

Study of neutrino textures for a non universal $U(1)$ model with 2 right-handed massive neutrinos

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Under the model the appearance of right-handed neutrinos is due to the additional $U(1)$ symmetry which extends the Yukawa lagrangian as:

$$L = \overline{q_L^0} \Gamma_a \Phi_a d_R^0 + \overline{q_L^0} \Delta_a \tilde{\Phi}_a u_R^0 + \overline{l_L^0} \Pi_a \Phi_a e_R^0 + \dots \quad (1)$$

$$\overline{l_L^0} \Sigma_a \tilde{\Phi}_a \nu_R + \frac{1}{2} \overline{\nu_R^c} (A + BS + CS^*) \nu_R + H.C \quad (2)$$

In which the neutrinos gain mass via the Type I see-saw mechanism[1]

The additional symmetry $U_x(1)$ transforms as:

$$\chi = e^{i\alpha X_x} \chi \quad (3)$$

Where

$$\chi = [q_{L_j}^0, d_{R_j}^0, u_{R_j}^0, l_{L_j}^0, e_{R_j}^0, \nu_{R_j}^0, \Phi_a, S] \quad (4)$$

This model also requires that the singlet charge under $U_x(1)$ and the charge of one of the Higgs doublet to be different of 0.[2][3]

After the symmetry breaking the leptonic masses are given by the following relations:

$$M_u = \frac{1}{\sqrt{2}}(\nu_1 \Delta_1 + \nu_2 \Delta_2) \quad (5)$$

$$M_d = \frac{1}{\sqrt{2}}(\nu_1 \Gamma_1 + \nu_2 \Gamma_2) \quad (6)$$

$$M_l = \frac{1}{\sqrt{2}}(\nu_1 \Pi_1 + \nu_2 \Pi_2) \quad (7)$$

$$m_D = \frac{1}{\sqrt{2}}(\nu_1 \Sigma_1 + \nu_2 \Sigma_2) \quad (8)$$

$$M_R = A + \frac{\nu_s}{\sqrt{2}}(B e^{i\alpha s} + C e^{-i\alpha s}) \quad (9)$$

Where Σ A and B are 3×2 and 2×2 matrices respectively and ν_1, ν_2 and ν_s are the values of the symmetry breaking of the Higgs doublets and the singlet.

Given the form of the model t the mass matrices are given by the relations:

$$\mathbb{M}_D = \frac{1}{\sqrt{2}}(\nu_1 \Sigma_1 + \nu_2 \Sigma_2) = \begin{pmatrix} h_e^1 & h_e^2 \\ h_\mu^1 & h_\mu^2 \\ h_\tau^1 & h_\tau^2 \end{pmatrix}$$

And

$$\mathbb{M}_R = A + \frac{\nu_s}{\sqrt{2}}(Be^{i\alpha s} + Ce^{-i\alpha s}) = \begin{pmatrix} h_e^1 & h_e^2 \\ h_\mu^1 & h_\mu^2 \end{pmatrix}$$

Where the couplings are to be determined in each case, and where the total mass matrix obtained applying the see-saw mechanism is given by the product of both matrices m_D and m_R in $m_D M_R^{-1} m_D^T$

The Lagrangian of the new Yukawa lagrangian can be compared to an effective lagrangian added to the known one [4]

Effective lagrangian

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{O}_{ij}^I + \mathcal{O}_{\tau\mu}^I + \mathcal{O}_{Ej}^I + \mathcal{O}_{E\mu}^I + \mathcal{O}_{\tau E}^I \\ &= \Omega_{ij}^I \left(\frac{\chi^*}{\Lambda}\right)^3 \bar{l}_L^i \phi_2 e_R^j + \Omega_{\tau\mu}^I \left(\frac{\chi}{\Lambda}\right)^3 \bar{l}_L^\tau \phi_2 e_R^\mu + \Omega_{Ej}^I \frac{\phi_2^\dagger \phi_1}{\Lambda} \bar{E}_L e_R^j \quad (10) \\ &\quad + \Omega_{E\mu}^I \frac{\phi_1^\dagger \phi_2 \chi}{\Lambda^2} \bar{E}_L e_\mu^j + \Omega_{\tau E}^I \left(\frac{\chi}{\Lambda}\right)^3 \bar{l}_L^\tau \phi_1 E_R\end{aligned}$$

where $i = e, \mu, j = e, \tau$ and Λ is the associated energy scale.

The diagonalization matrix for left-handed leptons to get the mass eigenstates $\mathbf{e} = (e, \mu, \tau, E)^T$ is given by:

$$\mathbf{E}_L = \mathbb{V}_L^E \mathbf{e}_L$$

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$$\mathbf{E}_L = \mathbb{V}_L^E \mathbf{e}_L$$

with $\mathbb{V}_L^E \approx \mathbb{V}_{L1}^E \mathbb{V}_{L2}^E$, where

$$\mathbb{V}_{L1}^E = \begin{pmatrix} 1 & 0 & 0 & \frac{q_{11}v_1}{\sqrt{2m_E}} \\ 0 & 1 & 0 & \frac{q_{21}v_1}{\sqrt{2m_E}} \\ 0 & 0 & 1 & r_3 \\ -\frac{q_{11}v_1}{\sqrt{2m_E}} & -\frac{q_{21}v_1}{\sqrt{2m_E}} & -r_3 & 1 \end{pmatrix} \quad (11)$$

$$\mathbb{V}_{L2}^E = \begin{pmatrix} c_{e\mu} & s_{e\mu} & r_1 & 0 \\ -s_{e\mu} & c_{e\mu} & r_2 & 0 \\ -r_1 c_{e\mu} + r_2 s_{e\mu} & -r_2 c_{e\mu} - r_1 s_{e\mu} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

in our case we only take the 3x3 matrix (there is no additional lepton E)

Due to $m_E^2 \approx g_{\chi_E}^2 v_\chi^2 / 2 \gg 1$, as a first approximation, the mass eigenvalue for the electron is given by:

$$m_e^2 \approx \frac{v_2}{4} \left(\frac{v_\chi}{\Lambda} \right)^6 [s_{e\tau}(\Omega'_{\mu\tau}s_{e\mu} - \Omega'_{e\tau}c_{e\mu}) + c_{e\tau}(\Omega'_{ee}c_{e\mu} - \Omega'_{\mu e}s_{e\mu})]^2,$$

and the parameters r_1, r_2 are at first approximation:

$$r_1 = \frac{s_{e\tau}\Omega'_{ee} + c_{e\tau}\Omega'_{e\tau}}{2\sqrt{2}} + \left(\frac{v_\chi}{\Lambda} \right)^3$$

$$r_2 = \frac{s_{e\tau}\Omega'_{\mu e} + c_{e\tau}\Omega'_{\mu\tau}}{2\sqrt{2}} \left(\frac{v_\chi}{\Lambda} \right)^3$$

The above matrix is diagonalized through the V_L^E matrix with the parameters mentioned above

The PMNS matrix is defined by:

$$U_{\text{PMNS}} = (\mathbb{V}_{L,3 \times 3}^E)^\dagger U_\nu \quad (13)$$

Thus, the free parameters are:

- $\{\nu_2, \nu_\chi, \Lambda, \theta_{e\mu}, \theta_{e\tau}, \Omega_{ee}^I, \Omega_{e\tau}^I\}$ with $\nu_2 = 5 \text{ GeV}$, $\Lambda = 17\nu_\chi$ and $\nu_\chi = 1.9 \text{ TeV}$.
- $\{h_{2e}^{\nu 1}, h_{2e}^{\nu 2}, h_{2\mu}^{\nu 1}, h_{2\mu}^{\nu 2}, h_{2\tau}^{\nu 1}, h_{2\tau}^{\nu 2}\}$

For symmetry reasons the only possible values for the couplings are given by the relation $X_{\Sigma_1} + X_{\Sigma_2} = X_a$

The process followed then corresponds to find a bunch of parameter values for the matrices that, after applying the see-saw mechanism and apply the rotation give a satisfactory PMNS matrix

Model D2

$\theta_{e\mu}$ (°)	$26 \rightarrow 36$
$h_{2e}^{\nu 1}$	$1 \times 10^{-3} \rightarrow 4 \times 10^{-3}$
$h_{2e}^{\nu 2}$	$3 \times 10^{-3} \rightarrow 1 \times 10^{-2}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constraints given by PMNS neutrino oscillation data reported by [5]

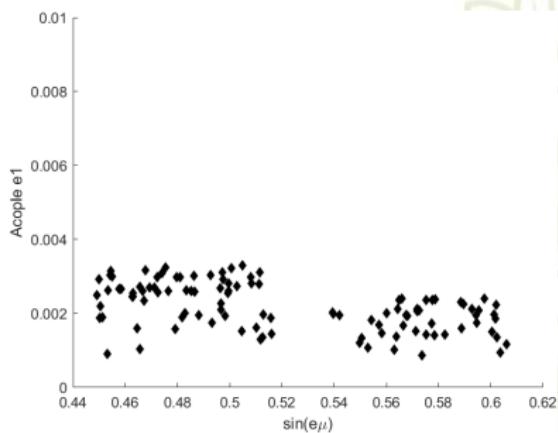
In this case the dirac matrix corresponds to:

$$\begin{pmatrix} X_{\Phi_1} & X_{\Phi_2} \\ X_{\Phi_2} & 0 \\ 0 & X_{\Phi_1} \end{pmatrix}$$

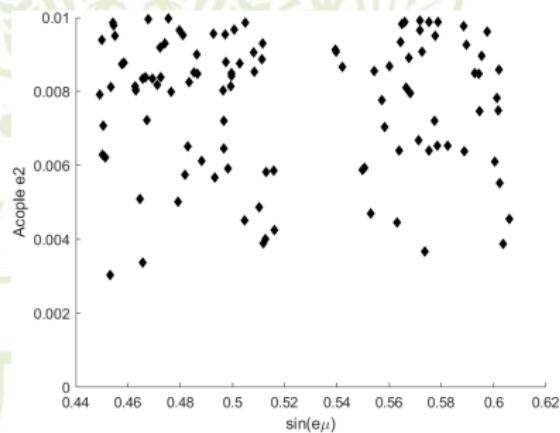
And the Majorana matrix corresponding to the 2 RH neutrinos is:

$$\begin{pmatrix} x & x \\ x & 0 \end{pmatrix}$$

Model D2



Coupling corresponding to X_{Φ_1} in the model D2



Coupling corresponding to X_{Φ_2} in the model D2

Model E2

$\theta_{e\mu}$ (°)	$26 \rightarrow 31$
$h_{2e}^{\nu 1}$	$2 \times 10^{-3} \rightarrow 4 \times 10^{-3}$
$h_{2e}^{\nu 2}$	$6 \times 10^{-3} \rightarrow 1 \times 10^{-2}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constraints given by PMNS neutrino oscillation data reported by [5]

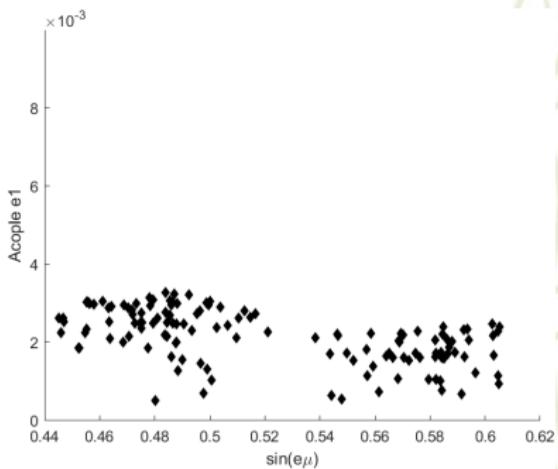
In this case the dirac matrix corresponds to:

$$\begin{pmatrix} X_{\phi_1} & X_{\phi_2} \\ X_{\phi_1} & X_{\phi_2} \\ 0 & X_{\phi_2} \end{pmatrix}$$

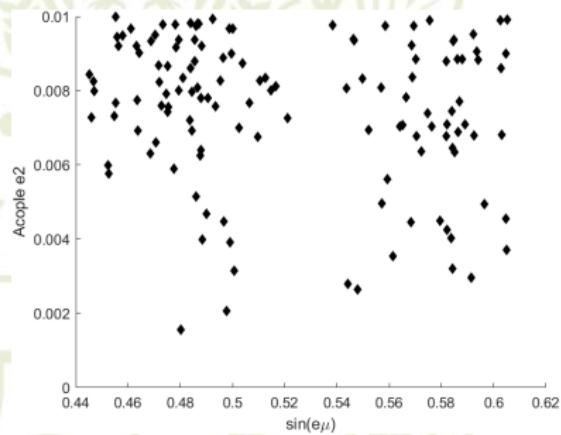
And the Majorana matrix corresponding to the 2 RH neutrinos is:

$$\begin{pmatrix} x & x \\ x & 0 \end{pmatrix}$$

Model E2



Coupling corresponding to X_{Φ_1} in
the model E2



Coupling corresponding to X_{Φ_2} in
the model E2

Model F1

$\theta_{e\mu}$ (°)	$27 \rightarrow 37$
$h_{2e}^{\nu 1}$	$2 \times 10^{-3} \rightarrow 4 \times 10^{-3}$
$h_{2e}^{\nu 2}$	$2 \times 10^{-3} \rightarrow 4 \times 10^{-3}$

Table: Yukawa coupling and $\theta_{e\mu}$ domain which fulfill the constraints given by PMNS neutrino oscillation data reported by [5]

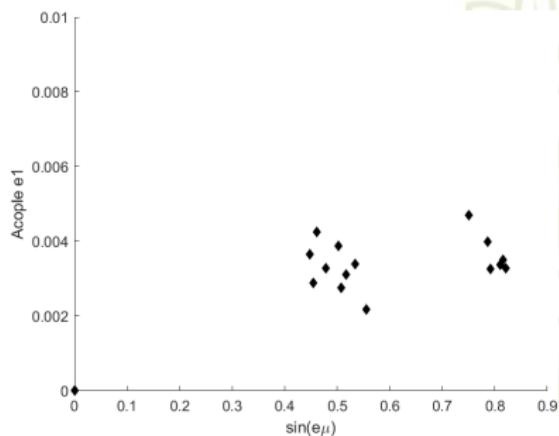
In this case the dirac matrix corresponds to:

$$\begin{pmatrix} X_{\phi_1} & X_{\phi_2} \\ X_{\phi_1} & X_{\phi_2} \\ X_{\phi_1} & X_{\phi_2} \end{pmatrix}$$

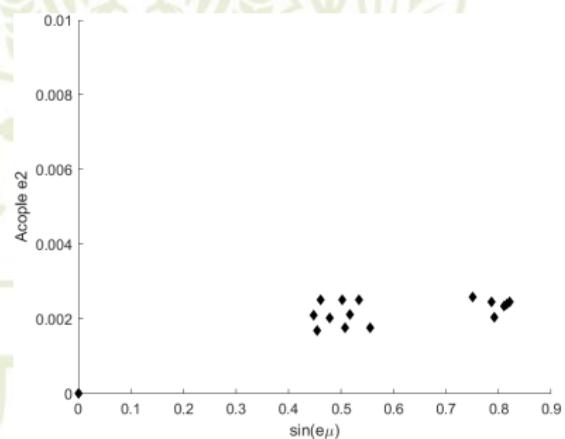
And the Majorana matrix corresponding to the 2 RH neutrinos is:

$$\begin{pmatrix} 0 & x \\ x & x \end{pmatrix}$$

Model F1



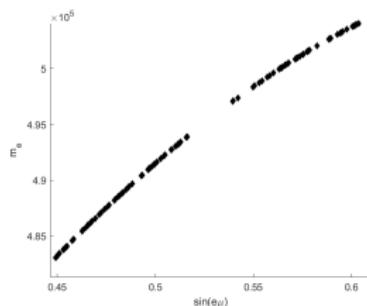
Coupling corresponding to X_{Φ_1} in the model F1



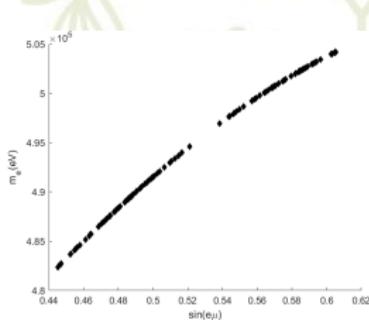
Coupling corresponding to X_{Φ_2} in the model F1

The electron mass

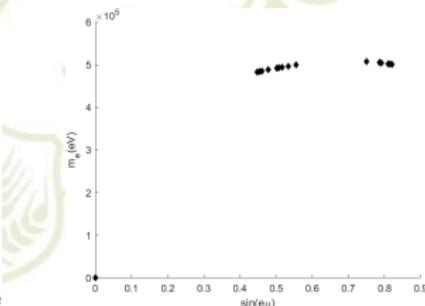
Following the expression for the mass of the electron, the mass of the electron for the run in each model is of the form:



Electron mass for the model D2 run

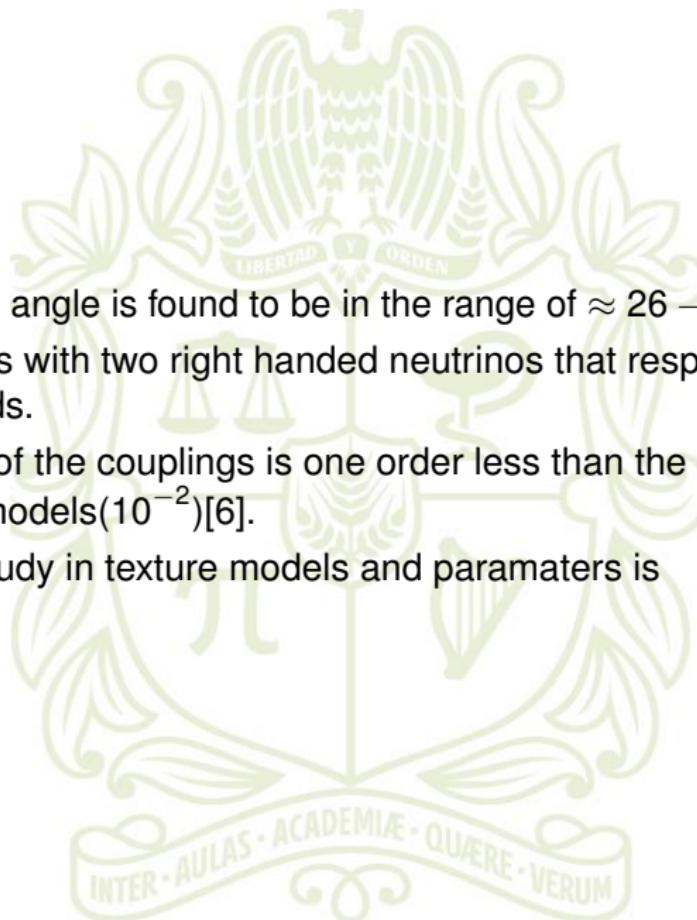


Electron mass for the model E2 run



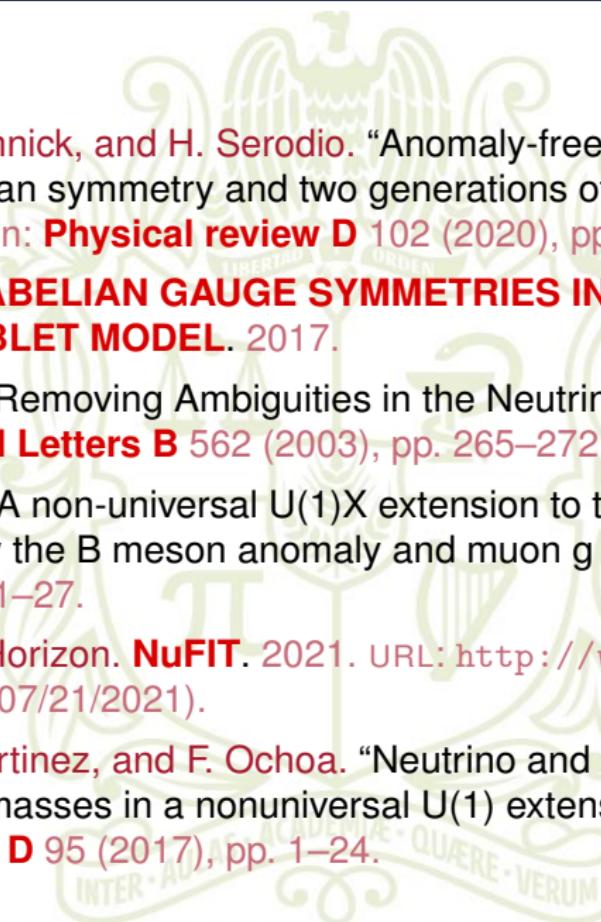
Electron mass for the model F1 run

Summary



- The neutrino mixing angle is found to be in the range of $\approx 26 - 36$
- It is possible models with two right handed neutrinos that respect experimental bounds.
- However the scale of the couplings is one order less than the theorized in 3 RH models(10^{-2})[6].
- A more profound study in texture models and parameters is needed

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- [1] A. Ordell, R. Pasechnick, and H. Serodio. “Anomaly-free 2HDMs with a gauged abelian symmetry and two generations of right-handed neutrinos”. In: **Physical review D** 102 (2020), pp. 1–14.
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 - [4] J.S Alvarado et al. “A non-universal $U(1)_X$ extension to the Standard Model to study the B meson anomaly and muon $g - 2$ ”. In: **Arxiv** 1 (2021), pp. 1–27.
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