The proton radius (puzzle?) and its relatives
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Workshop: Perceiving the Emergence of Hadron Mass through AMBER@CERN-VI

GENESIS 1. The beginning

- In the beginning God created the quarks (ordinary matter) and made them interact through the strong forces, the SU(3) group.
- And God said, “I do not understand a damn thing” so he said “Let there be light”, and there was light, the U(1) gauge group.

We will study the strong interactions using light at very low energies: $q^2 \to 0$.

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2}F_i' + \dots$$

Observables: $ep \to ep$, $\mu p \to \mu p$, $\gamma p \to \gamma p$, atomic physics, muonic atoms, ...

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Scales (and ratios)

$m_p \sim \Lambda_\chi$
$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p + m_\mu}$
$m_r \alpha \sim m_e$
$\ldots$
$Q^2 \to 0$

Tool: Effective Field Theories $\equiv$ Factorization
Why?: There is a hierarchy of different scales
EFTs are especially useful in these situations.
1) Perturbative calculations much easier and systematic.
2) Nonperturbative information is parameterized in a model independent way.
3) Power counting.

Effective Field Theory: Non-relativistic protons, photons and (non-relativistic) electron/muons.

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Caswell-Lepage

$iD_0 = i\partial_0 + Z_p e A^0$, $i\mathbf{D} = i\boldsymbol{\nabla} - Z_p e \mathbf{A}$

$$\mathcal{L}_{\rm NRQED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + + \frac{d_2}{m_p^2} F_{\mu\nu} D^2 F^{\mu\nu}$$

$$+\psi_p^\dagger \Big\{ iD_0 + \frac{c_k}{2m_p}\mathbf{D}^2 + \frac{c_4}{8m_p^3}\mathbf{D}^4 + \frac{c_F^{(p)}}{2m_p}\boldsymbol{\sigma}\cdot e\mathbf{B} + \frac{c_D^{(p)}}{8m_p^2}(\mathbf{D}\cdot e\mathbf{E} - e\mathbf{E}\cdot\mathbf{D})$$

$$+i\frac{c_S^{(p)}}{8m_p^2}\boldsymbol{\sigma}\cdot(\mathbf{D}\times e\mathbf{E} - e\mathbf{E}\times\mathbf{D}) + c_{A_1}^{(p)} e^2 \frac{\mathbf{B}^2 - \mathbf{E}^2}{8m_p^3} - c_{A_2}^{(p)} e^2 \frac{\mathbf{E}^2}{8m_p^3} \Big\} \psi_p$$

$$+(\text{leptons})$$

$$-\frac{c_3^{(pe)}}{m_p m_e}\psi_p^\dagger\psi_p\psi_e^\dagger\psi_e + \frac{c_4^{(pe)}}{m_p m_e}\psi_p^\dagger\boldsymbol{\sigma}\psi_p\psi_e^\dagger\boldsymbol{\sigma}\psi_e + \cdots .$$

Dictionary (relation Wilson coefficients with low energy constants):

$c_F^{(p)} \to \mu_p$ anomalous magnetic moment (low energy constant)

$c_D \to r_p$ proton radius (quasi low energy constant)

$c_{A_i}^{(p)} \to \alpha_E, \beta_M$ Proton polarizabilities (quasi low energy constant)

$c_{3/4}^{(pe)} \to$ Two-photon exchange ...

Definition of the proton radius

$$\langle p', s|J^\mu|p, s\rangle = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p}\right]u(p)\,,$$

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$$r_p^2 = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

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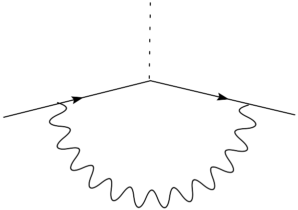
$$r_p^2(\nu) = 6\frac{d}{dq^2}G_{E,p}(q^2)|_{q^2=0}$$

Infrared divergent! $\rightarrow$ Wilson coefficient



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$$c_D(\nu) = 1 + 2F_2 + 8F_1' = 1 + 8m_p^2\left.\frac{dG_{p,E}(q^2)}{d\,q^2}\right|_{q^2=0}\,,$$

Standard definition (corresponds to the experimental number):

$$r_p^2 = \frac{3}{4}\frac{1}{m_p^2}\left(c_D(\nu) - c_{D,point-like}(\nu)\right)$$

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Theoretical setup (muonic hydrogen)

We use an effective field theory, Potential Non-Relativistic QED, which describes the muonic hydrogen dynamics and profits from the hierarchy $m_\mu \gg m_\mu\alpha \gg m_\mu\alpha^2$

$$\left. \begin{array}{l} \left(i\partial_0 - \frac{\mathbf{p}^2}{2m_r} - \frac{\alpha}{r}\right)\psi(\mathbf{r}) = 0 \\ + \text{corrections to the potential} \\ + \text{interaction with ultrasoft photons} \end{array} \right\} \text{potential NRQED} \qquad E \sim mv^2$$

$NRQED(m_\mu\alpha) \to pNRQED$

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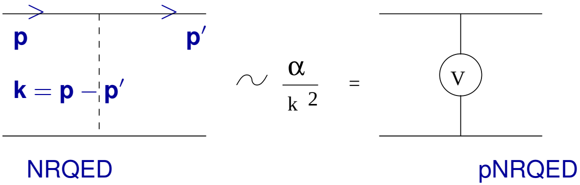
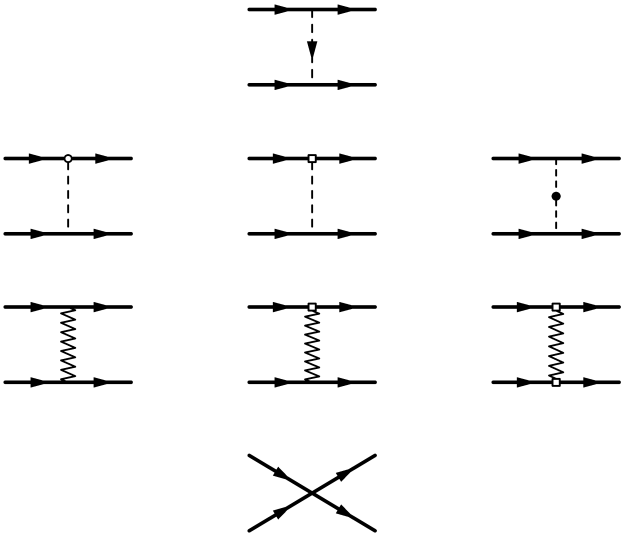
Matching NRQED to pNRQED



Hydrogen/Positronium/muonium
Tree level



Order $1/m^2$

$$\tilde{V}^{(b)} = \frac{\pi\alpha}{2}\left[Z_p \frac{c_D^{(\mu)}}{m_\mu^2} + Z_\mu \frac{c_D^{(p)}}{m_p^2}\right] ,$$

$$\tilde{V}^{(c)} = -i2\pi\alpha \frac{(\mathbf{p}\times\mathbf{k})}{\mathbf{k}^2}\cdot\left\{Z_p \frac{c_S^{(\mu)}\mathbf{s}_1}{m_\mu^2} + Z_\mu \frac{c_S^{(p)}\mathbf{s}_2}{m_p^2}\right\} ,$$

$$\tilde{V}^{(d)} = -Z_\mu Z_p 16\pi\alpha\left(\frac{d_2^{(\mu)}}{m_\mu^2} + \frac{d_2^{(\tau)}}{m_\tau^2} + \frac{d_{2,NR}}{m_p^2}\right) ,$$

$$\tilde{V}^{(e)} = -Z_\mu Z_p \frac{4\pi\alpha}{m_\mu m_p}\left(\frac{\mathbf{p}^2}{\mathbf{k}^2} - \frac{(\mathbf{p}\cdot\mathbf{k})^2}{\mathbf{k}^4}\right) ,$$

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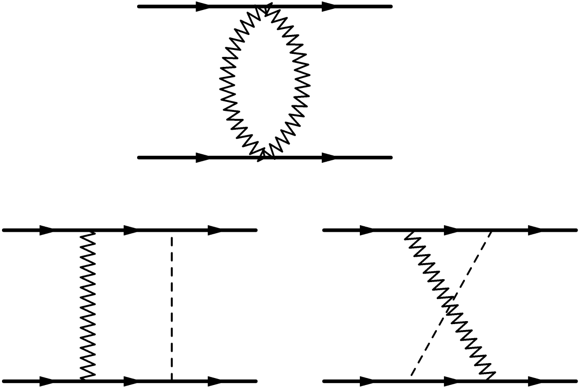
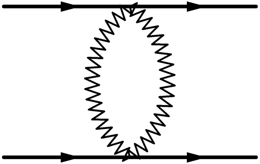
$$\tilde{V}^{(g)} = \frac{4\pi\alpha c_F^{(1)} c_F^{(2)}}{m_\mu m_p}\left(\mathbf{s}_1\cdot\mathbf{s}_2 - \frac{\mathbf{s}_1\cdot\mathbf{k}\mathbf{s}_2\cdot\mathbf{k}}{\mathbf{k}^2}\right) ,$$

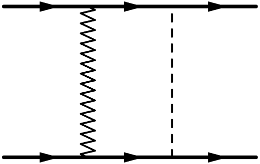
$$\tilde{V}^{(h)} = -\frac{1}{m_p^2}\left\{(c_3^{pl_i} + 3c_4^{pl_i}) - 2c_4^{pl_i}\mathbf{S}^2\right\} .$$

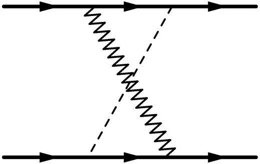


$$\tilde{V}^{(a)}_{1\,loop} = \frac{Z_\mu^2 Z_p^2 \alpha^2}{m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} - \frac{8}{3}\log 2 + \frac{5}{3}\right),$$

$$\tilde{V}^{(b,c)}_{1\,loop} = \frac{4 Z_\mu^2 Z_p^2 \alpha^2}{3 m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} + 2\log 2 - 1\right).$$







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Muonic Hydrogen: electron vacuum polarization

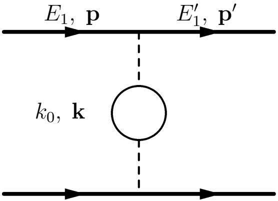

Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

$$\tilde{V}^{(0)} \equiv -4\pi Z_\mu Z_p \alpha_V(k) \frac{1}{\mathbf{k}^2},$$

$$\alpha_{eff}(k) = \alpha \frac{1}{1 + \Pi(-\mathbf{k}^2)},$$

where

$$\Pi(k^2) = \alpha \Pi^{(1)}(k^2) + \alpha^2 \Pi^{(2)}(k^2) + \alpha^3 \Pi^{(3)}(k^2) + \dots$$

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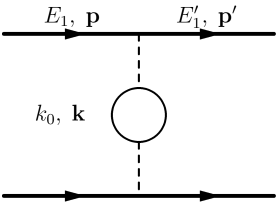

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Order $1/m^2$ from energy-dependent terms

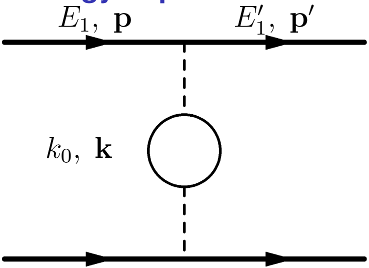


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E'_1$.

$$\delta \tilde{V}_E = -\frac{Z_\mu Z_p e^2}{4 m_\mu m_p} \frac{(\mathbf{p}^2 - \mathbf{p}'^2)^2}{\mathbf{k}^2} \frac{\alpha}{\pi} m_e^2 \int_4^\infty d(q^2) \frac{1}{(m_e^2 q^2 + \mathbf{k}^2)^2} u(q^2) \,.$$

$$u(q^2) = \frac{1}{3}\sqrt{1 - \frac{4}{q^2}}\left(1 + \frac{2}{q^2}\right) \,.$$

Theoretical setup.

Muonic hydrogen Lamb shift: $\Delta E_L \equiv E(2P_{3/2}) - E(2S_{1/2})$ and hyperfine splitting: $\Delta E_{HF} \equiv E(nS_{3/2}) - E(nS_{1/2})$

$$L_{pNRQED} = \int d^3\mathbf{r} d^3\mathbf{R} dt S^\dagger(\mathbf{r},\mathbf{R},t)\Big\{ i\partial_0 - \frac{\mathbf{p}^2}{2m_r}$$

$$-V(\mathbf{r},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) + e\mathbf{r}\cdot\mathbf{E}(\mathbf{R},t)\Big\} S(\mathbf{r},\mathbf{R},t) - \int d^3\mathbf{r}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$

$$V(\mathbf{r},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m_\mu} + \frac{V^{(2)}(r)}{m_\mu^2} + \ldots$$

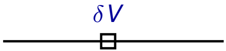
Observable: Spectrum or decays

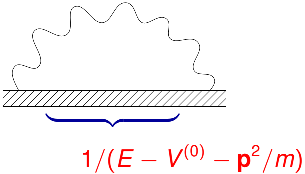
Corrections to the Green Function ($h_s^{(0)} = \mathbf{p}^2/m + V^{(0)}$)

$$G_s(E) = P_s \frac{1}{h_s^{(0)} - H_I - E} P_s = G_s^{(0)} + \delta G_s \qquad G_s^{(0)}(E) = \frac{1}{h_s^{(0)} - E}$$

A) Ultrasoft loops (lamb shift-like): $\mathbf{x} \cdot \mathbf{E} \leftarrow$

B) Quantum mechanics perturbation theory$\leftarrow$





Vacuum polarization effects: $\mathcal{O}(m_r\alpha^3)$

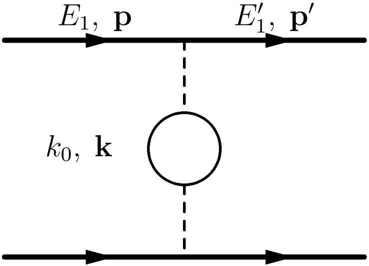


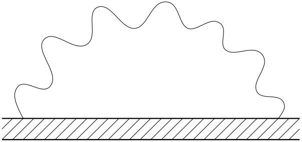
Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

1-loop static potential

$$E_{LO} = \langle n|\delta V|n\rangle = 205.0074\ \mathrm{meV} = \mathcal{O}(m_r\alpha^3)$$

$E_L \propto \beta_0$. Measure of (Non)-Asymptotically free theory!

Ultrasoft effects: $\mathcal{O}(m\alpha^5)$



$$\Delta E = -0.6677\ meV$$

$$\mathcal{O}(m\alpha^5 \frac{m_\mu}{m_p}) : \qquad \Delta E = -0.045\ meV$$

All (soft+ultrasoft):

$$\Delta E = -0.71896\ \mathrm{meV}.$$

Start the overlap with hadronic effects.

Hadronic corrections

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} \delta^3(\mathbf{r}) \to \Delta E \sim \frac{1}{m_p^2} D_d^{had.} (m_r \alpha)^3$$

$$D_d^{(p\mu)} = -c_3 - 16\pi\alpha d_2 + \frac{\pi\alpha}{2} c_D^{(p)}$$

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} (\mathbf{S}_1 + \mathbf{S}_2)^2 \delta^3(\mathbf{r})$$

$$D_s^{had.} = 2c_4$$

c_3, c_4, d_2, $c_D^{(p)}$, ... matching coefficients of NRQED.

$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2} F_{\mu\nu} D^2 F^{\mu\nu} - e\frac{c_D}{m_p^2} N_p^\dagger \boldsymbol{\nabla} \cdot \mathbf{E} N_p - \frac{c_3}{m_p^2} N_p^\dagger N_p \mu^\dagger \mu + \frac{c_4}{m_p^2} N_p^\dagger \boldsymbol{\sigma} N_p \mu^\dagger \boldsymbol{\sigma} \mu$$

Muonic hydrogen

$$
\begin{aligned}
\Delta E_L &= 206.0243\,\mathrm{meV} \\
&- \left[\frac{1}{\pi}\frac{m_r^3\alpha^3}{8}\right]\frac{\alpha}{M^2}\frac{r_p^2}{\mathrm{fm}^2}\left[47.3525 + 35.1491\alpha + 47.3525\alpha^2\ln(1/\alpha)\right] \\
&+ \left[\frac{1}{\pi}\frac{m_r^3\alpha^3}{8}\right]\frac{1}{M^2}\left[c_3^{\mathrm{had}} + 16\pi\alpha d_2^{\mathrm{had}}\right] \\
&+\mathcal{O}(m_r\alpha^6)\,.
\end{aligned}
$$

Hydrogen

$$E^{(\rm fs)}_{n\ell j} = m_r[f_{nj} - 1 + \frac{(Z\alpha)^2}{2n^2}] - \frac{m_r^2}{2(m_e+M)}[f_{nj}-1]^2 + E^{\rm EFT}_{n\ell j} + E^{(6)}_{n\ell j} + E^{(7)}_{n\ell j} + E^{(8)}_{n\ell j} ,$$

where $f_{nj} = [1+(Z\alpha)^2(n-\delta)^{-2}]^{-1/2}$ (with $\delta = j + \frac{1}{2} - [(j+\frac{1}{2})^2 - (Z\alpha)^2]^{1/2}$).

$$\begin{aligned}
E^{\rm EFT}_{n\ell j} &= \frac{m_r(Z\alpha)^4}{2n^3}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n} + \frac{m_r}{(m_e+M)}\frac{1}{4n}\right) \\
&+ \frac{(Z\alpha)^4}{8n^3}\left[m_r\left(\frac{3}{n} - \frac{8}{2\ell+1}\right) - \frac{m_r^3}{m_eM}\left(\frac{1}{n} + \frac{8}{2\ell+1} + \frac{32\alpha}{3\pi}\frac{(m_eZ+M)^2}{m_eM}\ln R(n,\ell)\right)\right] \\
&+ \left[\frac{\pi\alpha}{2m_eM}\left(\frac{Zc_D^{(e)}M}{m_e} + \frac{c_D^{(p)}m_e}{M}\right) - 16\pi Z\alpha\left(\frac{d_2^{(e)}}{m_e^2} + \frac{d_2^{(\mu)}}{m_\mu^2} + \frac{d_2}{M^2}\right) - \frac{c_3}{M^2}\right. \\
&+ \frac{2\pi Z\alpha}{m_eM}\left(1 + \frac{4\alpha Z}{3\pi} - \frac{7Z\alpha}{3\pi}\left(\frac{1}{2n} - H_n + \ln\frac{\nu n}{2\alpha m_r Z}\right)\right) \\
&\left.+ Z\alpha^2\left(\frac{1}{m_e} + \frac{Z}{M}\right)^2\left(\frac{10}{9} - \frac{4}{3}\ln\frac{\alpha^2 m_r Z^2}{\nu}\right)\right]\frac{(\alpha m_r Z)^3}{\pi n^3}\delta_{\ell 0} \\
&+ \left[X_{LS_e}\left(\frac{Zc_F^{(e)}}{m_eM} + \frac{Zc_S^{(e)}}{2m_e^2}\right) + \frac{Z}{2m_eM}\left((\ell^2+\ell) - \frac{7Z\alpha}{3\pi}\right)\right]\frac{2(1-\delta_{\ell 0})}{(\ell^2+\ell)(2\ell+1)}\frac{\alpha(Z\alpha m_r)^3}{n^3}
\end{aligned}$$

HOW to determine the Two-Photon Exchange correction?

- Dispersion relations + modelling
- lattice (not yet)
- Chiral perturbation theory ($\rightarrow$ (non-analytic) m_q dependence, N_c dependence)
- ....

$$HBET(m_\pi/m_\mu) \rightarrow NRQED(m_\mu\alpha) \rightarrow pNRQED$$

HBET (m_π)

$$\mathcal{L}_{HBET} = \mathcal{L}_\gamma + \mathcal{L}_l + \mathcal{L}_\pi + \mathcal{L}_{l\pi} + \mathcal{L}_{(N,\Delta)} + \mathcal{L}_{(N,\Delta)l} + \mathcal{L}_{(N,\Delta)\pi} + \mathcal{L}_{(N,\Delta)l\pi},$$

$$\mathcal{L}_\gamma = -\frac{1}{4}F^2 + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} + \cdots$$

$$\mathcal{L}_\pi = \frac{F_\pi^2}{4}\mathrm{Tr}\,[D_\mu U D^\mu U] + \cdots \qquad U = u^2 = e^{i\frac{\Pi}{F_\pi}}$$

$$\mathcal{L}_N = N^\dagger(iv^\mu\nabla_\mu + g_A u_\mu S^\mu)N + \cdots + (\Delta) + \cdots - e\frac{c_D}{m_p^2}N_p^\dagger \boldsymbol{\nabla}\cdot\mathbf{E}N_p$$

$$D_\mu = \partial_\mu + ieQA_\mu \qquad \nabla_\mu = \partial_\mu + \Gamma_\mu \qquad u_\mu = iu^\dagger(\nabla_\mu U)u$$

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu + ieQA_\mu)u + u(\partial_\mu + ieQA_\mu)u^\dagger\right\}$$

$$\mathcal{L}_{N,l} = \frac{1}{m_p^2}\sum_i c_{3,R}^{pl_i}\bar{N}_p\gamma^0 N_p \bar{l}_i\gamma^0 l_i + \frac{1}{m_p^2}\sum_i c_{4,R}^{pl_i}\bar{N}_p\gamma^j N_p \bar{l}_i\gamma_j l_i$$

$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} - e\frac{c_D}{m_p^2}N_p^\dagger\boldsymbol{\nabla}\cdot\mathbf{E}N_p - \frac{c_3}{m_p^2}N_p^\dagger N_p\mu^\dagger\mu + \frac{c_4}{m_p^2}N_p^\dagger\sigma N_p\mu^\dagger\sigma\mu$$

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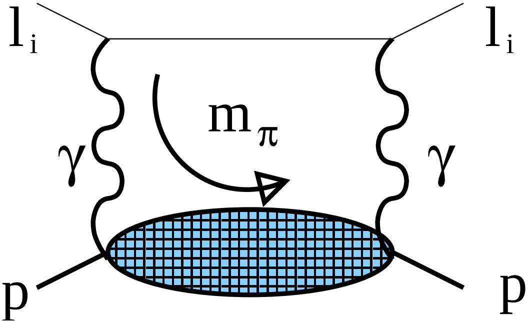
$$\delta\mathcal{L} = \cdots + \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} - e\frac{c_D}{m_p^2}N_p^\dagger\boldsymbol{\nabla}\cdot\mathbf{E}N_p - \frac{c_3}{m_p^2}N_p^\dagger N_p \mu^\dagger\mu + \frac{c_4}{m_p^2}N_p^\dagger\boldsymbol{\sigma}N_p\mu^\dagger\boldsymbol{\sigma}\mu$$

$c_3^{\rm had} =$

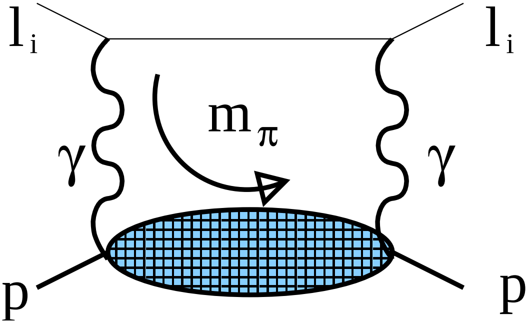


$$T^{\mu\nu} = i \int d^4x\, e^{iq\cdot x} \langle p, s | T J^\mu(x) J^\nu(0) | p, s \rangle ,$$

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) S_1(\rho, q^2) + \frac{1}{m_p^2} \left(p^\mu - \frac{m_p \rho}{q^2} q^\mu \right) \left(p^\nu - \frac{m_p \rho}{q^2} q^\nu \right) S_2(\rho, q^2)$$

$$S_1 = ?? \qquad S_2 = ??$$

$c_3^{\rm had} =$



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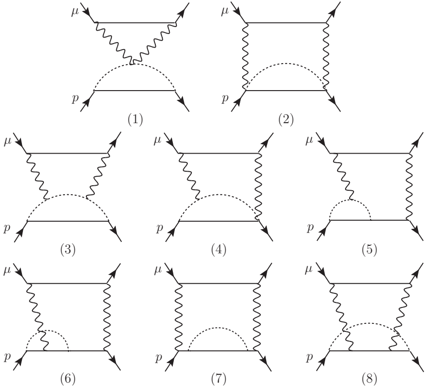
TWO-PHOTON EXCHANGE correction



m_μ extra suppression+χPT (Model independent)

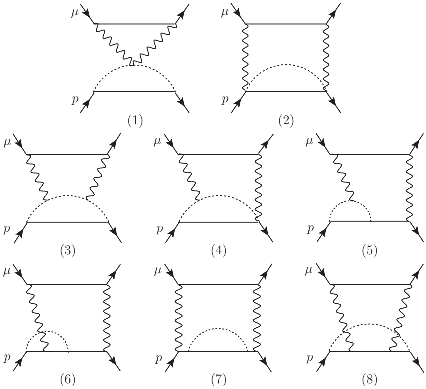
Power-like chiral enhanced ($\rightarrow$ χPT can predict the leading order!)

$$c_3^{\rm had} \sim \alpha^2 \frac{m_\mu}{m_\pi} + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) \qquad \delta E \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \frac{m_\mu}{m_\pi})$$

Error ($\Delta = M_\Delta - M_p \sim 300$ MeV): LO $\times \frac{m_\pi}{\Delta} \simeq$ LO $\times \frac{1}{2}$

$\rightarrow c_3^{\rm had} = \alpha^2 \frac{m_\mu}{m_\pi} 47.2(23.6)$

TWO-PHOTON EXCHANGE correction



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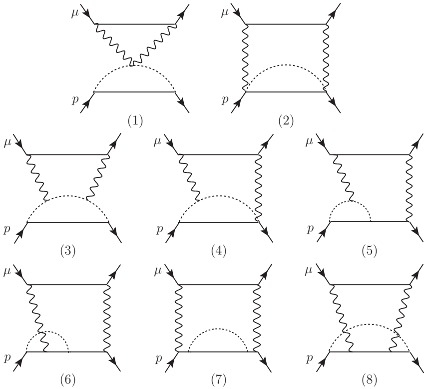

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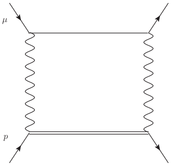
Large N_c. Including the Δ particle

Error:

$$\frac{m_\mu}{\Delta} \sim N_c \frac{m_\mu}{\Lambda_{QCD}} \to N_c \frac{m_\mu}{\Lambda_{QCD}} \sim \frac{1}{3}$$



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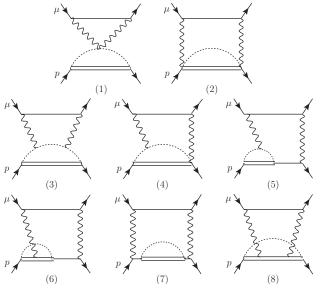

$$c_3^{\rm had} \sim \alpha^2 \frac{m_\mu}{m_\pi}\left[1 + \# \frac{m_\pi}{\Delta} + \cdots\right] + \mathcal{O}\left(\alpha^2 \frac{m_\mu}{\Lambda_{QCD}}\right) = \alpha^2 \frac{m_\mu}{m_\pi}\begin{cases} 47.2(23.6) & (\pi), \\ 56.7(20.6) & (\pi + \Delta), \end{cases}$$

$$\Delta E_{\rm TPE} = 28.59(\pi) + 5.86(\pi\&\Delta) = 34.4(12.5)\mu{\rm eV} \;\; ({\rm Peset\&AP})\,.$$

(Model dependent+DR: $\Delta E_{\rm TPE} = 33(2)\mu{\rm eV}$ (Birse-McGovern))

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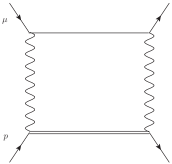

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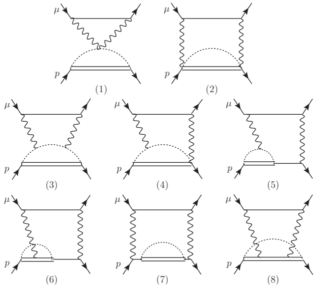

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$$\Delta E_{\rm TPE} \sim m_\mu \alpha^5 \times \frac{m_\mu^2}{(4\pi F_\pi)^2} \times \frac{m_\mu}{m_\pi} \sum_{n=0}^{\infty} c_n (N_c \sqrt{m_q})^n$$

$$\frac{\#}{\sqrt{m_q}} + ? + ?\sqrt{m_q} + \cdots$$

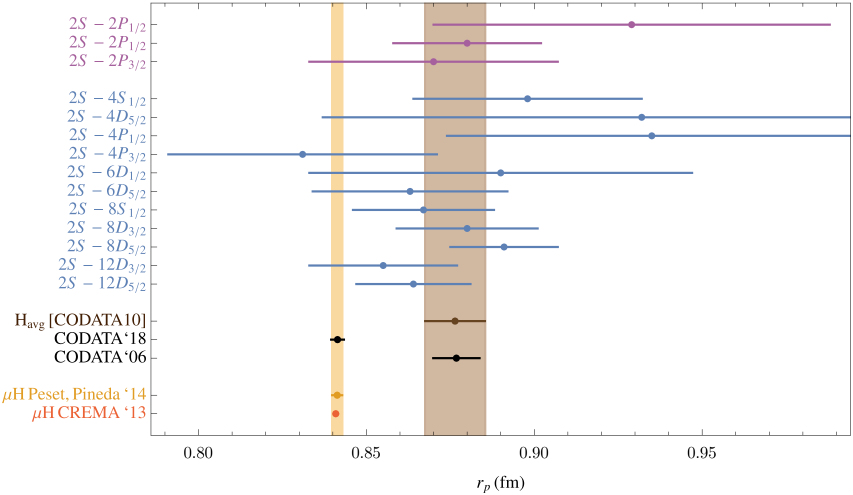
plus large N_c

$$\frac{\#}{\sqrt{m_q}} + \left[\#N_c + ? + \frac{?}{N_c} + \cdots\right] + \left[\#N_c^2 + ?N_c + ? + \cdots\right]\sqrt{m_q} + \cdots$$

? $\rightarrow$ Size of the counterterm in HBET

The proton radius in muonic and hydrogen spectroscopy (Lamb shift)

Old Experimental measurements



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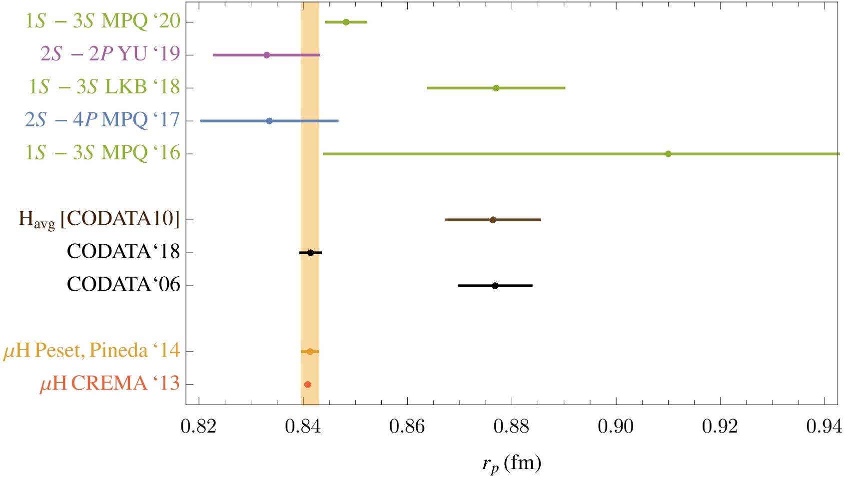

Hyperfine: Hydrogen and muonic hydrogen

Experiment:

$$E^{\rm exp}_{\rm hyd,HF}(1S) = 1420.405751768(1)\ {\rm MHz}\,,$$

$$E^{\rm exp}_{\mu p,{\rm HF}}(2S) = 22.8089(51)\ {\rm meV}\,.$$

Theory:

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.} (\mathbf{S}_1 + \mathbf{S}_2)^2 \delta^3(\mathbf{r})$$

$$D_s^{had.} = 2c_4$$

c_4, matching coefficient of NRQED.

$$HBET(m_\pi/m_\mu) \to NRQED(m_\mu \alpha) \to pNRQED$$

$$\delta\mathcal{L} = \cdots - \frac{c_4}{m_p^2} N_p^\dagger \boldsymbol{\sigma} N_p \mu^\dagger \boldsymbol{\sigma} \mu$$

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c_4, Spin-dependent effects

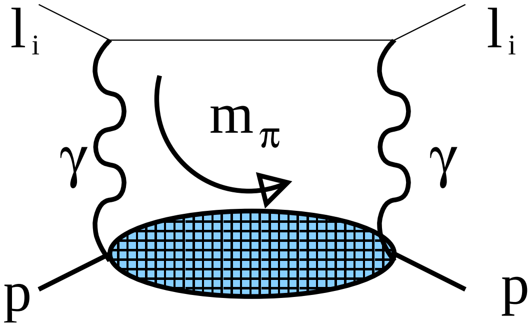


Figure: *Symbolic representation (plus permutations) of the spin-dependent correction.*

$$c_4^{pl} = -\frac{ig^4}{3}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2}\frac{1}{k^4-4m_l^2k_0^2}\left\{A_1(k_0,k^2)(k_0^2+2k^2)+3k^2\frac{k_0}{m_p}A_2(k_0,k^2)\right\}$$

Drell-Sullivan(67)

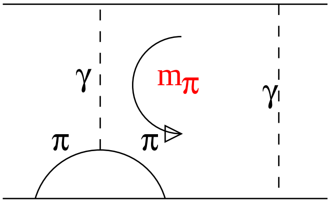
$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^\mu(x)J^\nu(0)|p,s\rangle\,,$$

which has the following structure ($\rho = q\cdot p/m$):

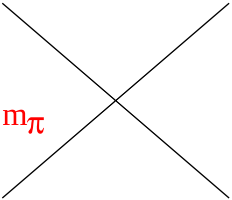
$$\begin{aligned} T^{\mu\nu} \quad = \quad & \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)S_1(\rho,q^2) \\ & +\frac{1}{m_p^2}\left(p^\mu-\frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu-\frac{m_p\rho}{q^2}q^\nu\right)S_2(\rho,q^2) \\ & -\frac{i}{m_p}\,\epsilon^{\mu\nu\rho\sigma}q_\rho s_\sigma A_1(\rho,q^2) \\ & -\frac{i}{m_p^3}\,\epsilon^{\mu\nu\rho\sigma}q_\rho\left((m_p\rho)s_\sigma-(q\cdot s)p_\sigma\right)A_2(\rho,q^2) \end{aligned}$$

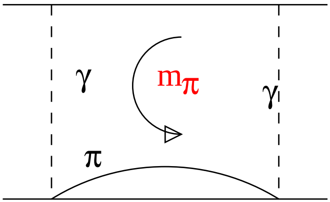
A_1, A_2 (χPT): Ji-Osborne; Peset-Pineda

Leading chiral logs to the hyperfine splitting

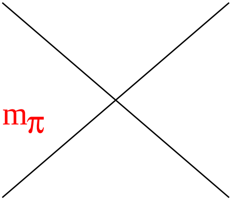


$$\sim \frac{1}{f_\pi^{\,2}} \ln m_\pi$$





$$\sim \frac{1}{f_\pi^{\,2}} \ln m_\pi$$



$$\delta V = 2\frac{c_4}{m_p^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r})\,.$$

$$\delta E_{HF} \sim \mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \ln m_\pi)$$

The leading chiral logs can be determined for Hydrogen and muonic hydrogen hyperfine splitting (AP).

$$\begin{aligned} c_4^{pl_i} \simeq & \left(1 - \frac{\mu_p^2}{4}\right) \alpha^2 \ln \frac{m_{l_i}^2}{\nu^2} + \frac{b_{1,F}^2}{18} \alpha^2 \ln \frac{\Delta^2}{\nu^2} \\ & + \frac{m_p^2}{(4\pi F_0)^2} \alpha^2 \frac{2}{3} \left(\frac{2}{3} + \frac{7}{2\pi^2} \right) \pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2} \\ & + \frac{m_p^2}{(4\pi F_0)^2} \alpha^2 \frac{8}{27} \left(\frac{5}{3} - \frac{7}{\pi^2} \right) \pi^2 g_{\pi N \Delta}^2 \ln \frac{\Delta^2}{\nu^2} \\ \overset{(N_c \to \infty)}{\simeq} & \alpha^2 \ln \frac{m_l^2}{\nu^2} + \frac{m_p^2}{(4\pi F_0)^2} \alpha^2 \pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2} . \end{aligned}$$

$$E_{\rm HF} = 4 \frac{c_4^{pl_i}}{m_p^2} \frac{1}{\pi} (\mu_{l_i p} \alpha)^3 \sim m_{l_i} \alpha^5 \frac{m_{l_i}^2}{m_p^2} \times (\ln m_q, \ln \Delta, \ln m_{l_i}) .$$

$$c_4^{pl_i} = c_{4,\rm R}^{pl_i} + c_{4,\rm point-like}^{pl_i} + c_{4,\rm Born}^{pl_i} + c_{4,\rm pol}^{pl_i} + \mathcal{O}(\alpha^3) .$$

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&\quad + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{2}{3}\left(\frac{2}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2} \\
&\quad + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2 \ln\frac{\Delta^2}{\nu^2} \\
&\overset{(N_c\to\infty)}{\simeq} \quad \alpha^2\ln\frac{m_l^2}{\nu^2} + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2\pi^2 g_A^2\ln\frac{m_\pi^2}{\nu^2}\,.
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Fixing c_4^{pe}. Hydrogen

Hydrogen. By fixing the scale $\nu = m_\rho$ we obtain the following number for the total sum in the SU(2) case:

$$E_{\mathrm{HF,logarithms}}(m_\rho) = -0.031\ \mathrm{MHz}\,,$$

which accounts for approximately 2/3 of the difference between theory (pure QED) and experiment.

$$E_{\mathrm{HF}}(QED) - E_{\mathrm{HF}}(exp) = -0.046\ \mathrm{MHz}.$$

What is left gives the expected size of the counterterm. Experimentally what we have is $c^{pe}_{4,NR} = -48.69(3)\alpha^2$ and $c^{pe}_{4,R}(m_\rho) \simeq c^{p}_{4,R}(m_\rho) \simeq -16\alpha^2$.

$$c_{4,\mathrm{TPE}}^{p\mu} = c_{4,\mathrm{TPE}}^{pe} + [c_{4,\mathrm{TPE}}^{p\mu} - c_{4,\mathrm{TPE}}^{pe}](\chi PT) + \mathcal{O}(\alpha)\,.$$

$$c_{4,\mathrm{point-like}}^{p\mu} - c_{4,\mathrm{point-like}}^{pe} = \left(1-\frac{\kappa_p^2}{4}\right)\ln\frac{m_\mu^2}{m_e^2} + \frac{m_\mu^2}{m_p^2}\left(1+\frac{\kappa_p}{2}\left(1-\frac{\kappa_p}{6}\right)\right)\ln\frac{m_\mu^2}{\nu_{\mathrm{pion}}^2}$$
$$\simeq 2.09 - 0.09 = 2.00(9)\,,$$

$$c_{4,\mathrm{pol}}^{p\mu} - c_{4,\mathrm{pol}}^{pe} = \begin{cases} 0.17(9) & (\pi), \\ 0.25(10) & (\pi\&\Delta)\,, \end{cases}$$

DR (Carlson et al) $\sim -0.3(1.4)$. RelativisticχPT 0.08(27)/0.11(55)(Hagelstein et al)

$$c_{4,\mathrm{Born}}^{p\mu} - c_{4,\mathrm{Born}}^{pe} = -\int_0^\infty dp\frac{1}{3p}G_M^{(1)}(-p^2)$$
$$\times\left[\left(\frac{p^2\kappa_p}{m_\mu^2} + \frac{32m_\mu^4 - 8m_\mu^2p^2(\kappa_p+2) - 2p^4\kappa_p}{m_\mu^2 p\left(\sqrt{4m_\mu^2+p^2}+p\right)} + 8\right) - (m_\mu \to m_e)\right],$$

$G_M^{(1)}(\chi$PT): Gasser et al.; Bernard et al.

$$c_{4,\mathrm{Born}}^{p\mu} - c_{4,\mathrm{Born}}^{pe} = \begin{cases} 0+1.11(55) & (\pi), \\ 0+1.42(53) & (\pi\&\Delta)\,, \end{cases}$$

$$c_{4,\mathrm{TPE}}^{p\mu} = c_{4,\mathrm{TPE}}^{pe} + [c_{4,\mathrm{TPE}}^{p\mu} - c_{4,\mathrm{TPE}}^{pe}](\chi PT) + \mathcal{O}(\alpha)\,.$$

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$$c^{p\mu}_{4,\mathrm{TPE}} = c^{pe}_{4,\mathrm{TPE}} + [c^{p\mu}_{4,\mathrm{TPE}} - c^{pe}_{4,\mathrm{TPE}}](\chi PT) + \mathcal{O}(\alpha)\,.$$

$$\begin{aligned} c^{p\mu}_{4,\mathrm{point-like}} - c^{pe}_{4,\mathrm{point-like}} &= \left(1 - \frac{\kappa_p^2}{4}\right) \ln \frac{m_\mu^2}{m_e^2} + \frac{m_\mu^2}{m_p^2}\left(1 + \frac{\kappa_p}{2}(1 - \frac{\kappa_p}{6})\right) \ln \frac{m_\mu^2}{\nu^2_{\mathrm{pion}}} \\ &\simeq 2.09 - 0.09 = 2.00(9)\,, \end{aligned}$$

$$c^{p\mu}_{4,\mathrm{pol}} - c^{pe}_{4,\mathrm{pol}} = \begin{cases} 0.17(9) & (\pi), \\ 0.25(10) & (\pi\&\Delta)\,, \end{cases}$$

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$$\begin{aligned} c^{p\mu}_{4,\rm Born} - c^{pe}_{4,\rm Born} &= -\int_0^\infty dp \frac{1}{3p} G_M^{(1)}(-p^2) \\ &\times \left[\left(\frac{p^2\kappa_p}{m_\mu^2} + \frac{32m_\mu^4 - 8m_\mu^2p^2(\kappa_p+2) - 2p^4\kappa_p}{m_\mu^2 p\left(\sqrt{4m_\mu^2+p^2}+p\right)} + 8\right) - (m_\mu \to m_e)\right]\,, \end{aligned}$$

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$$c^{p\mu}_{4,\rm Born} - c^{pe}_{4,\rm Born} = \begin{cases} 0 + 1.11(55) & (\pi), \\ 0 + 1.42(53) & (\pi\&\Delta)\,, \end{cases}$$

Overall, combining the three contributions, we obtain

$$[c_{4,\mathrm{TPE}}^{p\mu} - c_{4,\mathrm{TPE}}^{pe}](\chi PT) = 3.68(72)$$

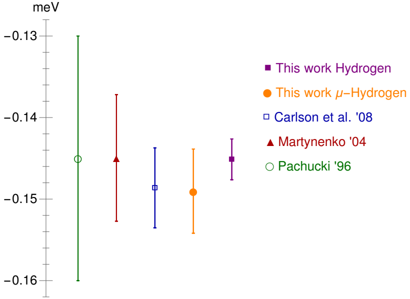


Figure: Two-photon exchange contribution to the hyperfine splitting of the 2S muonic hydrogen. Peset-Pineda

Variation of this idea has later been applied using DR (Tomalak). Error $\sim 1/2$.

Δ, (ppm)	Δ_Z	Δ_R^p	$\Delta_Z + \Delta_R^p$	Δ_0^{pol}	Δ_{HFS}
this work, μH r_E, r_M^W	$-7415(84)$	844(7)	$-6571(87)$	364(89)	$-6207(127)$
this work, electron r_E, r_M^W	$-7487(95)$	844(7)	$-6643(98)$	364(89)	$-6279(135)$
this work, μH r_E, r_M^e	$-7333(48)$	846(6)	$-6486(49)$	364(89)	$-6122(105)$
this work, electron r_E, r_M^e	$-7406(56)$	847(6)	$-6559(57)$	364(89)	$-6195(109)$
Hagelstein et al. [59]				-61^{+70}_{-52}	
Peset et al. [29]					$-6247(109)$
Carlson et al. [28, 39]	-7587	835	$-6752(180)$	351(114)	$-6401(213)$
Martynenko et al. [38]	-7180		-6656	410(80)	$-6246(342)$
Pachucki [7]	-8024		-6358	0(658)	$-6358(658)$

Figure: From Tomalak, 2017

The proton radius in $e-p$ scattering (Future $\mu-p$ scattering)

- Definition??
- very sensitive to low q^2 data:
  extrapolation from $|\mathbf{q}| \geq 100$ MeV to $|\mathbf{q}| = 0$
- dependence on the fitting functions: normalization factors, full data set ...
- Bonn group with dispersion relations:
  $r_p = 0.84^{+0.01}_{-0.01}$ fm.

$Q^2 \to 0$ INVOLVES COULOMB RESUMMATION $\to$ ATOMIC PHYSICS
$|\mathbf{q}| \sim m_\mu\alpha \sim m_e \sim 0.5$ MeV (muonic hydrogen)
$|\mathbf{q}| \sim m_e\alpha \sim 5.10^{-3}$ MeV (hydrogen)
New scales: $m_{\rm lepton}\alpha$, $m_{\rm lepton}\alpha^2$

Nonrelativistic proton and lepton

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{measured}} = Z^2 \left(\frac{d\sigma_{1\gamma}}{d\Omega}\right)_{\rm point-like} + \frac{d\sigma_{\text{Mott}}}{d\Omega}\left[\delta^{(p)}_{\rm soft} + Z^2\left(\delta^{(\mu)}_{\rm soft} + \delta_{\rm VP}\right) + Z^3\left(\delta^{(p\mu)}_{\rm soft} + \delta_{\rm TPE}\right) + \mathcal{O}(\alpha^2) + \mathcal{O}(\tau^2)\right].$$

$$\delta^{(p)}_{\rm soft} = \tau_p\left[\beta^2\left(c_F^{(p)\,2} - Z^2\right) - Z\left(c_D^{(p)\overline{\rm MS}}(\nu) - Z\right) + \frac{4}{3}\frac{Z^4\alpha}{\pi}\left(2\ln\frac{2\Delta E}{\nu} - \frac{5}{3}\right)\right],$$

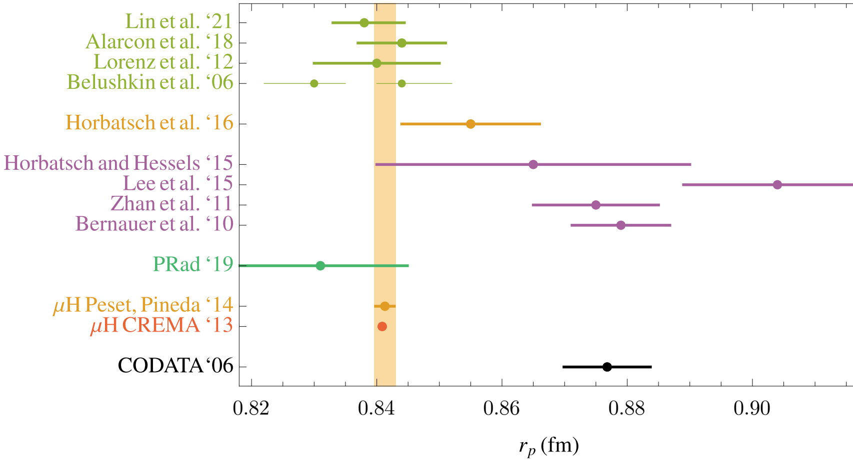
$$\delta_{\rm VP} = 32\tau_\mu\left[d_2^{(\mu)} + \frac{m_\mu^2}{M^2}d_2 + \frac{m_\mu^2}{m_\tau^2}d_2^{(\tau)}\right]$$

$$\delta^{\rm point-like}_{\rm TPE} = \delta_{\rm pot} + \delta_{\rm soft} + \delta^{\rm point-like}_{\rm hard}.$$

$$d_s(\nu) = -\frac{Z^2\alpha^2}{m_{l_i}^2 - M^2}\left[m_{l_i}^2\left(\ln\frac{M^2}{\nu^2} + \frac{1}{3}\right) - M^2\left(\ln\frac{m_{l_i}^2}{\nu^2} + \frac{1}{3}\right)\right],$$

$$\delta^{\rm point-like}_{\rm hard} \longrightarrow \delta_{\rm hard} = -\frac{Q^2}{2Mm_\mu}\left[\frac{d_s(\nu)}{\pi\alpha} - \frac{m_\mu}{M}\frac{c_3^{\rm had}}{\pi\alpha}\right].$$

r_p determinations using electron-proton elastic scattering data



The proton radius in *ep* scattering from χPT

Hessels, Horbatsch, AP

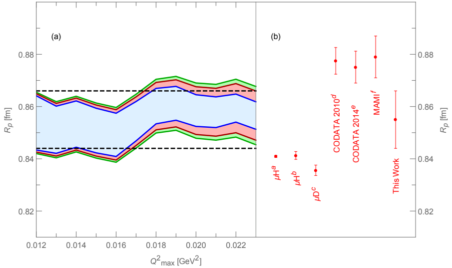
$$G_E(Q^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} Q^{2n} \langle r^{2n} \rangle$$

- Extrapolation from $|\mathbf{q}| \sim 100$ MeV to $|\mathbf{q}| = 0$
- dependence on the fitting functions: normalization factors, full data set ...

Higher moments diverge in the chiral limit

$$\langle r^{2k} \rangle \sim m_\pi^{2-2k}$$

Extrapolation controlled by χPT (at low Q^2): $r_p \sim 0.855$.
Bigger values for the moments produce larger values of r_p.



CONCLUSIONS

Effective Field Theories provide with a model independent, efficient and systematic (Power counting) approach to the dynamics of NR systems and a unified framework to determine the nonperturbative effects.

Rigorous connection between Quantum Field Theories (Wilson coefficients) and a NR Quantum-mechanical formulation of the NR systems (potentials). For instance. The proton radius is a Wilson coefficient of the effective theory. In general it is an scheme/scale dependent object.

The spin-independent TPE energy shift (and the associated error) is (and can only be) computed in a model independent way with χPT. Overall number consistent with determinations from a combined use of dispersion relations and models, but individual contributions are quite different. Unlike dispersion relations, no assumption on the high energy behavior.

χPT predicts the chiral logs of the hyperfine splitting and the difference between hydrogen and muonic hydrogen.

Analytic understanding of the QCD dynamics: m_q and N_c dependence.

$$\Delta E_L^{\rm th} = \left[206.0243(30) - 5.2270(7)\frac{r_p^2}{\rm fm^2} + 0.0455(125)\right]\ {\rm meV}\,.$$

$$E_{\rm HF}(1S) = 182.623(27)\,{\rm meV}, \qquad E_{\rm HF}(2S) = 22.8123(33)\,{\rm meV}$$

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Rigorous connection between Quantum Field Theories (Wilson coefficients) and a NR Quantum-mechanical formulation of the NR systems (potentials). For instance. The proton radius is a Wilson coefficient of the effective theory. In general it is an scheme/scale dependent object.

The spin-independent TPE energy shift (and the associated error) is (and can only be) computed in a model independent way with χPT. Overall number consistent with determinations from a combined use of dispersion relations and models, but individual contributions are quite different.
Unlike dispersion relations, no assumption on the high energy behavior.

χPT predicts the chiral logs of the hyperfine splitting and the difference between hydrogen and muonic hydrogen.

Analytic understanding of the QCD dynamics: m_q and N_c dependence.

$$\Delta E_L^{\rm th} = \left[206.0243(30) - 5.2270(7)\frac{r_p^2}{\rm fm^2} + 0.0455(125)\right] \rm meV\,.$$

$$E_{\rm HF}(1S) = 182.623(27)\,\rm meV, \qquad E_{\rm HF}(2S) = 22.8123(33)\,\rm meV$$

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Proton radius puzzle?

- r_p from muonic hydrogen. The most precise (TPE stopping factor).
- r_p from hydrogen $\rightarrow$ Most new measurements agree with muonic hydrogen. Only Paris remain ?! (1S-3S discrepancy with MPQ collaboration?)
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  Problems with extrapolations
  Better low energy (and add chiral constraints)
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