



# Charmonium Spectrum from Non-Relativistic Quantum Mechanics

Pedro Leal, Pablo Figueroa  
Diego Milanes, Carlos Sandoval  
34

November 29th, 6:35 pm

# Introduction

1. Dependence of the coupling constant on energy.
2. Complexity of effective theories.
3. Experimental approach using an effective potential.
4. Non-Relativistic frame.

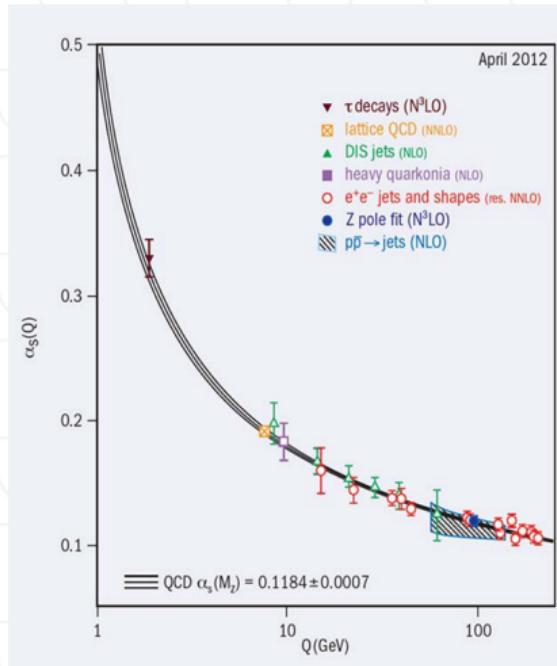


Figure: Asymptotic Freedom

## Variational Method

The function trial used was the tridimensional harmonic oscillator wave function. Such that b is the variational coefficient.

$$\psi_{nlm} = N_{nl} \left( \frac{r}{b} \right)^l L_n^{l+\frac{1}{2}} \left( \frac{r^2}{b^2} \right) \exp \left( \frac{-r^2}{2b^2} \right) Y_{lm}(\theta, \phi)$$

$\chi^2$

In order to find the parameters that best describe the experimental data it is necessary to minimize the  $\chi^2$  expression calculated as:

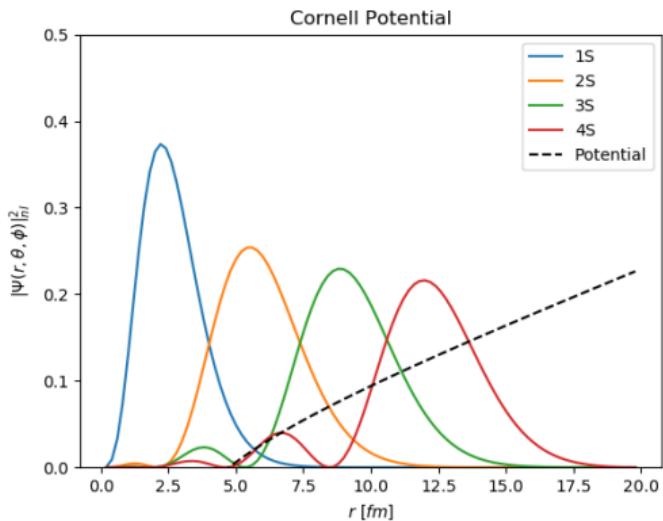
$$\chi^2 = \frac{1}{n} \sum_{i=1}^n \left( \frac{M_i^{exp} - M_i^{Num}}{\Delta_i} \right)^2$$

# Cornell Potential

$$V_{\text{Cornell}}(r) = -\frac{a}{r} + br$$

	[1]	This Document
a [GeV fm]	0.7089	1.0926
b [GeV/fm]	0.1497	0.04853
m_c [GeV]	1.4495	1.487

[1] M. Ali, Y. Mustafa, G. Hassan, and C. Moustakidis.  
"Spectra of quark-antiquark bound states via  
two derived qcd potentials."

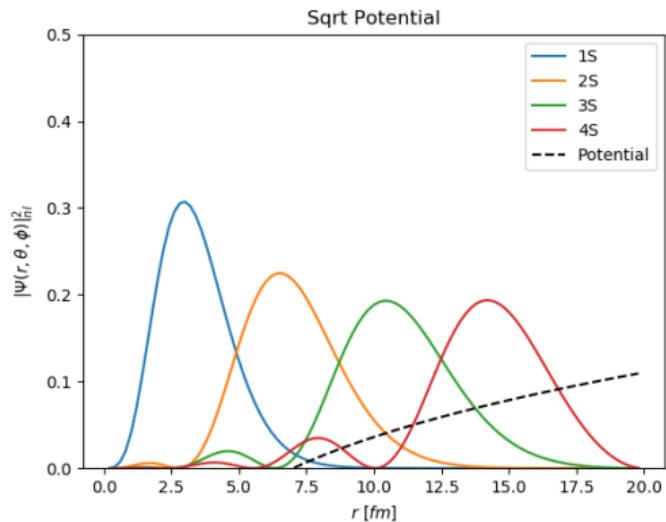


# Squared Root Potential

$$V_{\text{sqrt}}(r) = -\frac{a}{\sqrt{r}} + b\sqrt{r}$$

	[2]	This Document
a [GeV fm <sup>-1</sup> ]	0.41	1.064
b [GeV fm <sup>-1/2</sup> ]	1.15	0.152
m_c [GeV]	1.8	1.487

[2] S. Xiaotong and L. Hefen.  
"A new potential model for heavy  
quarkonium"

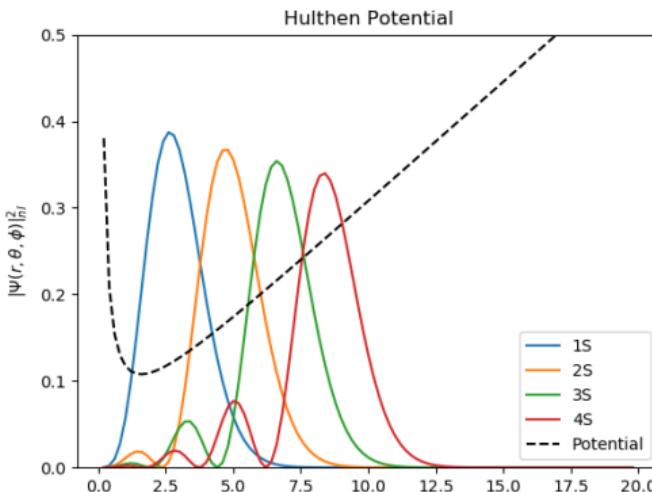


# Hulthen Potential

$$V_{\text{Effective Hulthen}}(r) = -\mu_0 \frac{e^{\frac{-r}{\mu}}}{1 - e^{\frac{-r}{\mu}}} + cr + V_0$$

$$\begin{aligned} \mu_0 &= 0.2 \text{ GeV} & \mu &= 2.5 \text{ GeV}^{-1} \\ c &= 0.193 \text{ GeV}^2 & V_0 &= -0.223 \text{ GeV} \\ m_c &= 1.4 \text{ GeV} \end{aligned}$$

Bhaghyesh, V. K. Kanti, and A. Monteiro.  
"Heavy quarkonium spectra and its decays  
in a nonrelativistic model with hulthen  
potential."



# Numerical and Experimental spectra of $[c\bar{c}]$

State	Name	Exp Mass[GeV]	Cornell Mass[GeV]	M. Ali	New	Sqrt Mass[GeV]	New	Hulthen Mass[GeV]	Bhaghyesh
$1^3S_1$	$J/\psi$	$3.096900 \pm 0.000006$		2.99475	2.97218	3.18711	2.98367		2.98581
$1^1S_0$	$\eta_c(1S)$	$2.9839 \pm 0.0004$		2.99475	2.97218	3.18711	2.98367		2.98581
$2^3S_1$	$\psi'(2S)$	$3.68610 \pm 0.00006$		3.64042	3.66515	3.53358	3.72015		3.7019
$2^1S_0$	$\eta'_c(2S)$	$3.6375 \pm 0.0011$		3.64042	3.66515	3.53358	3.72015		3.7019
$3^3S_1$	$\psi(3S)$	$4.039 \pm 0.001$		4.15717	4.03199	4.10575	4.0097		3.96522
$3^1S_0$	$\eta_c(3S)$			4.15717	4.03199	4.10575	4.0097		3.96522
$4^3S_1$	$\psi(4S)$	$4.421 \pm 0.0004$		4.43855	4.4104	4.35119	4.32082		4.55915
$4^1S_0$	$\eta_c(4S)$			4.43855	4.4104	4.35119	4.32082		4.55915

## Conclusions

- ▶ Two potentials with new parameters describing charmonium autoenergies were found.
- ▶ The new potentials found present changes in the domain of their wave function, which was expected because the change of the potential parameters.

## Ongoing

- ▶ Introduce spinorial terms corresponding to fine structure corrections.
- ▶ Add next leading order corrective parameters .
- ▶ Calculate the parameters for the Hulthen potential.
- ▶ Use the three differents potential to describe the Bottomonium states masses.

## References

-  M. Ali, Y. Mustafa, G. Hassan, and C. Moustakidis.  
Spectra of quark-antiquark bound states via two derived qcd potentials.  
*Quantum Physics Letters*, 5:7–14, 04 2016.
-  Bhaghyesh, V. K. Kanti, and A. Monteiro.  
Heavy quarkonium spectra and its decays in a nonrelativistic model with hulthen potential.  
*Journal of Physics G: Nuclear and Particle Physics*, 38:085001, 06 2011.
-  P. Z. et al.  
*Particle Data Group*, 06 2020.
-  D. J. Gross and F. Wilczek.  
Ultraviolet behavior of non-abelian gauge theories.  
*Phys. Rev. Lett.*, 30:1343–1346, Jun 1973.
-  S. Xiaotong and L. Hefen.  
A new potential model for heavy quarkonium.  
*International Atomic Energy Agency (IAEA)*, 1986.

# Why are the parameters different?

## Cornell Potential

- ▶ Different experimental data (2010).
- ▶ Only matrix calculations.

## Sqrt Potential

- ▶ The value of the charm mass  $m_c = 1.8 \text{ GeV}$ .
- ▶ Different experimental data(1986).
- ▶ Don't justify the parameters values.