

## Lepton masses in a non universal $U(1)$ model with three families

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Scalar bosons	X	$\mathbb{Z}_2$	Y
Doublets			
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1 + v_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}$	2/3	+	1
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2 + v_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$	1/3	-	1
Singlet			
$\chi = \frac{\xi_x + v_\chi + i\zeta_\chi}{\sqrt{2}}$	-1/3	+	0

**Table:** Boson particle content of the model, X-charge,  $\mathbb{Z}_2$  parity and hypercharge [1,2].

Quarks	X	$\mathbb{Z}_2$	Leptons	X	$\mathbb{Z}_2$
$q_L^1 = \begin{pmatrix} U^1 \\ D^1 \end{pmatrix}_L$	1/3	+	$I_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0	+
$q_L^2 = \begin{pmatrix} U^2 \\ D^2 \end{pmatrix}_L$	0	-	$I_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	0	+
$q_L^3 = \begin{pmatrix} U^3 \\ D^3 \end{pmatrix}_L$	0	+	$I_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1	+
$U_R^{1,3}$	2/3	+	$e_R^{e,\tau}$	-4/3	-
$U_R^2$	2/3	-	$e_R^\mu$	-1/3	-
$D_R^{1,2,3}$	-1/3	-			
Non-SM					
$T_L$	1/3	-	$\nu_R^{e,\mu,\tau}$	1/3	-
$T_R$	2/3	-	$N_R^{e,\mu,\tau}$	0	-
$J_L^{1,2}$	0	+	$E_L, \mathcal{E}_R$	-1	+
$J_R^{1,2}$	-1/3	+	$E_R, \mathcal{E}_L$	-2/3	+

Table: Fermion particle content of the model, X-charge and  $\mathbb{Z}_2$  parity [1,2].

$$[SU(3)_C]^2 U(1)_X \rightarrow A_C = \sum_Q [X_{Q_L} - X_{Q_R}],$$

$$[SU(2)_L]^2 U(1)_X \rightarrow A_L = \sum_I X_{I_L} + 3 \sum_Q X_{Q_L},$$

$$[U(1)_Y]^2 U(1)_X \rightarrow A_{Y^2} = \sum_{I,Q} [Y_{I_L}^2 X_{I_L} + 3 Y_{Q_L}^2 X_{Q_L}] - \sum_{I,Q} [Y_{I_R}^2 X_{I_R} + 3 Y_{Q_R}^2 X_{Q_R}],$$

$$U(1)_Y [U(1)_X]^2 \rightarrow A_Y = \sum_{I,Q} [Y_{I_L} X_{I_L}^2 + 3 Y_{Q_L} X_{Q_L}^2] - \sum_{I,Q} [Y_{I_R} X_{I_R}^2 + 3 Y_{Q_R} X_{Q_R}^2],$$

$$[U(1)_X]^3 \rightarrow A_X = \sum_{I,Q} [X_{I_L}^3 + 3 X_{Q_L}^3] - \sum_{I,Q} [X_{I_R}^3 + 3 X_{Q_R}^3],$$

$$[\text{Grav}]^2 U(1)_X \rightarrow A_G = \sum_{I,Q} [X_{I_L} + 3 X_{Q_L}] - \sum_{I,Q} [X_{I_R} + 3 X_{Q_R}]$$

## Lagrangian (charged leptons)

$$\begin{aligned}
 -\mathcal{L}_{Y,C} = & \eta \bar{l}_L^e \phi_2 e_R^\mu + h \bar{l}_L^\mu \phi_2 e_R^\mu + \zeta \bar{l}_L^\tau \phi_2 e_R^e + H \bar{l}_L^\tau \phi_2 e_R^\tau \\
 & + q_{11} \bar{l}_L^e \phi_1 E_R + q_{21} \bar{l}_L^\mu \phi_1 E_R + g_{\chi_E} \bar{E}_L \chi E_R \\
 & + g_{\chi_\varepsilon} \bar{\varepsilon}_L \chi^* \mathcal{E}_R + \text{h.c}
 \end{aligned} \tag{1}$$

The  $\mathcal{E}$  lepton is decoupled and gets mass:

$$m_{\mathcal{E}} = \frac{g_{\chi_\varepsilon} v_\chi}{\sqrt{2}}$$

After SSB, in the flavor basis  $\mathbf{E} = (e^e, e^\mu, e^\tau, E)^T$ :

$$\mathbb{M}_E^0 = \begin{pmatrix} 0 & \frac{\eta v_2}{\sqrt{2}} & 0 & \frac{q_{11} v_1}{\sqrt{2}} \\ 0 & \frac{h v_2}{\sqrt{2}} & 0 & \frac{q_{21} v_1}{\sqrt{2}} \\ \frac{\zeta v_2}{\sqrt{2}} & 0 & \frac{H v_2}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{g_{\chi_E} v_\chi}{\sqrt{2}} \end{pmatrix}$$

## Effective lagrangian

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} &= \mathcal{O}_{ij}^I + \mathcal{O}_{\tau\mu}^I + \mathcal{O}_{Ej}^I + \mathcal{O}_{E\mu}^I + \mathcal{O}_{\tau E}^I \\
 &= \Omega_{ij}^I \left(\frac{\chi^*}{\Lambda}\right)^3 \bar{l}_L^j \phi_2 e_R^i + \Omega_{\tau\mu}^I \left(\frac{\chi}{\Lambda}\right)^3 \bar{l}_L^\tau \phi_2 e_R^\mu + \Omega_{Ej}^I \frac{\phi_2^\dagger \phi_1}{\Lambda} \bar{E}_L e_R^j \\
 &\quad + \Omega_{E\mu}^I \frac{\phi_1^\dagger \phi_2 \chi}{\Lambda^2} \bar{E}_L e_\mu^i + \Omega_{\tau E}^I \left(\frac{\chi}{\Lambda}\right)^3 \bar{l}_L^\tau \phi_1 E_R
 \end{aligned} \tag{2}$$

where  $i = e, \mu, j = e, \tau$  and  $\Lambda$  is the associated energy scale.

Adding to  $\mathbb{M}_E^0$  one gets the new matrix mass:

$$\mathbb{M}_E = \begin{pmatrix} \Omega_{ee}^I \frac{v_2 v_X^3}{4\Lambda^3} & \frac{\eta v_2}{\sqrt{2}} & \Omega_{e\tau}^I \frac{v_2 v_X^3}{4\Lambda^3} & \frac{q_{11} v_1}{\sqrt{2}} \\ \Omega_{\mu e}^I \frac{v_2 v_X^3}{4\Lambda^3} & \frac{h v_2}{\sqrt{2}} & \Omega_{\mu\tau}^I \frac{v_2 v_X^3}{4\Lambda^3} & \frac{q_{21} v_1}{\sqrt{2}} \\ \frac{\zeta v_2}{\sqrt{2}} & \Omega_{\tau\mu}^I \frac{v_2 v_X^3}{4\Lambda^3} & \frac{H v_2}{\sqrt{2}} & \Omega_{\tau E}^I \frac{v_1 v_X^3}{4\Lambda^3} \\ \Omega_{Ee}^I \frac{v_1 v_2 v_X}{2\Lambda} & \Omega_{E\mu}^I \frac{v_1 v_2 v_X}{2\sqrt{2}\Lambda^2} & \Omega_{E\tau}^I \frac{v_1 v_2}{2\Lambda} & \frac{g_{\chi_E} v_X}{\sqrt{2}} \end{pmatrix}$$

The diagonalization matrix for left-handed leptons to get the mass eigenstates  $\mathbf{e} = (e, \mu, \tau, E)^T$  is given by:

$$\mathbf{E}_L = \mathbb{V}_L^E \mathbf{e}_L$$

with  $\mathbb{V}_L^E \approx \mathbb{V}_{L1}^E \mathbb{V}_{L2}^E$ , where

$$\mathbb{V}_{L1}^E = \begin{pmatrix} 1 & 0 & 0 & \frac{q_{11}v_1}{\sqrt{2}m_E} \\ 0 & 1 & 0 & \frac{q_{21}v_1}{\sqrt{2}m_E} \\ 0 & 0 & 1 & r_3 \\ -\frac{q_{11}v_1}{\sqrt{2}m_E} & -\frac{q_{21}v_1}{\sqrt{2}m_E} & -r_3 & 1 \end{pmatrix} \quad (3)$$

$$\mathbb{V}_{L2}^E = \begin{pmatrix} c_{e\mu} & s_{e\mu} & r_1 & 0 \\ -s_{e\mu} & c_{e\mu} & r_2 & 0 \\ -r_1 c_{e\mu} + r_2 s_{e\mu} & -r_2 c_{e\mu} - r_1 s_{e\mu} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$t_{e\mu} = \eta/h$  and  $t_{e\tau} = \zeta/H$ .

Due to  $m_E^2 \approx g_{\chi_E}^2 v_\chi^2 / 2 \gg 1$ , as a first approximation, the mass eigenvalues are given by:

$$m_e^2 \approx \frac{v_2}{4} \left( \frac{v_\chi}{\Lambda} \right)^6 [s_{e\tau}(\Omega'_{\mu\tau} s_{e\mu} - \Omega'_{e\tau} c_{e\mu}) + c_{e\tau}(\Omega'_{ee} c_{e\mu} - \Omega'_{\mu e} s_{e\mu})]^2,$$

$$m_\mu^2 \approx \frac{1}{2}(\eta^2 + h^2)v_2^2,$$

$$m_\tau^2 \approx \frac{1}{2}(\zeta^2 + H^2)v_2^2,$$

and the parameters  $r_1, r_2$  are:

$$r_1 = \frac{s_{e\tau}\Omega'_{ee} + c_{e\tau}\Omega'_{e\tau} + \sqrt{\eta^2 + h^2}s_{e\mu}\Omega'_{\tau\mu}}{2\sqrt{2}\sqrt{\zeta^2 + H^2}} \left( \frac{v_\chi}{\Lambda} \right)^3$$

$$r_2 = \frac{s_{e\tau}\Omega'_{\mu e} + c_{e\tau}\Omega'_{\mu\tau} + \sqrt{\eta^2 + h^2}s_{e\mu}\Omega'_{\tau\mu}}{2\sqrt{2}\sqrt{\zeta^2 + H^2}} \left( \frac{v_\chi}{\Lambda} \right)^3$$

## Lagrangian (neutral leptons)

$$\begin{aligned}
 -\mathcal{L}_{Y,N} = & h_{2e}^{\nu e} \bar{l}_L^e \tilde{\phi}_2 \nu_R^e + h_{2e}^{\nu \mu} \bar{l}_L^e \tilde{\phi}_2 \nu_R^\mu + h_{2e}^{\nu \tau} \bar{l}_L^e \tilde{\phi}_2 \nu_R^\tau \\
 & + h_{2\mu}^{\nu e} \bar{l}_L^\mu \tilde{\phi}_2 \nu_R^e + h_{2\mu}^{\nu \mu} \bar{l}_L^\mu \tilde{\phi}_2 \nu_R^\mu + h_{2\mu}^{\nu \tau} \bar{l}_L^\mu \tilde{\phi}_2 \nu_R^\tau \quad (5) \\
 & + h_{\chi^l}^{\nu j} \bar{\nu}_R^{iC} \chi^* N_R^j + \frac{1}{2} \bar{N}_R^{iC} M_N^{ij} N_R^j + \text{h.c}
 \end{aligned}$$

In the basis

$$\mathbf{N}_L = (\nu_L^{e,\mu,\tau}, (\nu_L^{e,\mu,\tau})^C, (N_R^{e,\mu,\tau})^C)^T$$

the neutrino mass matrix is:

$$\mathbb{M}_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_M \end{pmatrix}$$

Where the block matrices are defined as:

$$m_D^T = \frac{v_2}{\sqrt{2}} \begin{pmatrix} h_{2e}^{\nu e} & h_{2e}^{\nu \mu} & h_{2e}^{\nu \tau} \\ h_{2\mu}^{\nu e} & h_{2\mu}^{\nu \mu} & h_{2\mu}^{\nu \tau} \\ 0 & 0 & 0 \end{pmatrix}$$

$$M_D = \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} h_{N_{\chi e}} & 0 & 0 \\ 0 & h_{N_{\chi \mu}} & 0 \\ 0 & 0 & h_{N_{\chi \tau}} \end{pmatrix}$$

$$M_M = \mu_N \mathbb{I}_{3 \times 3}$$

Neutrino masses are generated via inverse seesaw mechanism by assuming the hierarchy  $M_M \ll m_D \ll M_D$ . The  $3 \times 3$  mass matrix containing the active neutrinos is given by:

$$m_{\text{light}} = m_D^T (M_D^T)^{-1} M_M (M_D)^{-1} m_D$$

$$\frac{\mu_N v_2^2}{h_{N_{xe}}^2 v_\chi^2} \begin{pmatrix} (h_{2e}^{\nu e})^2 + (h_{2\mu}^{\nu e})^2 \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu \mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \mu} \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu \mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \mu} \rho^2 & (h_{2e}^{\nu \mu})^2 + (h_{2\mu}^{\nu \mu})^2 \rho^2 & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu \mu} h_{2\mu}^{\nu \tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \tau} \rho^2 & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu \mu} h_{2\mu}^{\nu \tau} \rho^2 & (h_{2e}^{\nu \tau})^2 + (h_{2\mu}^{\nu \tau})^2 \rho^2 \end{pmatrix}$$

where  $\rho = h_{N_{xe}} / h_{N_{x\mu}}$ .

With  $\rho = 1$  and  $h_{2\mu}^{\nu e} = h_{2\mu}^{\nu \tau} = 0$ ,

$$m_{\text{light}} = \frac{\mu_N v_2^2}{h_{N_{xe}}^2 v_X^2} \begin{pmatrix} (h_{2e}^{\nu e})^2 & h_{2e}^{\nu e} h_{2e}^{\nu \mu} & h_{2e}^{\nu e} h_{2e}^{\nu \tau} \\ h_{2e}^{\nu e} h_{2e}^{\nu \mu} & (h_{2e}^{\nu \mu})^2 + (h_{2\mu}^{\nu \mu})^2 & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} \\ h_{2e}^{\nu e} h_{2e}^{\nu \tau} & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} & (h_{2e}^{\nu \tau})^2 \end{pmatrix} \quad (6)$$

The above matrix is diagonalized through,

$$U_\nu^T m_{\text{light}} U_\nu = m_{\text{light}}^{\text{diag}}$$

$$U_\nu = [v_1^T \ v_2^T \ v_3^T],$$

$$m_{\text{light}}^{\text{diag}} = \begin{pmatrix} m_1 = 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (7)$$

The matrix PMNS is defined by [3,4]:

$$U_{\text{PMNS}} = (\nabla_{L,3 \times 3}^E)^\dagger U_\nu \quad (8)$$

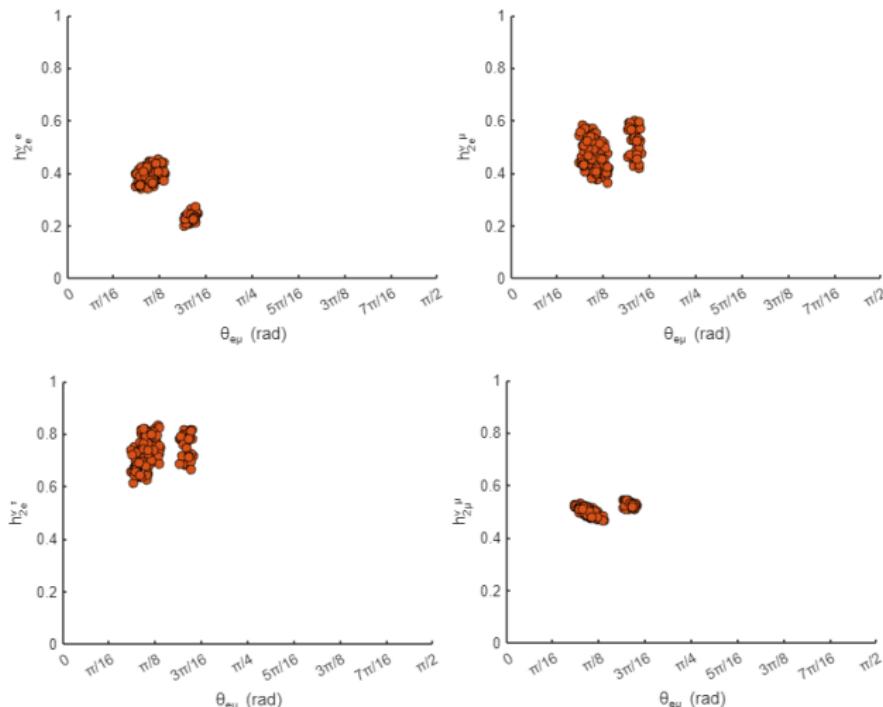
Thus, the free parameters are:

- $\{v_2, v_\chi, \Lambda, \theta_{e\mu}, \theta_{e\tau}, \Omega'_{ee}, \Omega'_{e\tau}, \Omega'_{\mu e}, \Omega'_{\mu\tau}, \Omega'_{\tau\mu}\}$  with  $v_2 = 2 \text{ GeV}$  and  $v_\chi = 7 \text{ TeV}$ .
- $\{h_{2e}^{\nu e}, h_{2e}^{\nu \mu}, h_{2e}^{\nu \tau}, h_{2\mu}^{\nu e}, h_{2\mu}^{\nu \mu}, h_{2\mu}^{\nu \tau}\}$  with  $h_{2\mu}^{\nu e} = h_{2\mu}^{\nu \tau} = 0$ .
- $\{\mu_N, h_{N_{\chi e}}\}$  with  $h_{N_{\chi e}} = \sqrt{0.02}$  and  $\frac{\mu_N v_2^2}{h_{N_{\chi e}}^2 v_\chi^2} = 50 \text{ meV}$   
 $\Rightarrow \mu_N = 12.25 \text{ keV}$

There are three different cases:

- 1  $\theta_{e\mu}, \theta_{e\tau} \ll 1; \Omega'_{ee} \approx \Omega'_{e\tau} \approx \Omega'_{\mu\tau} \approx 1$ .
- 2  $\theta_{e\mu}, \theta_{e\tau} \sim \pi/4; \Omega'_{\tau\mu} \approx \Omega'_{ee} \approx \Omega'_{e\tau} \approx 1$  or  $\Omega'_{\tau\mu} \approx \Omega'_{\mu e} \approx \Omega'_{\mu\tau} \approx 1$ .
- 3  $\theta_{e\mu} \ll 1, \theta_{e\tau} \sim \pi/4; \Omega'_{ee} \approx \Omega'_{e\tau} \approx 1$  or  $\Omega'_{\mu e} \approx \Omega'_{\mu\tau} \approx 1$ .

# Case 1



**Figure:** Yukawa couplings as a function of  $\theta_{e\mu}$  for NO scheme ( $\Lambda \approx 87$  TeV).

# Case 1

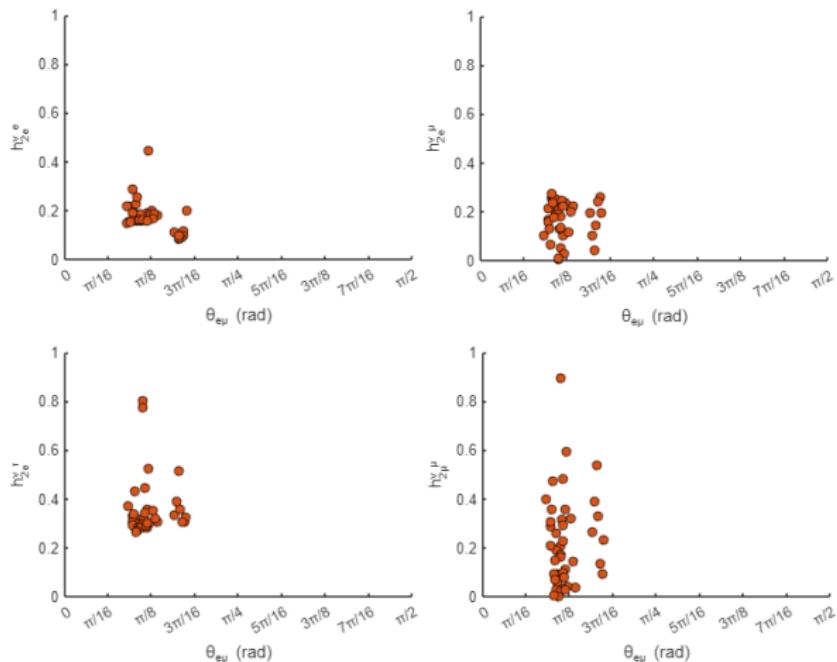
$\theta_{e\mu}$ (rad)*	$0.2895 \rightarrow 0.5553$
$h_{2e}^{\nu e}$	$0.1998 \rightarrow 0.4547$
$h_{2e}^{\nu \mu}$	$0.3637 \rightarrow 0.6018$
$h_{2e}^{\nu \tau}$	$0.6158 \rightarrow 0.8329$
$h_{2\mu}^{\nu \mu}$	$0.4666 \rightarrow 0.5468$

**Table:** Yukawa couplings and  $\theta_{e\mu}$  domain, which fulfill at  $3\sigma$  SK neutrino oscillation data for NO scheme, reported by [5,6].

	Data Available	Model
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$6.82 \rightarrow 8.04$	7.45
$\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}$	$2.431 \rightarrow 2.599$	2.50

**Table:** Difference of masses squared in the model compared to neutrino oscillation data ( $3\sigma$  SK) for NO scheme, reported by [5,6].

# Case 1



**Figure:** Yukawa couplings as a function of  $\theta_{e\mu}$  for IO scheme ( $\Lambda \approx 87$  TeV).

# Case 1

$\theta_{e\mu}$ (rad)*	$0.2888 \rightarrow 0.5467$
$h_{2e}^{\nu e}$	0.0872 → 0.4541
$h_{2e}^{\nu \mu}$	0.0091 → 0.2746
$h_{2e}^{\nu \tau}$	0.2640 → 0.8012
$h_{2\mu}^{\nu \mu}$	0.0004 → 0.8973

**Table:** Yukawa couplings and  $\theta_{e\mu}$  domain, which fulfill at  $3\sigma$  SK neutrino oscillation data for IO scheme, reported by [5,6].

	Data Available	Model
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	6.82 → 8.04	10.06
$\frac{\Delta m_{32}^2}{10^{-3} \text{ eV}^2}$	-2.584 → -2.413	-0.119

**Table:** Difference of masses squared in the model compared to neutrino oscillation data ( $3\sigma$  SK) for IO scheme, reported by [5,6].

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