Pion Generalised Partons Distributions at future Electron-Ion Colliders

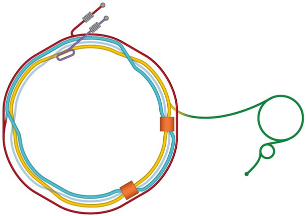
Cédric Mezrag

CEA Saclay, Irfu DPhN

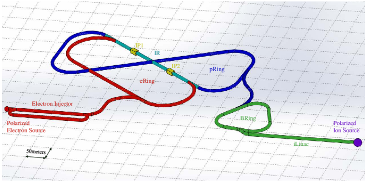
September 27^{th}, 2021

Pion Structure at Electron-Ion Colliders

US EIC



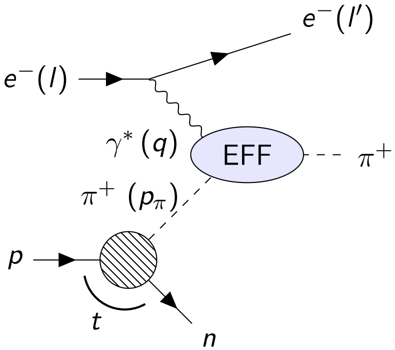
China EIC

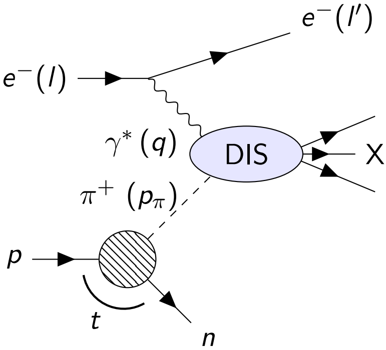


	EIC	EicC
Lepton beam energy (GeV)	5/10/18	3.5
Hadron beam energy (GeV)	41/100/275	20
Lepton polarization	70%	80%
Hadron polarization	70%	70%
Integrated luminosity (fb^{-1}/year)	10	50

Where is the pion?

Sullivan processes

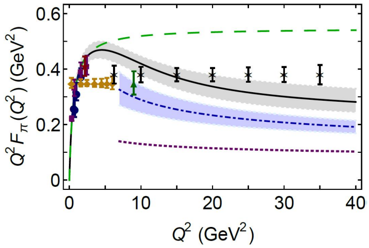


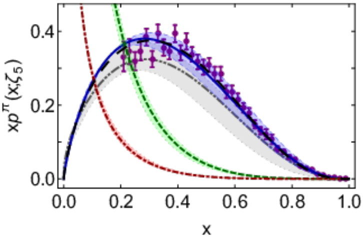


- Tested at JLab 6
  Huber *et al.*,PRC78, 045203
- Planned for JLab 12
  Aguilar *et al.*, EPJA 55 10, 190
- Envisioned at EIC and EicC
  see EIC Yellow Report and EicC white paper

- Not done at JLab 6
- Planned for JLab 12
  Aguilar *et al.*, EPJA 55 10, 190
- Envisioned at EIC and EicC
  see EIC Yellow Report and EicC white paper

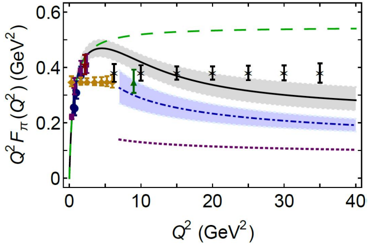
Pion Structure

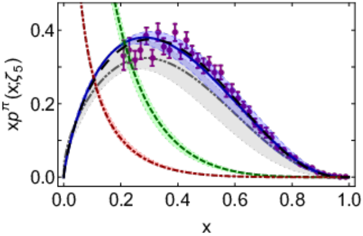




figures from Aguilar *et al.*, EPJA 55 10, 190

- pion EFF
  - 2D charge distribution in the transverse plane
  - Open questions on large momentum transfer behaviour
- pion PDFs
  - Consistency with Drell-Yan extraction
  - Question of the amount of gluons in the pion





figures from Aguilar *et al.*, EPJA 55 10, 190

- pion EFF
  - 2D charge distribution in the transverse plane
  - Open questions on large momentum transfer behaviour
- pion PDFs
  - Consistency with Drell-Yan extraction
  - Question of the amount of gluons in the pion

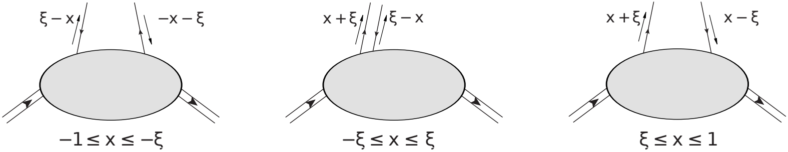
Can we go beyond and get access to the 3D structure?

Generalised Parton Distributions

Generalised Parton Distributions

- Generalised Parton Distributions (GPDs):

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,



  - ★ x: average momentum fraction carried by the active parton
  - ★ ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - ★ t: the Mandelstam variable

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

$$H_\pi^g(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

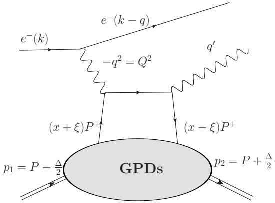
Generalised Parton Distributions

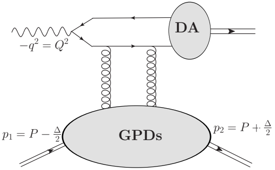
- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,
  - can be split into quark flavour and gluon contributions,

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,
  - can be split into quark flavour and gluon contributions,
  - are related to PDF in the forward limit $H^q(x, \xi = 0, t = 0; \mu) = q(x; \mu)$

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,
  - can be split into quark flavour and gluon contributions,
  - are related to PDF in the forward limit $H^q(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

Interpretation of GPDs I

2+1D structure of the nucleon

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

- Possibility to extract density from experimental data

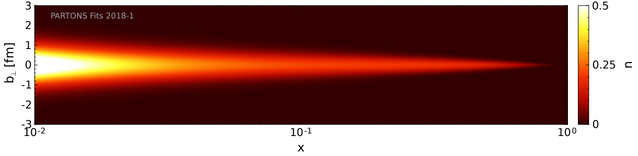


figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

Interpretation of GPDs I

2+1D structure of the nucleon

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

- Possibility to extract density from experimental data

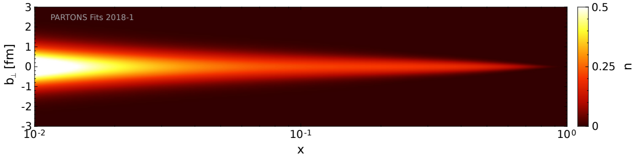


figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

- Correlation between x and $b_\perp$ $\to$ going beyond PDF and FF.

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

- Possibility to extract density from experimental data

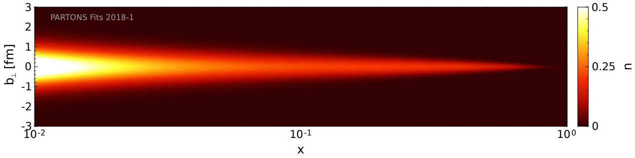
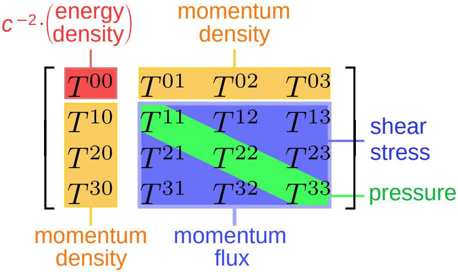


figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

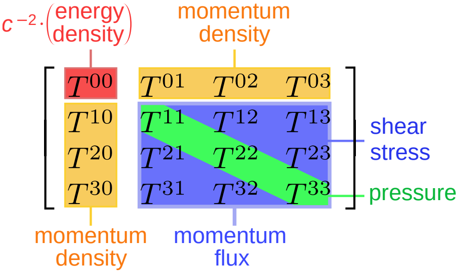
- Correlation between x and $b_\perp \to$ going beyond PDF and FF.
- Caveat: no experimental data at $\xi = 0$
  $\to$ extrapolations (and thus model-dependence) are necessary



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer,
IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

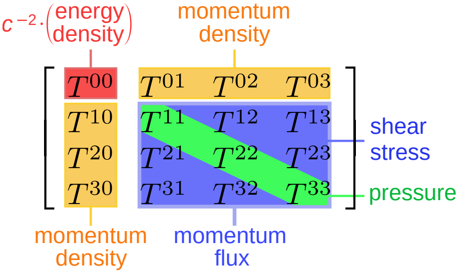


How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer,
IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = 2P^\mu P^\nu A_{q,g}(t;\mu) + \frac{1}{2}\left(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2\right) C_{q,g}(t;\mu) + 2M^2 g^{\mu\nu} \bar{C}_{q,g}(t;\mu)$$



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

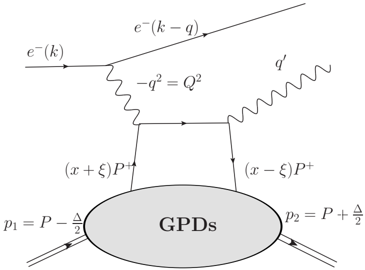
$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = 2P^\mu P^\nu A_{q,g}(t;\mu) + \frac{1}{2}\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right) C_{q,g}(t;\mu) + 2M^2 g^{\mu\nu}\bar{C}_{q,g}(t;\mu)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,t;\mu) = A_q(t;\mu) + \xi^2 C_q(t;\mu)$$

- Ji sum rule (nucleon)
- Fluid mechanics analogy

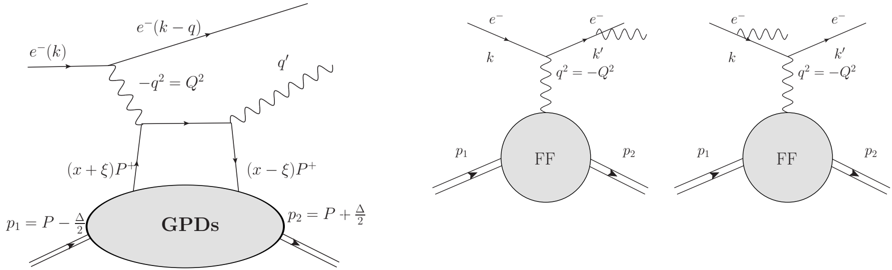
X. Ji, PRL 78, 610-613 (1997)
M.V. Polyakov PLB 555, 57-62 (2003)

Phenomenology of Pion GPDs



- Best studied experimental process connected to GPDs
  → Data taken at Hermes, Compass, JLab 6, JLab 12

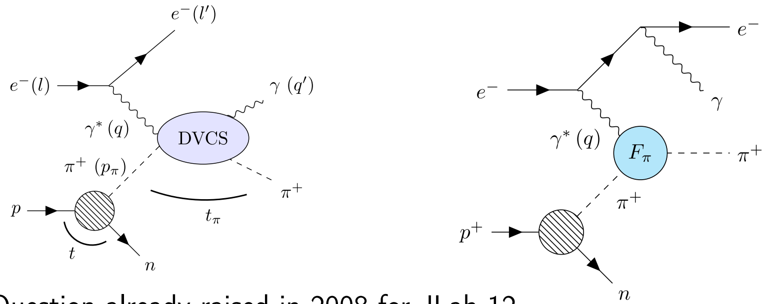
Deep Virtual Compton Scattering



- Best studied experimental process connected to GPDs
  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

DVCS on virtual Pion Target



- Question already raised in 2008 for JLab 12.
  Amrath *et al.*, EPJC 58, 179-192
- Would such processes be measurable at the future EIC and EicC?
  Answering the question of measurability of DVCS requires:
  - A pion GPD model
  - An evolution code
  - A phenomenological code able to compute amplitudes from GPDs
  - An event generator simulating how many events could be detected

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t;\mu) + mod(m,2)\xi^{m+1} C^q_{m+1}(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\ H^q(x,\xi,t;\mu) = F^q_1(t)$$

Lorentz Covariance

Pion GPD modelling I

Theoretical constaints

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$|H^q(x,\xi,t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Pion GPD modelling I

Theoretical constaints

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines
  $\rightarrow$ GPDs are continuous albeit non analytical at $x = \pm\xi$

  J. Collins and A. Freund, PRD 59 074009 (1999)

  Factorisation theorem

Pion GPD modelling I

Theoretical constaints

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

Factorisation theorem

- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

Axial-Vector WTI

Pion GPD modelling I

Theoretical constaints

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Soft pion theorem (pion GPDs only)

  Axial-Vector WTI

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

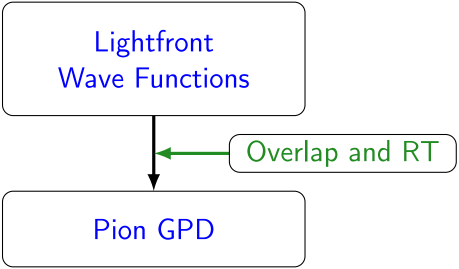
Pion GPD modelling II

Modelling strategy

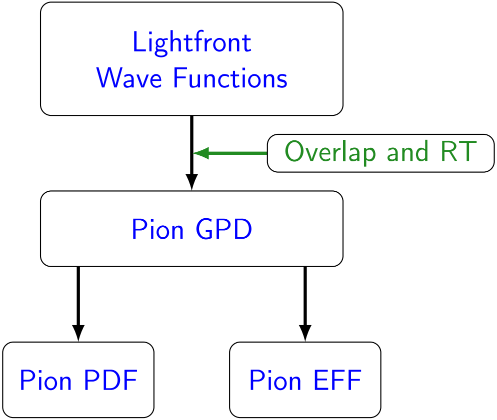
Lightfront
Wave Functions

Pion GPD modelling II

Modelling strategy

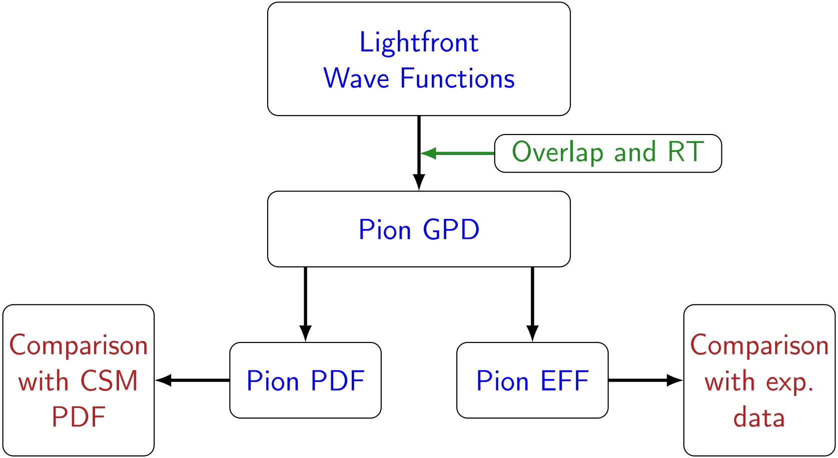


Pion GPD modelling II
Modelling strategy



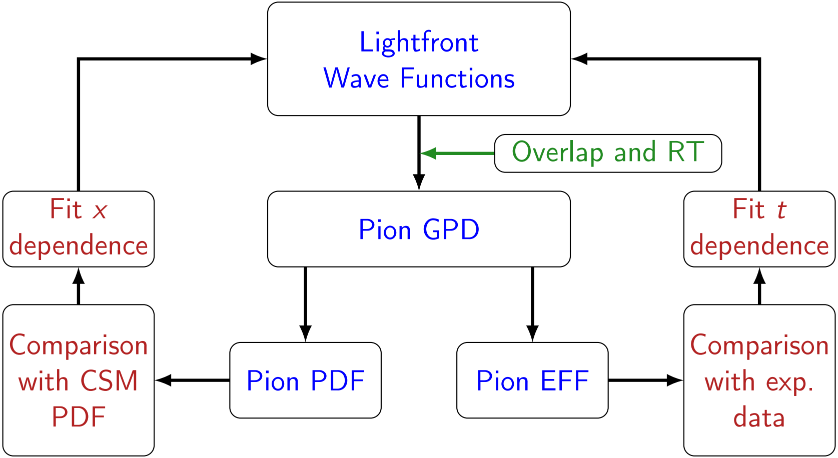
Pion GPD modelling II

Modelling strategy



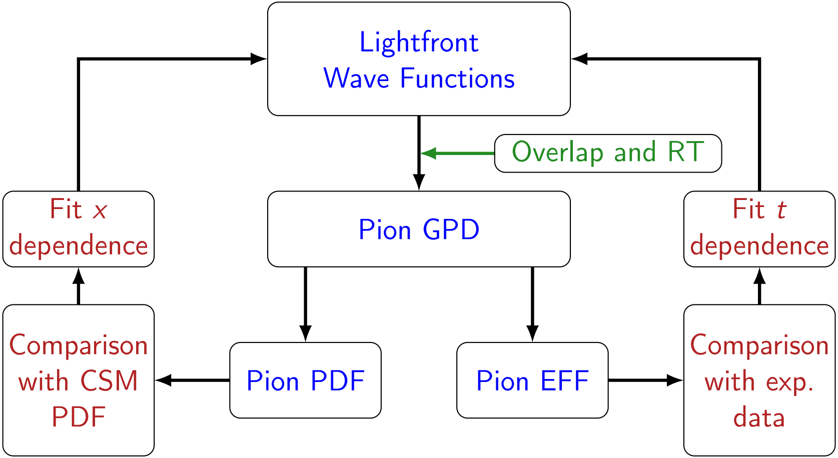
Pion GPD modelling II

Modelling strategy



Pion GPD modelling II

Modelling strategy



Model of LFWFs based on Continuous Schwinger Methods

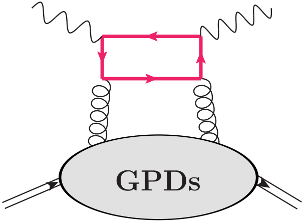
M. Ding *et al.*, PRD 101 5, 054014

Evolution of GPDs

- We adapted a PDF evolution code Apfel++ for GPDs
- It encodes the 1-loop GPDs splitting functions
- Benchmarked on the conformal space evolution (differences less than 0.1% when evolving from 1 GeV^2 to 10^4 GeV^2)

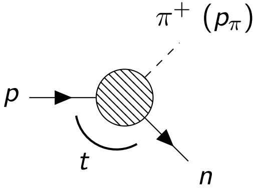
DVCS and Sullivan Amplitudes

DVCS Amplitude



Berthou *et al.*,EPJC 78 478

Sullivan Process



$$F(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t},$$

Amrath *et al.*, EPJC 58, 179-192

Generating and selecting events

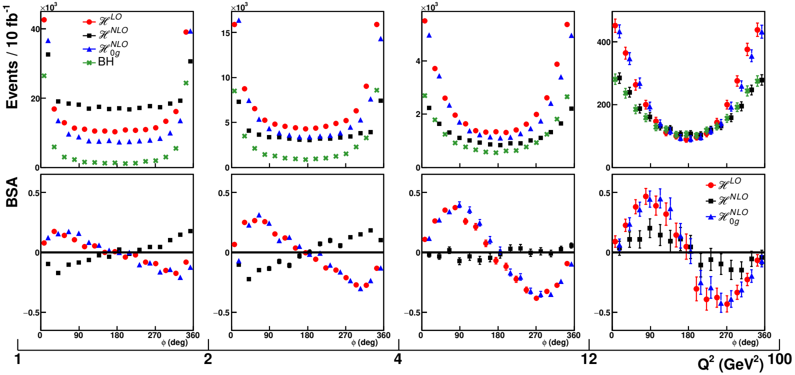
- We generate a large number of events in the center of mass of the process
- We select the events compatible with the geometry of the facilities and performances of the detectors as given in the Yellow report (EIC) or White Paper (EicC).
- We add kinematical cuts in s and the invariant mass of the pion-neutron system to avoid as much as possible contamination with resonnances.

see Amrath *et al.*, EPJC 58, 179-192

- We rescale with the one-year luminosity for the considered facility.

Assessing observable at EIC

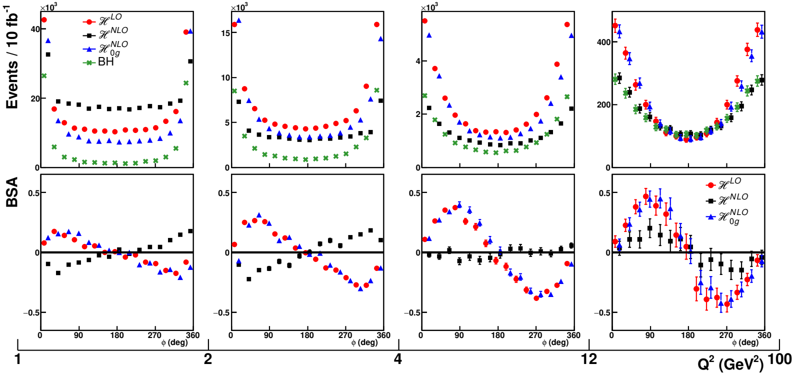
Number of $ep \to e\gamma\pi^+ n$ and Beam Spin Asymmetry $\mathcal{A}_{LU} = \frac{\sigma^{\rightarrow} - \sigma^{\leftarrow}}{\sigma^{\rightarrow} + \sigma^{\leftarrow}}$



J.M. Morgado Chavez *et al.*, in preparation

Assessing observable at EIC

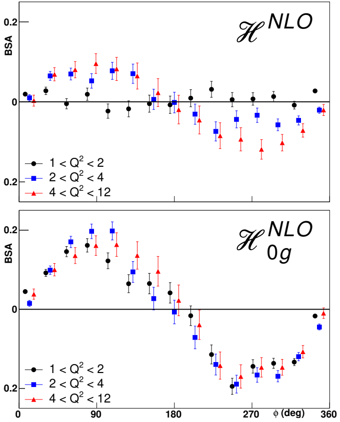
Number of $ep \to e\gamma\pi^+ n$ and Beam Spin Asymmetry $\mathcal{A}_{LU} = \frac{\sigma^{\rightarrow} - \sigma^{\leftarrow}}{\sigma^{\rightarrow} + \sigma^{\leftarrow}}$



J.M. Morgado Chavez *et al.*, in preparation

- Our GPD models yield a visible signal at EIC
- Sign change in the BSA due to gluons contributions

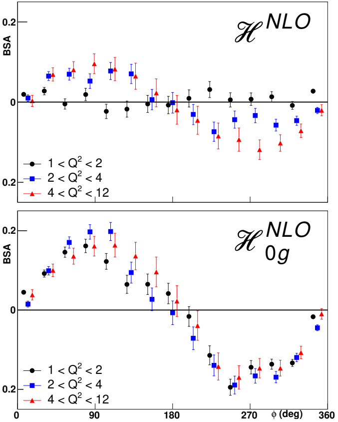
Assessing observable at EicC



J.M. Morgado Chavez *et al.*, in preparation

- BSA measurable at EicC
- No change of sign yet gluon important in the valence region
- Gluons strongly reduces the amplitude of the BSA (around a factor 2)
- Gluons introduce an important Q^2 dependence

Assessing observable at EicC



J.M. Morgado Chavez *et al.*, in preparation

- BSA measurable at EicC
- No change of sign yet gluon important in the valence region
- Gluons strongly reduces the amplitude of the BSA (around a factor 2)
- Gluons introduce an important Q^2 dependence

EicC can provide valuable information about the pion 3D structure in terms of gluons in the valence region

Summary and Conclusion

Summary

- Question: Is DVCS on virtual pion measurable at EIC and EicC ?
- Model of pion GPDs fulfilling all requiered properties
- Developed and benchmarked an evolution code
- Exploited the PARTONS framework to compute DVCS amplitudes
- Generated EIC and EicC events and selected them according to the Yellow Report and White paper specification

Conclusion

- Significant number of DVCS + BH events on virtual pion targets
- In principle enough to partly challenge our understanding of the pion
- Refined studies can be done in future