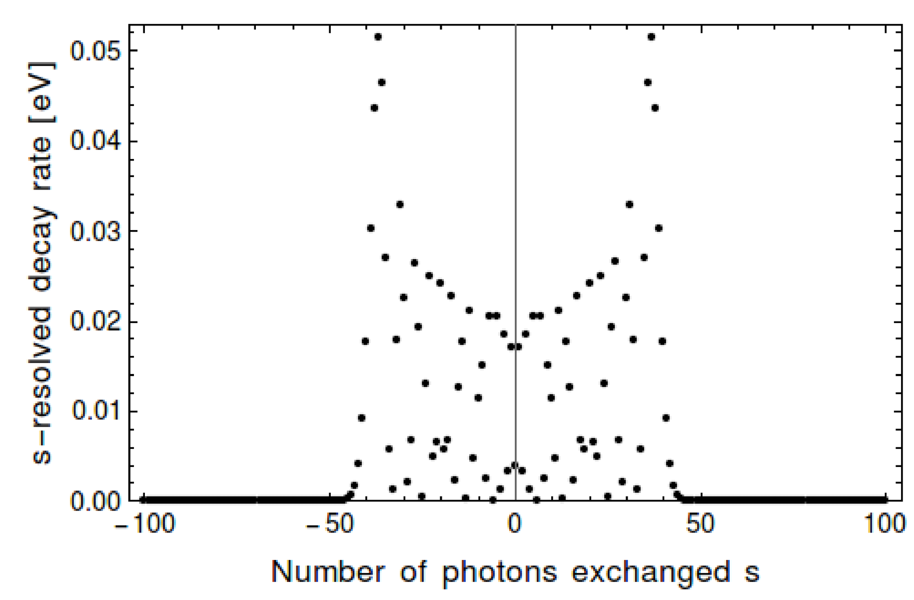

Figure 1: The multi-photon decay rate $W_s(\pi^- \to \mu^-\bar{\nu}_\mu)$ (11) (in units of 10^{-8}) as a function of the number of photons exchanged s in the rest frame of the pion, with the spherical coordinates $\theta = 90°$ and $\varphi = 0°$. The laser field amplitude and frequency are $\mathcal{E}_0 = 10^7$ V/cm and $\hbar\omega = 1.17$ eV.

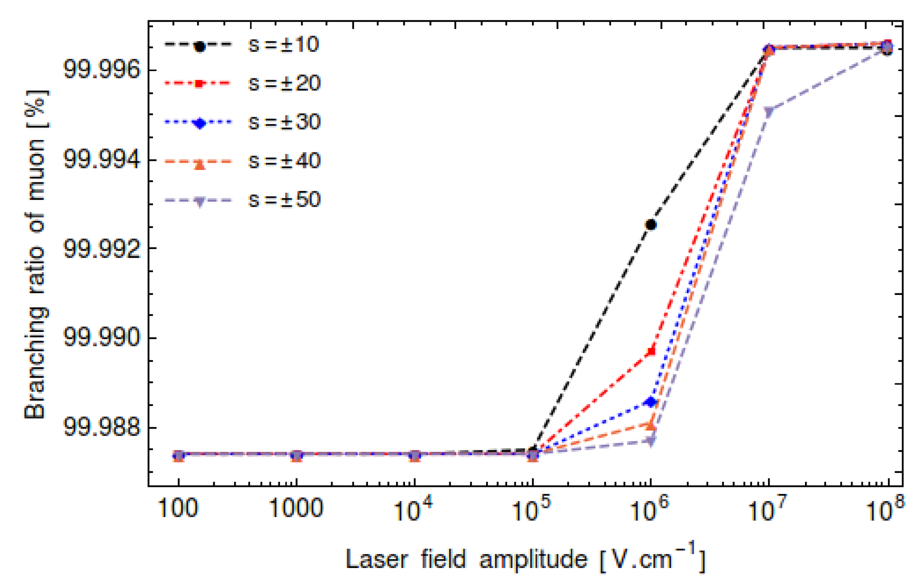

Figure 2: The behavior of the branching ratio (14) of the muonic decay channel as a function of the laser field amplitude strength for different numbers of photons exchanged. The frequency of laser field is $\hbar\omega = 1.17$ eV

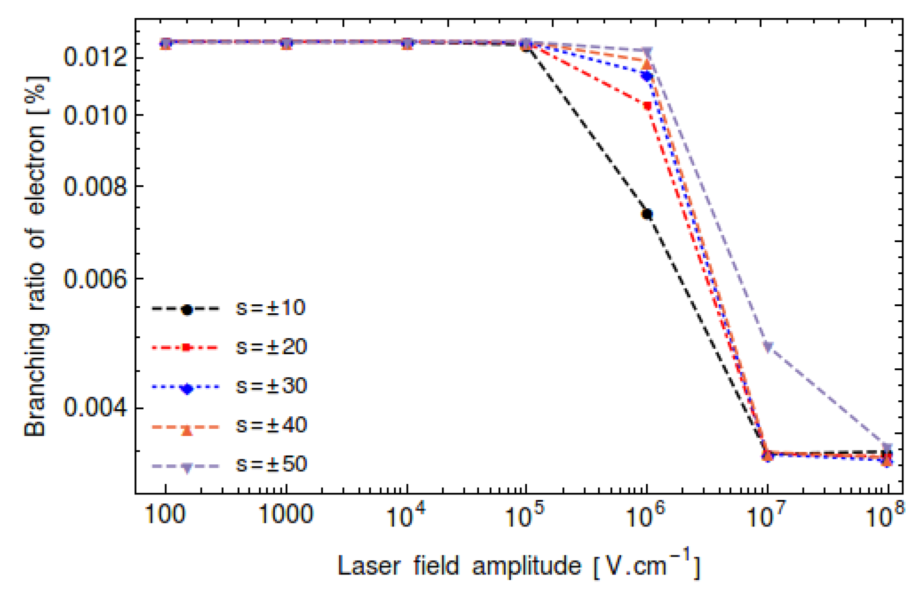

Figure 3: The behavior of the branching ratio (15) of the electronic decay channel as a function of the laser field amplitude for different numbers of photons exchanged. The frequency of laser field is $\hbar\omega = 1.17$ eV.

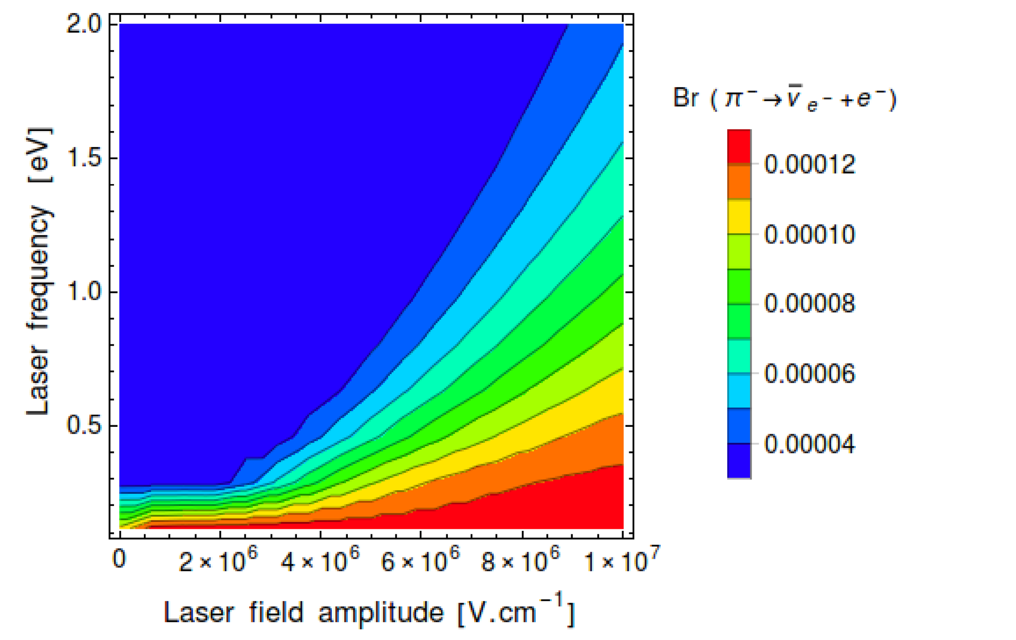

Figure 4: The behavior of the branching ratios (14) as a function of the laser field amplitude $\mathcal{E}_0$ and laser frequency ω for an exchange of $s = \pm 20$ photons.

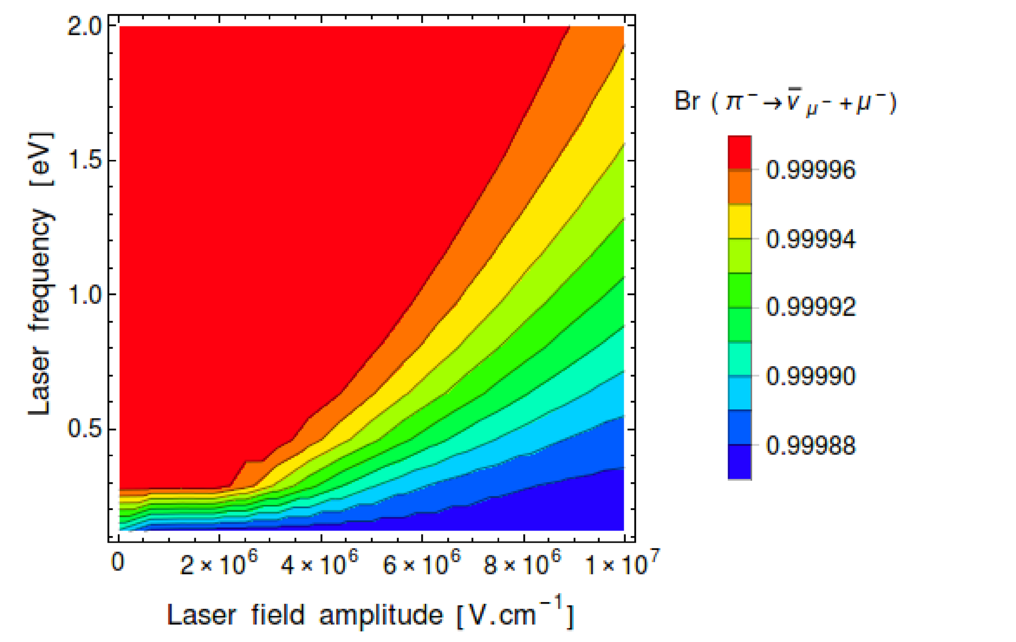

Figure 5: The behavior of the branching ratios of the muonic decay channel (15) as a function of the laser field amplitude $\mathcal{E}_0$ and laser frequency ω for an exchange of $s = \pm 20$ photons.

4 Conclusion

We have performed the analytical calculation for the negatively charged pion decay in the presence of a circularly polarized laser field. Summing up the results, it can be concluded that the pion lifetime (then the decay rate) can be affected by the laser field as long as the number of photons exchanged between the decaying system and the laser field is not sufficient to achieve the well-known sum-rule. Once the sum-rule is checked for a determined number of photons, the influence of the laser on the pion lifetime becomes zero. We have explained the modification of the pion lifetime in the presence of a laser field by considering the well-known quantum Zeno effect.

References

[1] Y. Attaourti, al, Phys. Rev. A **70**, 023404 (2004); B. Manaut, al, Phys. Rev. A **71**, 043401 (2005); C. Szymanowski, al, Phys. Rev. A **56**, 3846 (1997).

[2] S. V. al, Phys. Rev. Lett. **91**, 085001 (2003); S. Gordienko, al, Phys. Rev. Lett. **94**, 103903 (2005).

[3] W. Becker, G. T. Moore, R. R. Schlicher, and M. O. Scully, Phys. Lett. A **94**, 131 (1983);

[4] A.-H. Liu, S.-M. Li, and J. Berakdar, Phys. Rev. Lett. **98**, 251803 (2007).

[5] N. B. Narozhny and A. M. Fedotov, Phys. Rev. Lett.

[6] D. A. al, Phys. Rev. D **79**, 013004 (2009).

[7] A. Di Piazza, C. Müller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012).

[8] M. Tanabashi *et al.*, Phys. Rev. D **98**, 030001 (2018).

[9] W. Greiner and B. Müller, *Gauge Theory of Weak Interactions* (Springer, Berlin, 2000), 3rd ed.

[10] Mouslih S, Jakha M, Taj S, Manaut B and Siher E 2020 Laser-assisted pion decay *Phys. Rev. D* **102** 073006