

Strange quarks in pions – and up-down quarks A View from the Nambu-Jona-Lasinio model

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OUTLINE

- Motivations

* global properties (eg. Masses) usually are well described by NJL model/CQM = to assess strangeness (sea quarks) component of the pion mass

- NJL model and related issues: Quark-antiquark polarization and flavor dependent coupling constants Rather for global properties

- Results and different ways of weighting strange-quark/antiquark contribution for the light u,d quark and pion masses

- Final remarks

F.L.B. arXiv:2109.02203 [hep-ph]]. Phys. Rev. D **103** (2021) no.9, 094028



Pions/kaons as (quasi) Goldstone boson Probed in AMBER

How (in a broad way) do fundamental degrees of freedom (flavor)

survive and show up in hadron and nuclear observables?

NJL has an appeal to make possible an <u>analytical treatment</u> With clear identification of relation between degrees of freedom Many groups in the 1980-2010's Dynamical approach should help with link to QCD NJL ~ GCM ~SDE (RL) Constituent Quark Model (CQM) And extensions

Weinberg Large Nc CQM: F.L.B. Eur.Phys.J.A 52 (2016) 5, 13, arXiv:1601.04916

Quark masses (from the HIggs) are much smaller than needed to describe hadron masses

Sea quarks in the NJL

Are encoded in the quark-condensates

Difficult to generate asymmetry d-anti-d

 $\langle \bar{q}_R q_L + \bar{q}_L q_R \rangle \simeq -(250 MeV)^3$

From constituent quarks M*u ~ 360 MeV M*d ~ 370 MeV M*s ~ 510 MeV Most of Hadrons Spectra support or suggest the idea For global properties (pion may be an exception)

> It can receive corrections: Diquarks Virtual n-quarks states from Fock space etc

Constituent quarks: two main possible contributions Usually considered separatedly

Maybe "competing" contributions





Calculation based on Global Color Model For example by C.D. Roberts, R.T. Cahill, J. Praschifka, P.C. Tandy, Flavor (i,j = 0,... N_f^2) dependent couplings $\mathcal{L}_{NJL} = G_{ij}(\bar{\psi}\lambda_i\psi)(\bar{\psi}\lambda_j\psi) + [G_{ij} + \bar{G}_{ij}](\bar{\psi}\lambda_i i\gamma_5\psi)(\bar{\psi}\lambda_j i\gamma_5\psi)$

Flavor independent Contribution

+ Flavor dependent contribution

$$G_{ij} = G_{ji}, \quad G_{22} = G_{11}, \quad G_{55} = G_{44}, \quad G_{77} = G_{66}.$$

CP
+
U(1) invariance



Point of Reference

Mechanism for masses considered here Basically: Dynamical chiral symmetry breaking

Although it may involve other mechanism Since - coupling constants are assumed to be large - Gluon with effective masses

Solution for scalar condensat GAP / Schwinger Dyson eqs.

 $\frac{\partial S_{eff}}{\partial \phi_q} = 0.$

Chiral scalar condensate = it corrects quark mass = DChSB But it easily takes into account Other effects such as trace anomaly $\tilde{M^*} = m + \langle S \rangle (P_R \tilde{U} + P_L \tilde{U}^{\dagger})$

Vacuum becomes infinitely degenerated

Scalar field is eliminated by chiral rotation, With resulting non linear pion dynamics

Scalar mesons: Unsolved problem not considered - exotic states? Mixing: change of basis quark mass eigenstate X meson mass eigenstate

$$G_{ij}(\bar{\psi}\lambda_i\psi)(\bar{\psi}\lambda_j\psi) = G_{f_1f_2}(\bar{\psi}\psi)_{f_1}(\bar{\psi}\psi)_{f_2}$$

 $\pi^0 \leftrightarrow \eta \leftrightarrow \eta'$

 $G_{ij} = G_{ij}(M_u^*, M_d^*, M_s^*).$

 $\rightarrow G_{08}$

 $G_{f_1f_2} \rightarrow M^*_u(M_d, M_s)$

 $\rightarrow M_d^*(M_u, M_s)$

 $\rightarrow M_s^*(M_u, M_d)$

 $M_f^* - m_f = G_{ff} Tr (S_{0,f}(0)),$

 $\rightarrow G_{38}$

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 $G_{i\neq j} \rightarrow G_{03}$ Mixing between

Coupling constant that measure The strangeness-asymmetry

(M2) $x_s G_{88} = 10 \ GeV^{-2},$ (M3) $x_s G_{88} = G_{88} \ GeV^{-2},$

This is not Instanton induced mechanism

It is quark-antiquark-polarization

Gap or SDE equations

$$1 - 2G_{ij}I^{ij}_{f_1f_2}(P_0^2 = -M_{PS}^2, \vec{P}^2 = 0) = 0,$$

Bethe-Salpeter at the Born level

For Numerical estimates

These effective propagators include a Quark-gluon (running) coupling constant value (GAP eq. for DChSB)

$$D_2(k) = g^2 R_T(k) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2 \gamma_m E(k^2)}{\ln\left[\tau + (1 + k^2/\Lambda_{QCD}^2)^2\right]},$$

Transversal Propagator from Tandy- Maris / Chang-Xin

Fitting from SDE at RL level

where $\gamma_m = 12/(33 - 2N_f)$, $N_f = 4$, $\Lambda_{QCD} = 0.234 \text{GeV}$, $m_t = 0.5 \text{GeV}$, $D = 0.55^3/\omega$ (GeV²) and $\omega = 0.5 \text{GeV}$.

$$\tau = e^2 - 1, E(k^2) = [1 - exp(-k^2/[4m_t^2])/k^2],$$

$$D_{\alpha=5,6}(k) = g^2 R_{L,\alpha}(k) = \frac{K_F}{(k^2 + M_{\alpha}^2)^2},$$

Cornwall effective propagator - confining

 $K_F = (0.5\sqrt{2}\pi)^2/0.6 \text{ GeV}^2,$ $M_6 = M_6(k^2) = \frac{0.5}{1+k^2/\omega_6^2} \text{GeV for } \omega_6 = 1 \text{GeV}.$

Fitting of the parameters

set of	m_u	m_d	m_s	Λ	M_u	M_d	M_s
parameters	MeV	MeV	MeV	MeV	MeV	MeV	MeV
S	3	7	133	680	405	415	612
V_3	3	7	133	685	422	431	625

$$G_0 = 10 {\rm GeV}^{-2}$$

Point of Reference

Observable	S2	S5	$\mathbf{S6}$	G_0	V_3 -2	V_3 -5	V_3 -6	V_3 -G0	e.v.
$M_{\rm M0}({ m MeV})$ [M2]	135.0	135.3	135.1	136.4	135.5	136.1	135.6	137.1	135 [43]
$M_{\pi^0}({ m MeV})$ [M3]	133.5	134.2	133.7	136.4	134.15	134.9	134.4	137.1	
$M_{\pi^{\pm}}$ (MeV) [M2]	135.2	135.4	135.3	136.7	135.7	136.2	135.8	137.4	
$M_{\pi^{\pm}}$ (MeV) [M3]	133.7	134.4	133.9	136.7	134.4	135.0	134.5	137.4	
$M_{\pi^{\pm}} - M_{\pi^{0}} ({\rm MeV}) [{\rm M3}]$	0.2	0.2	0.2	0.3	0.1	0.1	0.1	0.3	0.1 [43, 45]
M_{K^0} (MeV) [M3]	498.5	498.5	498.5	499	498	498	499	498	498 [43]
$M_{K^{\pm}}$ (MeV) [M3]	490	491	493	490	486	487	488	490	494 [43]

Coupling constant Guu Up quark gap equation

 $\begin{array}{ll} (M2) & x_s G_{88} = 10 \; GeV^{-2}, \\ (M3) & x_s G_{88} = G_{88} \; GeV^{-2}, \end{array}$

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M_{s}^{*} M3-(G2) (MeV)	555	567	558	612	566	581	569	625	
M_s^* M2-(G2) (MeV)	560	570	563	612	600	595	604	625	
ch.lim. $M_{ch.L}^*$ (MeV)	381	381	381	381	415	415	415	415	

Chiral limit



Up dressed/constituent quark mass







Neutral-charged pion (strong) mass difference

Gasser-Leutwyler/Donogue: ~ 0.1 MeV

(M2) $x_s G_{88} = 10 \ GeV^{-2},$ (M3) $x_s G_{88} = G_{88} \ GeV^{-2},$



Role of the strangeness asymmetry G88

$$\Delta_s^{2,3} = T_{ff}(M3) - T_{ff}(M2),$$

(M2)
$$x_s G_{88} = 10 \ GeV^{-2},$$

(M3) $x_s G_{88} = G_{88} \ GeV^{-2},$

Observable [M	3] S2	S5	S6	$S-G_0$	V ₃ -2	V_3 -5	V_3 -6	V_3 -G0
$\Delta_s^{2,3}(M_{\pi}^*(s))$ (Me	eV) ∥ -1.5	-1.1	-1.4	0	-1.3	-1.2	-1.2	0

Other observables

	Observable	S2	S5	$\mathbf{S6}$	G_0	V ₃ -2	V_3 -5	V_3 -6	V_3 -G0	e.v.
	$M_{\pi^0}({\rm MeV})$ [M2]	135.0	135.3	135.1	136.4	135.5	136.1	135.6	137.1	135 [43]
	$M_{\pi^0}({ m MeV})$ [M3]	133.5	134.2	133.7	136.4	134.15	134.9	134.4	137.1	
	$M_{\pi\pm}$ (MeV) [M2]	135.2	135.4	135.3	136.7	135.7	136.2	135.8	137.4	
	$M_{\pi^{\pm}}$ (MeV) [M3]	133.7	134.4	133.9	136.7	134.4	135.0	134.5	137.4	
	$M_{\pi^{\pm}} - M_{\pi^0} ({\rm MeV}) [{\rm M3}]$	0.2	0.2	0.2	0.3	0.1	0.1	0.1	0.3	$0.1 \ [43, 45]$
	M_{K^0} (MeV) [M3]	498.5	498.5	498.5	499	498	498	499	498	498 [43]
	$M_{K^{\pm}}$ (MeV) [M3]	490	491	493	490	486	487	488	490	494 [43]
1	$F_{\pi} \; ({ m MeV})$	99	99	99	102	100	101	101	103	92
	$F_K (MeV)$	111	111	111	112	112	112	111	113	111
-	$(-<\bar{u}u>)^{1/3}$ MeV	331	334	332	343	336	338	336	347	240-260 [50, 51]
2	$(- < dd >)^{1/3} \text{ MeV}$	333	335	333	344	338	339	338	349	240-260 [50, 51]
	$(-<\bar{s}s>)^{1/3}$ MeV	348	353	349	366	352	356	353	369	290-300 [52]
3	$G_{qq\pi}$	3.3	3.2	3.2	3.4	3.3	3.3	3.3	3.6	
U	G_{qqK}	3.8	3.8	3.8	4.2	3.9	4.0	3.9	4.3	
	\bullet θ_{ps}	-3.7	-2.7	-3.6	0.0	- 3.4	2.6	3.4	0.0	(-11°)-(-24°) [43]
	Pr.s-content π	6%	10%	10%	0	16%	16%	16%	0	
	$\chi^2_{red} \text{ (with } < \bar{q}q > \text{)}$	103	110	103	146	122	129	123	164	
	χ^2_{red} (without $\langle \bar{q}q \rangle$)	29	30	27	51	37	41	39	56	

Quark condensates with too large values Because of the large coupling constant:

For low coupling constants: further difficulties with numerical convergence

Pion decay constant It depends very strongly on Ms* (probably this is a too much large variation – model-dependence)

Sigma terms

Renormalization of coupling constant involves different strange quark mass dependencies

For some set of parameters: positive or negative s-sigma terms (u,d quarks and pion) [Lattice calculations (Bali et al, 2012) indicated it should be small- large uncertanties

$$\mathcal{L}_{mix} = -\frac{M_{88}^2}{2}P_8^2 - \frac{M_{00}^2}{2}P_0^2 + 2G_{08}^n\bar{G}_{08}P_0P_8 + \mathcal{O}(P_3, P_3^2)...$$

$$|\eta \rangle = \cos\theta_{ps}|P_8 \rangle - \sin\theta_{ps}|P_0 \rangle,$$

$$|\eta' \rangle = \sin\theta_{ps}|P_8 \rangle + \cos\theta_{ps}|P_0 \rangle.$$
To estimate mesons mixings: use of auxiliary field method
$$\theta_{ps} = \frac{1}{2} \operatorname{arcsin}\left(\frac{8G_{08}^n\bar{G}_{08}}{(M_\eta^2 - M_\eta^2)}\right).$$

$$\frac{F_{\pi}(\operatorname{MeV})}{F_K(\operatorname{MeV})} \frac{111}{111} \frac{111}{111} \frac{112}{112} \frac{112}{112} \frac{111}{111} \frac{113}{111} \frac{92}{112} \frac{92}{111} \frac{92}{1$$

~ ~ ~ ~ ~ /

Chi-squared For 9 observables (2 fitting observables) - Dependence of pion observables on the strange quark (effective) mass \rightarrow to assess strange sea quarks contribution for the pion observables (mass, coupling constants and <u>decay constant</u>)

- eta and etaprime: they mix with pi0 \rightarrow need of higher precision processes with pi0 exchange

Quark-antiquark polarization offers another mechanism for mesons mixings

- By using the same method:

to assess up-quark contribution to K0 and down-quark contribution for K+-

- In this simpler level of calculation: (strange) quark-antiquark contents are the same

HOW TO ASSESS each component hadron/pion masses:

- momentum dependencies
- spatial distributions?

- are there pion interactions rather sensitive to strange quarks not to up/down quarks? (eg. pi0-eta-eta' mixing)

PLANNED OR ON-GOING

- parton strangeness content in the pion
- light mesons mixings
- etc

Thank you for your attention!

$$\begin{aligned} \Delta_{chL}^f &\simeq M_f - M_{ch.L.}, \\ \Delta_{G_{ij}}^f &\simeq M_f^* - M_f, \end{aligned}$$

Deviation from the chiral limit

up/down quark masses

Observable [M3]	S2	S5	S6	$S-G_0$	V_3 -2	V_3 -5	V_3 -6	V_3 -G0
Δ_u^{chL} (MeV)	24	24	24	24	7	7	7	7
Δ_d^{chL} (MeV)	34	34	34	34	16	16	16	16

							$\Delta_s^{2,3}$	$= T_f$	$_f(M3) - T_{ff}(M2)$),
							Δ_{s}^{0}	$= T_f$	$_{f}(M_{*}^{*'}) - T_{ff}(M_{*}^{*})$	i = 0,
							Λ^{m_0}	$= T_{e}$	$f(M^{*'}) - T_{ff}(M^{*})$	$(=m_{c})$
Observable	<u>S2</u>	S5	<u>S6</u>	G_0	V_{2} -2	Va-5	$ = \frac{\Delta_s}{\Lambda^{\infty}} $	-1_{J}	$J(M_s) = IJJ(M_s)$	- ms),
M_{π^0} (MeV) [M2]	135.0	135.3	135.1	136.4	135.5	136.1	Δ_s	$= I_{f_i}$	$f(M_s) - I_{ff}(M_s)$	$s \to \infty$
$M_{\pi^0}(\text{MeV})$ [M3]	133.5	134.2	133.7	136.4	134.15	134.9	134.4	137.1	100 [10]	
$M_{\pi^{\pm}}$ (MeV) [M2]	135.2	135.4	135.3	136.7	135.7	136.2	135.8	137.4		
$M_{\pi^{\pm}}$ (MeV) [M3]	133.7	134.4	133.9	136.7	134.4	135.0	134.5	137.4		
$M_{\pi^{\pm}} - M_{\pi^0} ({\rm MeV}) [{\rm M3}]$	0.2	0.2	0.2	0.3	0.1	0.1	0.1	0.3	0.1 [43, 45]	
M_{K^0} (MeV) [M3]	498.5	498.5	498.5	499	498	498	499	498	498 [43]	
$\frac{M_{K^{\pm}} (\text{MeV}) [\text{M3}]}{E (\text{MeV})}$	490	491	493	490	486	487	488	490	494 [43]	_
F_{π} (MeV) F_{K} (MeV)	99 111	99 111	99 111	$102 \\ 112$	$100 \\ 112$	101	101	103	92 111	
Observable/M3	S2	S5	S6	$S-G_0$	V ₃ -2	V_3 -5	V ₃ -6	V_3 -G0	e.v.	,
M_{*}^{*} M3-(G2) (MeV)	367	377	368	405	385	393	387	422		
M_{u}^{*} M2-(G2) (MeV)	386	394	387	405	401	406	402	422		
$M^*_u(M^*_s \to 0)$ MeV	618	512	595	405	651	537	626	422		
$M^*_u(M^*_s \to m_{0,s})$ MeV	563	491	547	405	579	508	563	422		
$M_u^*(M_s^* \to \infty)$ MeV	290	310	295	405	300	316	304	422		
M_d^* M3-(G2) (MeV)	375	384	378	415	394	402	395	431		
$M_d^* \text{M2-(G2)} (\text{MeV})$	396	399	396	415	410	415	411	431		
$M_d^*(M^*{}_s \to 0) \mathrm{MeV}$	625	520	602	415	657	544	632	431		
$M_d^*(M_s^* \to m_{0,s})$ MeV	555	491	541	415	585	514	568	431		
$M_d^*(M^*{}_s \to \infty) \text{ MeV}$	305	320	305	415	314	328	316	431		
M_{s}^{*} M3-(G2) (MeV)	555	567	558	612	566	581	569	625		
$M_s^* { m M2-(G2)}({ m MeV})$	560	570	563	612	600	595	604	625		
ch.lim. $M^*_{ch.L.}$ (MeV)	381	381	381	381	415	415	415	415		
M_{π^0} M3-(G2) (MeV)	133.5	134.2	133.7	136.7	134.2	134.9	134.4	137.4	0.135[43, 45]	
M_{π^0} M2-(G2) (MeV)	135.0	135.3	135.1	136.7	135.5	136.1	135.6	137.4		
$M_{\pi^0}(M_s^* \to 0)$ MeV	158	147	156	136.7	162	149	159	137.4		
$M_{\pi^0}(M_s^* \to m_{0,s})$ MeV	150	144	149	136.7	150	144	149	137.4		
$M_{\pi^0}(M_s^* \to \infty) \mathrm{MeV}$	129	129	129	136.7	129	129	129	137.4		

 $\begin{array}{lll} \Delta^f_{chL} &\simeq& M_f - M_{ch.L.}, \\ \Delta^f_{G_{ij}} &\simeq& M_f^* - M_f, \end{array}$

Observable	S2	S5	S6	G_0	V ₃ -2	V_3 -5		-6 V_3 -G	0	e.v.
$M_{\pi^0}(\text{MeV})$ [M2]	135.0	135.3	135.1	136.4	135.5	136.1	13	5.6 137.	1	135 [43]
$M_{\pi^0}({ m MeV})$ [M3]	133.5	134.2	133.7	136.4	134.15	134.9	134	4.4 137.	1	
$M_{\pi^{\pm}}$ (MeV) [M2]	135.2	135.4	135.3	136.7	135.7	136.2	135	5.8 137.	4	
$M_{\pi^{\pm}}$ (MeV) [M3]	133.7	134.4	133.9	136.7	134.4	135.0	134	4.5 137.	4	
$M_{\pi^{\pm}} - M_{\pi^{0}} ({\rm MeV}) [{\rm M3}]$	0.2	0.2	0.2	0.3	0.1	0.1	0.	1 0.3	0.	1 [43, 45]
M_{K^0} (MeV) [M3]	498.5	498.5	498.5	499	498	498	49	99 498	4	498 [43]
$M_{K^{\pm}}$ (MeV) [M3]	490	491	493	490	486	487	48	8 490	4	494 [43]
F_{π} (MeV)	99	99	99	102	100	101		1 103		92
F_K (MeV)		<u> </u>	111	112	112	112		1 113		
Observable [M3]		S2	S5	S6	$S-G_0$	V_3	-2	$V_{3}-5$	$V_{3}-6$	V_3 -G0
Δ_u^{chL} (MeV)		24	24	24	24		7	7	7	7
Δ_d^{chL} (MeV)		34	34	34	34	1	6	16	16	16
$\Delta_{G_{ii}}$ (MeV)	-	- 38	-28	-37	0	-3	37	- 29	-35	0
$\Delta_{G_{ij}}$ (MeV)		-40	-31	-37	0	-3	-37 -2		-36	0
$\Delta_s^{2,3}(M_u^*(s))$ (Me)	V)	-19	-17	-19	0	-1	.6	-13	-15	0
$\Delta_s^0(M_u^*(s))$ (MeV	7) -	251	-135	-227	0	-2	66	- 144	-239	0
$\Delta_s^{m_0}(M_u^*(s)) (\mathrm{Me})$	V) -	194	-114	-179	0	-1	94	-115	-176	0
$\Delta_s^\infty(M_u^*(s))$ (MeV	V)	77	67	73	0	8	5	77	83	0
$\Delta_s^{2,3}(M_{\pi}^*(s))$ (Me)	V) -	-1.5	-1.1	-1.4	0	-1	.3	-1.2	-1.2	0
$\Delta_s^0(M_\pi^*(s))$ (MeV	7)	-24	-13	-22	0	-2	28	-14	-25	0
$\Delta_s^{m_0}(M_\pi^*(s)) (\mathrm{Me})$	V)	-16	-10	-16	0	-1	.6	-9	-14	0
$\Delta_s^\infty(M_\pi^*(s))$ (MeV	V)	6	5	4	0	Į	5	6	5	0