

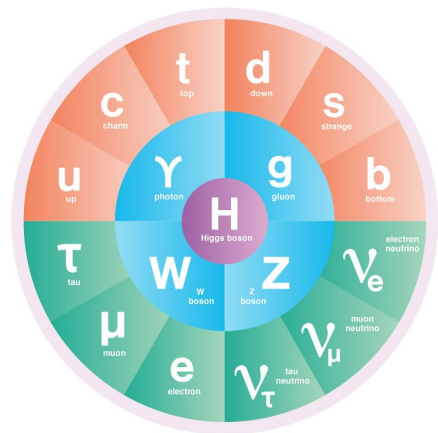
# Nuclear **Femtography** in the era of Jefferson Lab 12 GeV program and the EIC

Nobuo Sato

6th Colombian Meeting on High  
Energy Physics

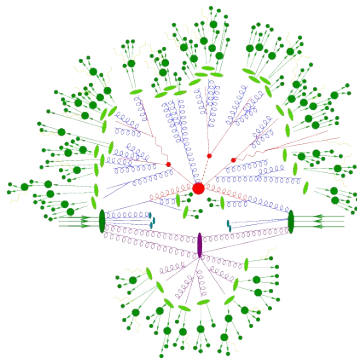
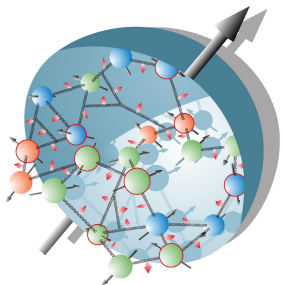


# Understanding the **emergent phenomena** of QCD



parts

*“In philosophy, systems theory, science, and art, emergence occurs when an **entity is observed** to have properties **its parts** do not have on their own, properties or behaviors which emerge only when the parts interact in a wider whole.” Wiki*



Observed entity

Hadron Structure

Hadron formation

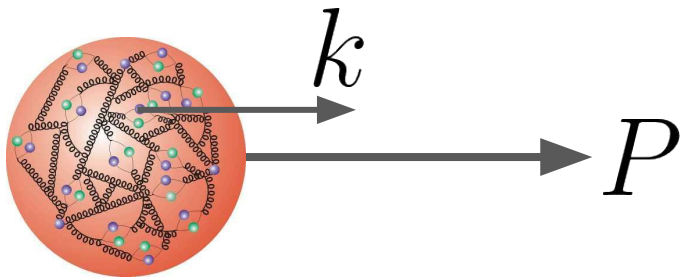
# An example of **hadron structure** (1D)

parton distribution  
function (PDF)

$$\xi = \frac{k^+}{P^+}$$

**Parton** momentum fraction relative to **parent hadron**

$$f_i(\xi) = \int \frac{dw^-}{4\pi} e^{-i\xi p^+ w^-} \langle N | \bar{\psi}_i(0, w^-, \mathbf{0}_T) \gamma^+ \psi_i(0) | N \rangle$$

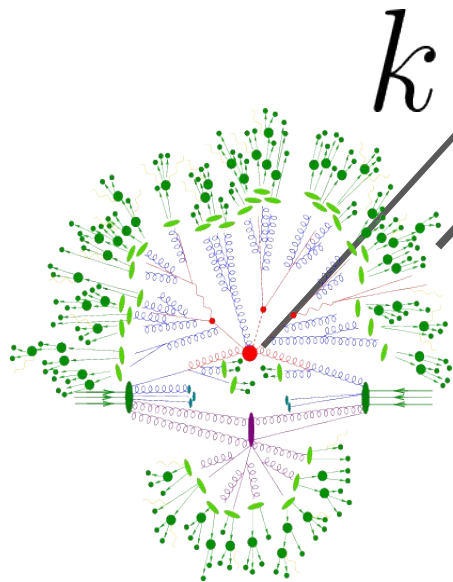


Interpretation in non-interacting QCD

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+) u_{k,\alpha} e^{-ik^+ x^- + ik_T \cdot x_T} + d_{k,\alpha}^\dagger(x^+) u_{k,-\alpha} e^{ik^+ x^- - ik_T \cdot x_T}$$

$$f_i(\xi) \sim \sum_{\alpha} \int d^2 k_T \langle N | \underbrace{b_{k,\alpha}^\dagger b_{k,\alpha}(\xi p^+, k_T, \alpha)}_{\text{number operator}} | N \rangle$$

# An example of **hadronization** (1D)



$$\zeta = \frac{p_h^+}{k^+}$$

**hadron** momentum fraction  
relative to **parent parton**

$$d_{h/j}(\zeta) \stackrel{!}{=} \frac{\text{Tr}_{\text{color, Dirac}}}{4N_{c,j}} \sum_X \zeta \int \frac{dw^+}{2\pi} e^{i(p_h^-/\zeta)w^+} \\ \times \gamma^- \langle 0 | \bar{\psi}_j(0, w^+, \mathbf{0}_T) | p_h, X \rangle \langle p_h, X | \psi_j(0) | 0 \rangle$$

**Fragmentation  
functions (FFs)**

**$X$**  = all states except detected hadron  **$h$**

# Hadron structure in **interacting** theory

Definition of PDFs in field theory requires renormalization

PDFs will depend on renormalization scale and its RGEs are the famous DGLAP equations

UV singularity when the field separation is zero

$$f_i(\xi) \stackrel{!}{=} \int \frac{dw^-}{4\pi} e^{-i\xi p^+ w^-} \langle N | \bar{\psi}_i(0, w^-, \mathbf{0}_T) \gamma^+ \psi_i(0) | N \rangle$$

Renormalization

$$f = Z_F \otimes f_{\text{bare}}$$
$$f(\xi) \rightarrow f(\xi, \mu)$$



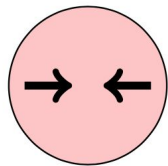
**D**okshitzer–**G**ribov–**L**ipatov–**A**ltarelli–**P**arisi

$$\frac{df_i(\xi, \mu^2)}{d \ln \mu^2} = \sum_j \int_{\xi}^1 \frac{dy}{y} P_{ij}(\xi, \mu^2) f_j\left(\frac{y}{\xi}, \mu^2\right)$$

aka **DGLAP**

# Other examples of hadron structures: **spin structures**

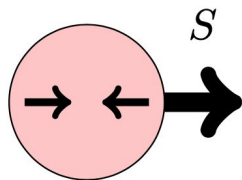
Unpolarized pdfs



$$f = f_{\rightarrow} + f_{\leftarrow}$$

$$\langle N | \bar{\psi}_i(0, w^-, \mathbf{0}_T) \gamma^+ \psi_i(0) | N \rangle$$

Helicity distribution

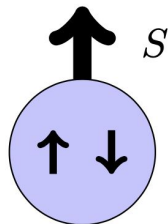


$$\Delta f = f_{\rightarrow} - f_{\leftarrow}$$

$$\langle N | \bar{\psi}_i(0, w^-, \mathbf{0}_T) \gamma^+ \gamma_5 \psi_i(0) | N \rangle$$

Spin  
crisis

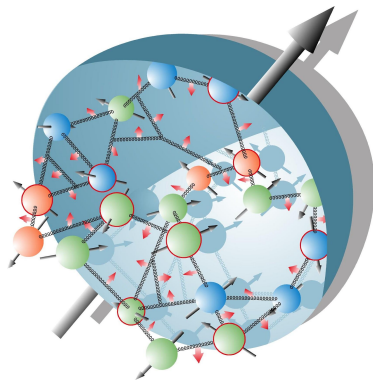
Transversity



$$\delta_T f = f_{\uparrow} - f_{\downarrow}$$

$$\langle N | \bar{\psi}_i(0, w^-, \mathbf{0}_T) \gamma^+ \gamma_{\perp} \gamma_5 \psi_i(0) | N \rangle$$

# Spin crisis

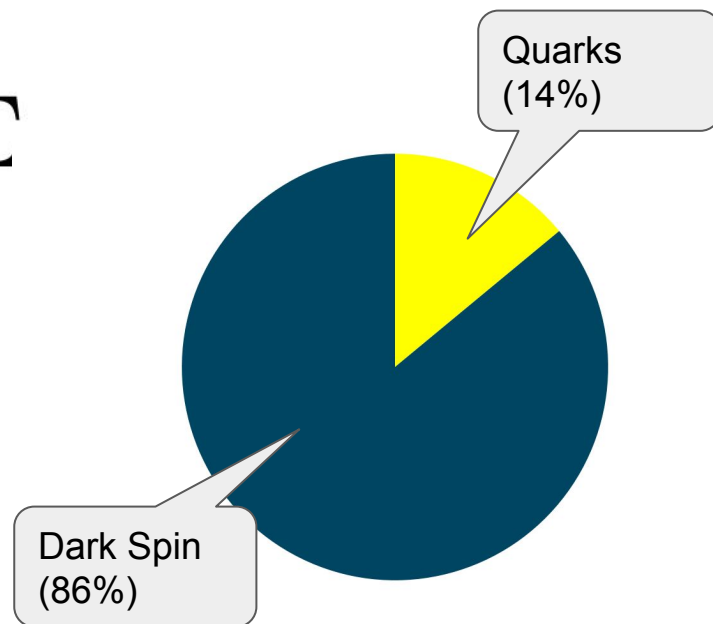


$$\frac{1}{2} = \overset{?}{\frac{1}{2}} \Delta \Sigma$$

$$\Delta \Sigma \sim 0.28(4)$$

NS, Melnitchouk, Kuhn, Ethier, Accardi ('15)

$$\Delta \Sigma = \sum_{i \in \text{quarks}} \int_0^1 d\xi \Delta q_i(\xi)$$



# Today's understanding

$$\frac{1}{2} = J_q + J_g$$

Accessible via moments of generalized parton distributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Moments of helicity pdfs



# Beyond 1D: Nuclear **femtography**

$$\xi = \frac{k^+}{P^+}$$

Parton distribution  
functions (PDFs)

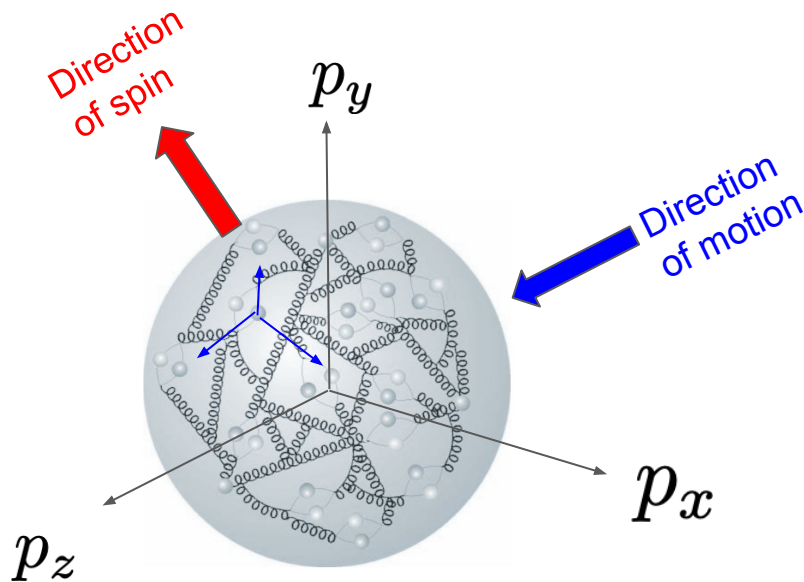
$$f(\xi)$$

Transverse  
momentum  
distributions (TMDs)

$$f(\xi, k_T)$$

Generalized parton  
distributions (GPDs)

$$f(\xi, b_T)$$



# 3D structures

TMDs

RGEs of TMDs are more complex -> Collins, Soper, Sterman (CSS)

$$f(\xi, k_T) = \int \frac{dw^- d^2 w_T}{16\pi^3} e^{-i\xi p^+ w^- + i k_T \cdot w_T} \langle N | \bar{\psi}_i(0, w^-, w_T) \gamma^+ \psi_i(0) | N \rangle$$

GPDs

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+ z^-} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^+ q(\frac{1}{2}z) | p \rangle \Big|_{z^+=0, \mathbf{z}=0} \\ &= \frac{1}{2P^+} \left[ H^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p) \right] \end{aligned}$$



F.T.  $f(\xi, b_T)$

# Ok, so how do we get these structures?

**Route 1:** Solve QCD on a supercomputer (lattice QCD)

eg.

$$\langle 0 | T \phi(x_1) \dots \phi(x_N) | 0 \rangle = \mathcal{N} \int [d\phi] e^{iS[\phi]} \phi(x_1) \dots \phi(x_N).$$

possible , but still in its infancy

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**Route 2:** Use high energy experimental reactions and QCD factorization



This talk

# High energy scattering

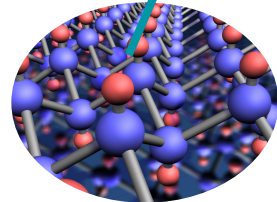
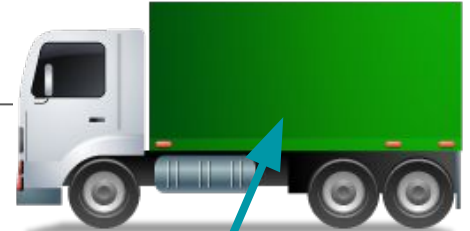
Want to see  
**internal structure**



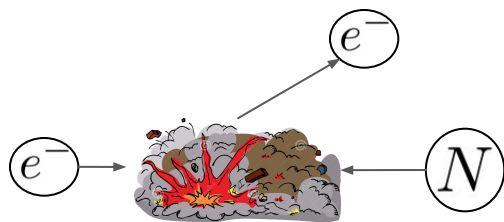
But we only see **debris**

What part of this is the  
**"internal structure"**?

**Factorization**

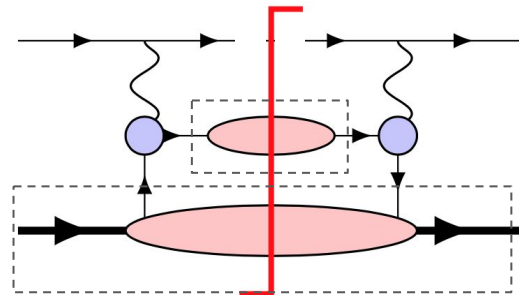


# Example: Deep-inelastic scattering (DIS)



Interpretation

$$E' \frac{d\sigma}{d^3l'}$$



Collision dependent factor

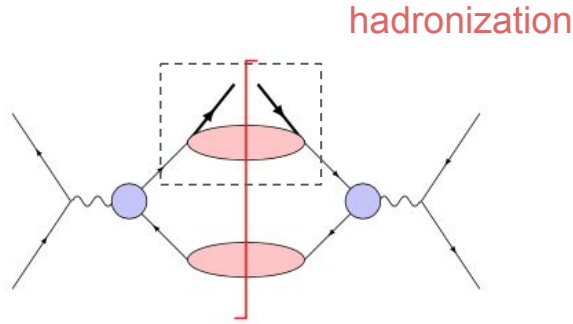
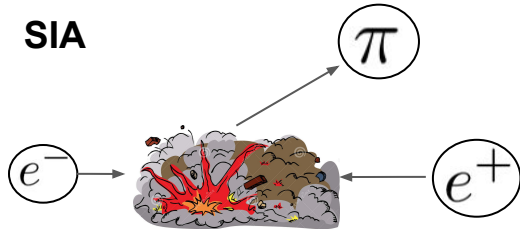
Internal structure

$$\sum_i \int_x^1 \frac{d\xi}{\xi} H_i(\xi) f_i\left(\frac{x}{\xi}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Error of approximations

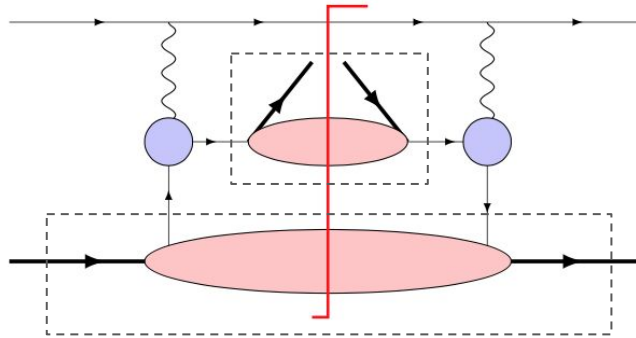
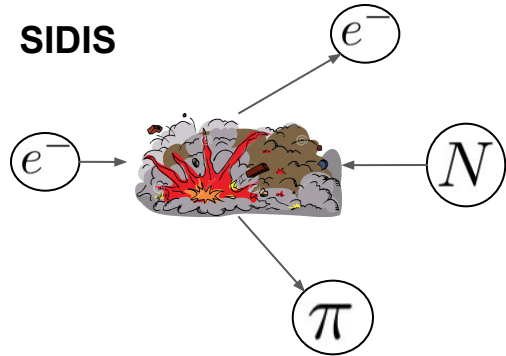
# Factorization in other reactions

**SIA**



$$d\sigma = \sum_i H_i^{\text{SIA}} \otimes d_i$$

**SIDIS**

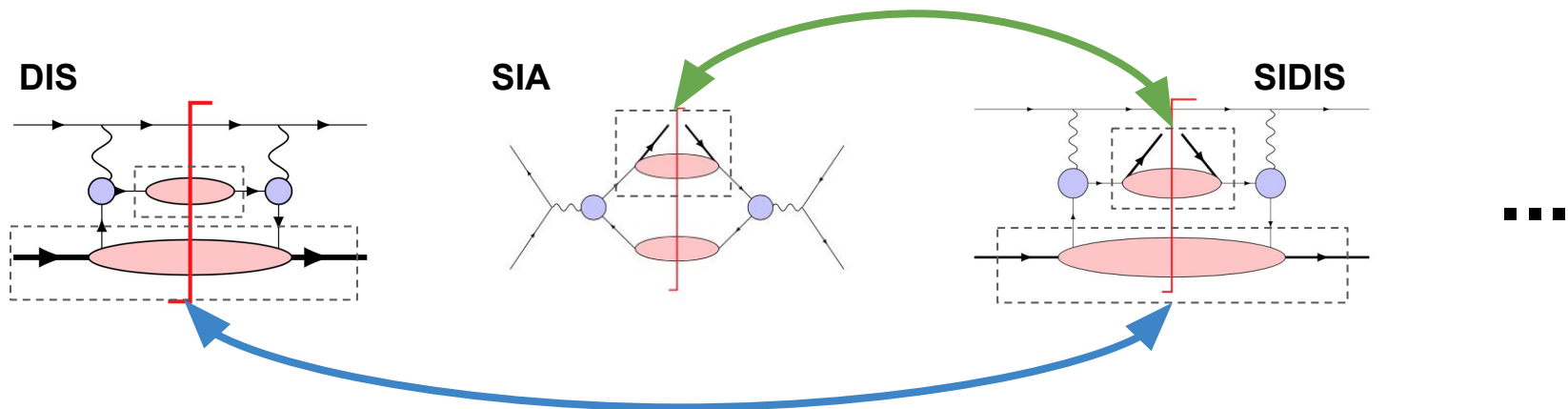


$$d\sigma = \sum_{ij} H_{ij}^{\text{SIDIS}} \otimes f_i \otimes d_j$$

structure + hadronization

..and many more

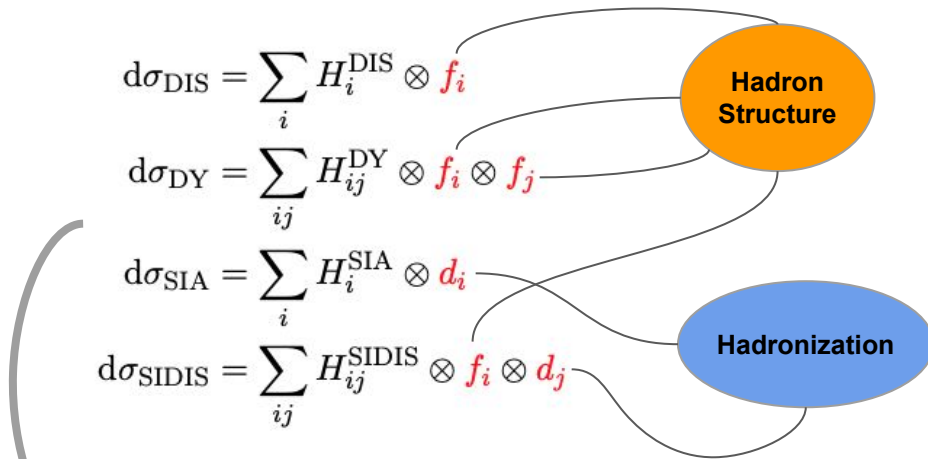
# Universality



cross sections described by **universal**  
**non-perturbative** functions, e.g. PDFs, FFs

# QCD global analysis

Experiments = theory + errors



RGE boundary conditions

$$f_i(\xi, \mu_0^2) = N_i \xi^{a_i} (1 - \xi)^{b_i} (1 + \dots)$$

$$d_i(\zeta, \mu_0^2) = N_i \zeta^{a_i} (1 - \zeta)^{b_i} (1 + \dots)$$

$$\mathbf{a} = (N_i, a_i, b_i, \dots)$$

Posterior distribution

Prior distribution

$$\rho(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data}) \pi(\mathbf{a})$$

Likelihood

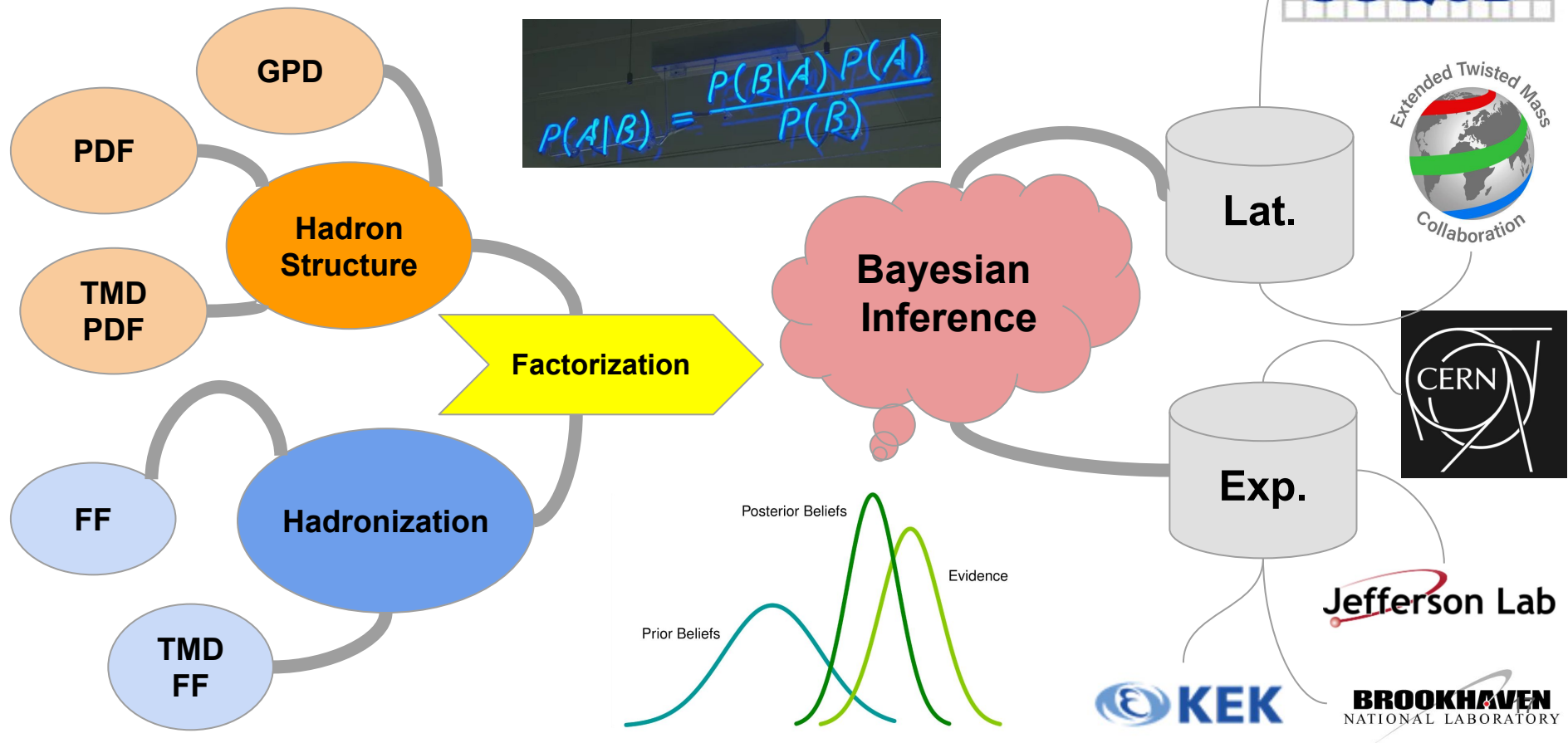
$$\mathcal{L}(\mathbf{a}, \text{data}) = \exp \left[ -\frac{1}{2} \chi^2(\mathbf{a}, \text{data}) \right]$$

$$\text{E}[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) f_i(\xi, \mu^2; \mathbf{a})$$

$$\text{V}[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) [f_i(\xi, \mu^2; \mathbf{a}) - \text{E}[f_i(\xi, \mu^2)]]^2$$

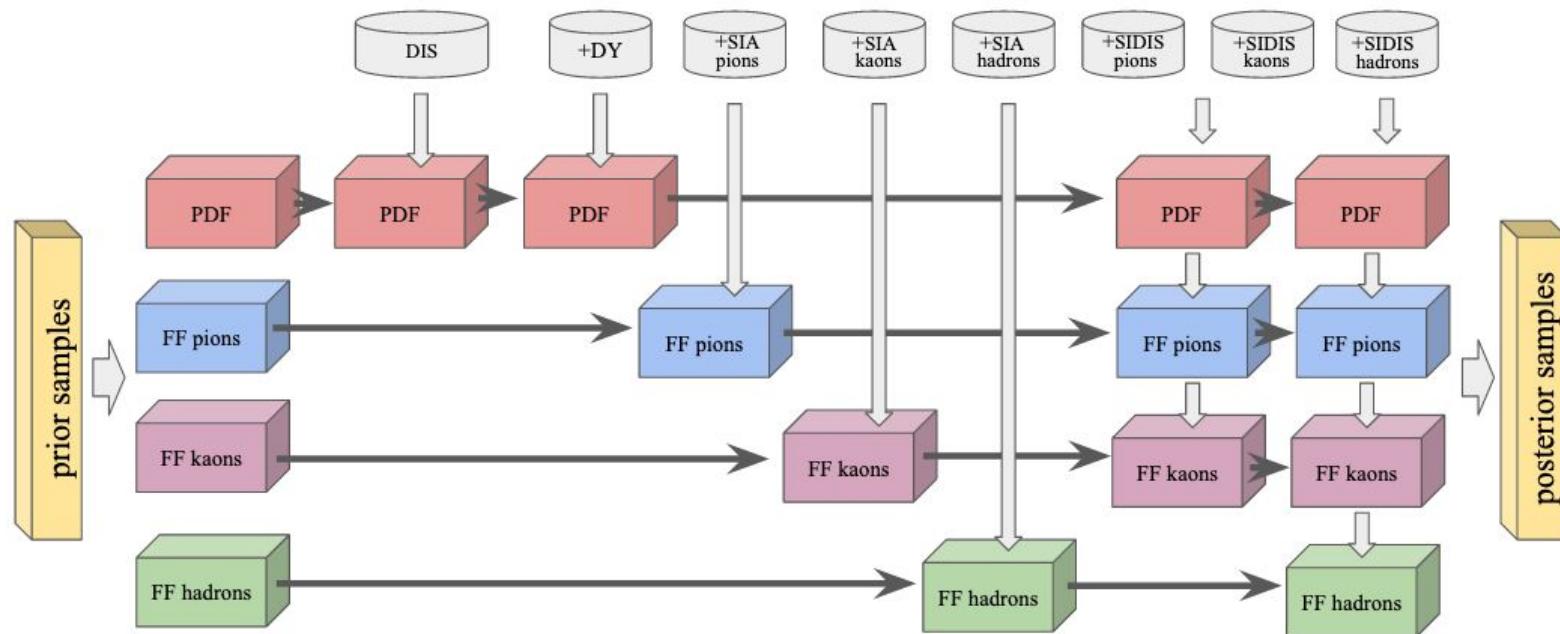


# The QCD global analysis paradigm



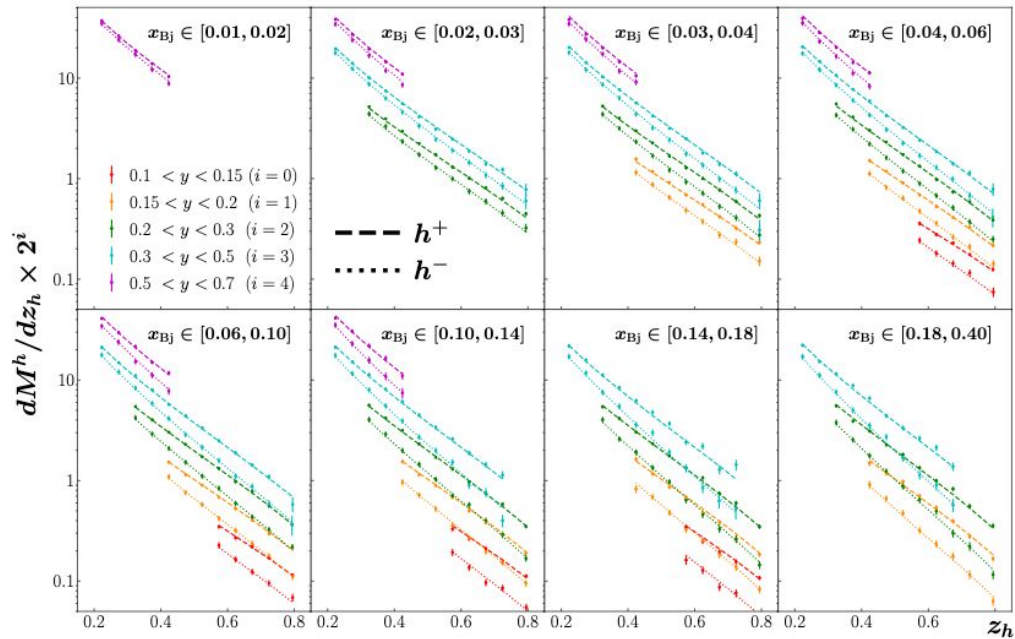
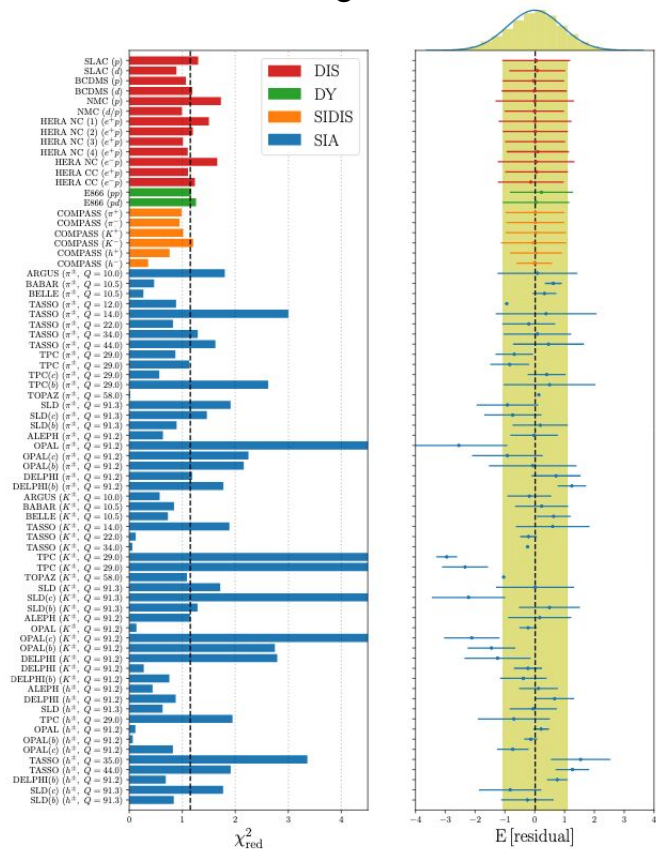
# An example: JAM20-SIDIS

Moffat, Melnitchouk, Rogers, NS



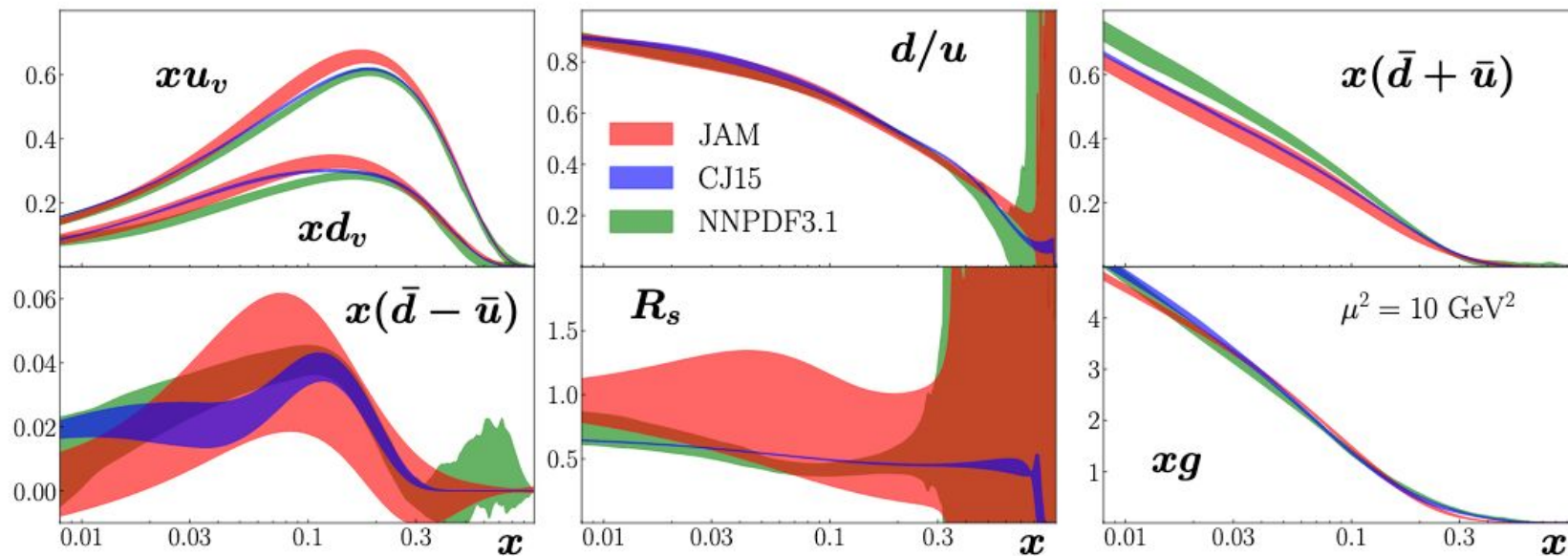
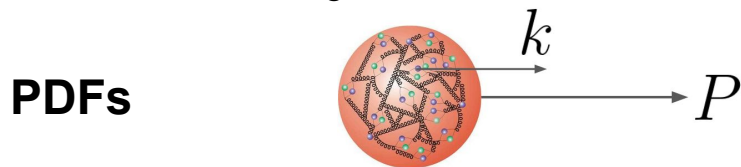
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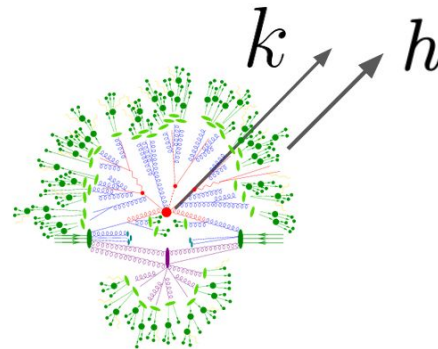
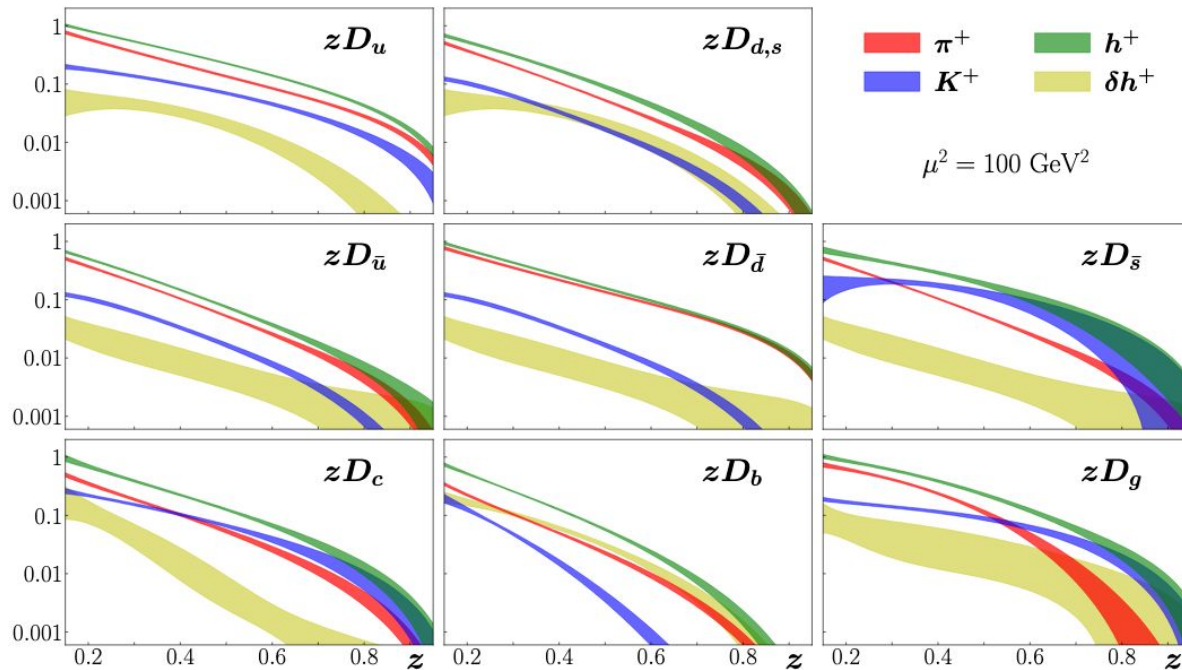
Moffat, Melnitchouk, Rogers, NS



# An example: JAM20-SIDIS

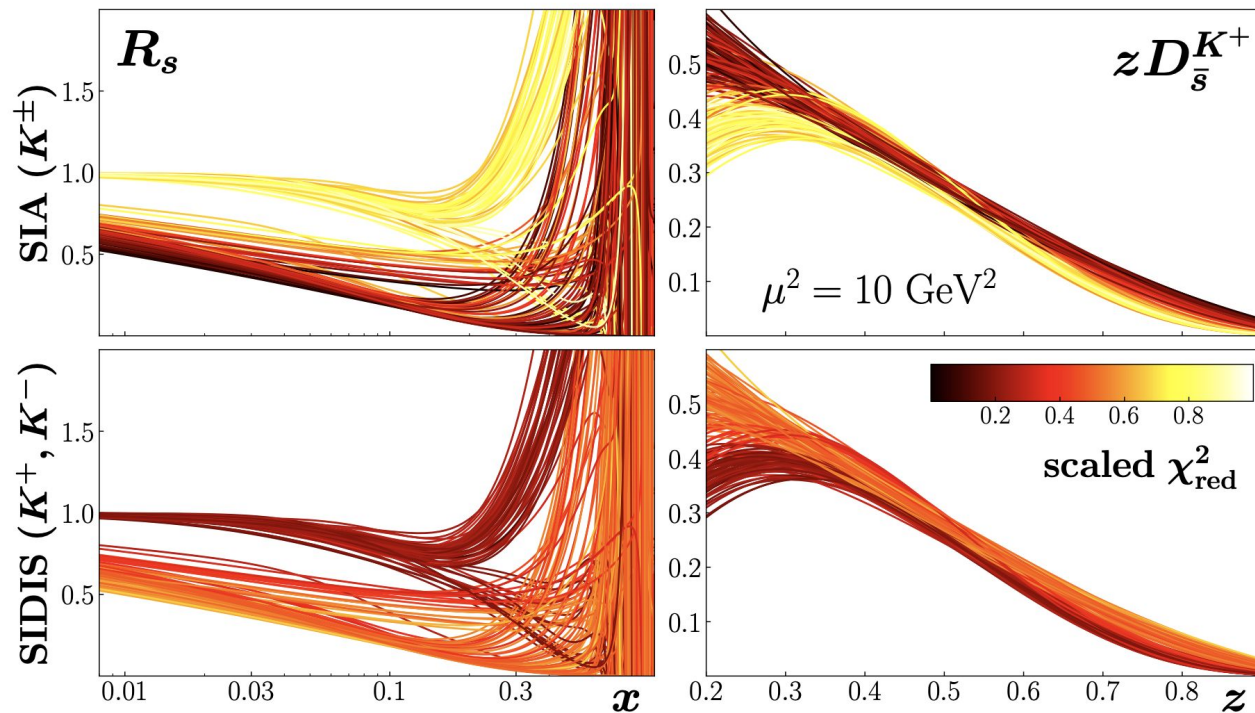
Moffat, Melnitchouk, Rogers, NS

FFs



# An example: JAM20-SIDIS

Moffat, Melnitchouk, Rogers, NS

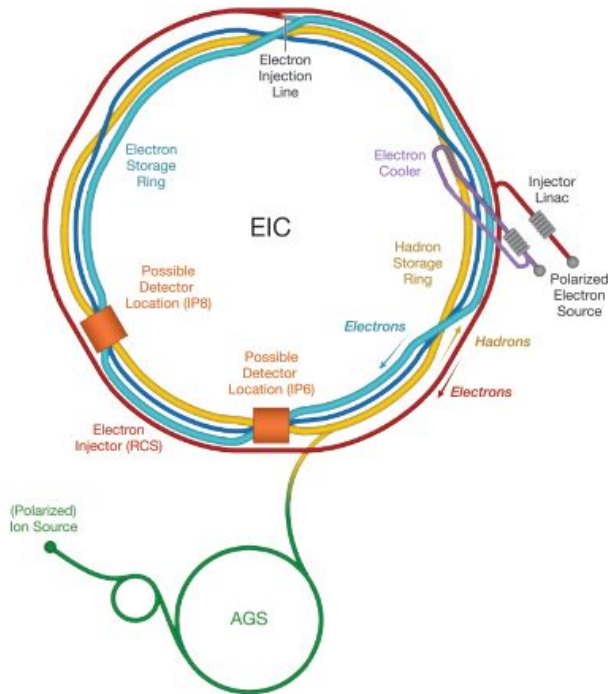
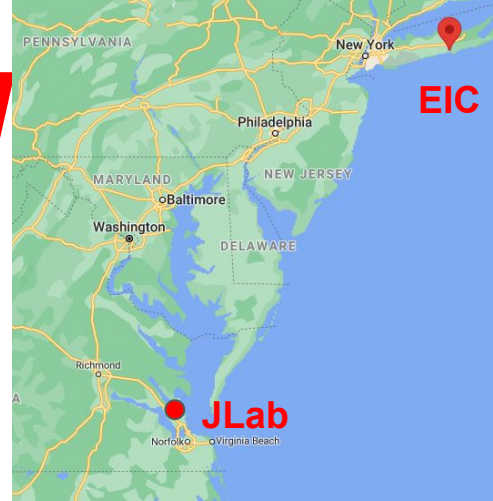


$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

The simultaneous fit of PDFs and FFs provides new insights on nucleon strangeness



# Summary: Nuclear **femtography** a worldwide effort



Jefferson Lab

**BROOKHAVEN**  
NATIONAL LABORATORY

