Effective potential and dynamical symmetry breaking up to five loops in a massless abelian Higgs model

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Coleman-Weinberg (CW) and Minimal subtraction (MS) schemes

$$L = \ln\left(\frac{\sigma^2}{\mu^2}\right) \text{ for CW scheme} \qquad \tilde{L} = \ln\left(x\frac{\sigma^2}{\tilde{\mu}^2}\right) \text{ for MS scheme}$$

Renormalization group functions

$$\beta_{\text{CW}}^{\text{one loop}} = \tilde{\beta}_{MS}^{\text{one loop}} \tag{1}$$

Effective potential

F. A. Chishtie, et al. PRD 83 (2011)

$$V_{\text{eff(CW)}}^{\text{one loop}} = \tilde{V}_{\text{eff}(MS)}^{\text{one loop}} \tag{2}$$

These relations are more complicated when higher loops are considered.

We need to use multiscale methods.

Renormalization group equation (RGE) and dynamical symmetry breaking (DSB)

$$\left[\mu\frac{\partial}{\partial\mu}+\beta_x\frac{\partial}{\partial x}-\gamma_\varphi\phi\frac{\partial}{\partial\phi}\right]\mathcal{V}_{eff}(\phi;\mu,x,\mathcal{L})=0 \quad (3)$$

where

$$\mathcal{V}_{\text{eff}}(\phi;\mu,x,\mathcal{L})=\phi^4\mathcal{S}_{\text{eff}}(\mu,x,\mathcal{L}), \quad (4)$$

$$\mathcal{L}=\ln\left[\frac{\phi^2}{\mu^2}\right] \quad (5)$$

$$\left[-(2+2\gamma_\phi)\frac{\partial}{\partial\mathcal{L}}+\beta_x\frac{\partial}{\partial x}-4\gamma_\varphi\right]\mathcal{S}_{\text{eff}}(\mu,x,\mathcal{L})=0 \quad (6)$$

with $\mathcal{S}_{\text{eff}}(x,\mathcal{L})=\mathcal{S}_{\text{eff}}^{\text{LL}}(x,\mathcal{L})+\mathcal{S}_{\text{eff}}^{\text{NLL}}(x,\mathcal{L})+\cdots,$ (7)

$$\left[-2\frac{\partial}{\partial\mathcal{L}}+\beta_x^{(2)}\frac{\partial}{\partial x}\right]\mathcal{S}_{eff}^{\text{LL}}(x,\mathcal{L})=0 \quad (8)$$

$$\left[-2\frac{\partial}{\partial\mathcal{L}}+\beta_x^{(2)}\frac{\partial}{\partial x}\right]\mathcal{S}_{eff}^{\text{NLL}}(x,\mathcal{L})+\left[\beta_x^{(3)}\frac{\partial}{\partial x}-4\gamma_\varphi^{(2)}\right]\mathcal{S}_{eff}^{\text{LL}}(x,\mathcal{L})=0 \quad (9)$$

A. G. Quinto, et al.
NPB 907 (2016)

Renormalization group equation (RGE) and dynamical symmetry breaking (DSB)

The renormalization group functions are usually calculated in the MS echeme and they should be adapted to the CW scheme,

$$\tilde{\mathcal{L}} = \ln\left[\frac{x\phi^2}{2\mu^2}\right] \tag{10}$$

Both schemes can be related by a redefinition of the mass scale μ,

C. Ford and D. R. T. Jones PLB 274 (1992)

$$\mu_{MS}^2 = f(x)\mu_{CW}^2 \tag{11}$$

The beta functions in both schemes

$$\beta_{CW} = \beta_{MS}\left(1 - \frac{1}{2}\beta_{MS}\partial_x \ln f\right)^{-1} \tag{12}$$

In four spacetime dimensions, divergences usually start at one loop,

$$\begin{aligned}\beta_{CW} &= \left(\beta_{MS}^{(2)} + \beta_{MS}^{(3)} + \cdots\right)(1 + \mathcal{O}(x)) \\ &= \beta_{MS}^{(2)} + \mathcal{O}(x^3)\end{aligned} \tag{13}$$

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Renormalization group equation (RGE) and dynamical symmetry breaking (DSB)

In the MS and CW schemes, the effective potential would be calculated at one loop level in the forms

$$V_{MS} = \phi^4(\tilde{A}(x) + \tilde{B}(x)\tilde{\mathcal{L}}) \quad (14)$$

$$V_{CW} = \phi^4(A(x) + B(x)\mathcal{L}) \quad (15)$$

From $\tilde{\mathcal{L}} = \ln\left[\frac{x\phi^2}{2\mu^2}\right]$ and $\mathcal{L} = \ln\left[\frac{\phi^2}{\mu^2}\right]$ $\Rightarrow$ $\tilde{\mathcal{L}} = \mathcal{L} + \ln\left[\frac{x}{2}\right]$ (16)

$$V_{MS} = \phi^4\left[\left(\tilde{A}(x) + \ln\frac{x}{2}\tilde{B}(x)\right) + \tilde{B}(x)\mathcal{L}\right] \quad (17)$$

At the two loop level, the RGE can be used to relate renormalization group functions and the effective potential in the CW and the MS scheme, but not interchangeably.

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Massless Abelian Higgs model

The renormalized Lagrangian of the massless AH model

$$\mathcal{L}' = Z_\phi |\underbrace{D_\mu \phi}_{= \left(\partial_\mu - ie\tilde{\mu}^{e/2} A_\mu\right)\phi}|^2 + Z_{\phi^4} \lambda \tilde{\mu}^e \left(|\phi|^2\right)^2 + \frac{Z_A}{4} F_{\mu v}^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \quad (15)$$

Renormalized coupling constants as

$$\begin{aligned} \alpha \equiv e^2 &= e_0^2 \tilde{\mu}^{-e} Z_A \\ \lambda &= \lambda_0 \tilde{\mu}^{-e} Z_\phi^2 Z_{\phi^4}^{-1} \\ \xi &= \xi_0 Z_A^{-1} \end{aligned} \quad (16)$$

In the MS scheme, the beta and gamma functions of the model are

$$\begin{aligned} \tilde{\beta}_\alpha &= \tilde{\beta}_\alpha^{(2)} + \tilde{\beta}_\alpha^{(3)} + \tilde{\beta}_\alpha^{(4)} + \tilde{\beta}_\alpha^{(5)} \\ \tilde{\beta}_\xi &= \tilde{\beta}_\xi^{(2)} + \tilde{\beta}_\xi^{(3)} + \tilde{\beta}_\xi^{(4)} + \tilde{\beta}_\xi^{(5)} \\ \tilde{\beta}_\lambda &= \tilde{\beta}_\lambda^{(2)} + \tilde{\beta}_\lambda^{(3)} + \tilde{\beta}_\lambda^{(4)} + \tilde{\beta}_\lambda^{(5)} \\ \tilde{\gamma}_\phi &= \tilde{\gamma}_\phi^{(1)} + \tilde{\gamma}_\phi^{(2)} + \tilde{\gamma}_\phi^{(3)} + \tilde{\gamma}_\phi^{(4)} \end{aligned} \quad (17)$$

Bernhard Ihrig, et al.
PRB 100 (2019)

Renormalization group function in the CW scheme

Remember!

$$L = \ln\left(\frac{\sigma^2}{\mu^2}\right) \text{ for } \boxed{\text{CW}} \text{ scheme}$$

$$\tilde{L} = \ln\left(x\frac{\sigma^2}{\tilde{\mu}^2}\right) \text{ for } \boxed{\text{MS}} \text{ scheme}$$

$$\underbrace{\begin{aligned}\tilde{L}_1 &= \ln\left(\lambda\frac{\sigma^2}{k_1^2}\right)\\ \tilde{L}_2 &= \ln\left(\alpha\frac{\sigma^2}{k_2^2}\right)\\ \tilde{L}_3 &= \ln\left(\xi\frac{\sigma^2}{k_3^2}\right)\end{aligned}}_{\text{MS scheme}} \tag{18}$$

$$\underbrace{\begin{aligned}k_1 &= \lambda^{1/2}\mu\\ k_2 &= \alpha^{1/2}\mu\\ k_3 &= \xi^{1/2}\mu\end{aligned}}_{\text{Relation between MS and CW scheme}} \tag{19}$$

The CW RG functions through an order by order

$$\underbrace{\begin{aligned}\gamma_\phi^{(1)} &= -3\tilde{\gamma}_\phi^{(1)}\\ \beta_\lambda^{(2)} &= -3\tilde{\beta}_\lambda^{(2)}\\ \beta_\alpha^{(2)} &= -3\tilde{\beta}_\alpha^{(2)}\\ \beta_\xi^{(2)} &= -3\tilde{\beta}_\xi^{(2)}\end{aligned}}_{\text{Expected!}} \tag{20}$$

$$\underbrace{\begin{aligned}\beta_\lambda^{(3)} &= -3\tilde{\beta}_\lambda^{(3)} + 3\tilde{\beta}_\lambda^{(2)}\left(\frac{\tilde{\beta}_\lambda^{(2)}}{2\lambda} + \frac{\tilde{\beta}_\alpha^{(2)}}{2\alpha} + \frac{\tilde{\beta}_\xi^{(2)}}{2\xi}\right)\\ \beta_\alpha^{(3)} &= -3\tilde{\beta}_\alpha^{(3)} + 3\tilde{\beta}_\alpha^{(2)}\left(\frac{\tilde{\beta}_\lambda^{(2)}}{2\lambda} + \frac{\tilde{\beta}_\alpha^{(2)}}{2\alpha} + \frac{\tilde{\beta}_\xi^{(2)}}{2\xi}\right)\\ \beta_\xi^{(3)} &= -3\tilde{\beta}_\xi^{(3)} + 3\tilde{\beta}_\xi^{(2)}\left(\frac{\tilde{\beta}_\lambda^{(2)}}{2\lambda} + \frac{\tilde{\beta}_\alpha^{(2)}}{2\alpha} + \frac{\tilde{\beta}_\xi^{(2)}}{2\xi}\right)\\ \gamma_\phi^{(2)} &= -3\tilde{\gamma}_\phi^{(2)} + 3\tilde{\gamma}_\phi^{(1)}\left(\frac{\tilde{\beta}_\lambda^{(2)}}{2\lambda} + \frac{\tilde{\beta}_\alpha^{(2)}}{2\alpha} + \frac{\tilde{\beta}_\xi^{(2)}}{2\xi}\right)\end{aligned}}_{\text{Two loops}} \tag{21}$$

$\underbrace{\text{A. G. Quinto, et al. arXiv:2111.08865 (2021)}}_{\text{Up to four loops}}$

The effective potential and Dynamical symmetry breaking

We consider a shift in the N-th component of $\phi_N = \underbrace{\phi_N^q}_{= \frac{1}{\sqrt{2}}(\phi_{1N} + i\phi_{2N})} + \sigma$ (22) $\Rightarrow$ $\boxed{V_{\text{eff}}^{(0\ell)}(\sigma) = \frac{1}{4}\lambda\sigma^4}$ (23)

classical level

Spontaneous symmetry breaking and the corresponding generation of mass as

$$
\begin{aligned}
m_{\phi_{1N}}^2 &= \frac{3}{2}\lambda\sigma^2 \\
m_{\phi_{2N}}^2 &= \frac{1}{2}\lambda\sigma^2 - \xi m_A^2 \\
m_A^2 &= \frac{1}{2}e^2\sigma^2
\end{aligned}
\tag{24}
$$

$$\boxed{A(\lambda, \alpha, \xi) = A^{(1)} = \frac{1}{4}\lambda} \tag{26}$$

fixed by the CW condition

we shall use the ansatz

$$V_{\text{eff}}(\sigma_{cl}) = \sigma^4 \underbrace{S_{\text{eff}}(\sigma, \lambda, \alpha, \xi, L)}_{} \tag{25}$$

$$= \boxed{A(\lambda, \alpha, \xi)} + B(\lambda, \alpha, \xi)L + C(\lambda, \alpha, \xi)L^2 + D(\lambda, \alpha, \xi)L^3 + \cdots$$

The RGE for our model is

$$[2(-1 + \gamma_\phi)\partial_L + \beta_\lambda\partial_\lambda + \beta_\alpha\partial_\alpha + \beta_\xi\partial_\xi + 4\gamma_\phi]S_{\text{eff}}(\sigma; \lambda, \alpha, \xi, L) = 0 \tag{27}$$

The effective potential and Dynamical symmetry breaking

Separating the resulting expression by orders of L, we obtain a series of equations,

$$\boxed{2(-1+\gamma_\phi)B(\lambda,\alpha,\xi)+\beta_\lambda\partial_\lambda A(\lambda,\alpha,\xi)+4\gamma_\phi A(\lambda,\alpha,\xi)=0}$$
$$2(-1+\gamma_\phi)C(\lambda,\alpha,\xi)+\{\beta_\lambda\partial_\lambda+\beta_\alpha\partial_\alpha+\beta_\xi\partial_\xi+4\gamma_\phi\}B(\lambda,\alpha,\xi)=0$$
$$2(-1+\gamma_\phi)D(\lambda,\alpha,\xi)+\{\beta_\lambda\partial_\lambda+\beta_\alpha\partial_\alpha+\beta_\xi\partial_\xi+4\gamma_\phi\}C(\lambda,\alpha,\xi)=0 \qquad (28)$$

Expanded form

$$-2\left(B^{(1)}+B^{(2)}+B^{(3)}+\ldots\right)+2\left(\gamma_\phi^{(1)}+\gamma_\phi^{(2)}+\ldots\right)\left(B^{(1)}+B^{(2)}+B^{(3)}+\ldots\right)$$
$$+\left(\beta_\lambda^{(2)}+\beta_\lambda^{(3)}+\ldots\right)\partial_\lambda \boxed{A^{(1)}}+4\left(\gamma_\phi^{(1)}+\gamma_\phi^{(2)}+\ldots\right)\boxed{A^{(1)}}=0 \qquad (29)$$

We can find

$$\boxed{A^{(1)}=\frac{1}{4}\lambda}$$

$$B^{(2)}=\frac{3}{8}\left(6\alpha^2+2\alpha\lambda+(N+4)\lambda^2\right).$$
$$B^{(3)}=b_1\alpha\lambda\xi+b_2\alpha^3+b_3\alpha^2\lambda+b_4\alpha\lambda^2+b_5\lambda^3+b_6\alpha^4\lambda^{-1}$$
$$B^{(4)}=b_7\alpha^3\xi+b_8\alpha^4+b_9\alpha\lambda^2\xi+b_{10}\alpha^3\lambda+b_{11}\alpha^2\lambda\xi+b_{12}\alpha^2\lambda^2+b_{13}\alpha\lambda^3+b_{14}\lambda^4$$
$$+b_{15}\alpha^5\lambda^{-1}+b_{16}\alpha^6\lambda^{-2} \qquad (30)$$
$$B^{(5)}=b_{17}\lambda^3\alpha\xi+b_{18}\lambda^3\alpha^2+b_{19}\lambda^2\alpha^3+b_{20}\lambda^2\alpha^2\xi+b_{21}\lambda\alpha^2\xi^2+b_{22}\lambda\alpha^4+b_{23}\lambda\alpha^3\xi$$
$$+b_{24}\alpha^5+b_{25}\alpha^4\xi+b_{26}\alpha\lambda^4+b_{27}\lambda^5+b_{28}\alpha^5\xi\lambda^{-1}+b_{29}\alpha^6\lambda^{-1}+b_{30}\alpha^7\lambda^{-2}+b_{31}\alpha^8\lambda^{-3}.$$

A. G. Quinto, et al. arXiv:2111.08865 (2021)

The effective potential and Dynamical symmetry breaking

The effective potential up to five loops which was calculated using the renormalization group equation

$$V^{(5\ell)}_{\text{eff},R}(\sigma) = \sigma^4\left[A^{(1)} + \sum_{n=2}^{5} B^{(n)} L + \sum_{n=3}^{5} C^{(n)} L^2 + \sum_{n=4}^{5} D^{(n)} L^3 + \rho\right] \tag{31}$$

The constant ρ is fixed using the CW normalization condition,

$$\left.\frac{d^4}{d\sigma^4} V_{\text{eff},R}(\sigma)\right|_{\sigma=\mu} = \frac{4!}{4}\lambda \tag{32}$$

The effective potential has a minimum at σ = μ means imposing that

$$\left.\frac{d}{d\sigma} V_{\text{eff},R}(\sigma)\right|_{\sigma=\mu} = 0 \tag{33}$$

which can be used to determine the value of λ as a function of free parameters α, ξ and N.

we look for real and positive values for λ, and correspond to a minimum of the potential. i.e.,

$$\left.\frac{d^2}{d\sigma^2} V_{\text{eff},R}(\sigma)\right|_{\sigma=\mu} > 0 \tag{34}$$

The effective potential and Dynamical symmetry breaking

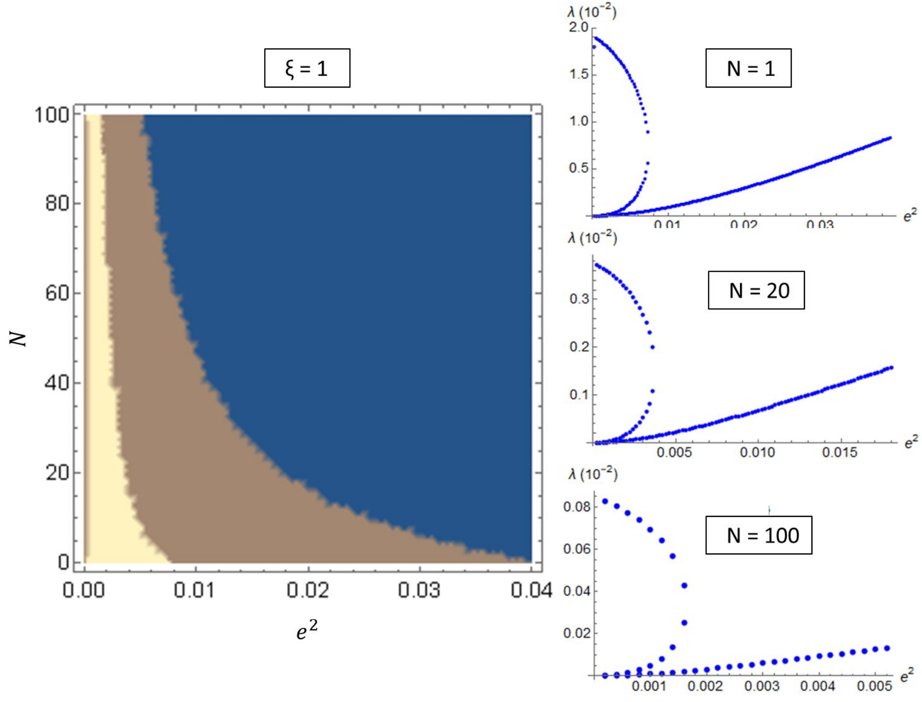

Figure 1. In the left hand side we show a region plot corresponding to the result of scanning for DSB for different values of the free parameters e^2 and N, with $\xi = 1$. In our model we find three regions: in the yellow region we have three possible solutions for λ, in the brown one we have only one solution and in the blue region we do not have solution for λ, meaning DSB is not operational. In the right hand side, we show a set of plots explicitly showing the behavior of the solutions for λ as a function of the e^2 parameter, for specific values of N = 1, 20, 100 and $\xi = 1$.

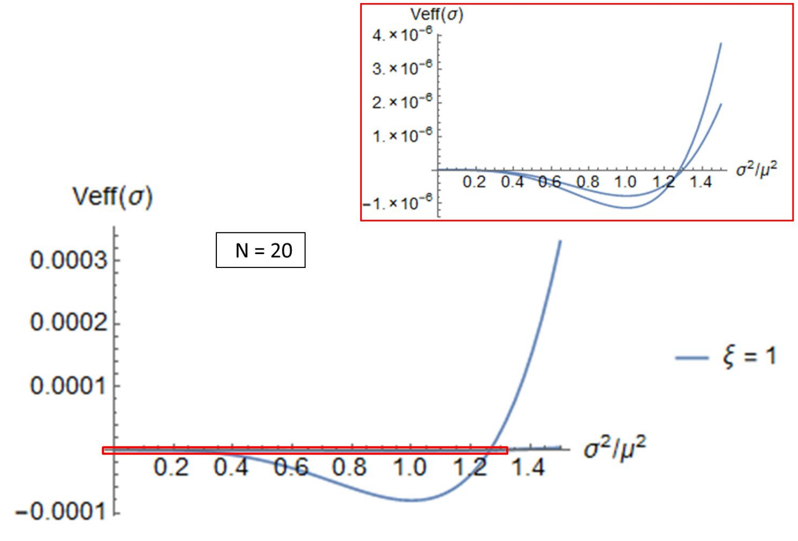

Figure 2. This graph of Veff (σ) vs. σ^2/μ^2 shows the potential minimum for the gauge parameter, **ξ = 1**. The effective potential was evaluated using **e^2 = 0.001**, **N = 20** and the values of $\boxed{\lambda}$. The red rectangle in the bottom-left graph is shown in a different scale in the top-right one.

$$\boxed{\lambda_1 = 0.003553, \lambda_2 = 0.00003590, \lambda_3 = 0.00001210}$$

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Thanks for your attention