Recent challenges in LHC phenomenology

Fabrizio Caola
Rudolf Peierls Centre for Theoretical Physics & Wadham College

NNPDF Collaboration & N3PDF Meeting, Gargnano, 7/9/2021

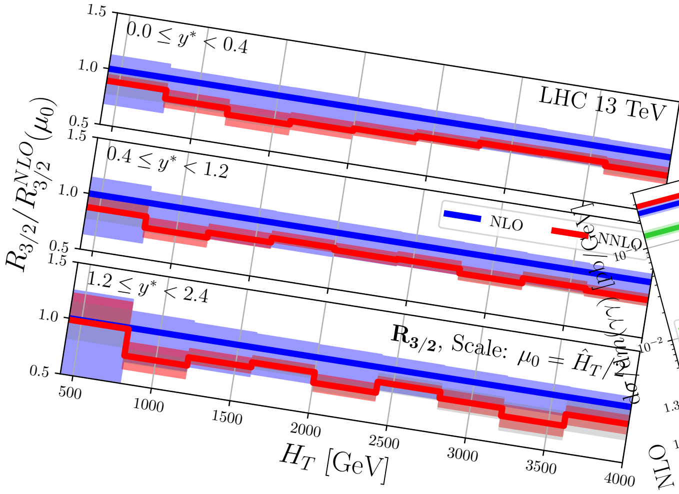
SOME recent challenges in LHC phenomenology

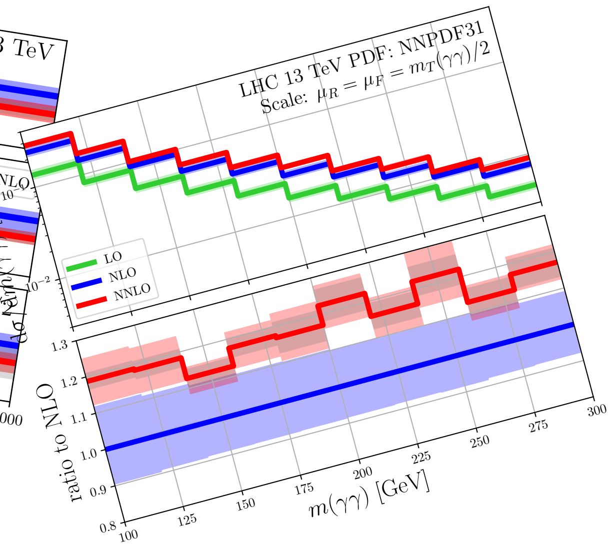
Fabrizio Caola
Rudolf Peierls Centre for Theoretical Physics & Wadham College

NNPDF Collaboration & N3PDF Meeting, Gargnano, 7/9/2021

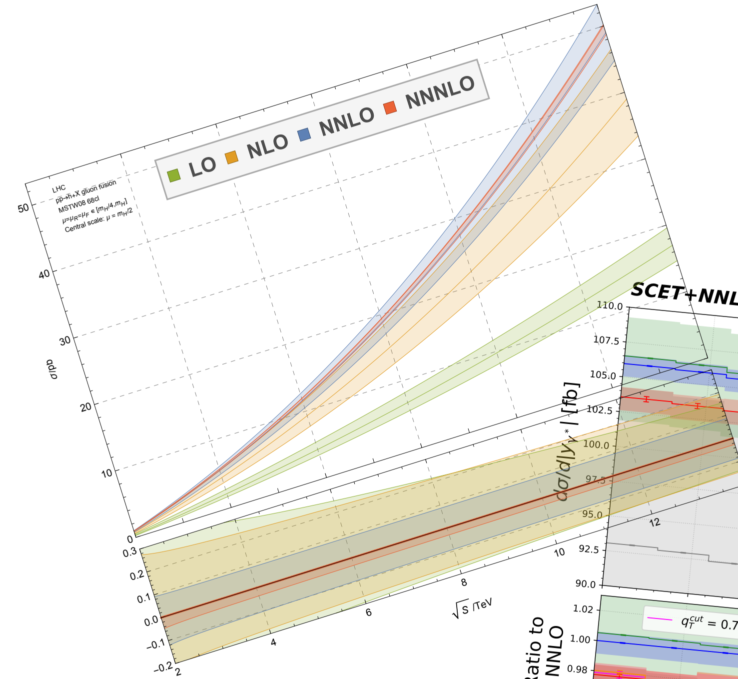
High-precision phenomenology

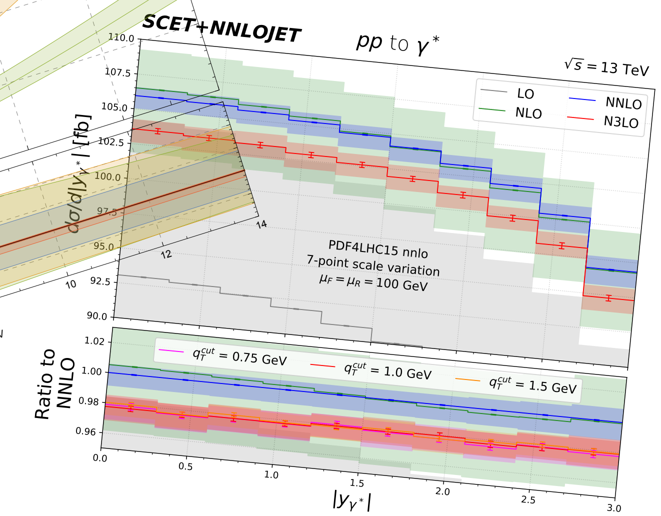
2→3 @ NNLO

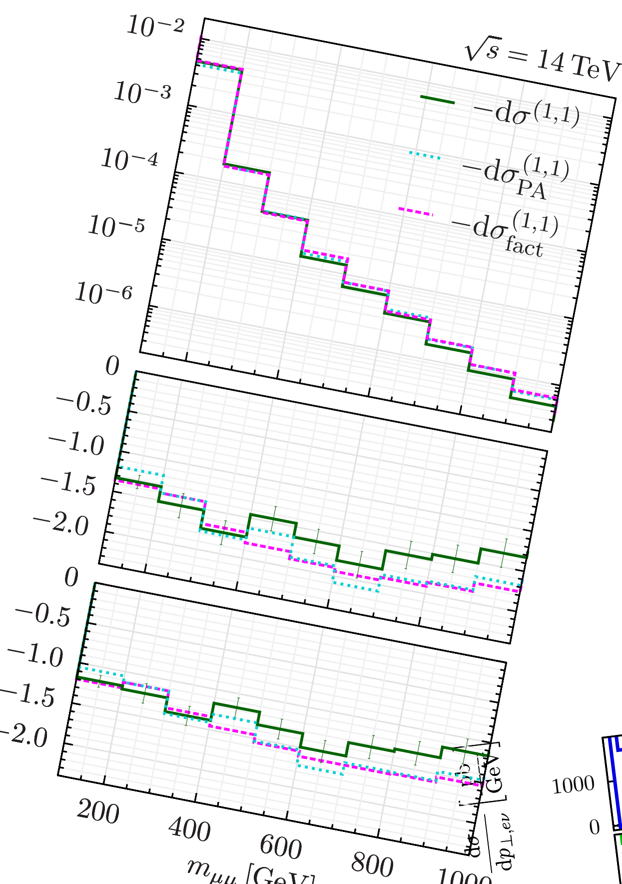


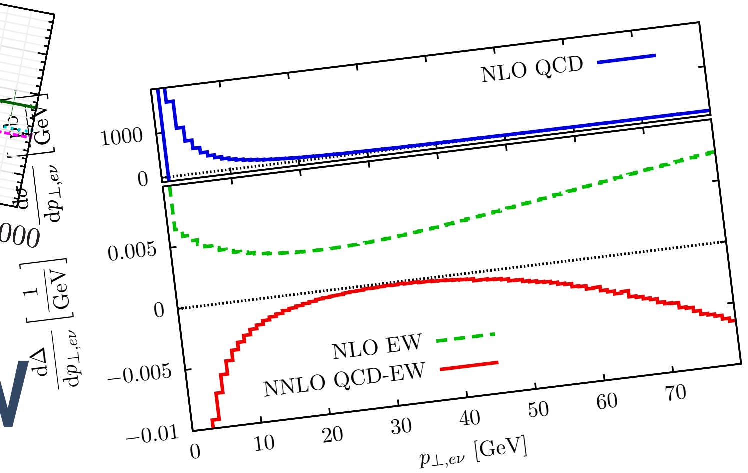


N³LO

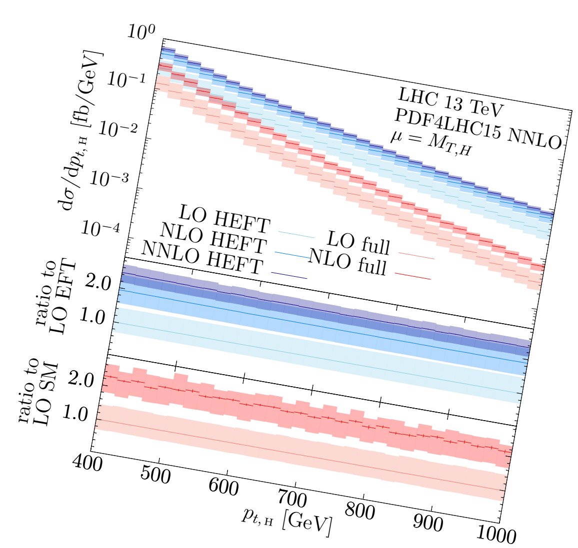




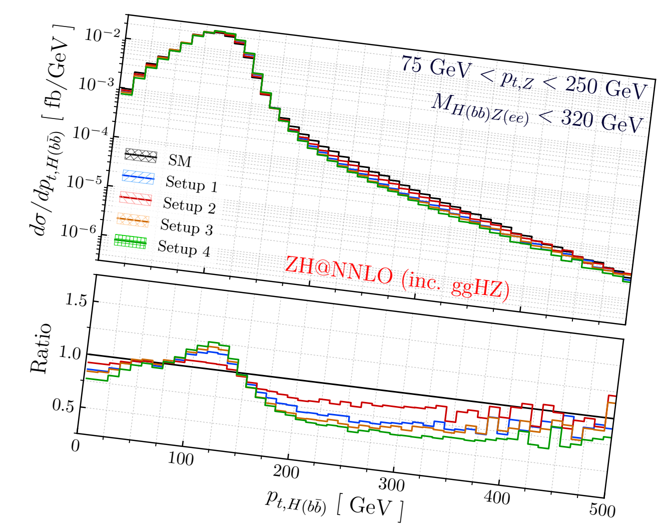




Mixed QCD-EW



Exploring the TeV region



Anomalous couplings vs h.o.

The punch line

A lot of progress on the phenomenological side:

- Few percent accuracy: becoming possible for a wide range of reactions
- Standard candles: even higher precision could be reached
- Gaining more control on the TeV region
- Gaining more control on complex final states → rich phenomenology

Few percent: opportunities

- No direct NP at the TeV-scale: $\delta_{NP} \sim Q^2/\Lambda_{NP}^2 \sim$ few percent
- $\alpha_{EW} \sim 0.01$ → study the SM at the quantum level (e.g. Higgs couplings)

However:

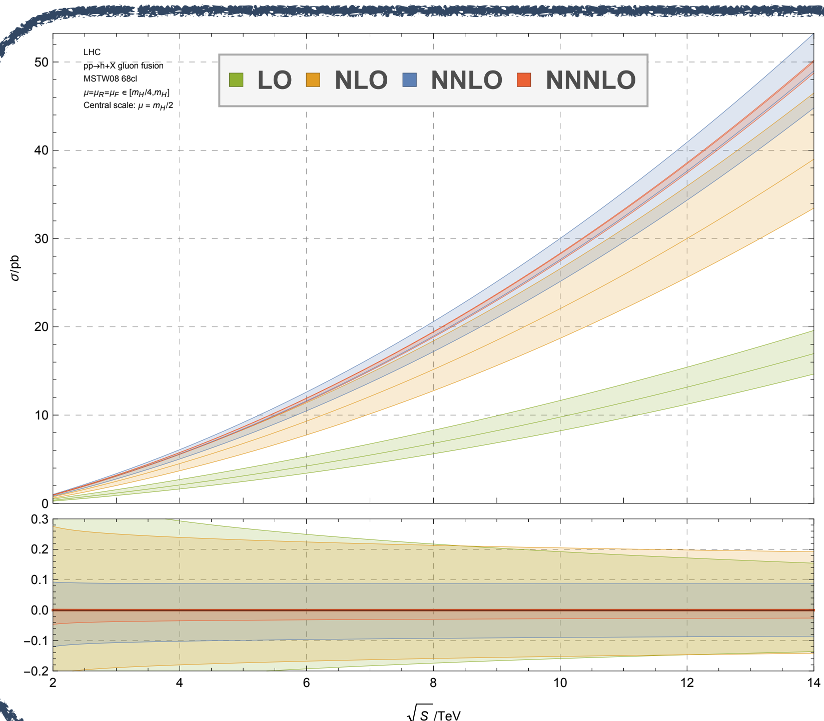
- Technical issues (CPU is not infinite…)
- Input parameters
- Non-perturbative QCD: $Q^n/\Lambda_{QCD}^n \sim 0.01^n$ → need to control (at least n=1)
- Physics at the few percent: interesting theoretical challenges…

The rise of the N^3LO era

N^{3}LO: inclusive results

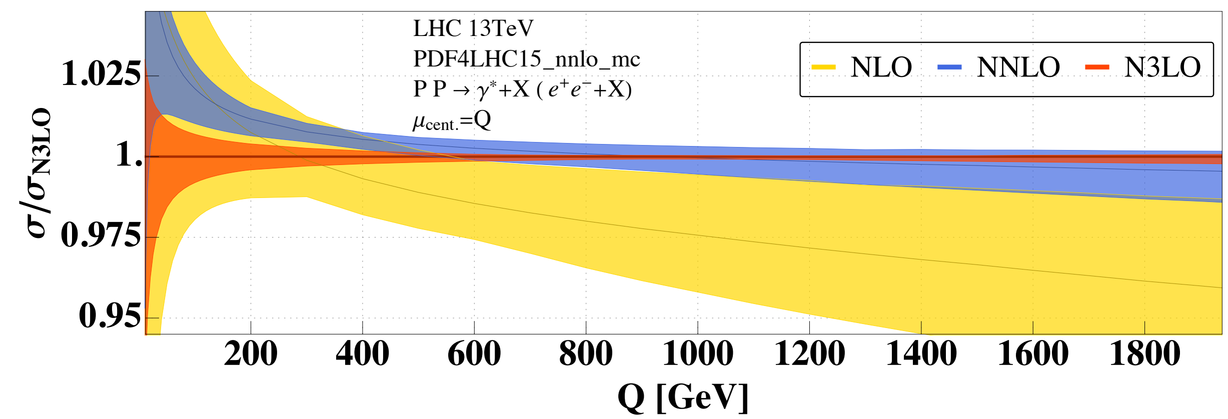
To a large extent, inclusive N^{3}LO for 2 → 1 processes has been solved

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger (2016-…);
Duhr, Dulat, Mistlberger (2020-21)]

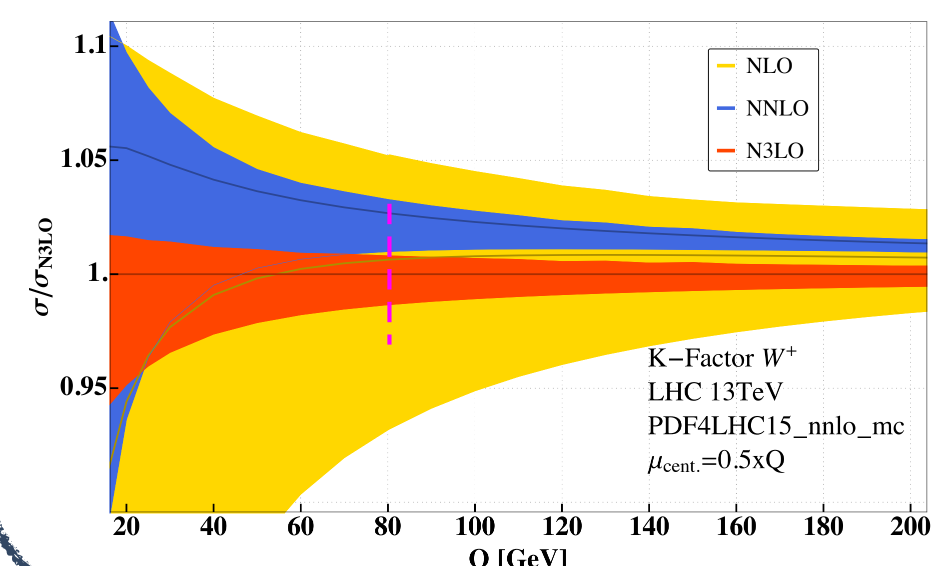


ggH

Drell-Yan

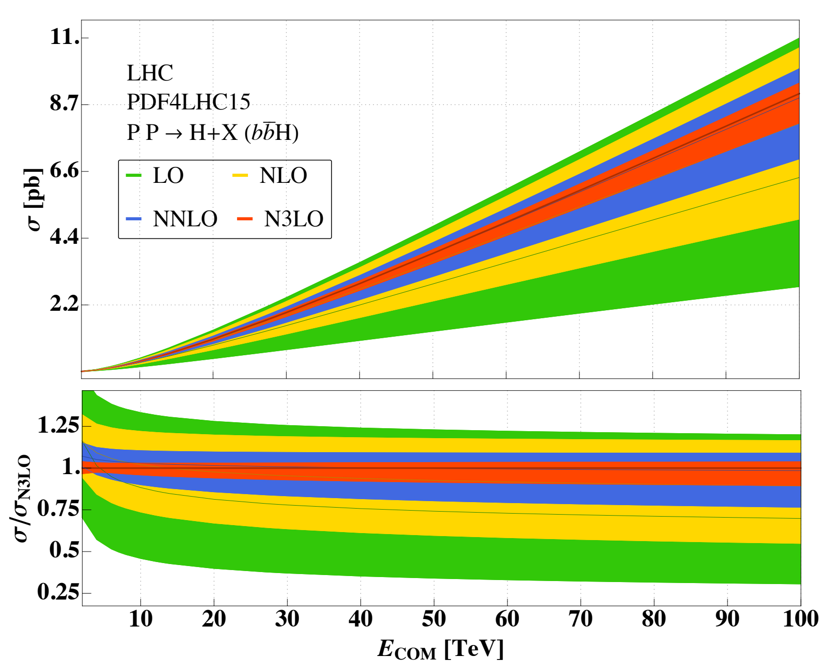


γ*



W

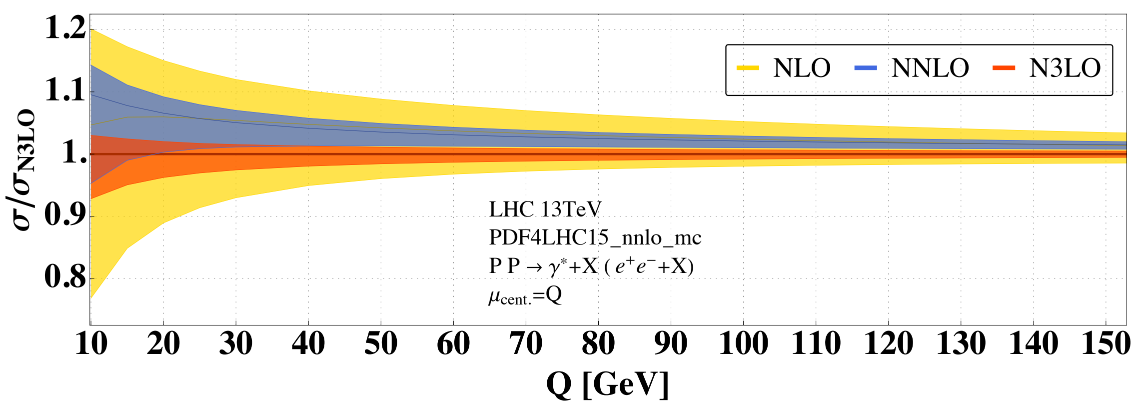
bbH

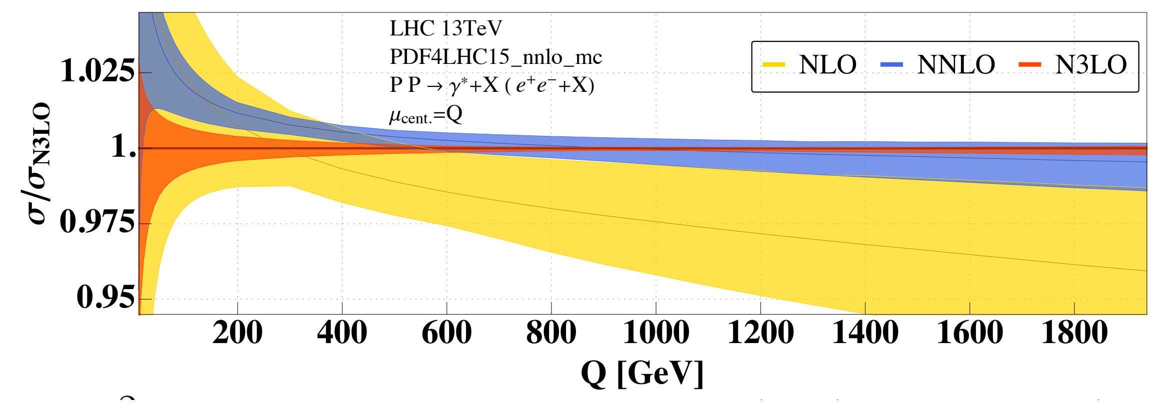


Inclusive Drell-Yan at N³LO

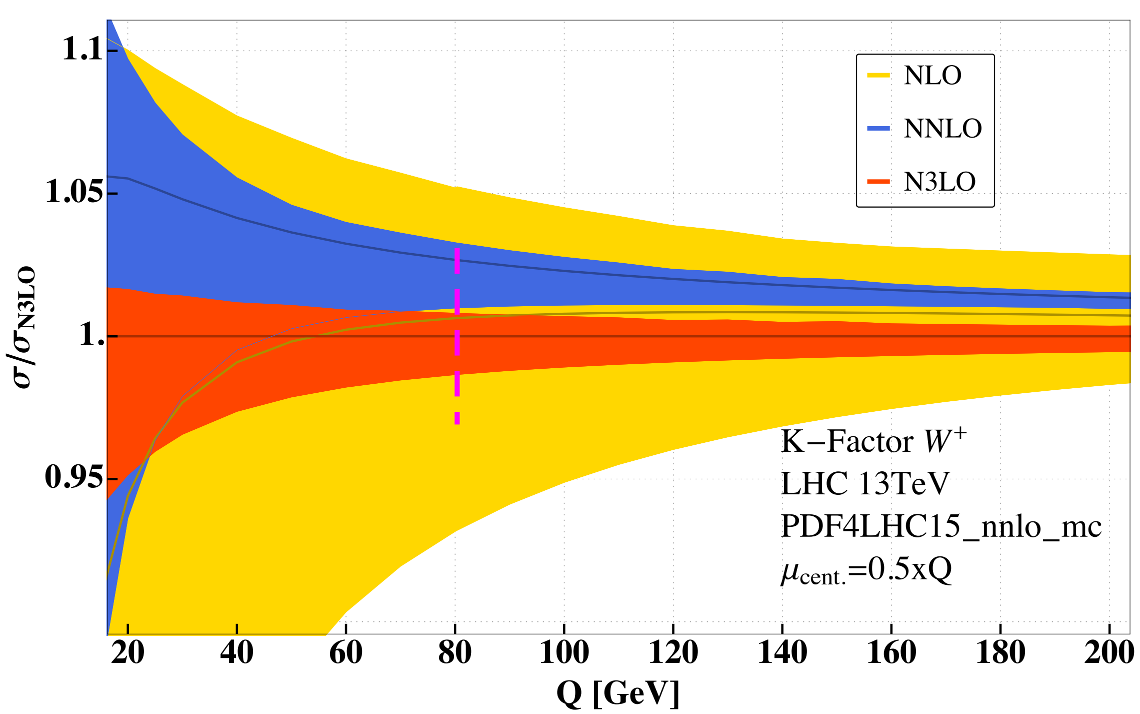
In the EW region Q~100 GeV: ~2-3% N³LO vs per-mill NNLO

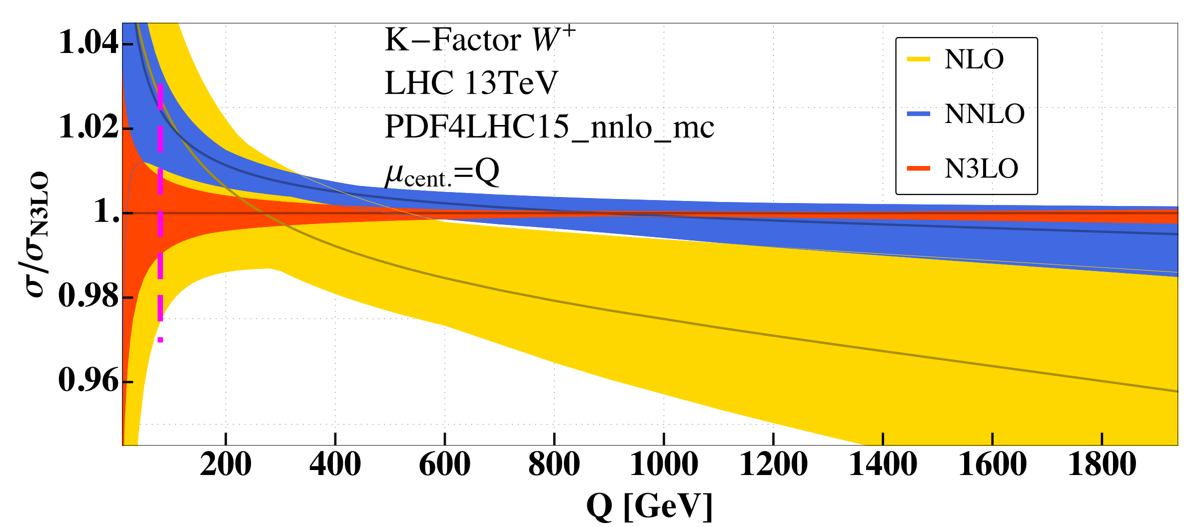
γ^*





W+

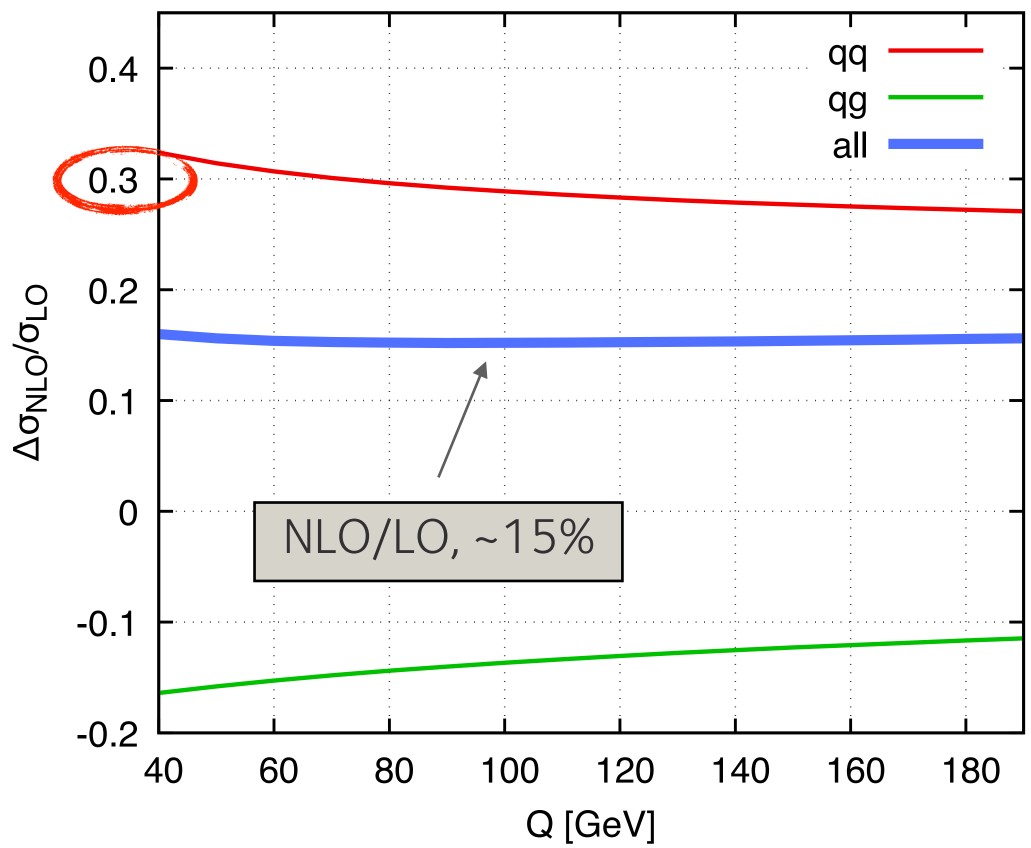


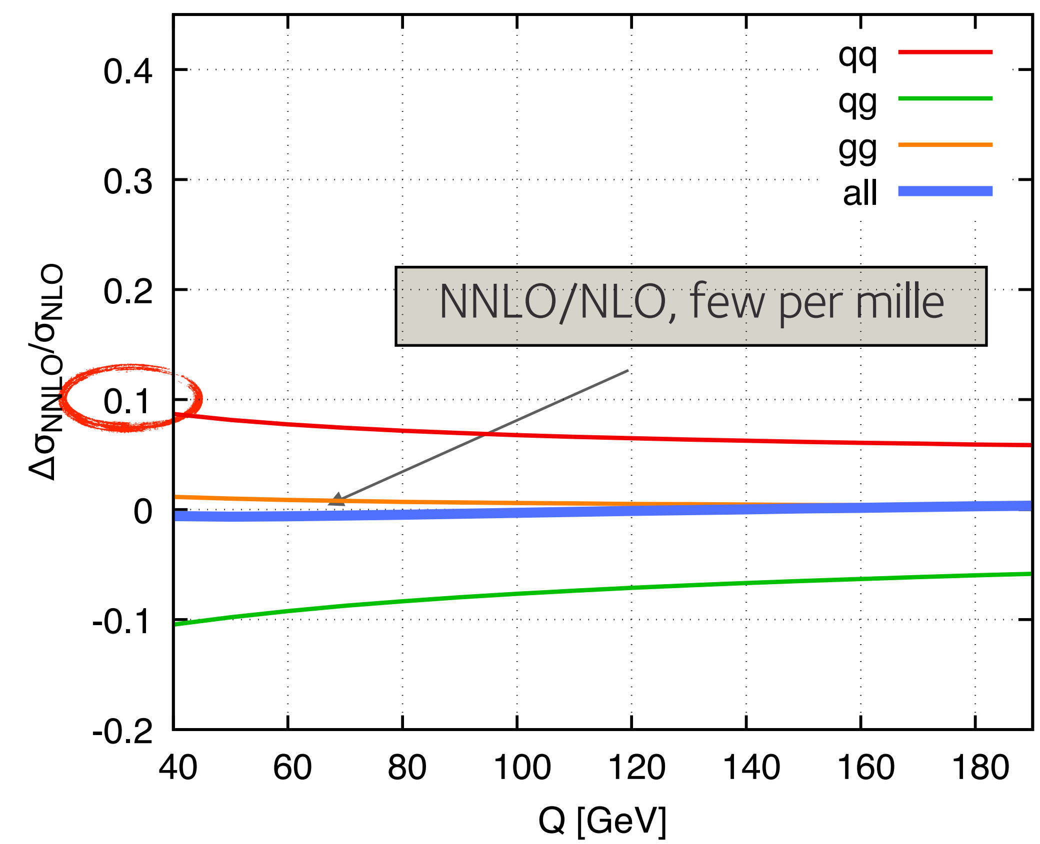


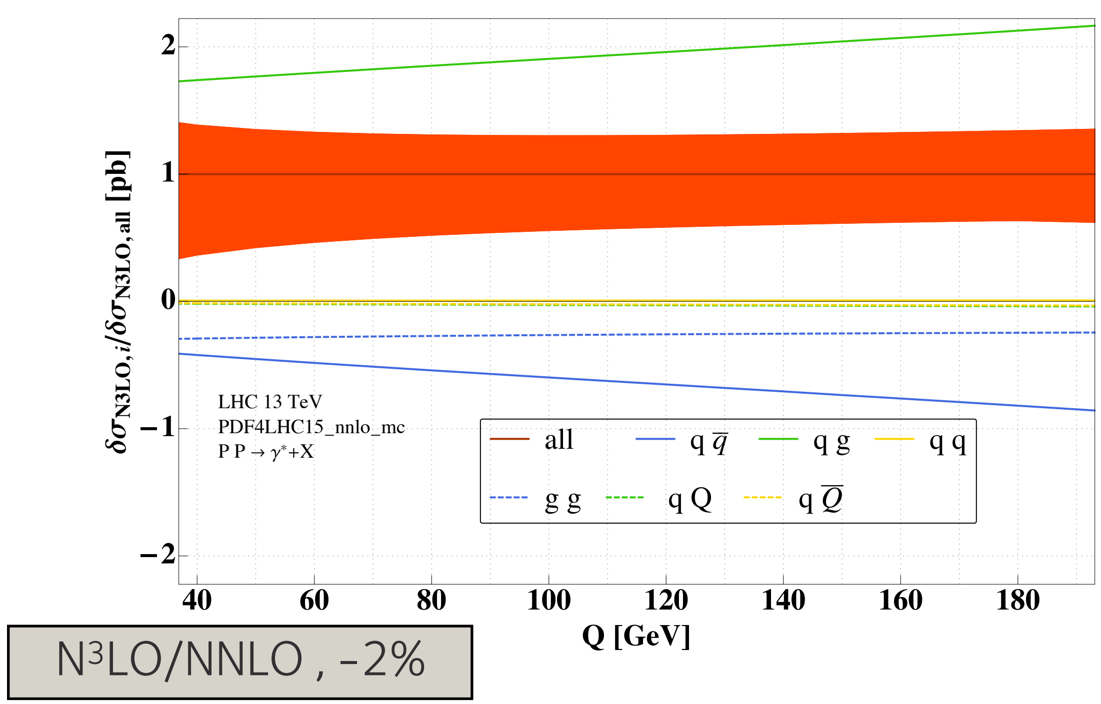
Band only overlap at large Q^2 → trouble in the high-precision region?

Neutral-current DY: flavour decomposition

Per-mille NNLO: unnaturally small. Very large cancellations







- Individual channels (μ=Q) much larger than final result, delicate cancellation pattern
- Individual channels: perturbative convergence
- N^3LO ``natural'', tiny PDFs changes can significantly affect this picture

N³LO: PDFs

N³LO PDFs not available → order mismatch

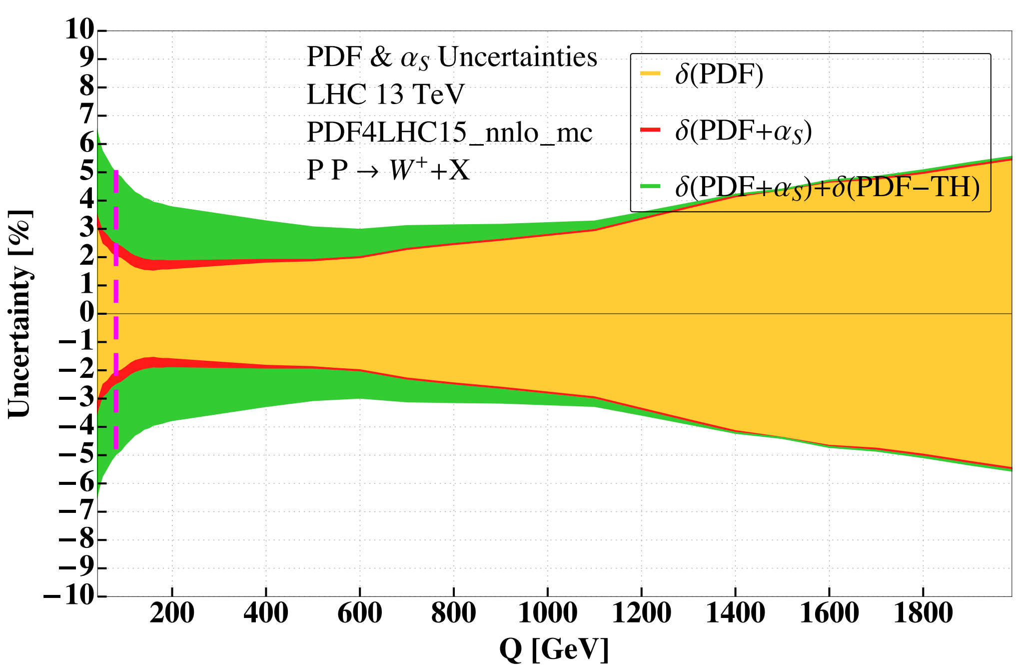
Q/GeV	$K_{\rm QCD}^{\rm N^3LO}$	δ(scale)	δ(PDF+α_S)	δ(PDF-TH)
30	0.952	$^{+1.5\%}_{-2.5\%}$	$\pm4.1\%$	$\pm2.7\%$
50	0.966	$^{+1.1\%}_{-1.6\%}$	$\pm3.2\%$	$\pm2.5\%$
70	0.973	$^{+0.89\%}_{-1.1\%}$	$\pm2.7\%$	$\pm2.4\%$
90	0.978	$^{+0.75\%}_{-0.89\%}$	$\pm2.5\%$	$\pm2.4\%$
110	0.981	$^{+0.65\%}_{-0.73\%}$	$\pm2.3\%$	$\pm2.3\%$
130	0.983	$^{+0.57\%}_{-0.63\%}$	$\pm2.2\%$	$\pm2.2\%$
150	0.985	$^{+0.50\%}_{-0.54\%}$	$\pm2.2\%$	$\pm2.2\%$

γ^*

Error: estimate from previous orders

$$\delta(\text{PDF-TH}) = \frac{1}{2}\left|\frac{\sigma^{(2),\ \text{NNLO-PDFs}}_{W^\pm} - \sigma^{(2),\ \text{NLO-PDFs}}_{W^\pm}}{\sigma^{(2),\ \text{NNLO-PDFs}}_{W^\pm}}\right|.$$

W+



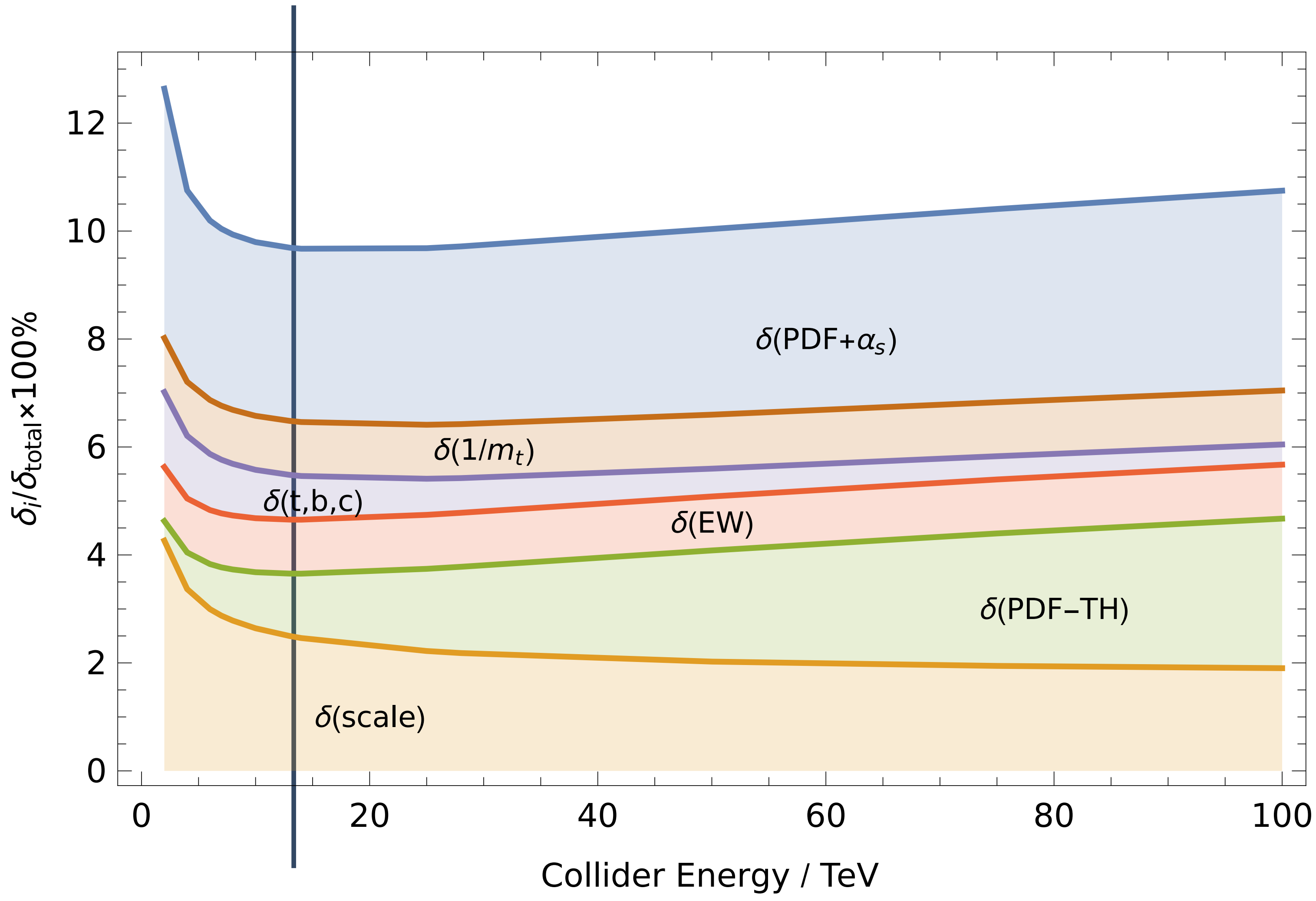
- ~ 2% PDF-TH error in the EW region
- significant fraction of the error budget
- same order of ``standard'' PDF+α_s

N^{3}LO: PDFs

N^{3}LO PDFs not available → order mismatch

ggH

[HE-HL LHC report, 2019]



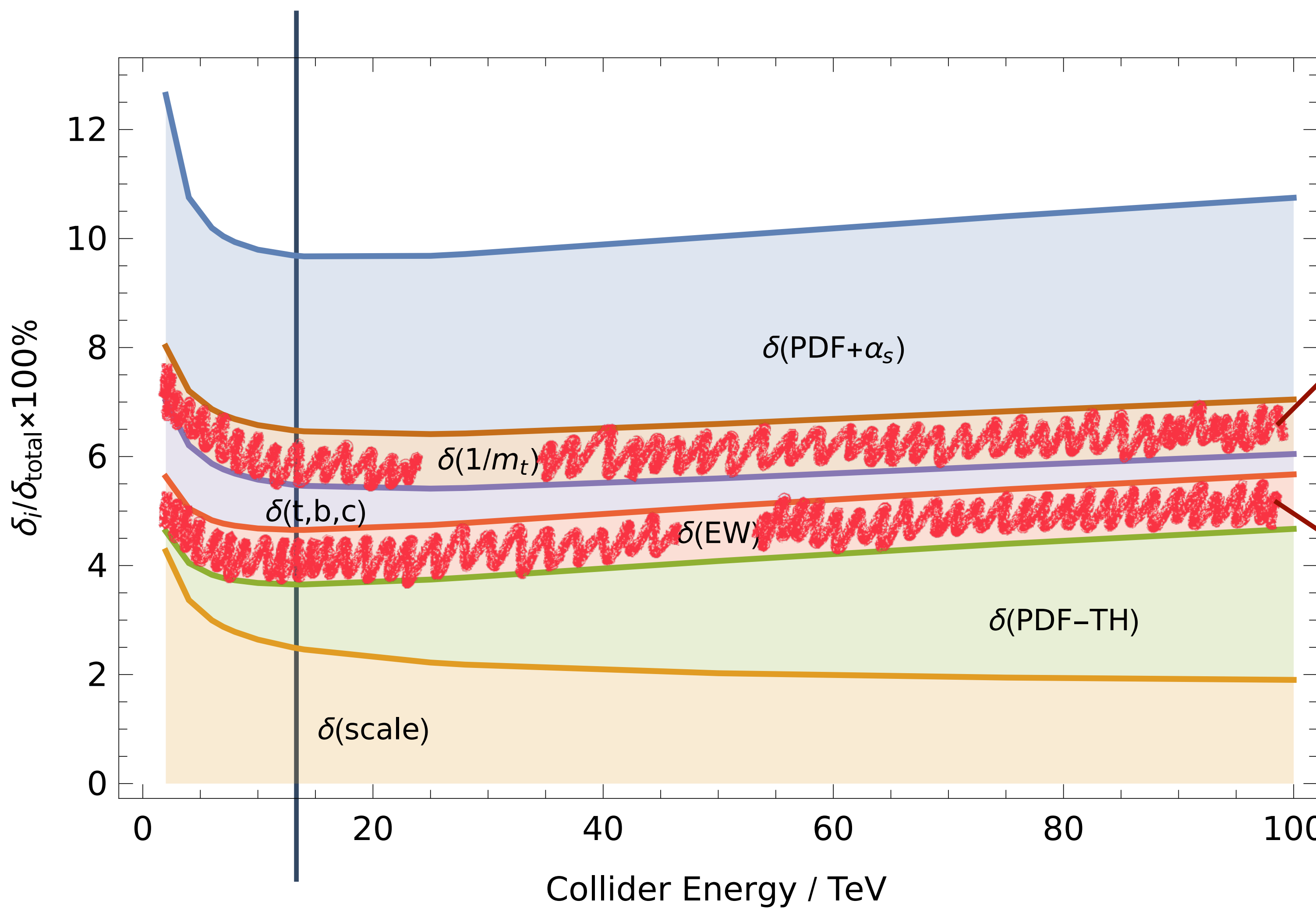
A sizeable source of the error budget

N³LO: PDFs

N³LO PDFs not available → order mismatch

ggH

[HE-HL LHC report, 2019]



[Czakon, Harlander, Klappert, Niggetiedt (2021)]

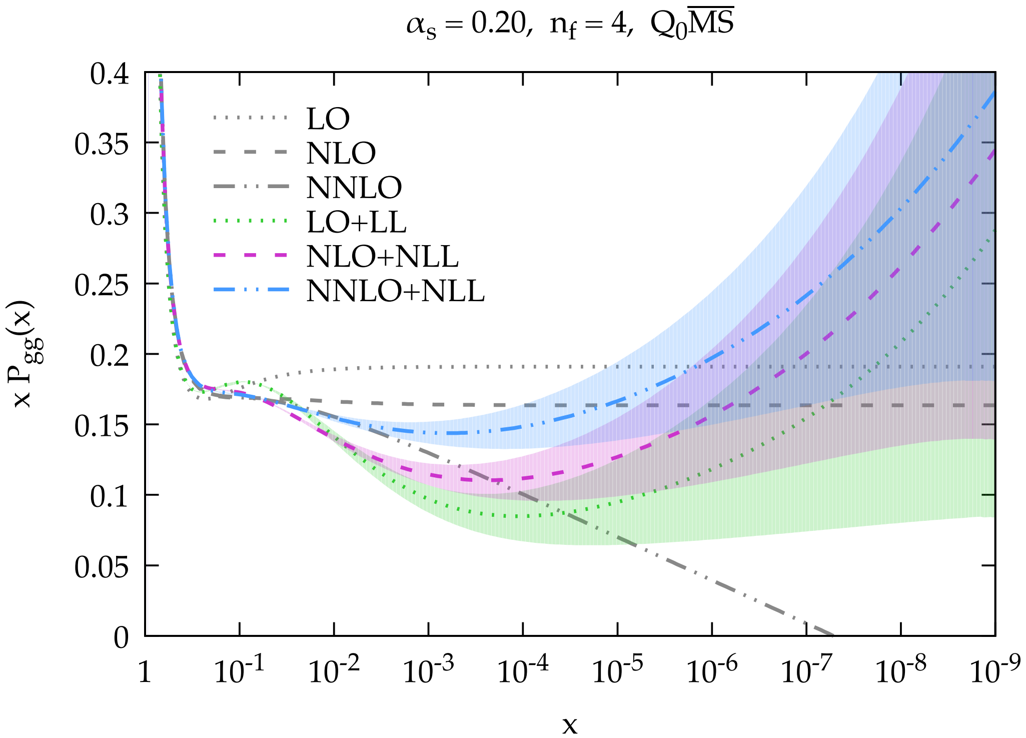
[Becchetti, Bonciani, del Duca, Hirschi, Moriello, Schweitzer (2020); Bonetti, Panzer, Smirnov, Tancredi, Melnikov (2019-2020)]

A sizeable source of the error budget… even more so

N³LO PDFs issues: evolution

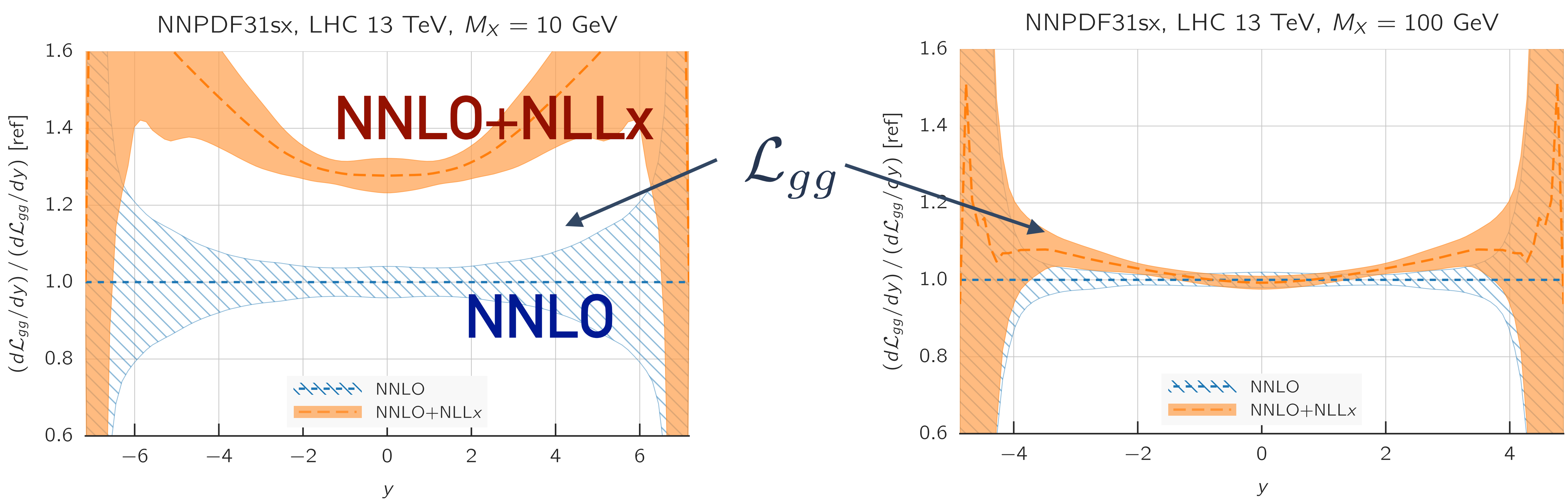
N³LO: evolution and the problems of small-x

[Bonvini, Marzanni, Muselli (2017-18)]



- N³LO calculation underway [Herzog, Moch, Ruijl, Ueda, Vermaseren, Vogt, in progrress]
- N³LO: rapid small-x growth → perturbative instabilities@N³LO
- NLL resummation known, but large subleading effects [Bonvini, Marzani (2018)]

[Ball, Bertone, Bonvini, Marzani, Rojo, Rottoli (2018)]



NNLO: an issue at low-mass, not quite so at the EW scale

N³LO PDFs issues: evolution

N³LO: evolution and the problems of small-x

NNLO: an issue at low-mass, not quite so at the EW scale. N³LO?

$$\chi_0(M) = \frac{C_A}{\pi}\left[2\psi(1) - \psi(M) - \psi(1-M)\right] \to$$

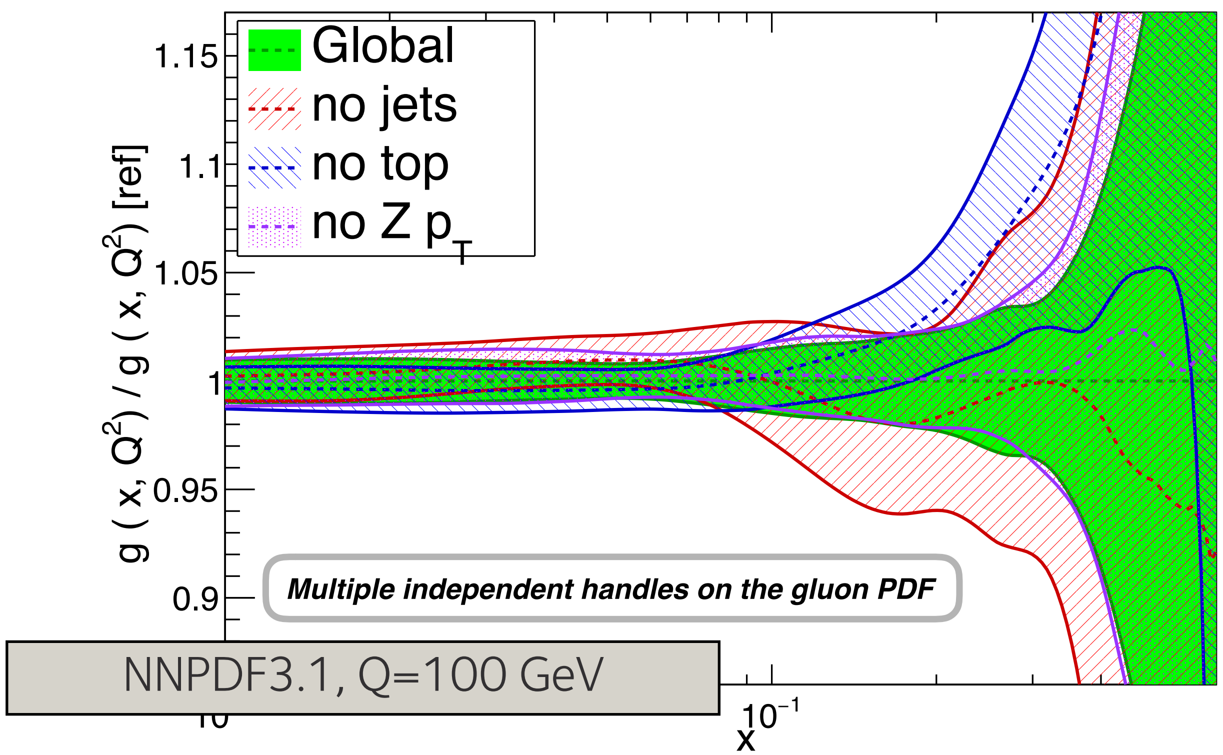
$$\gamma_{\rm LL}(N) = \frac{\bar{\alpha}_s}{N} + 0 \cdot \alpha_s^2 + 0 \cdot \alpha_s^3 + 2\zeta_3 \frac{\bar{\alpha}_s^4}{N^4} \quad , \qquad \bar{\alpha}_s = \alpha_s C_A/\pi$$

Spurious leading pole in 0, starting at N³LO (vs pole at N~0.3).

Is this an issue for precision physics (at the EW scale and beyond)?

- How dangerous is the spurious N³LO growth?
- Are subleading terms under control?
- To which extent DGLAP evolution washes out small-x effects?
- Control-sample with effectively no evolution (i.e. LHC-only fits)?

N^3LO PDFs issues: data



- Collider data crucial to reduce perturbative uncertainty → fully-consistent N^3LO fit would require top, Z p_t, jets @ N^3LO

N^3LO for PDFs: status and prospects

- DIS ✔
- DY ✔
- Z p_t: ~ (unknown, but should be possible)
- Top: ~ (unknown, but should be possible given current understanding)
- Jets: ✘ (unknown, and there may be serious problems…)

The problem with N^3LO calculations

<u>Factorization theorem:</u>

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\ \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J\ (1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p))$$

with $\mathrm{d}\sigma_{\mathrm{part}}$ = R + V is insensitive to IR physics (reabsorbed, to LP, in PDFs)

At higher order, this may not be enough…

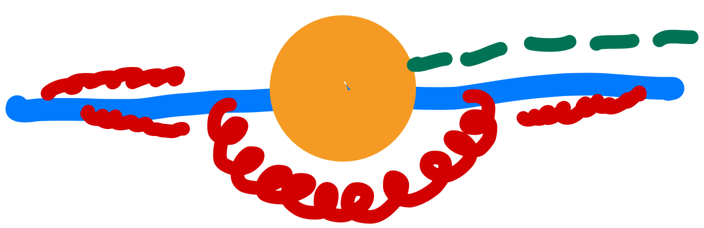
- top @ N^3LO and beyond: R + V is not enough. ``Non-perturbative'' bound state singularities that need to be accounted for [Beneke, Ruiz-Femenia (2016)]

…or may be badly violated

- massive initial state: from NNLO, non-trivial space-like vs time-like analytic continuation lead to factorization violation (non-abelian Coulomb phases…) (see [FC, Melnikov, Napoletano, Tancredi (2020)] for a modern-language derivation)
- a similar mechanism seems to be present from N^3LO for processes with non-trivial color → ``standard'' collinear factorization may be broken [Forshaw, Seymour, Kyrieleis, Siodmok (2006-2012); Catani, de Florian, Rodrigo (2012)]

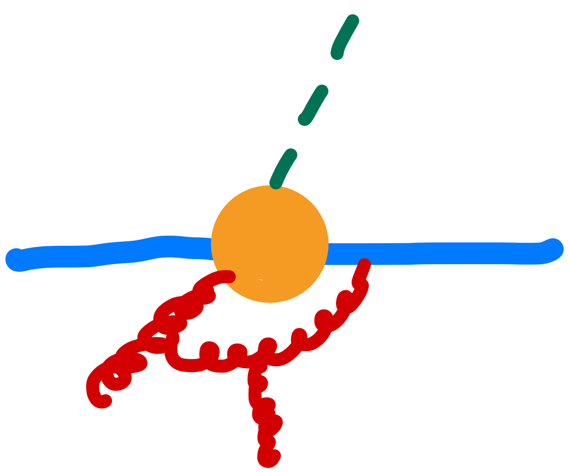
N^3LO: going differential

Colour-singlet production at order α_s^3:

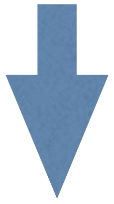


Soft/collinear (+virtual) effects at vanishingly small p_t

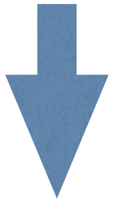
+



If $p_t \neq 0$: at least one hard emission



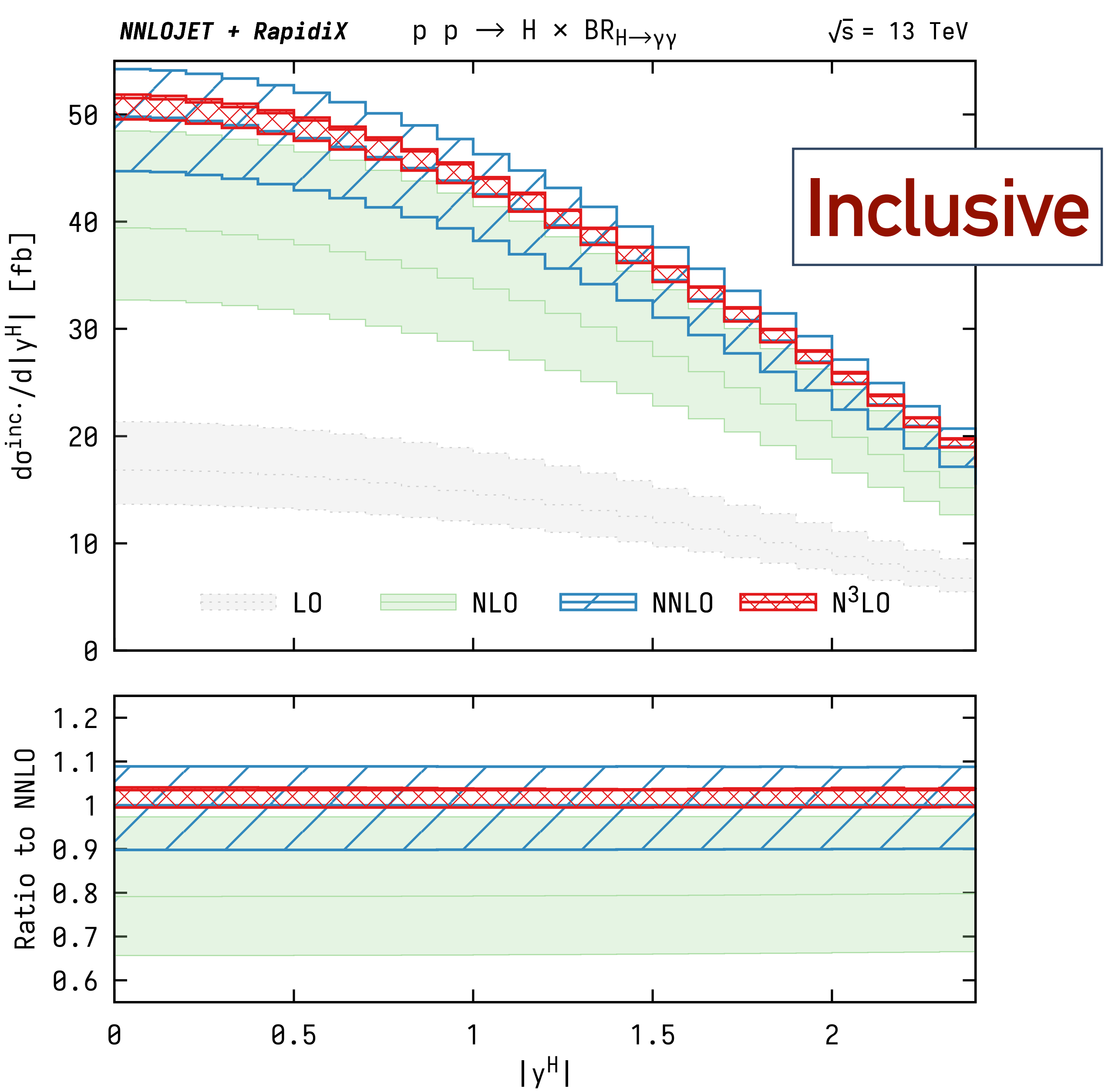
Rapidity distribution at vanishingly small p_t

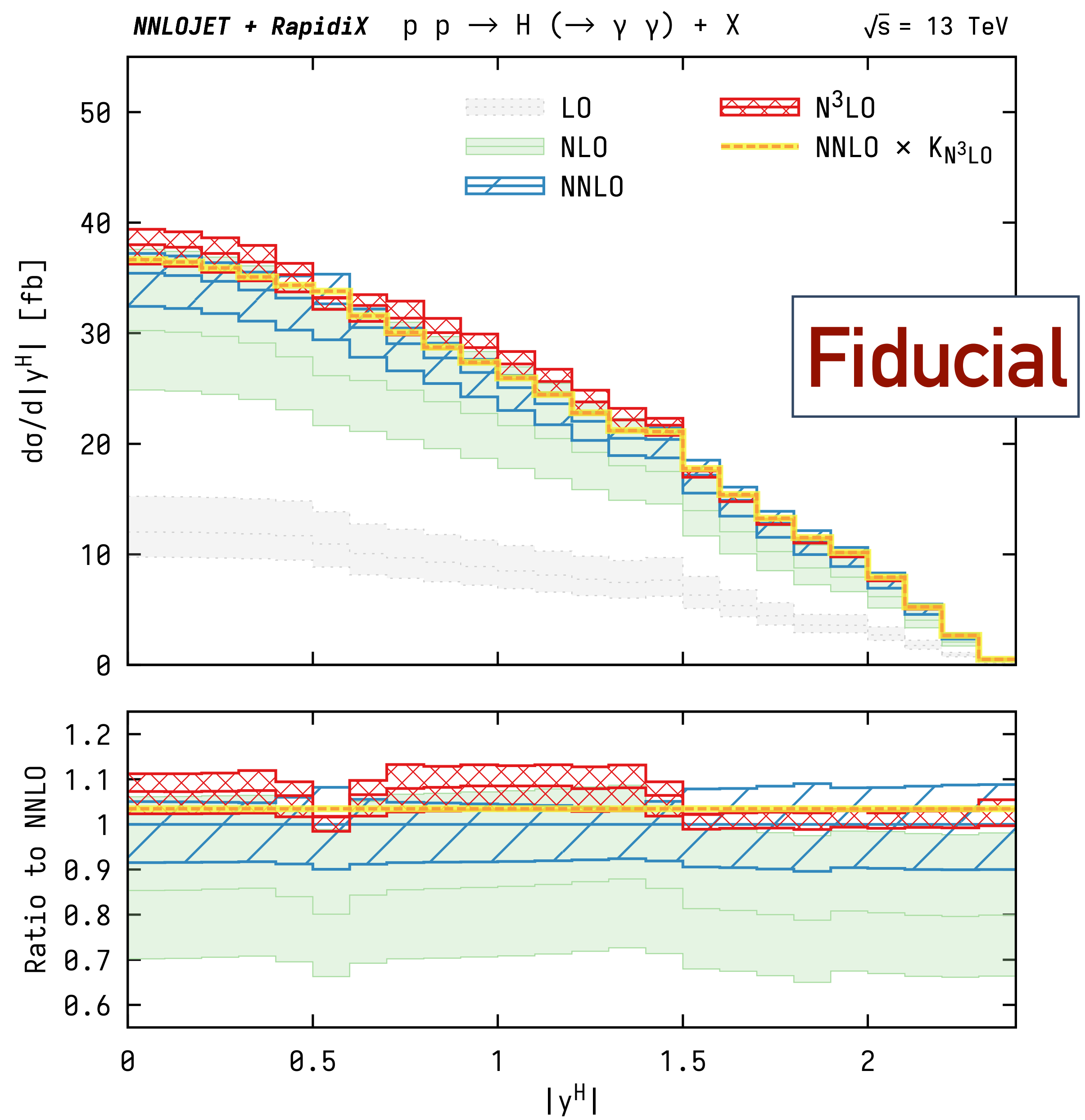


V+J@NNLO

Fully-differential Higgs @ N³LO: P2B

[Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni (2021)]

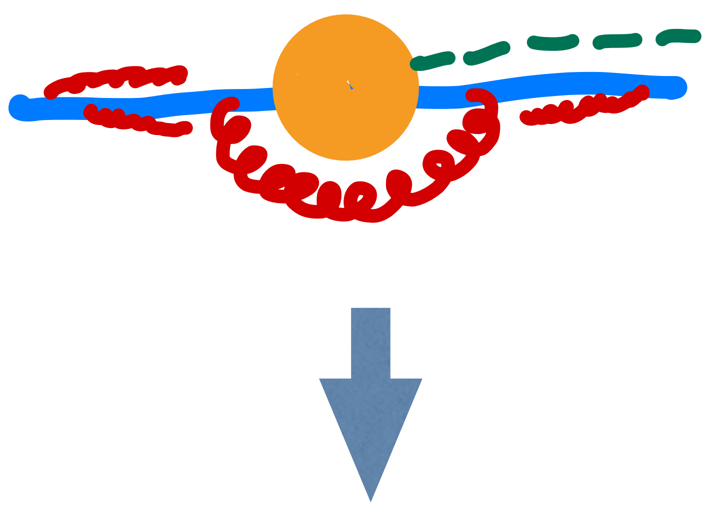




- Higgs rapidity distribution [Dulat, Mistlberger, Pelloni (2018)]
- Exquisite numerical control of H+j@NNLO [NNLOjet, 2015-2021]
- Combined using P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]

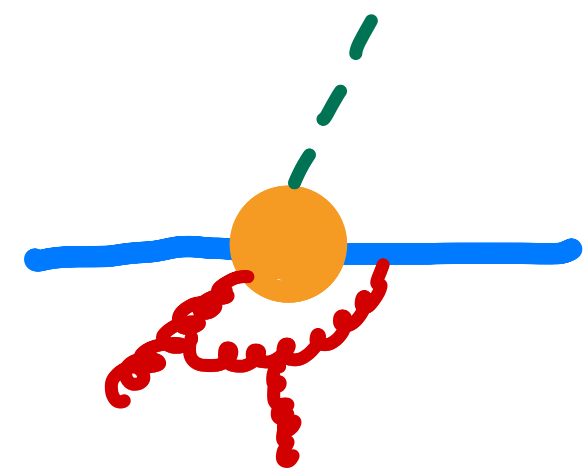
N^3LO without full rapidity distribution

Colour-singlet production at order α_s^3:



Up to power corrections: from resummation

+

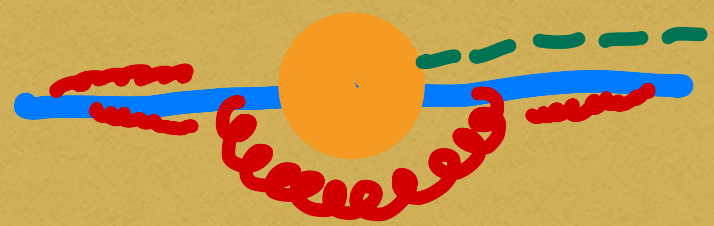


V+J@NNLO

$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t}\mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t}\mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

N^3LO without full rapidity distribution

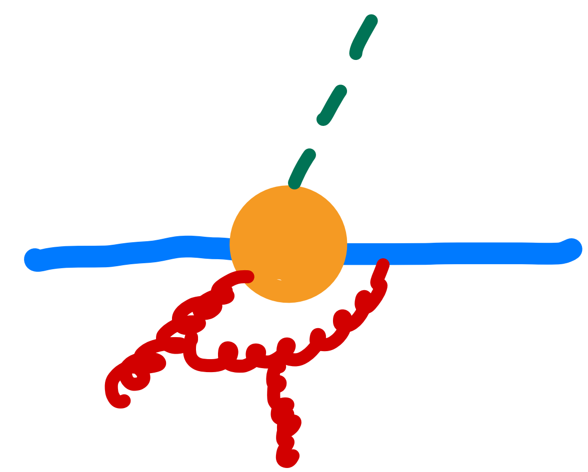
Colour-singlet production at order $\alpha_s{}^3$:



Last missing ingredient for N^3LL resummation: ``N^3LO beam function"

[Behring, Melnikov, Rietkerk, Tancredi, Wever (2019); Luo, Yang, Zhu, Zhu (2020); Ebert, Mistlberger, Vita (2020)]

+



V+J@NNLO

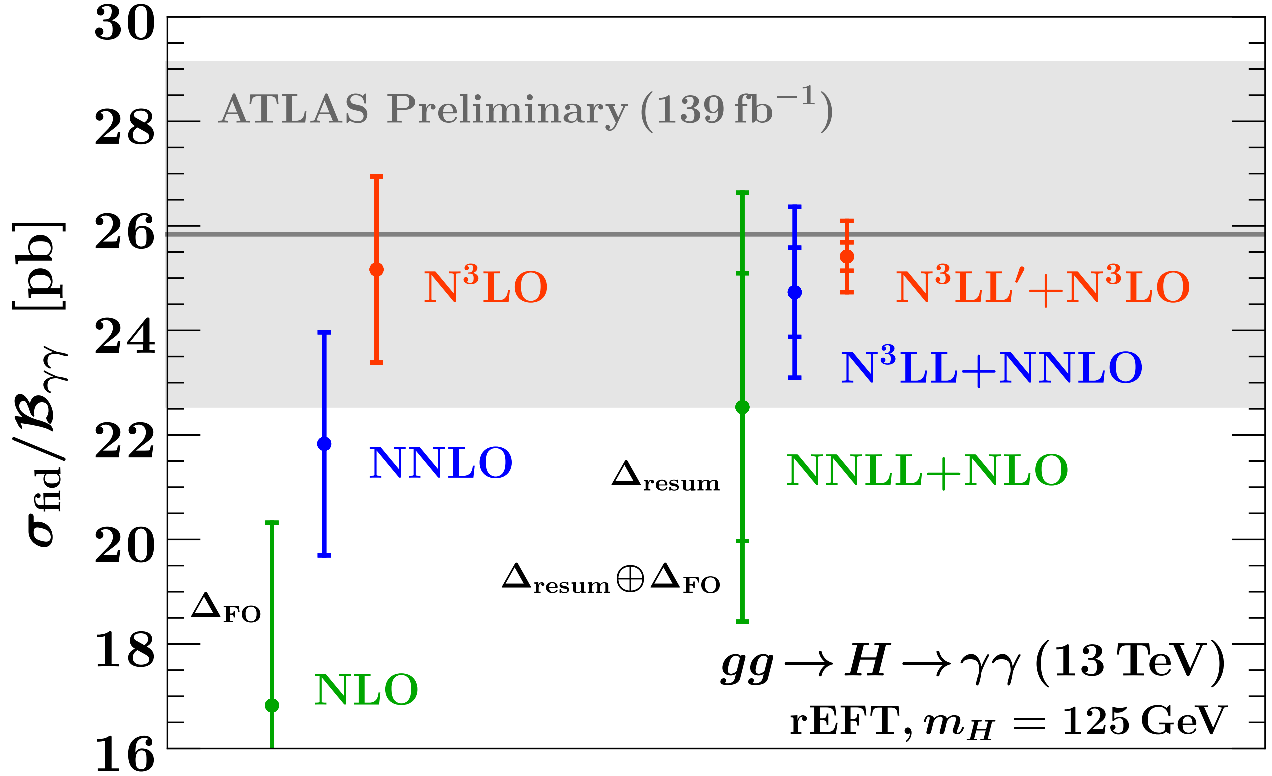
$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t}\mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t}\mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

Easy to go from N^3LO to N^3LO + N^3LL

N³LO+N³LL: recent results

[V+jet@NNLO: NNLOjet, extremely stable down to $p_t \sim 0.5$ GeV]

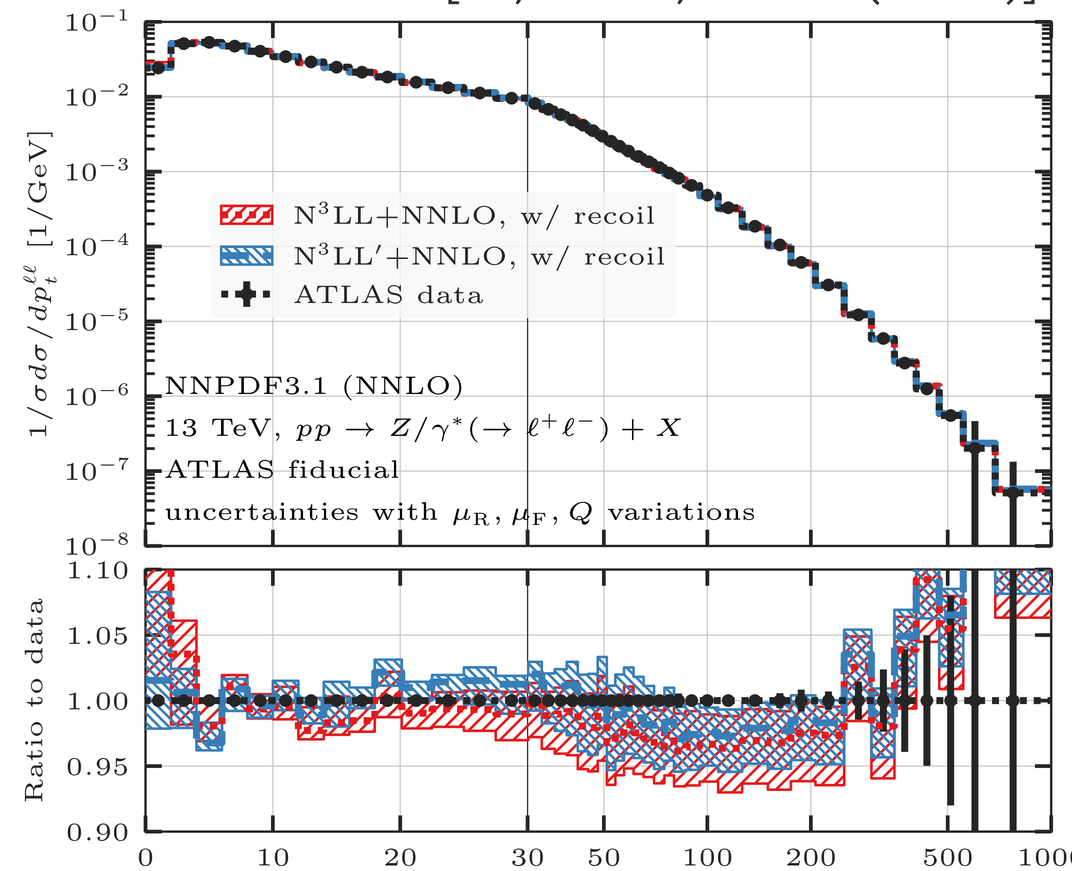
H



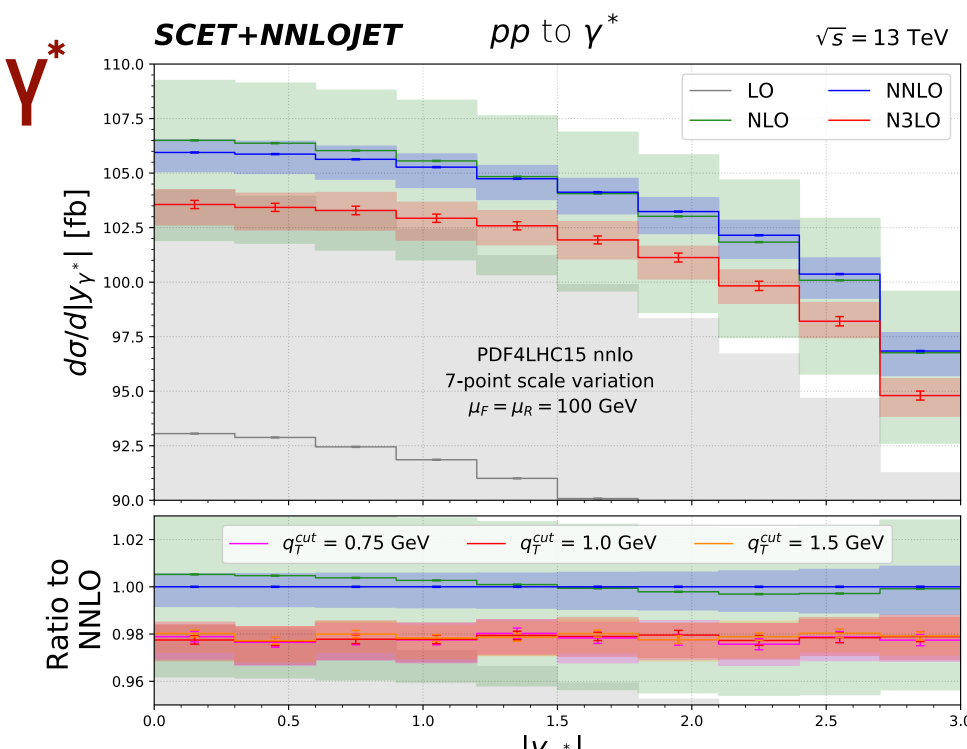
[Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

H/Z

[Re, Rottoli, Torrielli (2021)]



γ*



[Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)]

Z

Order	NLO	NNLO	N³LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	766.3 ± 0.5	757.4 ± 1.9	746.1 ± 1.9
Order	NLL+NLO	NNLL+NNLO	N³LL+N³LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	773.7 ± 0.5	759.8 ± 1.9	749.6 ± 2.0

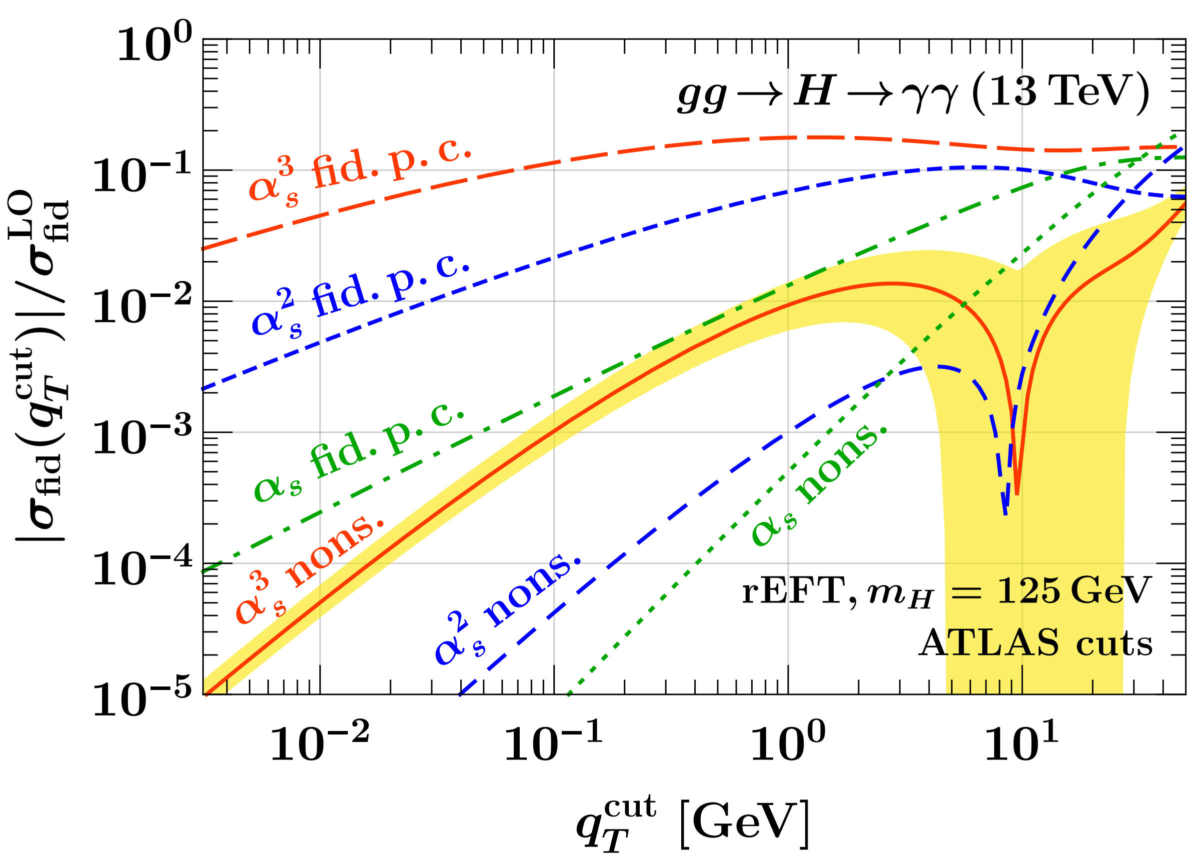
[Camarda, Cieri, Ferrera (2021)]

N^{3}LO from resummation: a word of caution

To extract N^{3}LO: subleading power must be under control

$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t}\mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t}\mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

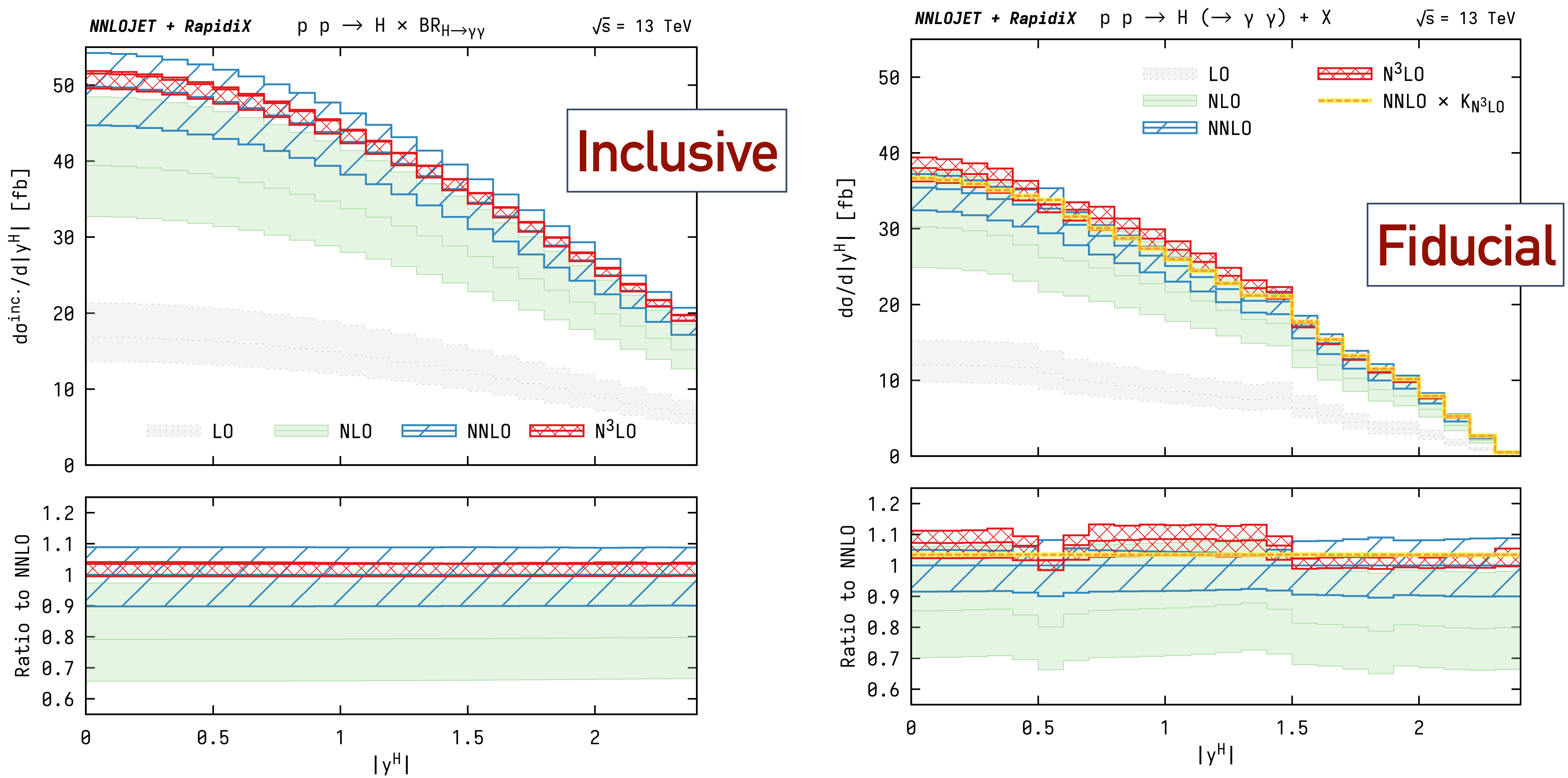
- Subleading power ~ $\alpha_s^n (p_t/Q)^2 \ln^{2n-1}(p_t/Q)$ → much lower cutoff w.r.t. NNLO
- Naive estimate: NNLO V+j down to ~1-0.5 GeV → error up to order 1%



- For Higgs, confirmed by (and included in).
  [Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]
- Good news: first subleading is enough
- N^{3}LO+N^{3}LL: less severe, but more ambiguities

Fiducial N³LO: a more serious problem

Higgs fiducial: [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni (2021)]



- Inclusive: flat K-factor (as for inclusive), tiny error, no structure
- Fiducial: large corrections, large error, non-trivial shapes

?

Fiducial N^{3}LO: a more serious problem

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\, \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J \,\left(1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)\right)$$

``observable F_J must be insensitive to IR regions''

violated by ATLAS/CMS experimental cuts

- Drell-Yan: $p_{t,l} > 25$ GeV, $|y_l| < 2.5$ → the infamous ``symmetric cuts''. Well-known source of troubles [Frixione, Ridolfi (1997)]
- Higgs: asymmetric cuts. $p_{t,\gamma 1(2)} < 0.35(0.25)\, m_H$, $|y_\gamma| < 2.37$, with gap

Unfortunately, both symmetric and asymmetric cuts share the same feature: introduce linear p_t dependence on the acceptance at small p_t

[Catani, Cieri, de Florian, Ferrera, Grazzini (2018); Ebert, Michel, Tackmann + Billis, Dehnadi (2017-2021); Salam + Slade (2015, 2021)]

Spoiling R+V cancellations: a cartoon

Inclusive calculations:

$$\sigma_{\rm inc} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V =$$

Spoiling R+V cancellations: a cartoon

Inclusive calculations:

$$\sigma_{\mathrm{inc}} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V =$$

$$\left(\int^{p_{t,H}^{\mathrm{IR}}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\mathrm{IR}} \right) + \left(\int_{p_{t,H}^{\mathrm{IR}}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\mathrm{fin}} \right)$$

Spoiling R+V cancellations: a cartoon

Inclusive calculations:

$$\sigma_{\rm inc} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V =$$

$$\left(\int^{p_{t,H}^{\rm IR}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\rm IR} \right) + \left(\int_{p_{t,H}^{\rm IR}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\rm fin} \right)$$

unitarity

insensitive to IR physics

Spoiling R+V cancellations: a cartoon

Inclusive calculations:

$$\sigma_{\rm inc} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V =$$

$$\left(\int^{p_{t,H}^{\rm IR}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\rm IR} \right) + \left(\int_{p_{t,H}^{\rm IR}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\rm fin} \right)$$

unitarity

insensitive to IR physics

Fiducial: non-trivial acceptance may weight the real integral

$$\sigma_{\rm fid} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} \, f(p_{t,H}) + V f(p_{t,H} = 0)$$

Spoiling R+V cancellations: a cartoon

Inclusive calculations:

$$\sigma_{\mathrm{inc}} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V =$$

$$\left(\int^{p_{t,H}^{\mathrm{IR}}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\mathrm{IR}} \right) + \left(\int_{p_{t,H}^{\mathrm{IR}}} \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H} + V^{\mathrm{fin}} \right)$$

unitarity

insensitive to IR physics

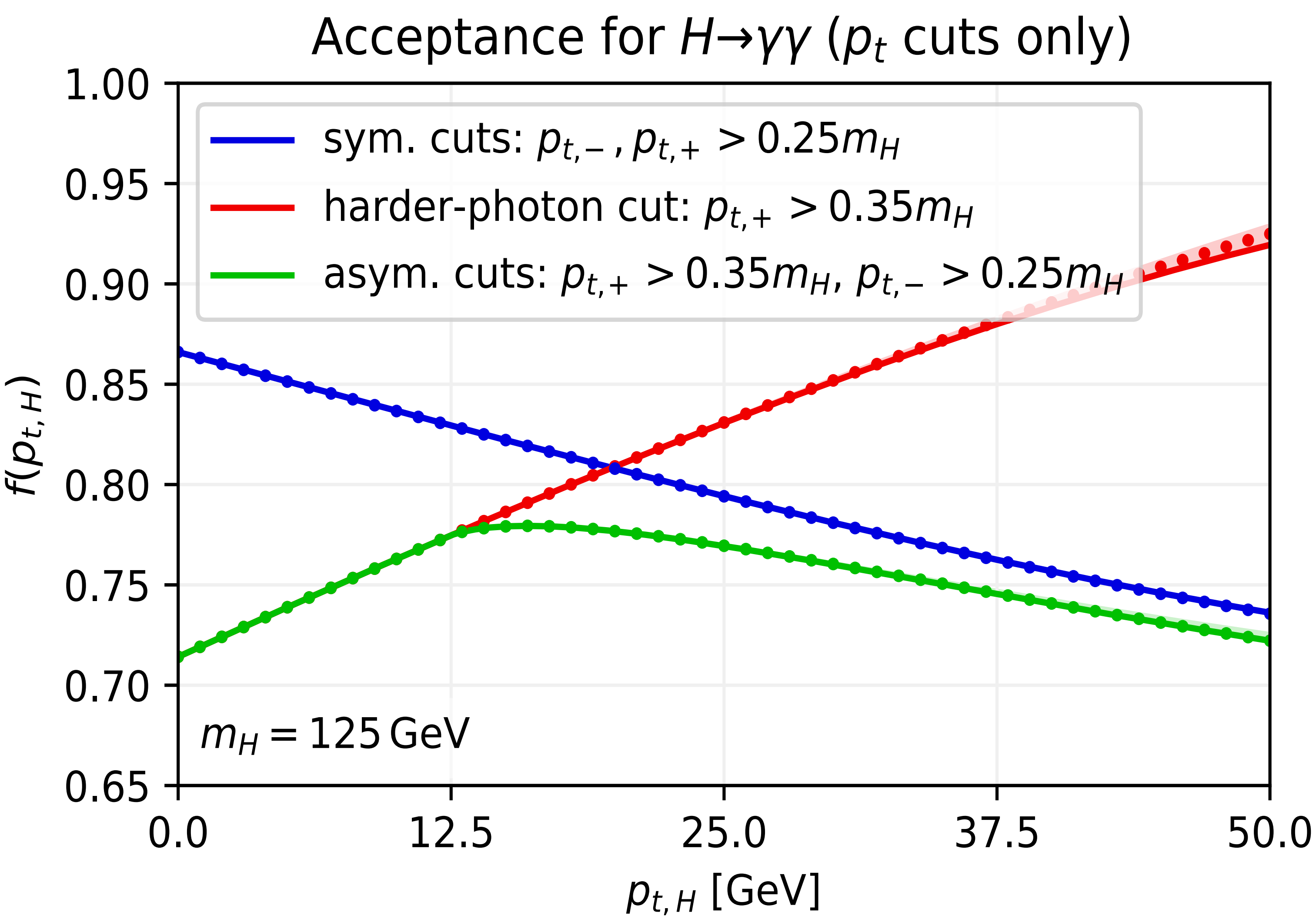
Fiducial: non-trivial acceptance may weight the real integral

$$\sigma_{\mathrm{fid}} = R + V = \int \frac{d\sigma_{H+j}}{dp_{t,H}} dp_{t,H}\, f(p_{t,H}) + V f(p_{t,H} = 0)$$

If f changes strongly at low p_t: contamination from IR physics.

Serious problem for fixed-order perturbation theory

LHC typical acceptances:



For small $p_{t,H}$:

$$f^{\mathrm{symm}}(p_{t,H}) \sim f_0 + f_1^{\mathrm{symm}} \cdot \frac{p_{t,H}}{m_H}$$

$$f^{\mathrm{asymm}}(p_{t,H}) \sim f_0 + f_1^{\mathrm{asymm}} \cdot \frac{p_{t,H}}{m_H}$$

In the IR region (=small $p_{t,H}$), LHC cuts have a linear dependence on the Higgs transverse momentum → spoil R+V cancellation in fixed-order calculations

Linear acceptances: how bad?

A cartoon: double-logarithmic approximation [$L=\ln(m_H/2p_{t,H})$]

$$\frac{d\sigma}{dp_{t,H}} = \frac{4C_A\alpha_s L}{\pi p_{t,H}} e^{-\frac{2C_A\alpha_s}{\pi}L^2}\sigma_{\rm tot} = \frac{\sigma_{\rm tot}}{p_{t,H}}\sum_{n=1}^{\infty}(-)^{n-1}\frac{2L^{2n-1}}{(n-1)!}\left(\frac{2C_A\alpha_s}{\pi}\right)^n$$

The fixed-order series is then:

~ n!, factorial growth

$$\sigma_{\rm fid} = \int \frac{d\sigma}{dp_{t,H}}dp_{t,H}\left(f_0 + f_1\frac{p_{t,H}}{m_H}\right) = \sigma_0\left[f_0 + f_1\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n)!}{2(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n + \ldots\right]$$

Pert.theory does not improve after $n \sim 1/2 + \pi/(8\, C_A\, \alpha_s) \sim 1.5$

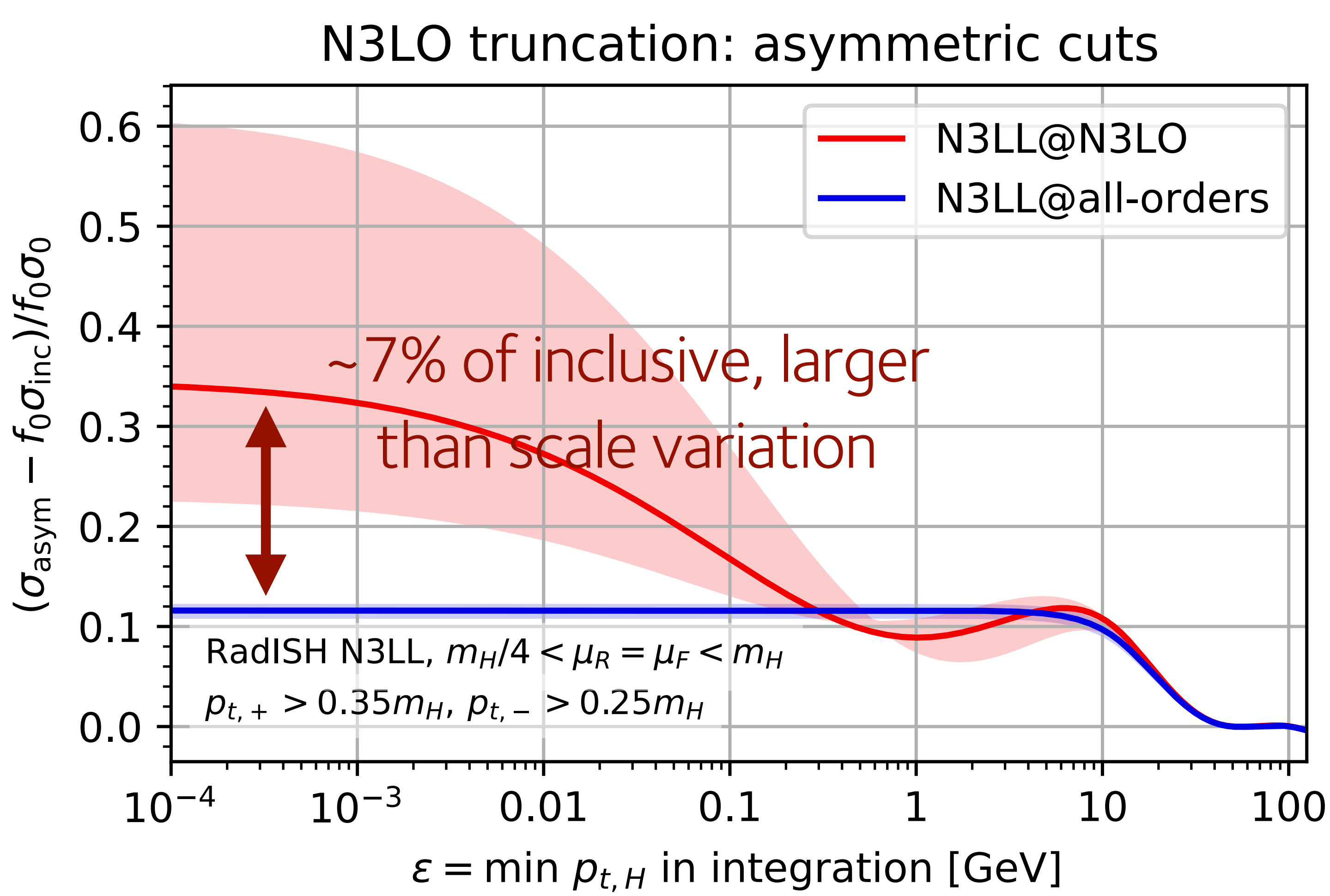
$$\frac{\sigma^{\rm DL}_{\rm fid,sym}}{f_0\sigma_{\rm tot}} - 1 = \frac{f_1^{\rm sym}}{f_0}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n)!}{2(n!)}\left(\frac{2C_A\alpha_s}{\pi}\right)^n + \cdots \tag{2.11a}$$

$$\simeq \frac{f_1^{\rm sym}}{f_0}\left(\underbrace{0.24}_{\alpha_s} - \underbrace{0.34}_{\alpha_s^2} + \underbrace{0.82}_{\alpha_s^3} - \underbrace{2.73}_{\alpha_s^4} + \underbrace{11.72}_{\alpha_s^5} + \ldots\right) \simeq \frac{f_1^{\rm sym}}{f_0} \times \underbrace{0.12}_{\rm resummed}\,, \tag{2.11b}$$

Linear acceptances: how bad?

Realistic scenario:

$$\frac{\sigma_{\text{asym}} - f_0\sigma_{\text{inc}}}{\sigma_0 f_0} \simeq 0.15_{\alpha_s} - 0.29_{\alpha_s^2} + 0.71_{\alpha_s^3} - 2.39_{\alpha_s^4} + 10.26_{\alpha_s^5} + \dots \quad \simeq 0.06 \quad \text{@DL},$$
$$\simeq 0.15_{\alpha_s} - 0.23_{\alpha_s^2} + 0.44_{\alpha_s^3} - 1.15_{\alpha_s^4} + 3.83_{\alpha_s^5} + \dots \quad \simeq 0.06 \quad \text{@LL},$$
$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.29_{\alpha_s^3} + \dots \quad \simeq 0.10 \quad \text{@NNLL},$$
$$\simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \dots \quad \simeq 0.12 \quad \text{@N3LL}.$$



F.o. calculations: sensitive to unphysically low values of $p_{t,H}$

(DL: ~10^{-2} GeV to cover 95% of the cross-section)

Fiducial cross-sections: possible ways out

With current experimental setup: f.o. results unreliable.

[Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

$$\sigma^{\rm FO}_{\rm incl} = 13.80\,[1 + 1.291 + 0.783 + 0.299]\,{\rm pb}\,,$$

$$\begin{aligned}\sigma^{\rm FO}_{\rm fid}/\mathcal{B}_{\gamma\gamma} = 6.928\,[1 &+ (1.300 + 0.129_{\rm fpc}) \\ &+ (0.784 - 0.061_{\rm fpc}) \\ &+ (0.331 + 0.150_{\rm fpc})]\,{\rm pb}\,.\end{aligned}$$

Starting from N3LO: spurious effect can be as large as correction itself

A possible option: always match with resummation. However:

- f.o. provides a very clean, solid and robust framework. Should be careful to let it go without thinking
- Resummation: a whole plethora of new ambiguities…

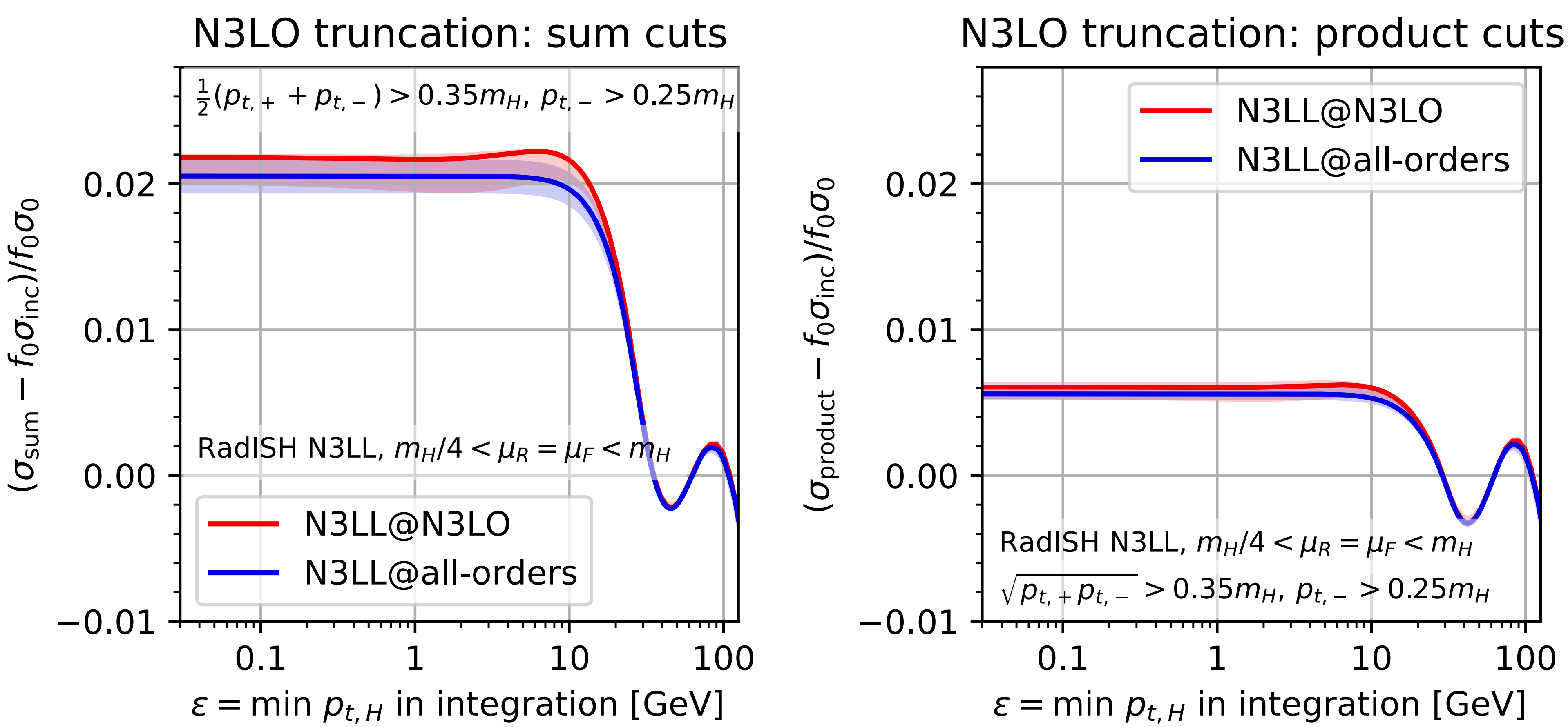
Fiducial cross-sections: possible ways out

A better option: change cuts to remove linear dependence. Rules of the game:

- only use data on tape / do not ask for ``bad'' exp. regions
- do not significantly affect S/B

Simplest case: Higgs →γγ (pure kinematics)

- very simple solution: $p^{\gamma}_{t,min} > p_{t,cut\ 1}$, $p^{\gamma}_{t,1} + p^{\gamma}_{t,2}$ // $p^{\gamma}_{t,1} \times p^{\gamma}_{t,2} > p_{t,\ cut\ 2}$



[Salam, Slade (2021)]

What about DY?

Same issue, but situation less severe (C_A vs C_F)

$$\begin{aligned}\frac{\sigma^{(\mathrm{u})}_{\mathrm{sym}} - f_0\sigma_{\mathrm{inc}}}{\sigma_0 f_0} &\simeq -0.074_{\alpha_s} + 0.051_{\alpha_s^2} - 0.057_{\alpha_s^3} + 0.090_{\alpha_s^4} - 0.180_{\alpha_s^5} + \dots &&\simeq -0.047 \quad \text{@DL},\\ &\simeq -0.074_{\alpha_s} + 0.027_{\alpha_s^2} - 0.014_{\alpha_s^3} + 0.010_{\alpha_s^4} - 0.010_{\alpha_s^5} + \dots &&\simeq -0.055 \quad \text{@LL},\\ &\simeq -0.118_{\alpha_s} + 0.012_{\alpha_s^2} - 0.016_{\alpha_s^3} + \dots &&\simeq -0.114 \quad \text{@NNLL},\\ &\simeq -0.118_{\alpha_s} + 0.012_{\alpha_s^2} - 0.016_{\alpha_s^3} + \dots &&\simeq -0.114 \quad \text{@N3LL}.\end{aligned}$$

Solution more tricky, V couples production / decay

[see Salam, Slade (2021) for a discussion]

Other processes? In principle, could be a problem any time you have an essentially symmetric configuration at LO (e.g. top, some configurations for Z+j, jets, photons)…. A lot to explore…

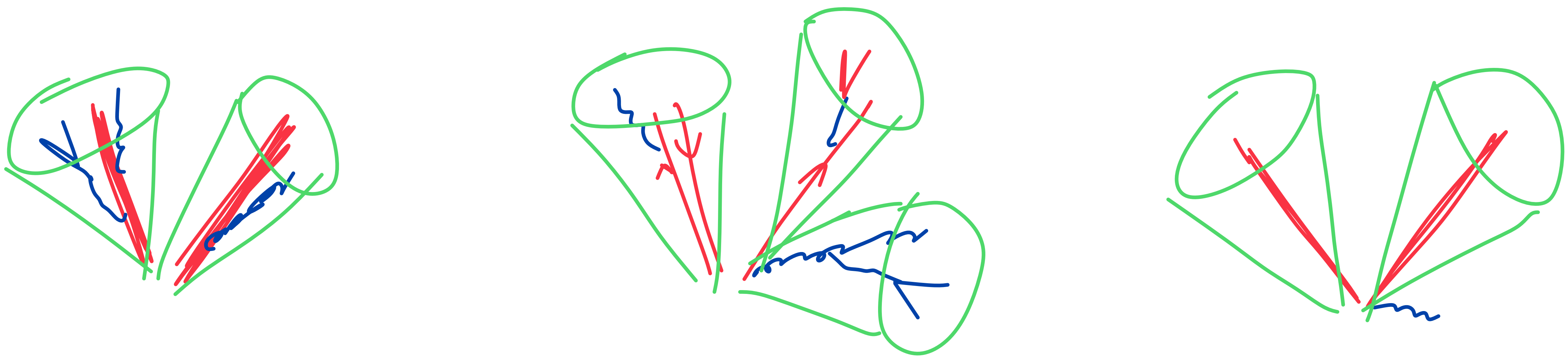
Back to NNLO:
1. Heavy flavour

X+b/c: more and more prominent…

The problem: TH vs EXP have a quite different definition of ``flavour”

EXP: displaced vertices, hadron tagging…

TH: ``what is the net flavour of a jet?”



2 b-jets — light - b - light — b b

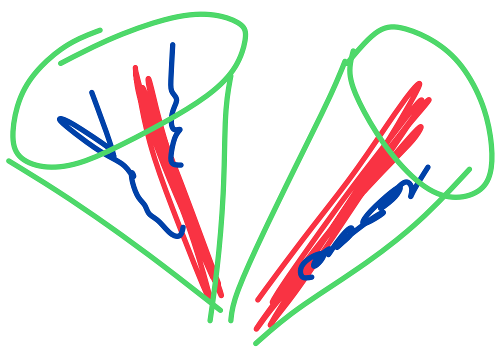
$b\bar{b}$ must behave like
a gluon [coll. safety]

X+b/c: more and more prominent…

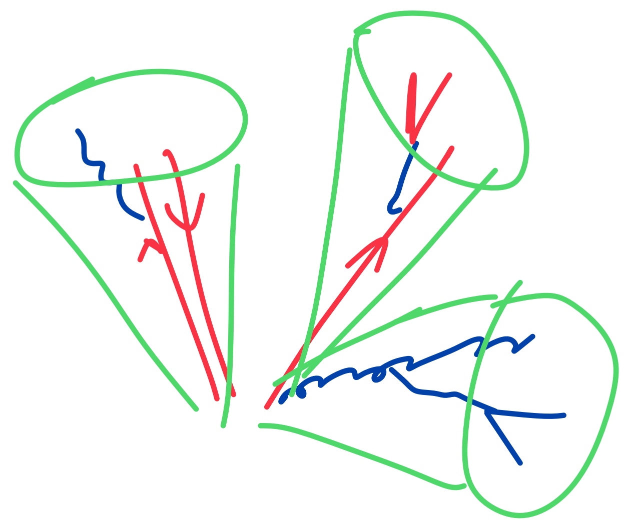
The problem: TH vs EXP have a quite different definition of ``flavour''

EXP: displaced vertices, hadron tagging…

TH: ``what is the net flavour of a jet?''

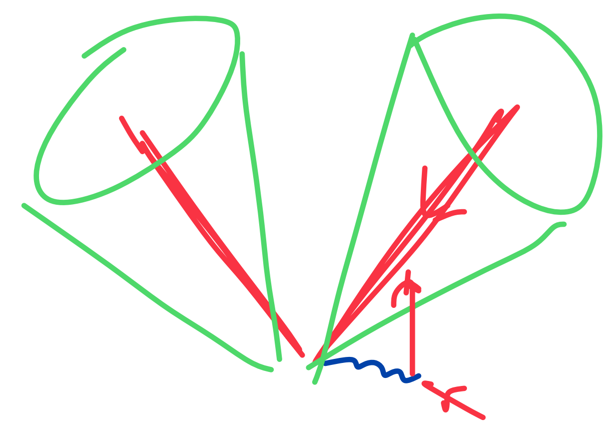


2 b-jets



light – b – light

$b\bar{b}$ must behave like a gluon [coll. safety]



b b

b light

IR unsafe

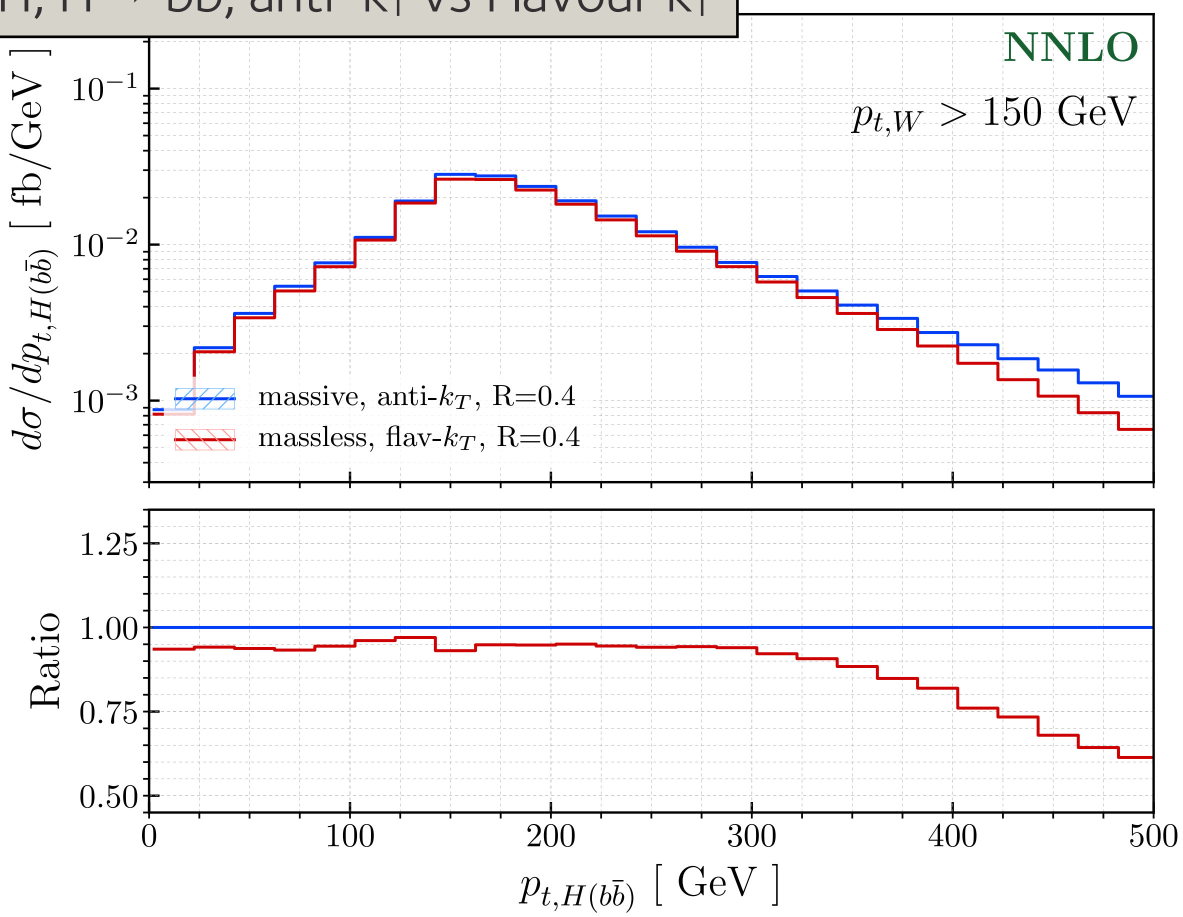
X+b/c: more and more prominent…

Solution: use a different jet algorithm, ``flavour k_T”

[Banfi, Salam, Zanderighi (2006)]

The problem: very different behaviour w.r.t. anti-k_T. Cannot compare with exp!

VH, H→ bb, anti-k_T vs Flavour k_T



[Behring, Bizon, FC, Melnikov, Röntsch (2020)]

Possible solutions:

- if process dominated by g→bb: let a shower take care of it [ask Maria & Fabio….]
- if g → bb is subdominant: massive calculation (possible at NNLO, but for simple processes….)
- complex scenarios? One would need a jet algo that is flavour safe + same behaviour of anti-k_T (work in progress…)

X+b/c: more and more prominent…

Solution: use a different jet algorithm, ``flavour k_T”

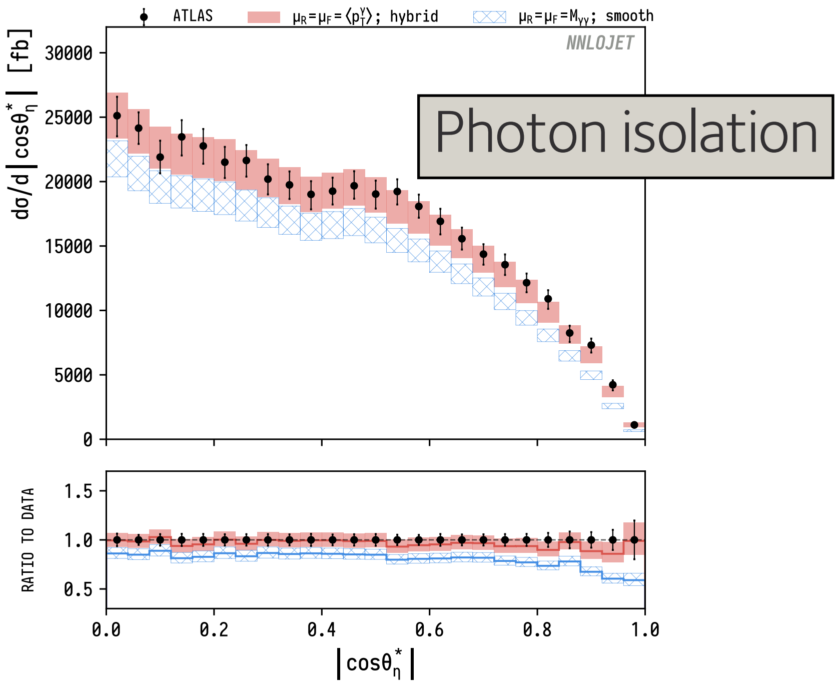
- To which extent this is an issue e.g. for W+c?
- How relevant is this for PDFs extraction?
- What are collaboration actually measuring (D-mesons, charm ``jet”, mixture)? How relevant are corrections? How do they massage the data?

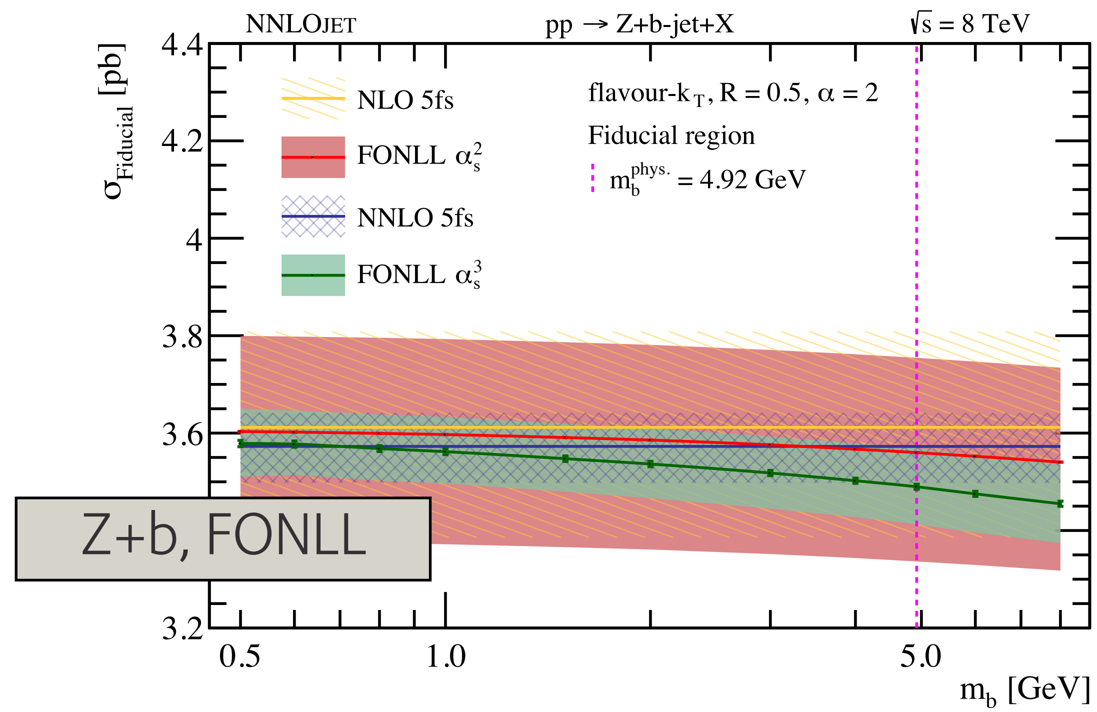
Back to NNLO:
2. Status and prospects

2→2 NNLO is well-understood

NNLO: from proof of concept to detailed phenomenology

[Gehrmann, Glover, Huss, Whitehead (2020)]

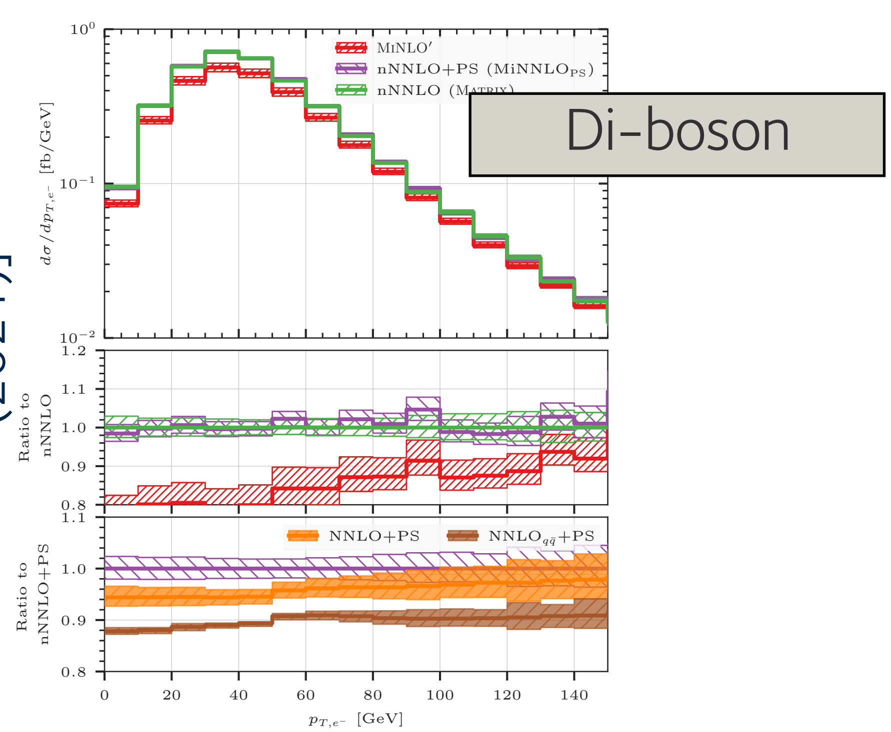


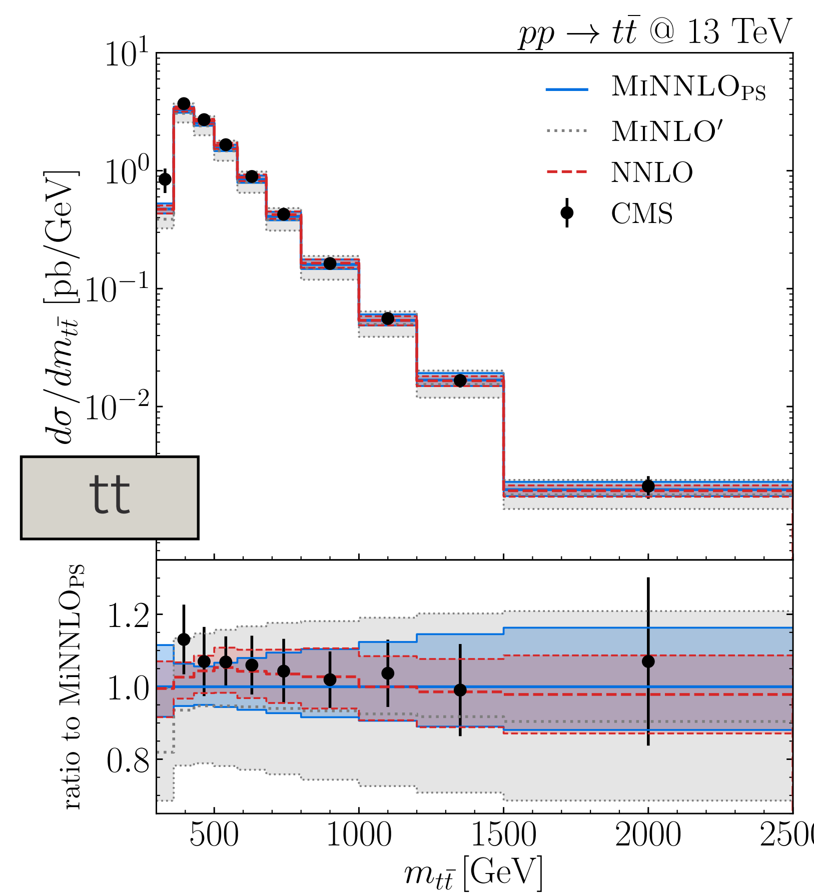


[Gauld, Gehrmann-de Ridder, Glover, Huss, Majer (2020)]

NNLLO + PS becoming a reality

[Buonocore, Koole, Lombardi, Rottoli, Wieseemann, Zanderighi (2021)]

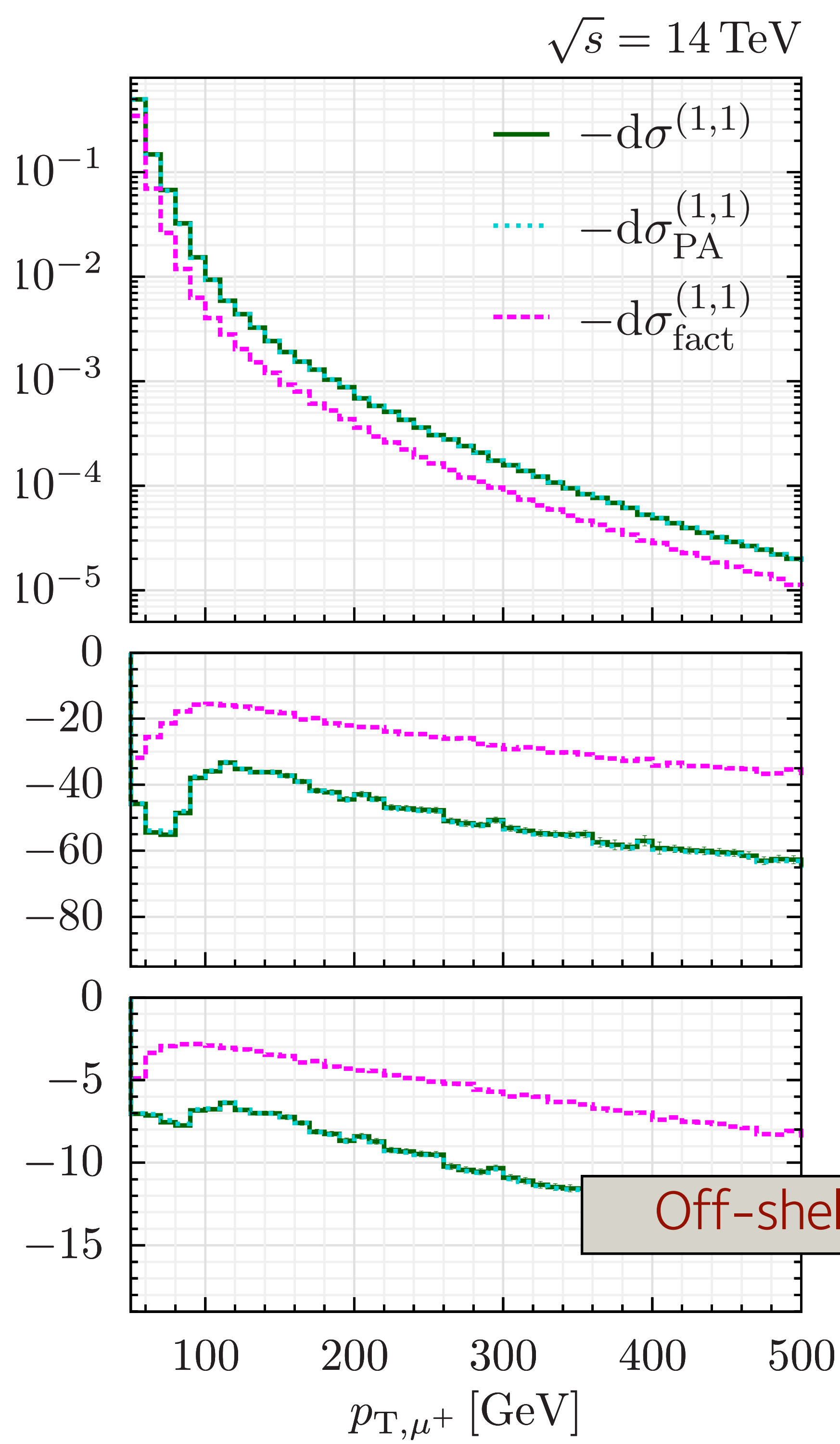




[Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi (2021)]

2→2 NNLO is well-understood

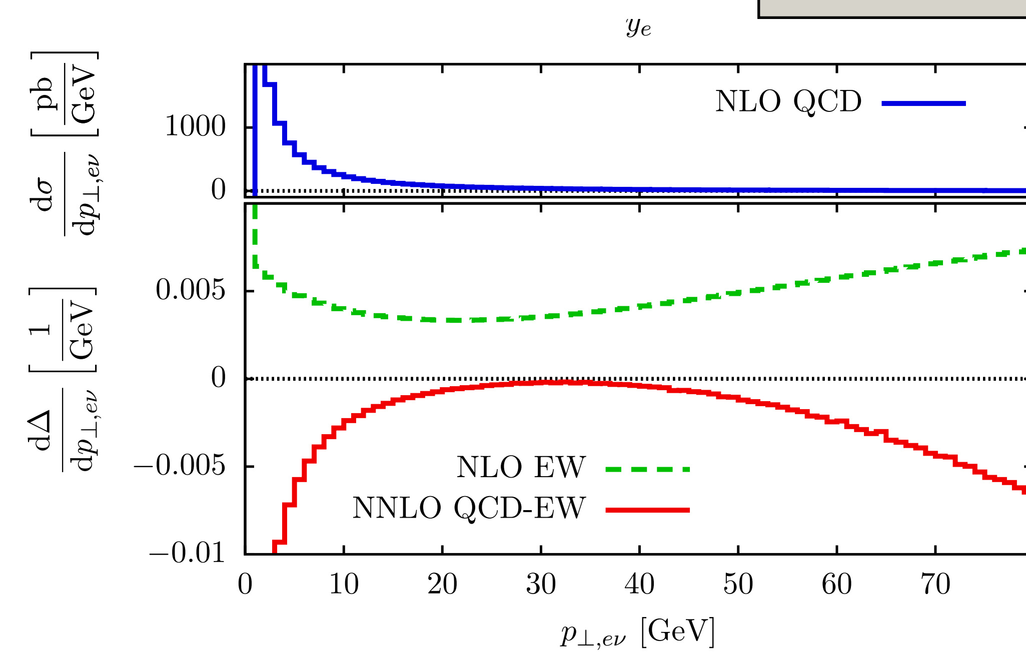
Recent development: NNLO QCD+EW



[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)]

Off-shell NC

On-shell CC



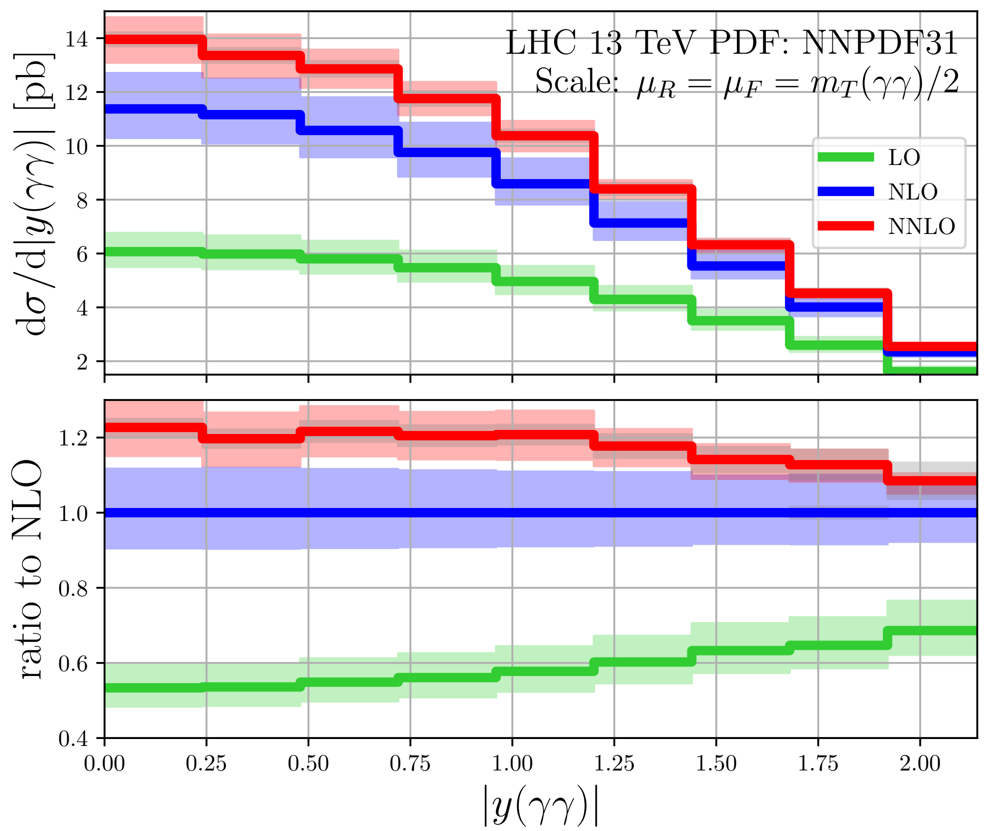
[Behring, Buccioni, FC, Delto, Jaquier, Melnikov, Röntsch (2020)]

- W-mass studies
- QCD-EW PDFs
- High-mass studies

2→3 NNLO is coming

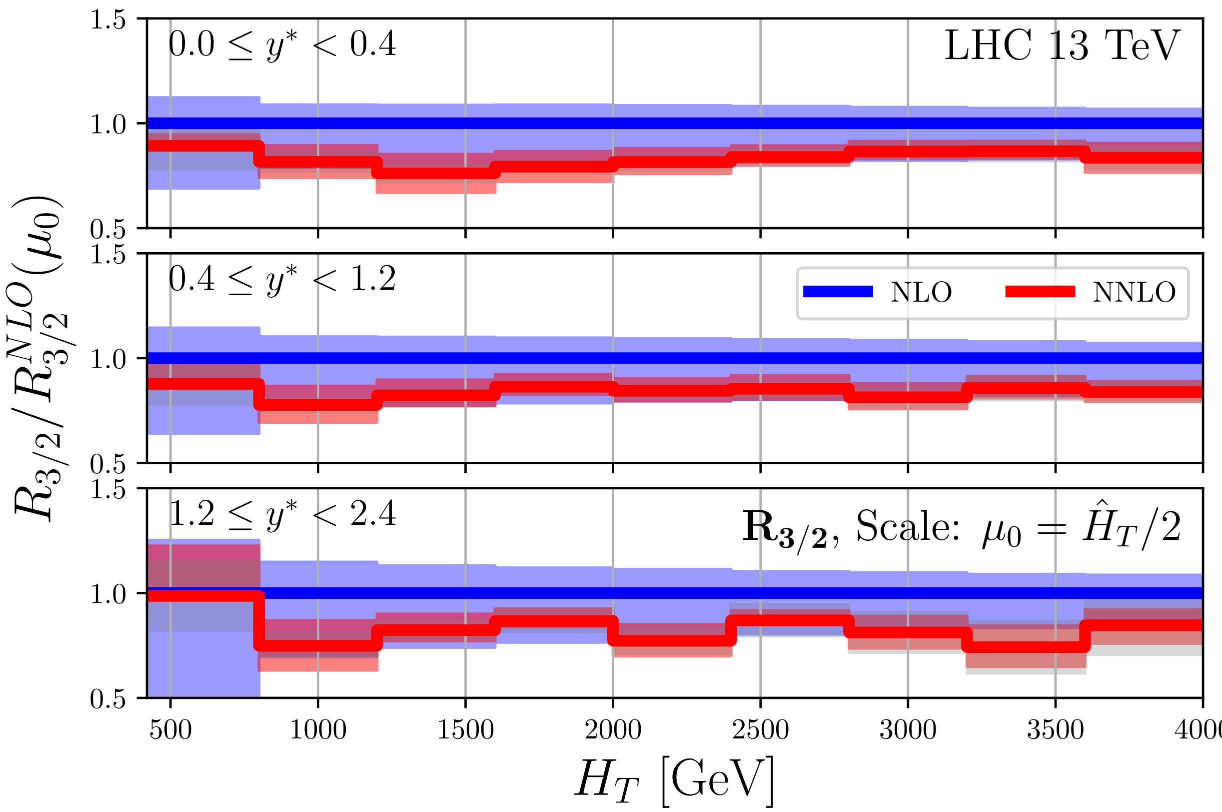
γγj

[Chawdhry, Czakon, Mitov, Poncelet (2021)]



jjj

[Czakon, Mitov, Poncelet (2021)]

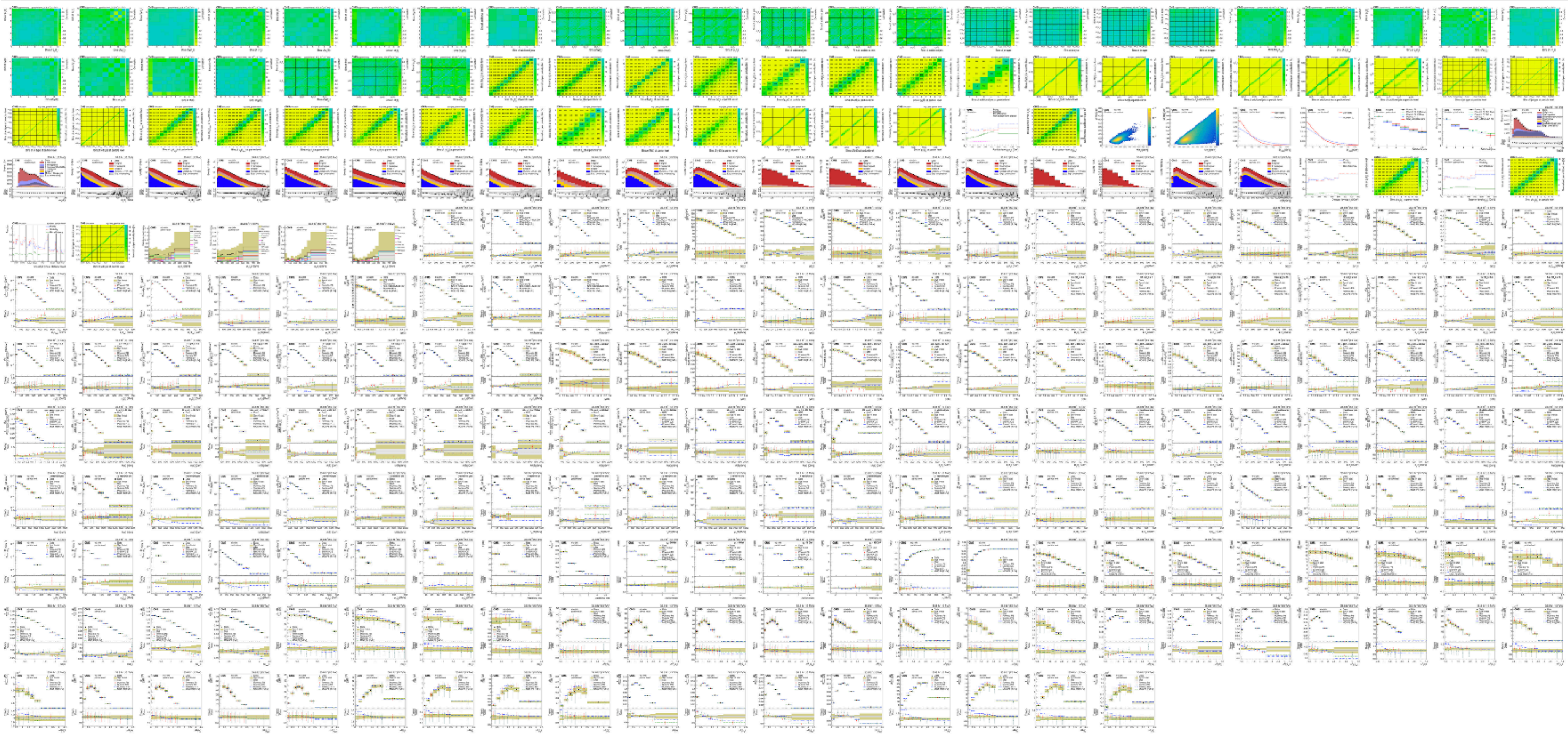


- jjj: ``Tour de force in QCD”.
- still very much in the exploratory phase
- much richer phenomenology → a lot to study / understand, beyond standard distributions
- 1/2L amplitudes are slow... efficient interpolation/learning of multi-dimensional functions?

Understanding complex events/ kinematics: tt at high scale

[FC, Dreyer, McDonald, Salam (2021)]

A lot of data is available



CMS 1803:08856 (semileptonic tt): 270 plots

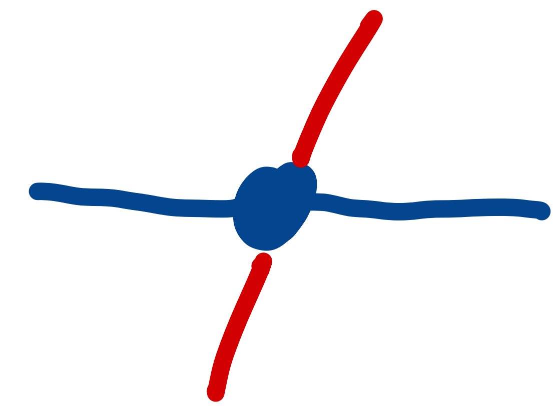
A lot of information, not always obvious

"Energetic" tops

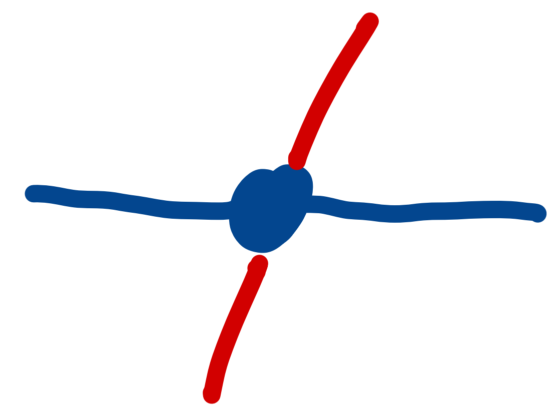
Hardness variable	explanation
$p_T^{\text{top,had}}$	transverse momentum of hadronic top candidate
$p_T^{\text{top,lep}}$	transverse momentum of leptonic top candidate
$p_T^{\text{top,max}}$	p_T of the top (anti-)quark with larger $m_T^2 = p_T^2 + m^2$
$p_T^{\text{top,min}}$	p_T of the top (anti-)quark with smaller $m_T^2 = p_T^2 + m^2$
$p_T^{\text{top,avg}}$	$\frac{1}{2}(p_T^{\text{top,had}} + p_T^{\text{top,lep}})$
$\frac{1}{2}H_T^{t\bar{t}}$	with $H_T^{t\bar{t}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}}$
$\frac{1}{2}H_T^{t\bar{t}+\text{jets}}$	with $H_T^{t\bar{t}+\text{jets}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}} + \sum_i p_T^{j_{\not{t}},i}$
$m_T^{J,\text{avg}}$	average m_T of the two highest m_T large-R jets (J_1, J_2)
$\frac{1}{2}m^{t\bar{t}}$	half invariant mass of $p^{t\bar{t}} = p^{\text{top,had}} + p^{\text{top,lep}}$
$p_T^{t\bar{t}}$	transverse component of $p^{t\bar{t}}$
$p_T^{j_{\not{t}},1}$	transverse momentum of the leading small-R non-top jet

"Energetic" tops

Hardness variable	explanation
$p_T^{\text{top,had}}$ $p_T^{\text{top,lep}}$ $p_T^{\text{top,max}}$ $p_T^{\text{top,min}}$ $p_T^{\text{top,avg}}$	Identical at LO
$\frac{1}{2}H_T^{t\bar{t}}$ $\frac{1}{2}H_T^{t\bar{t}+\text{jets}}$ $m_T^{J,\text{avg}}$	with $H_T^{t\bar{t}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}}$ with $H_T^{t\bar{t}+\text{jets}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}} + \sum_i p_T^{j_{\not t},i}$ average m_T of the two highest m_T large-R jets (J_1, J_2)
$\frac{1}{2}m^{t\bar{t}}$	half invariant mass of $p^{t\bar{t}} = p^{\text{top,had}} + p^{\text{top,lep}}$
$p_T^{t\bar{t}}$ $p_T^{j_{\not t},1}$	transverse component of $p^{t\bar{t}}$ transverse momentum of the leading small-R non-top jet

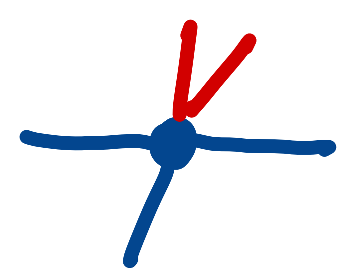


"Energetic" tops

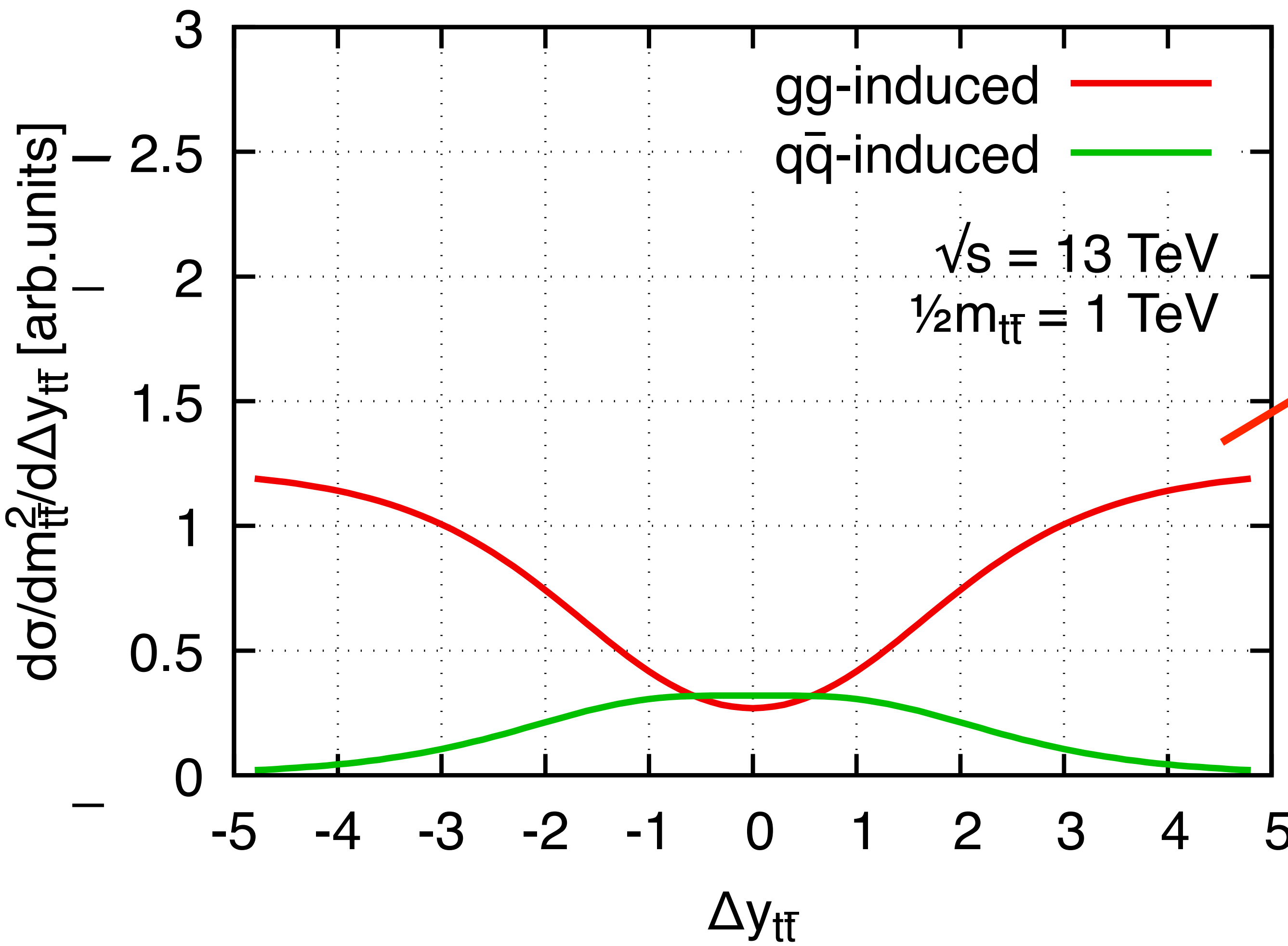
Hardness variable	explanation
$p_T^{\text{top,had}}$ $p_T^{\text{top,lep}}$ $p_T^{\text{top,max}}$ $p_T^{\text{top,min}}$ $p_T^{\text{top,avg}}$	Identical at LO & high p_t 
$\frac{1}{2}H_T^{t\bar{t}}$	with $H_T^{t\bar{t}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}}$
$\frac{1}{2}H_T^{t\bar{t}+\text{jets}}$	with $H_T^{t\bar{t}+\text{jets}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}} + \sum_i p_T^{j_{\not t},i}$
$m_T^{J,\text{avg}}$	average m_T of the two highest m_T large-R jets (J_1, J_2)
$\frac{1}{2}m^{t\bar{t}}$	half invariant mass of $p^{t\bar{t}} = p^{\text{top,had}} + p^{\text{top,lep}}$
$p_T^{t\bar{t}}$	transverse component of $p^{t\bar{t}}$
$p_T^{j_{\not t},1}$	transverse momentum of the leading small-R non-top jet

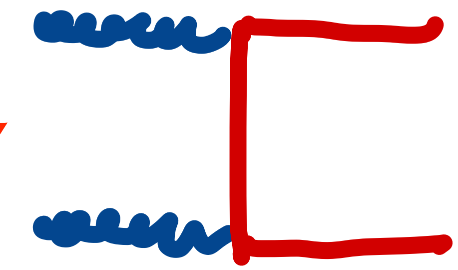
“Energetic” tops

Hardness variable	explanation
$p_T^{\text{top,had}}$	transverse momentum of hadronic top candidate
$p_T^{\text{top,lep}}$	transverse momentum of leptonic top candidate
$p_T^{\text{top,max}}$	p_T of the top (anti-)quark with larger $m_T^2 = p_T^2 + m^2$
$p_T^{\text{top,min}}$	p_T of the top (anti-)quark with smaller $m_T^2 = p_T^2 + m^2$
$p_T^{\text{top,avg}}$	$\frac{1}{2}(p_T^{\text{top,had}} + p_T^{\text{top,lep}})$
$\frac{1}{2}H_T^{t\bar{t}}$	with $H_T^{t\bar{t}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}}$
$\frac{1}{2}H_T^{t\bar{t}+\text{jets}}$	with $H_T^{t\bar{t}+\text{jets}} = m_T^{\text{top,had}} + m_T^{\text{top,lep}} + \sum_i p_T^{j_{\not{t}},i}$
$m_T^{J,\text{avg}}$	average m_T of the two highest m_T large-R jets (J_1, J_2)
$\frac{1}{2}m^{t\bar{t}}$	half invariant mass of $p^{t\bar{t}} = p^{\text{top,had}} + p^{\text{top,lep}}$
$p_T^{t\bar{t}}$ $p_T^{j_{\not{t}},1}$	α_s suppressed, starts at NLO



"Energetic" tops





$$\Delta y_{t\bar{t}}^{\max} \approx 2 \ln m_{t\bar{t}}/m_{\rm top}$$

$$\frac{d\sigma}{dm_{t\bar{t}}^2} = \frac{\alpha_s^2 \pi}{m_{t\bar{t}}^4} \left[\left(\frac{1}{3} \ln \frac{m_{t\bar{t}}^2}{m_t^2} - \frac{7}{12} \right) \mathcal{L}_{gg} + \frac{8}{27} \mathcal{L}_{q\bar{q}} \right]$$

$\frac{1}{2} m^{t\bar{t}}$	Very delicate observable at high scales

- Logarithmic enhancement (theoretically delicate beyond LO)
- Contributions from large-y, low-p_t tops (issue for boosted reco...)
- Plus: gluon/quark separation → good handle for PDF studies?

"Energetic" tops: expectations vs reality

$$\frac{d\sigma}{dV}$$

$m_{tt}/2$

$p_{t,top}$

$p_{t,jet}$

"Energetic" tops: expectations vs reality

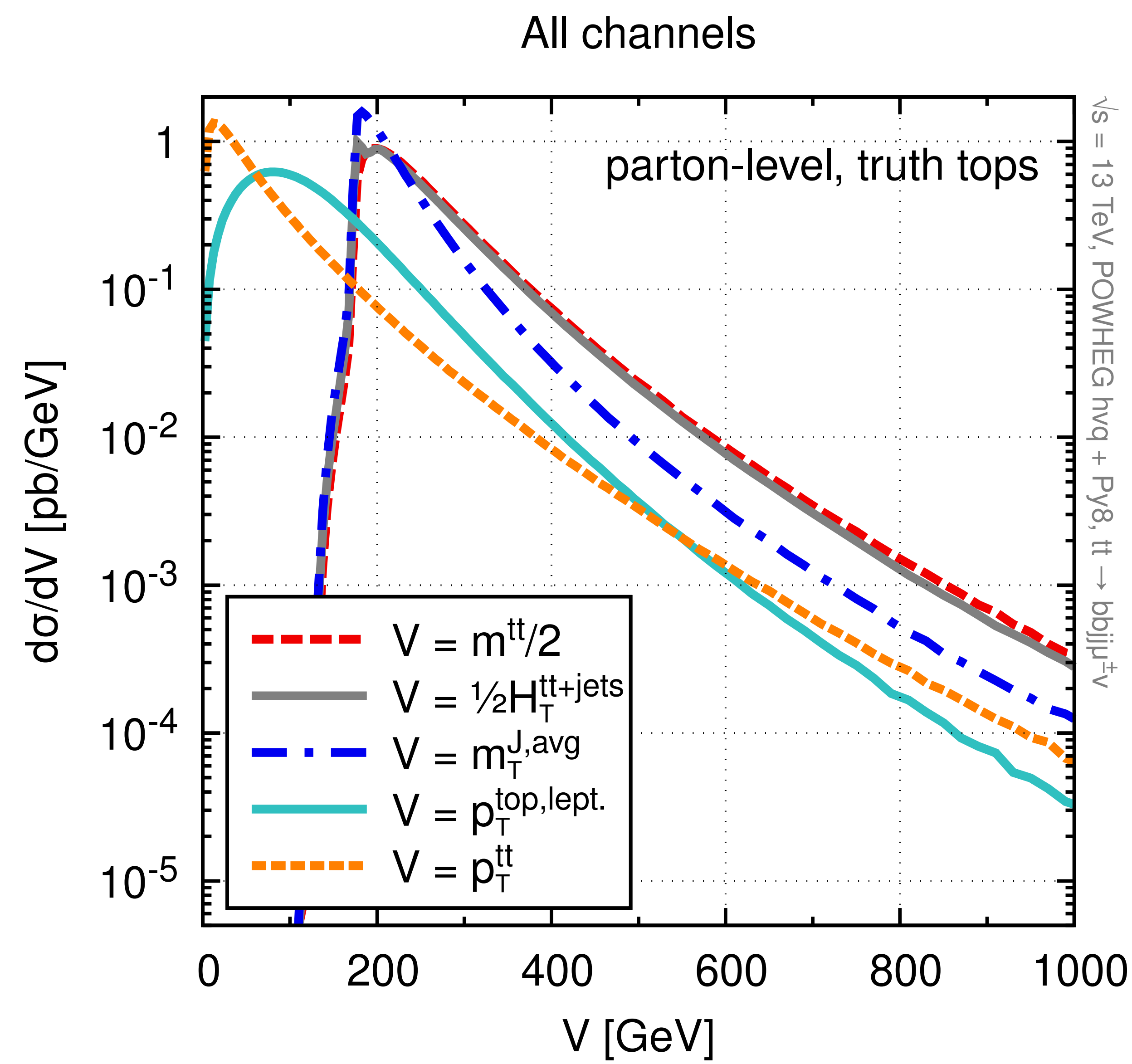
$$\frac{d\sigma}{dV}$$

$m_{tt}/2$

$p_{t,top}$

$p_{t,jet}$

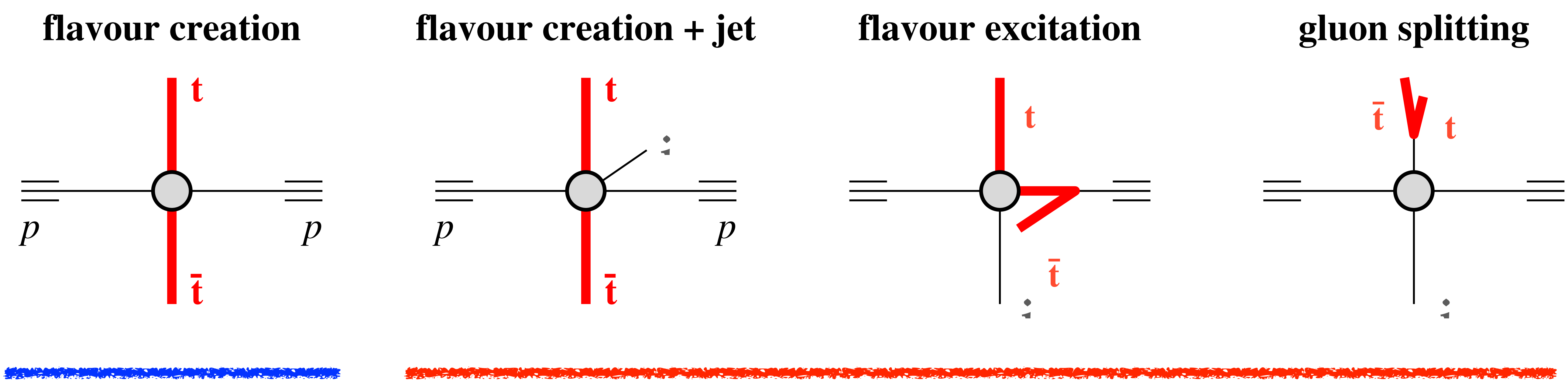
vs



[POWHEG NLO + Pythia8, see backup for full setup]

"LO" expectations do not borne out

Understanding energetic tops: 1-topologies



$\mathcal{O}(\alpha_s^2)$

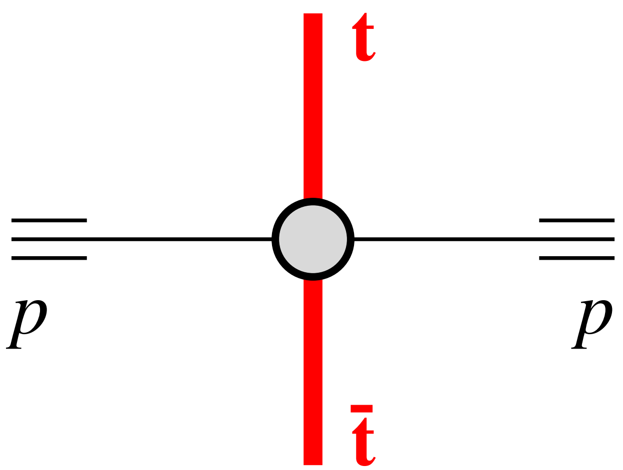
$\mathcal{O}(\alpha_s^3)$

- ``NLO"-topologies suppressed by $\alpha_s(1\ \text{TeV}) \sim 0.09$
- $\ln(p_t/m_t) \sim 2$, not large enough to compensate for α_s
- However…

Very different underlying 2→2 scattering

Consider high-p_t 2→2 scattering, i.e. p_t = 1TeV, θ=π/2

flavour creation

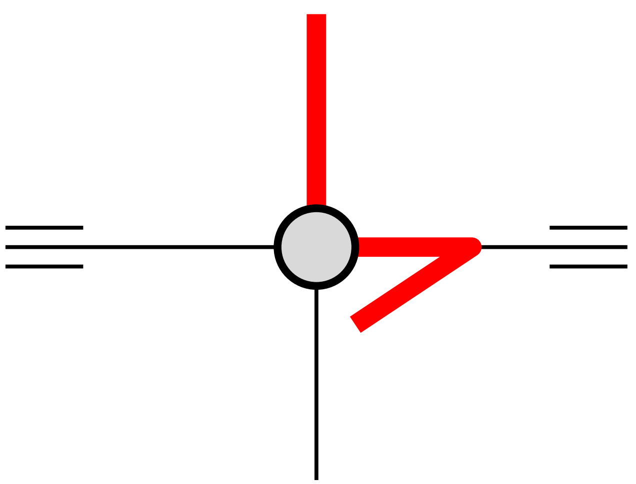


$$\sum_i \mathscr{L}_{q_i\bar{q}_i} \simeq 0.13$$

$$\times |\mathscr{M}_{q\bar{q}\to t\bar{t}}|^2 = g_s^4 \frac{C_F}{N_C} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} = g_s^4 \frac{C_F}{N_C} \cdot \frac{1}{2}$$

$$\simeq g_s^4 \cdot 0.028$$

flavour excitation



$$\mathscr{L}_{\Sigma t} + \mathscr{L}_{\Sigma \bar{t}} \simeq 0.0170 \quad \left[\Sigma \equiv \sum_i (q_i + \bar{q}_i)\right]$$

$$\times |\mathscr{M}_{qt\to qt}|^2 = g_s^4 \frac{C_F}{N_C} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} = g_s^4 \frac{C_F}{N_C} \cdot 5$$

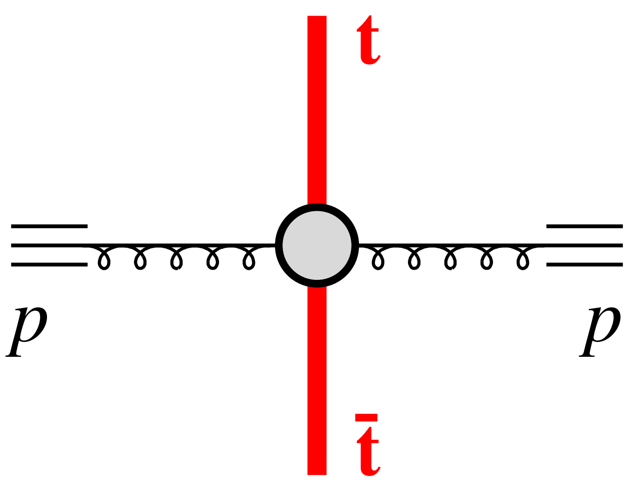
$$\simeq g_s^4 \cdot 0.038$$

Comparable results, t-channel exchange compensates for α_s

Very different underlying 2→2 scattering

Consider high-p_t 2→2 scattering, i.e. p_t = 1TeV, θ=π/2

flavour creation

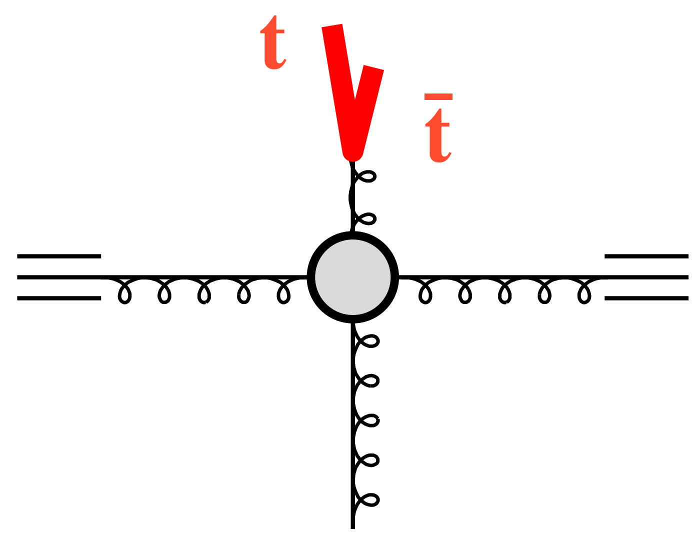


$$\mathscr{L}_{gg} \simeq 0.16$$

$$\times |\mathscr{M}_{gg\to t\bar{t}}|^2 = g_s^4 \cdot \mathbf{0.15}$$

$$\simeq g_s^4 \cdot 0.024$$

gluon splitting



$$\mathscr{L}_{gg} \simeq 0.16$$

$$\times |\mathscr{M}_{gg\to gg}|^2 = g_s^4 \cdot \mathbf{30.4}$$

$$\times \mathscr{P}_{g\to t\bar{t}} \simeq \mathbf{0.004}$$

$$\simeq g_s^4 \cdot 0.020$$

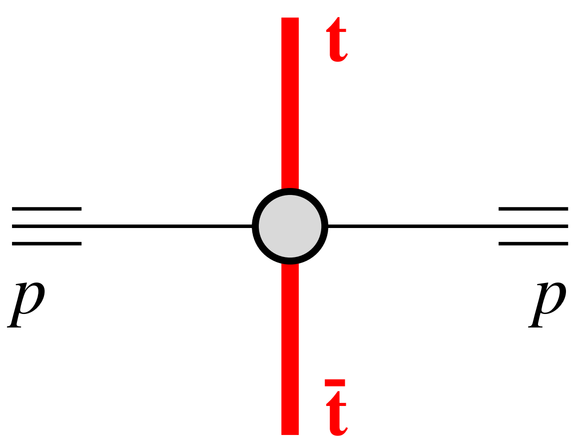
Again, ME enhancement compensates for α_s

Very different "hard" scale

$2 \rightarrow 2$ cross section decreases very fast, $\sigma(p_t^{2\rightarrow 2} > X) \sim 1/X^7$

Example: $p_t^{top,\,min}$ If $p_t^{top,\,min} = 1$ TeV, then

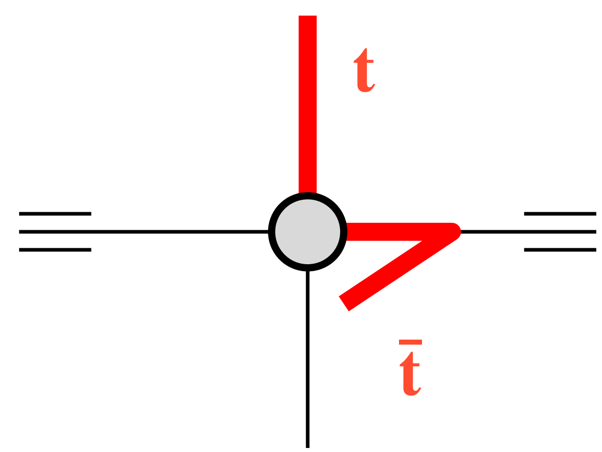
flavour creation



$p_t^{2\rightarrow 2} = 1$ TeV

✔

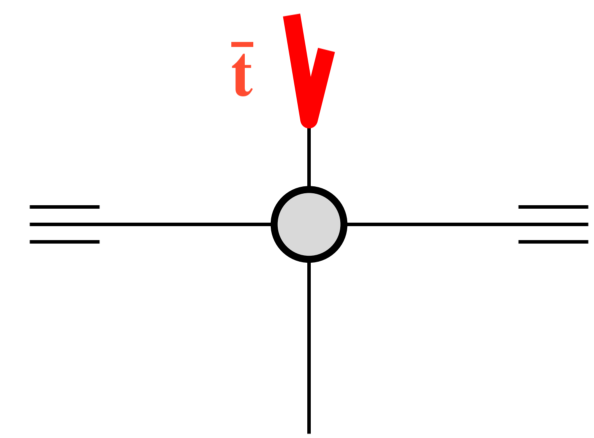
flavour excitation



$p_t^{2\rightarrow 2} \gtrsim 2$ TeV

Suppressed by $(1/2)^7$

gluon splitting



$p_t^{2\rightarrow 2} \gtrsim 2$ TeV

Suppressed by $(1/2)^7$

Take this info into account, separate topologies

Algorithm 2 Event analysis algorithm at hadron (particle) level

Require: at least one lepton (we require it to have a transverse momentum of at least 25 GeV), missing transverse momentum and hadrons.

1: Cluster the hadronic part of the event with the anti-k_t algorithm with $R = 0.4$ and discard any jets below some p_t threshold, $p_{T,\min}$, as one would normally (we take $p_{T,\min} = 30$ GeV).

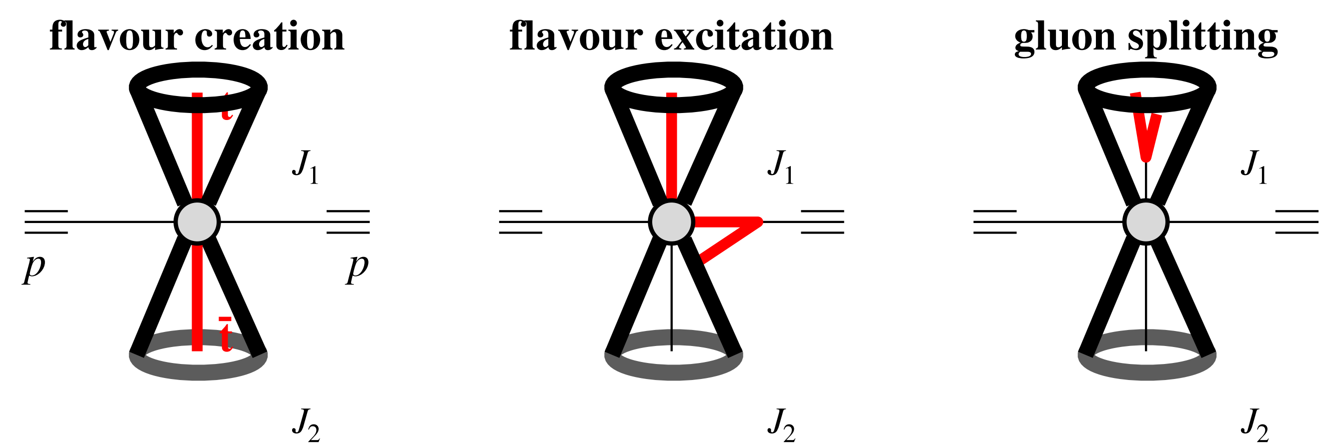
2: Optionally, e.g. if subject to finite detector acceptance, exclude jets and leptons with an absolute rapidity beyond some $y_{\max}$. The remaining set of jets is referred to as $\{j\}$ and the hadrons contained within that set of jets is $\{H\}$.

3: For each jet j, recluster its constituents with the exclusive longitudinally invariant ($R = 1$) k_t algorithm [61] with a suitable $d_{\rm cut}$ (we use $(20\ \text{GeV})^2$), thus mapping the $R = 0.4$ jets $\{j\}$ to a declustered set $\{j_d\}$. One applies b-tagging to the $\{j_d\}$ (sub)jets to aid with the subsequent top identification.

4: Use a resolved top-tagging approach to identify the hadronic and leptonic top-quark candidates from the lepton(s) and from the jets $\{j_d\}$ obtained in step 3. Here, we will adopt the algorithm outlined in Section 4.2.

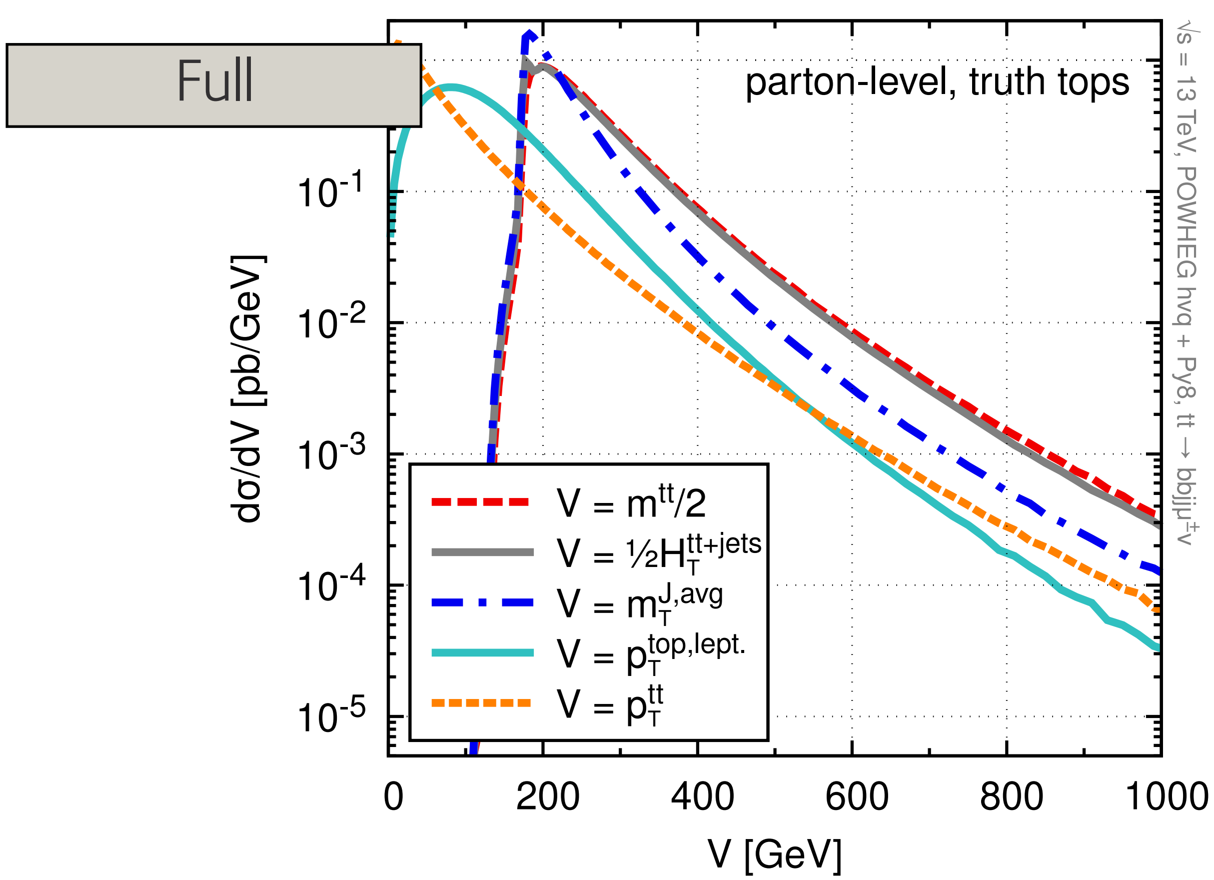
5: Identify all particles from the set $\{H\}$ that do not belong to either of the top-quark candidates. Refer to this subset as $\{H_{\not t}\}$. Cluster the $\{H_{\not t}\}$ with the original jet definition (anti-k_t, $R = 0.4$) and apply a transverse momentum threshold $p_{T,\min}$ to obtain the set of non-top $R = 0.4$ jets, $\{j_{\not t}\}$, ordered in decreasing p_T.

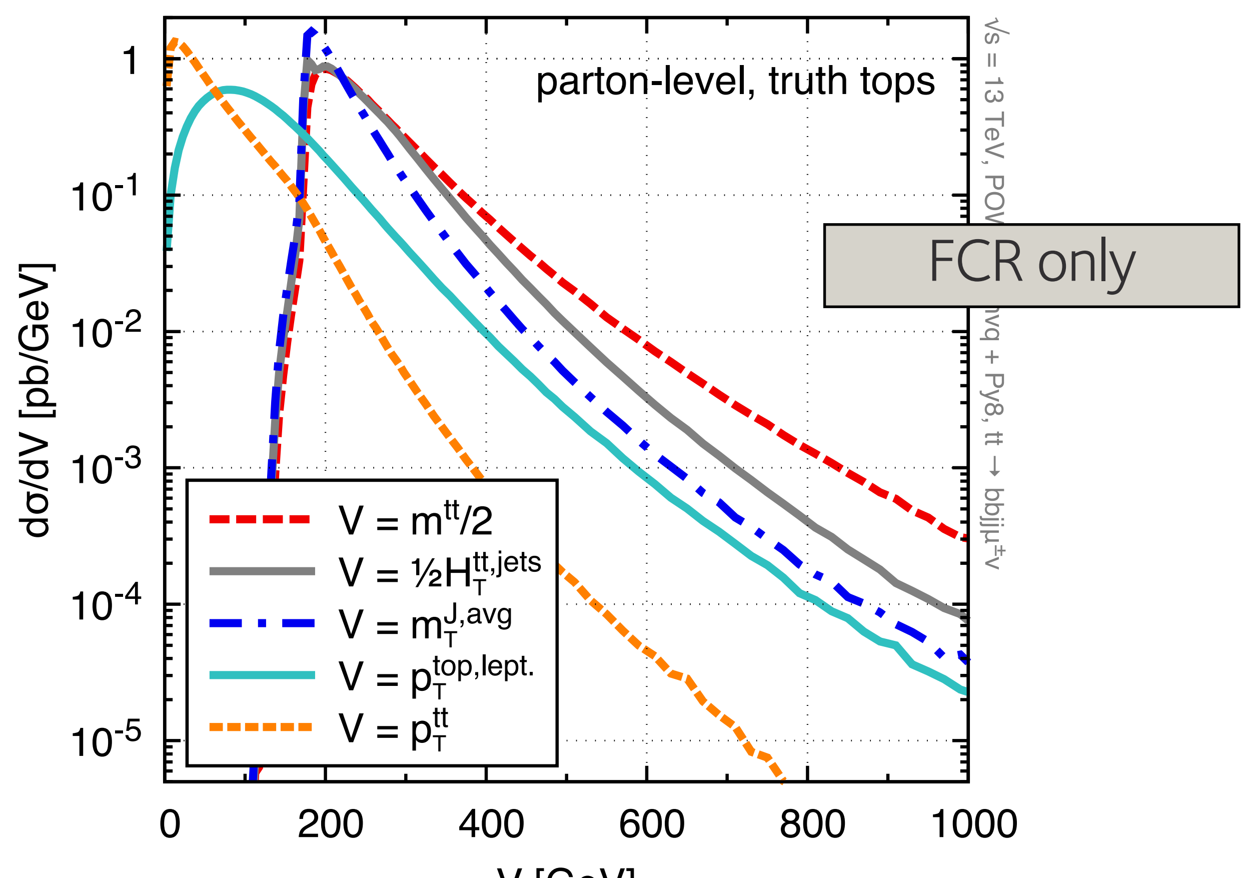
6: Apply step 3 of Algorithm 1 using $\{j_{\not t}\}$ and the reconstructed top and anti-top candidates as the inputs.



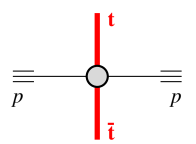
Relatively easy to separate these contribution, in a safe and practical way

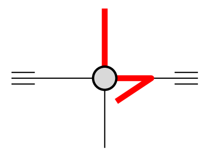
``Perturbative" expectations recovered

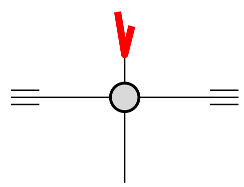




Why is this useful?







topology	channel	$\lvert\mathrm{ME}\rvert^2$	luminosity	FS splitting	product
FCR	$gg \to t\bar{t}$	0.15	0.16	1	0.024
	$q_i\bar{q}_i \to t\bar{t}$	0.22	0.13	1	0.028
FEX	$tg \to tg$	6.11	0.0039	1	0.024
	$t\Sigma \to t\Sigma$	2.22	0.0170	1	0.038
GSP	$gg \to gg(\to t\bar{t})$	30.4	0.16	$\mathcal{P}_{g\to t\bar{t}} \simeq 0.004$	0.020
	$g\Sigma \to g(\to t\bar{t})\Sigma$	6.11	1.22	$\mathcal{P}_{g\to t\bar{t}} \simeq 0.004$	0.031
	$q\bar{q} \to gg(\to t\bar{t})$	1.04	0.13	$\mathcal{P}_{g\to t\bar{t}} \simeq 0.004$	0.001

- One process really contains multiple, different information → non-trivial to extract
- Each topology has different features → sensitivity to different EFTs operators/kinematics regions
- For PDFs: FEX involves g→tt IS splitting, higher-x than processes with similar p_t and ``safer"
- Understanding variables crucial when TH is incomplete (e.g.: $m_{jj,avg}$ largely insensitive to FONLL logs

<u>As we collect more data and get access to more ``exotic" regions, perhaps these kinds of analyis will become quite useful</u>

(otherwise: risk of endless discussion on ``large K-factors", "outside the scale band", ``α_s at the TeV scale" on events with hard scale of ~200 GeV. ...)

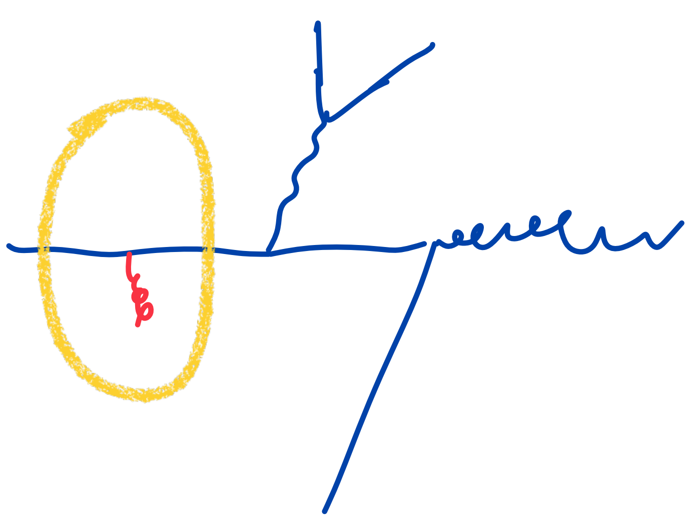
Is perturbation theory enough?

Beyond pQCD

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\, \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J\, \left(1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)\right)$$

Everything we discuss is valid only provided that we can neglect $(\Lambda_{QCD}/Q)^p$ terms. At the percent level, this may not be the case if p=1 contributes

- For DIS: solid proof that $p \gtrsim 2$
- For inclusive quantities (e.g. DY total xsec): leading NP corrections have p=2 (non-trivial!)
- For more exclusive quantities: potential sources of linear power corrections.
- Top, Jets are known to have linear power corrections. What about color singlet?



Asymmetric color configuration: linear dependence on small gluon ``kick"

Vanishes upon azimuthal integration → not affecting the total xsec

Beyond pQCD

The obvious problem: at colliders, we cannot deal with QCD non-perturbatively

However: we know one source of NP that ``creeps” into perturbative results.

When integrating over soft momenta → Landau pole ambiguity

Must cancel against NP corrections → use it as an estimate of the latter (it turns out that many other sources of NP corrections are suppressed, e.g. instantons)

To probe Landau pole: give the gluon a small mass (tricky…) and use it as an IR probe

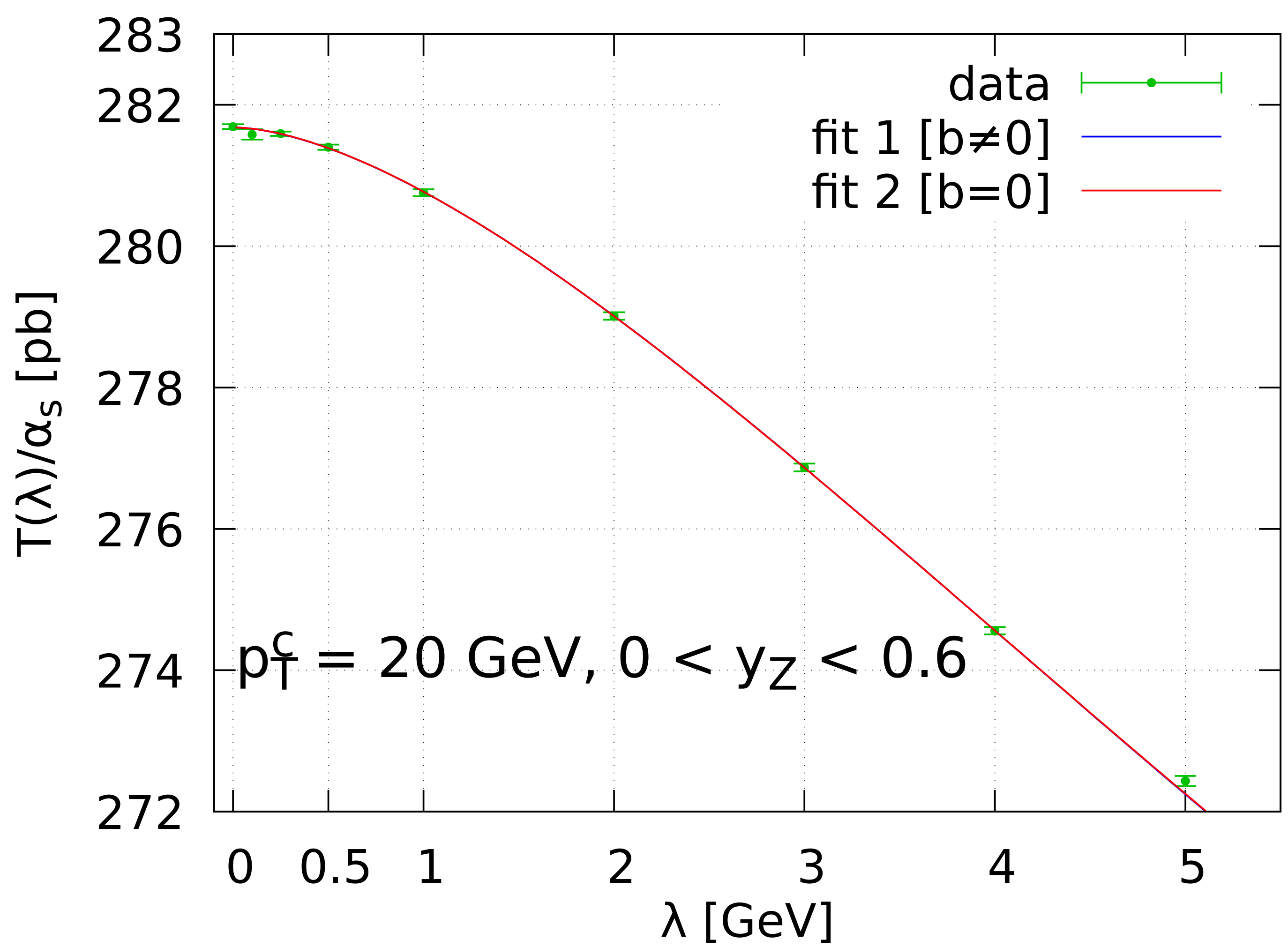
This can be made precise, and it has a solid QFT foundation.

NP ↔ non-analytic terms in m_g^2. ``Large $n_f$ approximation”, ``IR Renormalons”

Caveat: cannot deal with processes involving gluons at Born level.
Results I’ll show have some hidden assumptions…

Z p_t and linear renormalons

[Ferrario Ravasio, Limatola, Nason (2020)]: Numerical study based on renormalon calculus



Compute Z p_t with massive gluons, and extrapolate to $m_g \to 0$

$$f(\lambda) = a\left[1 + b\left(\frac{\lambda}{p_T^c}\right) + c\left(\frac{\lambda}{p_T^c}\right)^2 \log^2\left(\frac{\lambda}{p_T^c}\right) + d\left(\frac{\lambda}{p_T^c}\right)^2 \log\left(\frac{\lambda}{p_T^c}\right)\right]$$

Fit consistent with b=0 → no linear power corrections

Z p_t and linear renormalons

Can we understand and generalise this result?

[FC, Ferrario Ravasio, Limatola, Melnkov, Nason (2021)]

With some caveats:

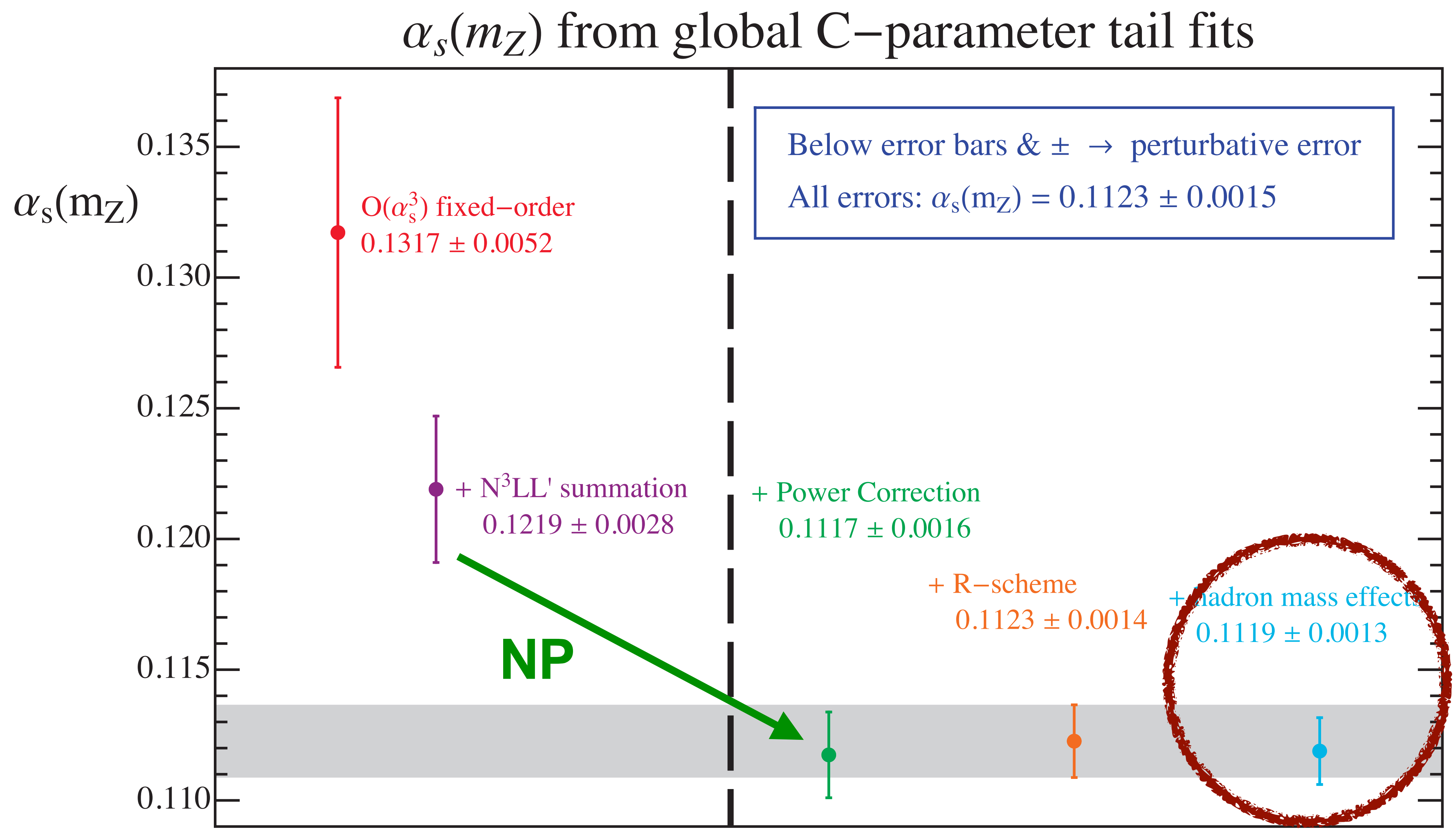
- it is remarkably complicated for QCD to generate linear power corrections
- only IR regions contribute (obvious)
- virtual corrections: only HQ mass renormalization
- collinear region: always quadratic
- soft region can lead to linear power corrections. Need next-to-eikonal analysis

Two immediate results

- no linear power corrections for ``inclusive'' enough color singlet distributions (total xsec, rapidity distribution, p_t distribution)
- relatively easy to introduce linear PC from observable definition… In several cases, easy to compute linear correction…

Example: the α_s saga

The strong coupling can be determined from fits to e^+e^- event shapes



[Hoang, Kolodrubetz, Mateu, Stewart (2015)]

Long-standing issue of ``weirdly low" value

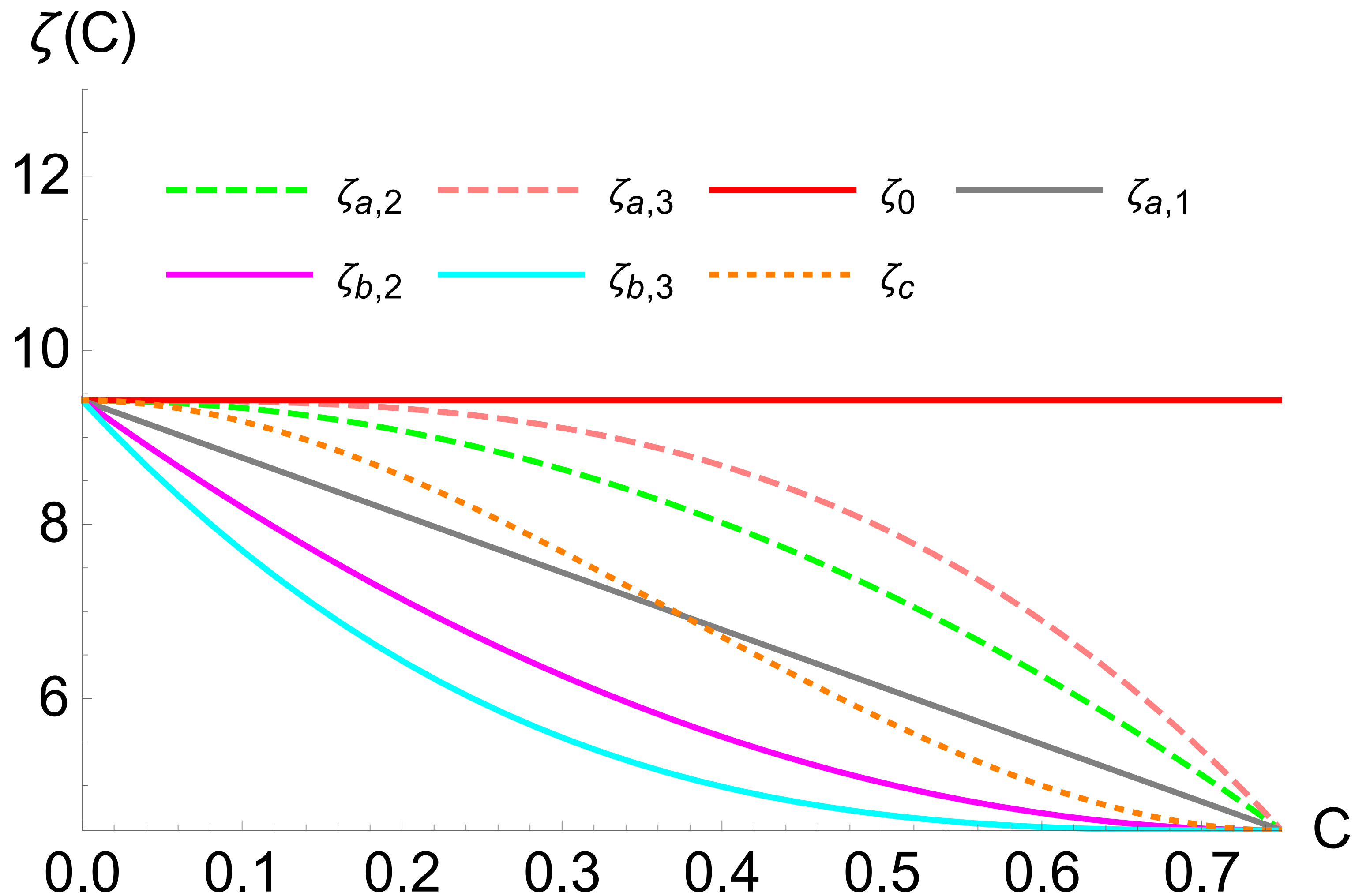
NP corrections important, and included with some assumption

Example: the α_s saga

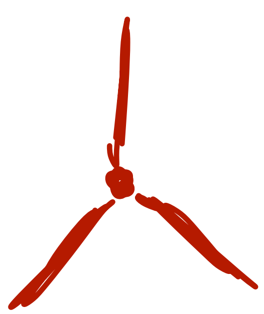
Are the assumptions justified? [Luisoni, Monni, Salam (2020)]

- C=0: degenerate configuration, easy to compute NP. Standard approach: extrapolate them to all values of C
- But also C=3/4 is degenerate → also here easy to compute NP. Different result! [Luisoni, Monni, Salam (2020)]

C=0



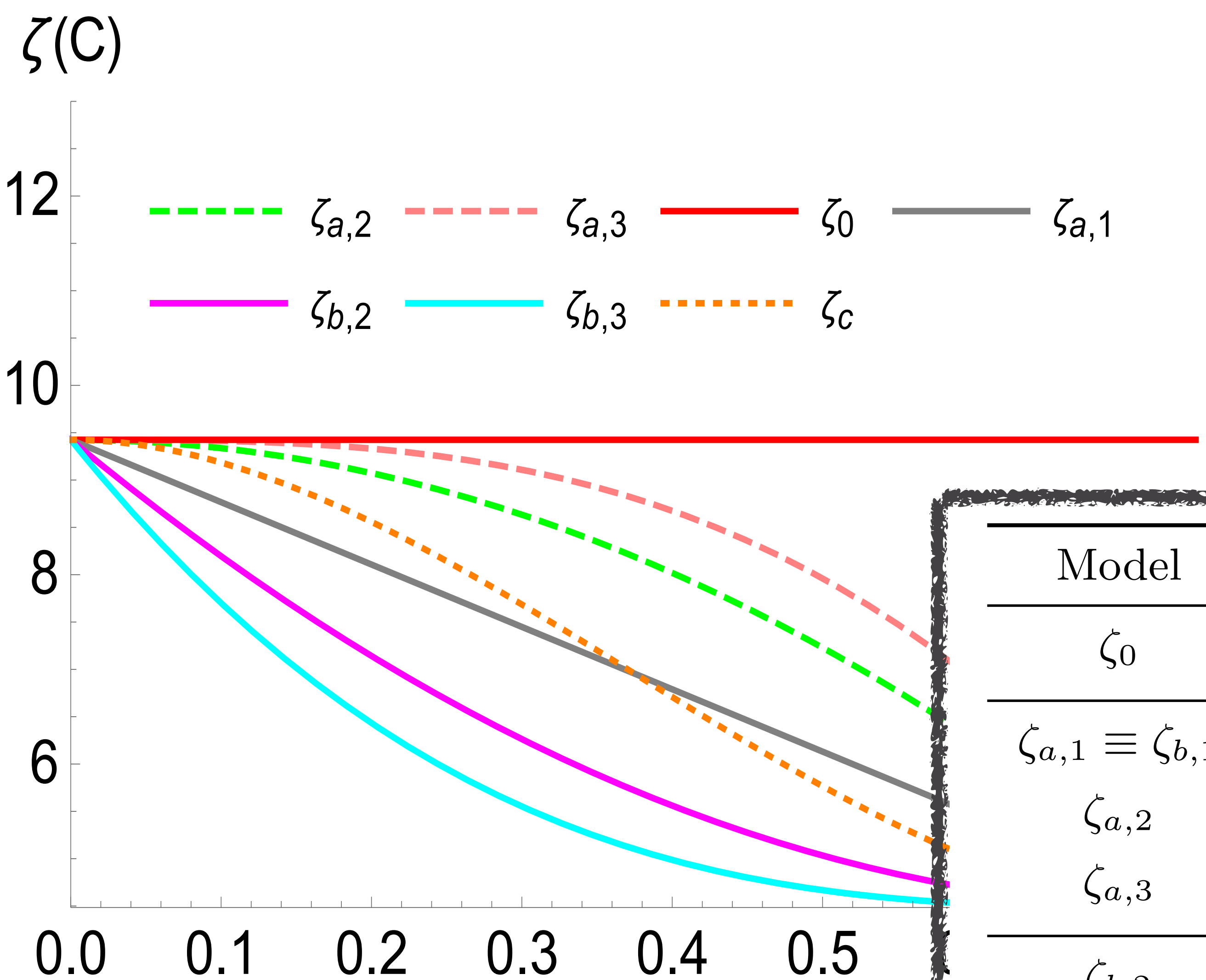
C=3/4



Example: the α_s saga

Are the assumptions justified? [Luisoni, Monni, Salam (2020)]

LMS approach: we know NP at two points. Interpolate between them and see



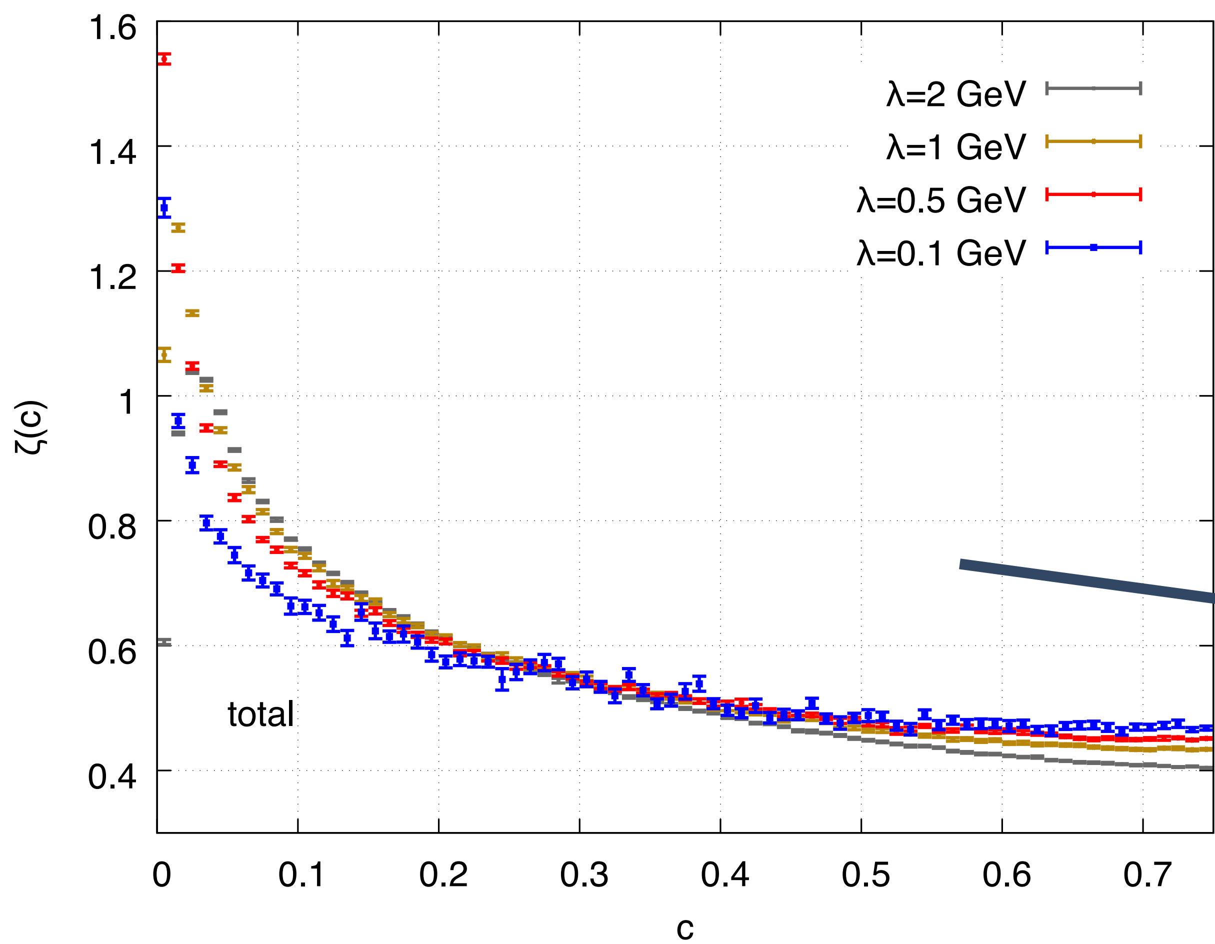
Wide spread of results...

Model	$\alpha_s(M_Z^2)$	$\alpha_0(\mu_I^2)$	χ^2/d.o.f.
ζ_0	$0.1121 \pm 0.0006^{+0.0023}_{-0.0014}$	$0.53 \pm 0.01^{+0.07}_{-0.04}$	1.076
$\zeta_{a,1} \equiv \zeta_{b,1}$	$0.1142 \pm 0.0005^{+0.0026}_{-0.0015}$	$0.52 \pm 0.01^{+0.06}_{-0.04}$	1.045
$\zeta_{a,2}$	$0.1121 \pm 0.0006^{+0.0024}_{-0.0015}$	$0.52 \pm 0.01^{+0.07}_{-0.04}$	1.033
$\zeta_{a,3}$	$0.1099 \pm 0.0007^{+0.0022}_{-0.0014}$	$0.54 \pm 0.01^{+0.07}_{-0.05}$	1.116
$\zeta_{b,2}$	$0.1163 \pm 0.0005^{+0.0028}_{-0.0017}$	$0.51 \pm 0.01^{+0.06}_{-0.04}$	1.079
$\zeta_{b,3}$	$0.1167 \pm 0.0004^{+0.0028}_{-0.0018}$	$0.53 \pm 0.01^{+0.06}_{-0.04}$	1.143
ζ_c	$0.1156 \pm 0.0005^{+0.0027}_{-0.0016}$	$0.48 \pm 0.01^{+0.05}_{-0.03}$	1.074

Example: the α_s saga

Using our results, we can compute NP corrections for arbitrary C
(with some caveats)

[FC, Ferrario Ravasio, Limatola, Melnkov, Nason (2021)]



Model	$\alpha_s(M_Z^2)$
ζ_0	$0.1121 \pm 0.0006^{+0.0023}_{-0.0014}$
$\zeta_{a,1} \equiv \zeta_{b,1}$	$0.1142 \pm 0.0005^{+0.0026}_{-0.0015}$
$\zeta_{a,2}$	$0.1121 \pm 0.0006^{+0.0024}_{-0.0015}$
$\zeta_{a,3}$	$0.1099 \pm 0.0007^{+0.0022}_{-0.0014}$
$\zeta_{b,2}$	$0.1163 \pm 0.0005^{+0.0028}_{-0.0017}$
$\zeta_{b,3}$	$0.1167 \pm 0.0004^{+0.0028}_{-0.0018}$
ζ_c	$0.1156 \pm 0.0005^{+0.0027}_{-0.0016}$

Preliminary pheno investigations: right direction $\alpha_s \sim 0.117(1)$!

Conclusions and outlook

- Progress in precision SM phenomenology keeps proceeding at a remarkable pace
  - N^3LO, complex NNLO, QCD-EW, EW…
  - More and more elaborate resummations, non-leading logs…
  - Parton shower…
  - Computational tools ($\rightarrow$ ingredients for N^3LL resummation)
  - SM/BSM interplay: EFTs…

- This is necessary but not sufficient for physics at the few percent. Many unexpected issues that keep popping up

- A better understanding of NP corrections may be required

- Future ahead: not only computations. Very interesting analysis, from hardcore pheno to subtle QFT…

Thank you very much for your attention!