Standard Model and Beyond

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Set of ten lectures on SM available on "indiacms.res.in" at 2020_LECTURE_SERIES

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Standard Model

- It is a gauge theory, based on the symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$, with Spontaneous Symmetry Breaking.
- It is a renormalizable Quantum Field Theory.
- Of the four fundamental forces, it explains strong, electromagnetic and weak interactions.
- Some of its important predictions are
  1. Logarithmic scaling violations
  2. Weak Neutral Currents
  3. Weakening of Strong force at higher energies
  4. $W^{\pm}$ and Z^0 gauge bosons
  5. Fundamental scalar Higgs boson

  All these predictions are experimentally verified.

Global Phase Transformation

- To understand what is meant by Gauge Theory, let us look at the simpler theory of Quantum Electrodynamics (QED).
- Consider $\psi(x)$, which denotes the quantum field for an electron.
- Its Lagrangian is given by

$$\mathcal{L} = \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x).$$

- It is easy to see that this Lagrangian is invariant under the phase transformation

$$\psi(x) \to \psi'(x) = e^{i\alpha}\psi(x) \;\; (\alpha \text{ a real constant}).$$

- Since α is a constant, the electron field $\psi(x)$, at every space-time point, undergoes the same phase transformation
- Such a transformation is called global phase transformation.

Local Phase (or Gauge) Transformation

- Global phase transformations are restrictive. The phase of the electron field has to be set to be the same at all points in the universe.
- We demand the freedom to set the phase of the electron in our neighbourhood.
- That is: We want α to be $\alpha(x)$ (NOT a constant but a function of space-time).
- The mass term in the Lagrangian is still invariant but the kinetic term changes because ∂_μ acting on $\psi'(x)$ leads to an extra term $-(\partial_\mu \alpha(x))\psi(x)$.
- The invariance of the Lagrangian, which was present when α was a constant, is now lost when α is made a function of space-time.

Introduction of a Vector Field

- We now insist that the Lagrangian should be invariant under local phase transformations also.
- Obviously, the original $\mathcal{L}$ is not, so we need to modify it.
- We saw that in the local phase transformation of the original $\mathcal{L}$, we get an extra term containing $\partial_\mu \alpha$.
- We need to introduce a new term in the Lagrangian, whose change under local phase transformation, cancels the above term.
- The modified Lagrangian is

$$\mathcal{L}' = \bar{\psi}(x)[i\gamma^\mu \partial_\mu + eA_\mu(x)]\psi(x) - m\bar{\psi}(x)\psi(x),$$

where a vector field $A_\mu(x)$ is introduced.

Restoring the Invariance

- The modified Lagrangian $\mathcal{L}'$ is invariant under local phase transformations, if we make the transformation $A'_\mu(x) = A_\mu(x) + (1/e)\partial_\mu \alpha(x)$, while transforming $\psi(x)$.
- The field $A_\mu(x)$ and its transformation properties are precisely those electromagnetic field of classical electrodynamics.
- We require the existence of electromagnetism by demanding local phase invariance of the Lagrangian.
- The transformation on $\psi(x)$ is a unitary transformation, involving one real function $\alpha(x)$.
- Hence, the symmetry transformation in QED is called $U(1)$ symmetry.

Lagrangian of QED

- We want the electromagnetic field to be dynamic field.
- We need to add a kinetic term for it, which must be invariant under the $U(1)$ symmetry.
- We define field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- It is trivial to see that $F_{\mu\nu}$ is invariant under the $A_\mu(x) \to A'_\mu(x)$ transformation.
- The Lagrangian for QED is

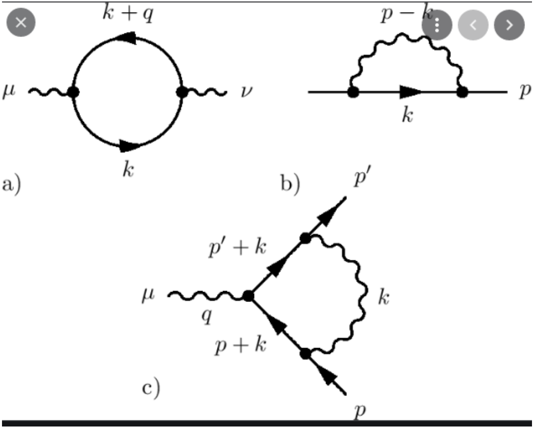
$$\mathcal{L}_{\text{QED}} = \bar{\psi}(x)[i\gamma^\mu \partial_\mu + eA_\mu(x)]\psi(x) - m\bar{\psi}(x)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}.$$

- As Dirac once boasted, it explains most of Physics and all of Chemistry.
- Symmetry does not allow a mass term for $A_\mu(x)$.

Success of QED Lagrangian

- The term $e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$ describes the local interaction of an electron field with a photon field at the space-time point x.
- Using this interaction as the basis, Dirac was able to explain a number features in atomic physics.
- An important prediction is that electron spin magnetic moment has $g = 2$.
- It also predicted that electron energy levels in hydrogen atom should depend only on total j (not separately on l and s).
- In other words, ${}^2S_{1/2}$ and ${}^2P_{1/2}$ levels of hydrogen atom should be degenerate.
- Since QED is a quantum theory, we must consider higher order corrections, which occur due to multiple interactions.

One-Loop Effects in QED



The diagrams show one-loop corrections to photon propagator, electron propagator, vertex correction.

Infinite and Finite Corrections in QED

- The vertex correction diagram leads to two interesting predictions:
  1. The 1-loop corrected electric charge of the electron is predicted to be

     $$e_{1-\mathrm{loop}} = e_{\mathcal{L}}[1 + (\alpha/\pi)\ln(\Lambda/m_e)]$$

     where Λ is a high scale which $\to \infty$.
  2. The magnetic moment of the electron receives a finite correction of the form $g[1 + \alpha/(2\pi)]$.
- The finite correction in g was predicted by Julian Schwinger and was measured by Polykarp Kusch.
- But, infinite correction in electric charge??? It was argued that this is not a serious problem.
- There will definitely be new physics at Planck scale $\Lambda = M_{\mathrm{Pl}} = 10^{22}$ MeV, meaning $(\alpha/\pi)\ln(\Lambda/m_e) \sim 0.05$.
- To match $e_{1-\mathrm{loop}}$ to experiment, need to adjust the value of $e_{\mathcal{L}}$ by 5%.

Effective Theory of Weak Interactions

- Fermi constructed an effective theory of β decay, which described $\Delta J = 0$ nuclear transitions.
- Gamow-Teller introduced a modification which could describe β decays $\Delta J = \pm 1$ nuclear transitions.
- Later, parity violation is incorporated into theory in the form of $(V - A)$ theory.
- All these theories were parametrized by Fermi Coupling constant G_F, which has dimensions M^{-2}.
- Is it possible to construct a fundamental theory of weak interaction based on exchange of vector bosons (as we did with QED)?

Intermediate Vector Boson Theory

- Intermediate Vector Boson hypothesis attempted to do that.
- It immediately encountered three problems:
  1. Exchange particle must be charged and hence must interact with photon.
  2. Weak interactions are parity violating and electromagnetic interactions are parity conserving.
  3. Exchange particles must be massive. But, local symmetry transformations require the vector bosons to be massless.
- Electroweak model (Weinberg-Salam model) could overcome all three problems:
  1. Expand the symmetry to $SU(2)$ which has three free parameters and can lead to three vector bosons interacting with one another.
  2. Further expand the symmetry to $SU(2)_L \times U(1)$ and arrange the details so that weak is PV and EM is PC.
  3. Introduce an $SU(2)$ doublet of complex scalars and generate Spontaneous Symmetry Breaking through Higgs Mechanism.

Weinberg-Salam (Electroweak) Model

- It is based on $SU(2)_L \times U(1)_Y$ symmetry.
- Left-chiral fermions are in $SU(2)_L$ doublets and right-chiral fermions are $SU(2)_L$ singlets.
- The hyper-charges Y of left-chiral and right-chiral fermions are adjusted such that both chiral projections of a fermion have the same electric charge.
- There are four free parameters (called symmetry generators) in the symmetry and hence there are four vector bosons.
- The off-diagonal generators of $SU(2)_L$ mix to form a pair of charged vector bosons $W_\mu^\pm$.
- The diagonal generator of $SU(2)_L$ and that of $U(1)_Y$ mix to form the photon A_μ and a new weak boson Z_μ.
- One of the fields of the scalar doublet acquires non-zero vacuum expectation value leading to Spontaneous Symmetry Breaking which in turn gives rise to masses for $W^\pm$ and Z.

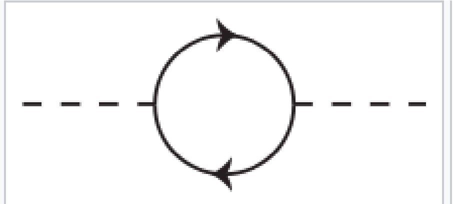
Predictions of Standard Model

- Electroweak model made the following four important predictions:
  1. Weak neutral current interactions mediated by Z (discovered in 1973 at CERN)
  2. Existence of $W^{\pm}$ and Z (discovered in 1983 at CERN)
  3. One-loop corrections to the relationship between masses of M_W and M_Z (verified during 1995-2005 at CERN and Fermilab)
  4. A fundamental scalar called Higgs boson (discovered in 2012 at CERN)
- The strong interaction part Quantum Chromodynamics is based on the symmetry group $SU(3)_c$. It made the following predictions:
  1. Logarithmic scaling violations (discovered in 1967 at SLAC).
  2. Existence of colour quantum number (confirmed at SLAC, CESR and DESY)
  3. Existence of mediators of $SU(3)_c$ symmetry called gluons (discovered in 1978 at DESY)
  4. Weakening of the strength of the strong force at higher energies (confirmed at SLAC, DESY and CERN).

Most Severe Problem with Standard Model

- Concerns the one-loop correction to Higgs boson mass.
- In QED, we talked about infinite correction to the electric charge of electron.
- But, we argued that it need not bother us too much.
- Similar arguments can be made for almost all of the quantities in the Standard Model, such as masses of fermions, masses of vector bosons and various coupling constants.
- The one-loop corrections to all these quanties have logarithmic divergences because the quantities are protected by approximate symmetries.
- No such symmetry exists for fundamental scalars. Hence, the one-loop corrections to the Higgs boson pose a severe problem to Standard Model.

One-Loop Corrections to Higgs Boson Mass



- The 1-loop correction to the Higgs boson propagator, comes from the above Feynman diagram and similar diagrams.
- It leads to the prediction

$$m_H^2(1 - \mathrm{loop}) = m_H^2(\mathcal{L}) + \Lambda^2.$$

- Such a correction is called Quadratic divergence.
- Once again, let us imagine $\Lambda = M_{\mathrm{Pl}} = 10^{19}$ GeV.
- To match $m_H(1 - \mathrm{loop})$ with the experimental value 126 GeV, we have to do a ridiculous fine-tuning of $m_H(\mathcal{L})$ to 1 part in 10^{34} !!!.

Two Solutions to Higgs Mass Problems

- First idea is to assume that the Higgs boson is NOT fundamental scalar but is a bound state of a fermion and an anti-fermion.
- This is called Technicolor theory. It is a nice idea but the details do not work out.
- Moreover, we have seen the Higgs boson and it seems to be a fundamental scalar.
- The other idea is to impose a symmetry which relates the Higgs boson to a fermion.
- Postulate Supersymmetry which introduces a scalar for every fermion and a fermion for every scalar.
- If we insist that the rescaling of m_H should be moderate (may be of order 10 or so), then the supersymmetric partners must have masses ~ 1000 GeV. None seen at LHC.
- It is expected that the study of self-interactions of Higgs boson will throw light on the nature of the Higgs Mechanism.

More Problems with Standard Model

- Neutrinos are predicted to be massless in the Standard Model.
- Observation of neutrino oscillations showed that they have tiny masses (more than a million times smaller than the mass of electron).
- Developing a theory of neutrino masses is one of the priorities.
- There are literally hundreds of models of neutrino masses.
- How to make a distinction between any of them? Take more data.
- Different models have widely different predictions for charged lepton flavour violation (CLFV).
- Intense effort going on to observe signals of CLFV.

Some Popular Beyond Standard Model Ideas

- Parity violation and CP violation are put in by hand in the SM.
- It will be nicer if we have a model where they are generated through Spontaneous Symmetry Breaking.
- Left-right symmetric model (LRSM) does that. It also predicts right-handed weak interactions. Not seen so far.
- SM contains three different coupling constants, one each for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$.
- An ideal theory should have only one. Grand Unified Theories (GUTs) have that property at high energy and they reduce to SM at the scale of 100 GeV.
- They predict the proton to decay with a life-time of about 10^{30} years. Not seen so far.

Have We Seen Anything Beyond SM?

- At the moment, flavour physics seems to offer glimpses of physics beyond standard model.
- Standard Model has the property of lepton flavour universality (LFU).
- That is: the couplings of electrons, muons and taus to $W^{\pm}$ and Z are the same.
- Data from BaBar, Belle and LHC-b shows tantalizing glimpses of violation of LFU, in the form of ratios R_D/R_{D^*} and R_K/R_{K^*}.
- Measurement of $(g-2)$ of muon also shows a deviation from the prediction of SM.
- Intense experimental and theoretical effort going on to understand the source of these deviations and unravel the physics behind them.