Extension of LCSR for the pion TFF to low momenta via RG summation of radiative corrections

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in collaboration with
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based on
PRD103 (2021) 096003, EPJ Web Conf. 222 (2019) 0301, PRD 98 (2018) 096017

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OUTLINE

1. Intro: Experimental and Theoretical motivations to modify fixed order pQCD (FOPT) calculation of transition FF for $\gamma\gamma^*(Q^2) \to \pi^0$ at low Q^2.
2. Dispersive form for pion TFF + RG generates a “New” perturbation theory - Fractional APT.
3. Intro/Review to APT $\Rightarrow$ Generalized “Fractional” **APT**= **FAPT**, coupling behavior.
4. Light cone sum rules with **FAPT**:
   Determination of twist-2 DA and twist-4,6
   new prediction for the pion-photon TFF down to 0.35 GeV^2
5. Pion DA and μ-pair pion induced Drell-Yan production
6. Conclusions

Experimental status of pion transition FF

Why it is interesting for QCD?
The measurements of TFF is the clean experiment with a single pion in final state that possesses the best accuracy (BESIII) among others exclusive hard reactions ($\boldsymbol{q_2^2 \sim 0}$).

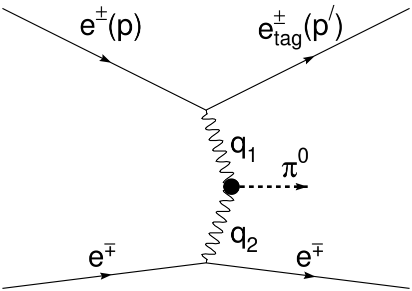
BESIII (2019) $Q^2 : 0.3 - 3.1\ \mathrm{GeV}^2$
Promising very precise data
(preliminary, **arXiv:1810.00654**)

CELLO (1991) $Q^2 : 0.7 - 2.2\ \mathrm{GeV}^2$

CLEO (1998) $Q^2 : 1.6 - 8.0\ \mathrm{GeV}^2$
agrees with collinear QCD

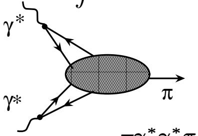
BaBar (2009) $Q^2 : 4 - 40\ \mathrm{GeV}^2$
TFF has growing tendency with Q^2
creating the “BaBar puzzle”, **[MS2009]**

Belle (2012) $Q^2 : 4 - 40\ \mathrm{GeV}^2$
returns to collinear QCD



Factorization $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD

$$\int d^4x e^{-iq_1\cdot z}\langle\pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot \boldsymbol{F}^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$



where $-q_1^2 = Q^2 > 0, \ \ -q_2^2 = q^2 \geq 0$

$$\boldsymbol{F}^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \boldsymbol{T(Q^2,q^2,\mu_F^2;x)} \otimes \boldsymbol{\varphi_\pi(x;\mu_F^2)} + O(\frac{1}{Q^4})\,,$$

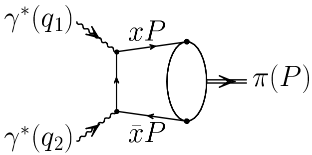
Collinear factorization at $Q^2, q^2 \gg$ (a hadron scale)^2, for the leading twist, μ_F^2 – boundary between large scale Q^2 and hadronic m_ρ^2. At the parton level

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) \quad = \quad \frac{\sqrt{2}}{3}f_\pi\int_0^1 dx\,\frac{\mathbf{1}}{\boldsymbol{Q^2x+q^2\bar{x}}}\,\boldsymbol{\varphi_\pi(x)}$$

$+$ **radiative corrections**$(\alpha_s, \beta_0\alpha_s^2)$(-20 %)

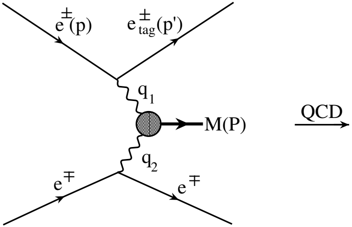
$+$ twist-4

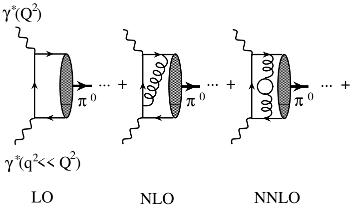
$+$ twist-6



The **radiative corrections** and its sum will be the Main subject of our consideration.

Theoretical status of the pion TFF: hard and hadronic parts





Our Theoretical advances in both parts of **QCD factorization**:

- high order NNLO$_\beta$ contribution $O(\alpha_s^2\beta_0)$ **[Melic et.al. 2003,MS2009]** to the hard part;
- distribution amplitude **[BMS2001]** of twist-2 for pion part, obtained in NLC QCD SR;
- contributions of twist-4 and corrections a'la twist-6 **[Agaev et al,2012]** .

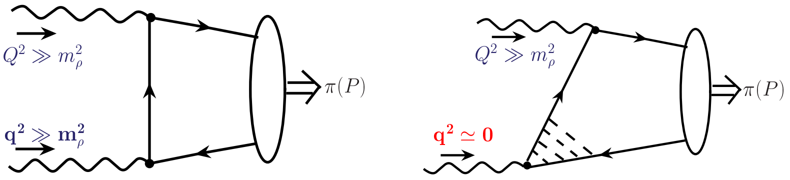
3 steps to LCSR for TFF:

Perturbative & twist expansion;
Dispersive representation in q^2;
LCSR with duality interval s_0 for $q^2 \to 0$.

$\gamma^*\gamma \to \pi$: Light-Cone Sum Rules consideration at $q^2 \sim 0$

LCSR effectively accounts for long-distance effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2 **[Khodjamirian EJPC(1999)]**,

$$F_{\gamma\gamma^*\pi}(Q^2, q^2 \to 0) \quad = \quad \int_{s_0}^{\infty} \rho^{\rm PT}(Q^2, s)\frac{ds}{s} + \int_0^{s_0} \rho^{\rm PT}(Q^2, s) e^{(m_\rho^2 - s)/M^2} \frac{ds}{m_\rho^2},$$

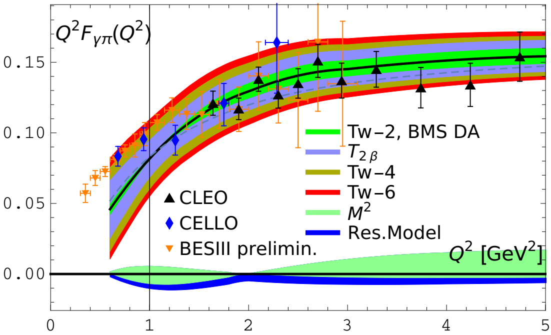


FOPT/twist contributions are given in form of convolution with pion DAs:

$$\rho^{\rm PT} \sim \frac{1}{\pi}\mathbf{Im}\left[T^{(2)}_{\rm LO} + a_s T^{(2)}_{\rm NLO} + a_s^2 T^{(2)}_{{\rm NNLO}_{\beta_0}} + \ldots\right] \otimes \varphi_\pi^{\rm tw2} + \frac{1}{\pi}\mathbf{Im}\left[T^{(4)}\right] \otimes \varphi_\pi^{\rm tw4} + \ldots$$

Pion TFF in LCSR in QCD FOPT vs exp. data

Challenge for low energy discription
Total rad. corrections $\quad -18$ % at 3 GeV^2 .



Mikhailov&AP&Stefanis, PRD93(2016)114018

Source	Uncertainty (%) at 3 GeV^2
Unknown NNLO term $\mathcal{T}_c^{(2)}$	∓ 5
Range of Tw-2 BMS DAs	$-3.4 \div 4.1$
Tw-4 $\delta^2 = [0.152 - 0.228]$ GeV^2	± 3.0
Tw-6 $\langle \bar{q}q \rangle^2 = (0.24 \pm 0.01)^6$ GeV^6	$-2.4 \div 3.0$
Total	$-13.6 \div 14.9$

Motivation, method of solution, phenomenological goals

- Description of the low BESIII momentum domain is inaccessible in FOPT QCD. The new perturbation theory is required.
  - Sensitivity to the values of higher twists parameters.
- We rearrange the QCD perturbation theory following to RG and dispersion relation.
  - The **new PT** appears naturally as a generalization of **FAPT**.
  - The domain of applicability is extended down to Λ^2_{QCD} and below.
- Extracting the hadronic parameters of twists 2, 4, 6 from BESIII data.
  - Reconciling with the current results of lattice simulation and other estimations.

FAPT – Fractional Analitic Perturbation Theory

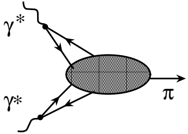
Road-map for LCSR with RG summed perturbative terms

3 steps to LCSR for FF within FOPT:

1) Perturbative & twist expansion;
2) Dispersive representation in q^2;
3) LCSR with duality interval s_0 for $q^2 \to 0$.

4 steps to LCSR for FF with RG summation:

1) Perturbative & twist expansion;
2) **(New!) RG summation of perturbative term;**
3) Dispersive representation in q^2 $\to$ **FAPT;**
   Limitation on FAPT coupling from asymptotic behavior.
   Additional image of coupling in FAPT
4) LCSR with duality interval s_0 for $q^2 \to 0$.



We apply this scheme for pion TFF $\gamma(q^2 \simeq 0)\gamma^*(Q^2) \to \pi^0$

2nd step: RG improvement for pion TFF

$$
\begin{aligned}
F^{(\text{tw=2})}_{\text{FOPT}}(Q^2,q^2) &= N_{\text{T}}\left(T_{\text{LO}} + a_s T_{\text{NLO}} + a_s^2 T_{\text{NNLO}} + \ldots\right) \otimes \varphi_\pi^{(2)}\,,\\
T_{\text{LO}} &= a_s^0(\mu_F^2)\ T_0(y) \equiv 1/\left(q^2\bar{y} + Q^2 y\right)\\
a_s T_{\text{NLO}} &= a_s^1(\mu_F^2)\ T_0(y) \otimes \left[\mathcal{T}^{(1)} + \underline{L\ V_0}\right](y,x)\,,\quad L \equiv \ln\left[(q^2\bar{y} + Q^2 y)/\mu_F^2\right]\,,\\
a_s^2 T_{\text{NNLO}} &= a_s^2(\mu_F^2)\ T_0(y) \otimes \Bigg[\mathcal{T}^{(2)} - \underline{L\,\mathcal{T}^{(1)}\beta_0} + \underline{L\,\mathcal{T}^{(1)} \otimes V_0} - \underline{\frac{L^2}{2}\,\beta_0 V_0}\\
&\qquad + \underline{\frac{L^2}{2}\ V_0 \otimes V_0} + \underline{\underline{L\ V_1}}\Bigg](y,x)\,,
\end{aligned}
$$

The "underlined" <u>terms</u> of RG-evolution can be resumed to $a_s(\mu_F^2) \to \bar{a}_s(y) \equiv \bar{a}_s\left(q^2\bar{y} + Q^2 y\right)$ and ERBL–factor **[Ayala et al. PRD98 2018, 096017]**:

$$
F^{(\text{tw=2})}(Q^2,q^2) = N_{\text{T}}\, T_0(y) \underset{y}{\otimes} \left\{\left[1 + \bar{a}_s(y)\mathcal{T}^{(1)}(y,x) + \bar{a}_s^2(y)\mathcal{T}^{(2)}(y,x) + \ldots\right] \underset{x}{\otimes} \exp\left[-\int_{a_s(\mu_F^2)}^{\bar{a}_s(y)} d\alpha\, \frac{V(\alpha;x,z)}{\beta(\alpha)}\right]\right\} \underset{z}{\otimes} \varphi_\pi^{(2)}(z,\mu_F^2)\,,
$$

To restore FOPT expression one could use LO approximations:

$$
\frac{V(\alpha;x,z)}{\beta(\alpha)} \to \frac{V_0}{\beta_0\alpha}\,, \qquad \bar{a}_s(y) = \frac{a_s(\mu^2)}{1 + a_s(\mu^2)\beta_0 L} \to a_s(\mu^2) - L\beta_0 a_s^2(\mu^2)\,.
$$

2nd step: RG improvement for pion TFF in one loop.

Collecting all of the "underlined" terms of RG-evolution into $\bar{a}_s(y)$ and ERBL–factor **[Ayala et al. PRD98 2018, 096017]**

$$F^{(\text{tw=2})}(Q^2,q^2) = N_{\text{T}} T_0(y) \underset{y}{\otimes} \left\{ \left[1 + \bar{\boldsymbol{a}}_{\boldsymbol{s}}(\boldsymbol{y}) \mathcal{T}^{(1)}(y,x) + \bar{\boldsymbol{a}}^{\boldsymbol{2}}_{\boldsymbol{s}}(\boldsymbol{y}) \mathcal{T}^{(2)}(y,x) + \ldots \right] \underset{x}{\otimes} \right.$$

$$\left. \exp\left[- \int_{a_s}^{\bar{a}_s(y)} d\alpha \, \frac{V(\alpha; x, z)}{\beta(\alpha)} \right] \right\} \underset{z}{\otimes} \varphi_{\pi}^{(2)}(z,\mu^2)\,,$$

$$\varphi_{\pi}^{(2)}(x,\mu^2) = \psi_0(x) + \sum_{n=2,4,\ldots}^{\infty} b_n(\mu^2)\, \psi_n(x) - \text{Gegenbauer basis}$$

$$F^{(\text{tw=2})}(Q^2,q^2) = F_0^{\text{RG}}(Q^2,q^2) + \sum_{n=2,4,\ldots}^{\infty} b_n(\mu^2) F_n^{\text{RG}}(Q^2,q^2)$$

One loop resumed result ($\nu_n = \gamma_n/2\beta_0$):

$$F_n^{\text{RG}}(Q^2,q^2) = N_{\text{T}} T_0(y) \underset{y}{\otimes} \left\{ \left[1 + \bar{\boldsymbol{a}}_{\boldsymbol{s}}(\boldsymbol{y}) \mathcal{T}^{(1)}(y,x) \right] \left(\frac{\bar{\boldsymbol{a}}_{\boldsymbol{s}}(\boldsymbol{y})}{\boldsymbol{a}_{\boldsymbol{s}}(\boldsymbol{\mu}^{\boldsymbol{2}})} \right)^{\nu_n} \right\} \underset{x}{\otimes} \psi_n(x)$$

- For $q^2 = 0$, $y \ll 1$, the coupling $\bar{a}_s(y) \equiv \bar{a}_s\left(q^2\bar{y} + Q^2 y\right)$ is inapplicable within factorization.
  Resumed formula fall out from the PT applicability domain.

3rd step: Dispersive form of TFF leads to FAPT, twist-2 term

$$\Big[F(Q^2,q^2)\Big]_{\text{an}} = \int_{m^2}^{\infty} \frac{\rho_F(Q^2,\sigma)}{\sigma+q^2-i\epsilon}\,d\sigma,\;\; \rho_F(Q^2,\sigma) = \frac{\textbf{Im}}{\pi}\Big[F(Q^2,-\sigma)\Big]\,.$$

For $m^2=0$, the dispertion relation of RG improved TFF expressed in terms of FAPT couplings $\mathcal{A}_\nu, \mathfrak{A}_\nu$, ($\nu_n=\gamma_n/2\beta_0$):

$$n=0;\; \boldsymbol{F_0^{\text{FAPT}}}(Q^2,q^2) = N_T\,T_0(Q^2,q^2;y)\underset{y}{\otimes}\Big\{\mathbb{1}+\mathbb{A}_{\mathbf{1}}(\boldsymbol{y})\mathcal{T}^{(1)}(y,x)\Big\}\underset{x}{\otimes}\psi_0(x)$$

$$n\neq 0;\; \boldsymbol{F_n^{\text{FAPT}}}(Q^2,q^2) = \frac{N_T}{a_s^{\nu_n}(\mu^2)}\,T_0(Q^2,q^2;y)\underset{y}{\otimes}\Big\{\mathbb{A}_{\boldsymbol{\nu_n}}(\boldsymbol{y})\mathbb{1}+\mathbb{A}_{\mathbf{1}+\boldsymbol{\nu_n}}(\boldsymbol{y})\mathcal{T}^{(1)}(y,x)\Big\}\underset{x}{\otimes}\psi_n(x)$$

$\{\mathbb{A}_\nu\}$ – nonpower series, where

$$\mathbb{A}_\nu(y) = \mathcal{A}_\nu(q^2\bar{y}+Q^2y)-\mathfrak{A}_\nu(0)$$

- The same expression as for RG-case, $\mathbb{A}_\nu(y) \Leftrightarrow \bar{a}_s^\nu(y)$
- For correspondence with PT asymptotics we need to put condition on FAPT coupling: $\boxed{\boldsymbol{\mathcal{A}_\nu(0)=\mathfrak{A}_\nu(0)=0 \text{ for } 0<\nu\leqslant 1}}$

Dispersive "Källen–Lehmann" representation

Different coupling images in Euclidean, $\mathcal{A}_n$, and Minkowsk., $\mathfrak{A}_n$, regions $\overline{\alpha}_s^n \to \{\mathcal{A}_n, \mathfrak{A}_n\}$ **[Shirkov&Solovtsov1997-07]** - nonpower series

$$\mathcal{A}_n[L] = \int_0^\infty \frac{\rho_n(\sigma)}{\sigma + Q^2 - i\epsilon}\, d\sigma, \quad \rho_n(\sigma) = \frac{\mathrm{Im}}{\pi}\left[\overline{a}_s^n(-\sigma)\right]\beta_0$$

For 1 loop run, $L = \ln(Q^2/\Lambda^2)$, $L_s = \ln(s/\Lambda^2)$:

$$\rho_1(\sigma) \overset{1l}{=} \frac{1}{L_\sigma^2 + \pi^2}$$

$$\mathcal{A}_1[L] = \int_0^\infty \frac{\rho_1(\sigma)}{\sigma + Q^2}\, d\sigma \overset{1l}{=} \frac{1}{L} - \frac{1}{e^L - 1}$$

$$\mathfrak{A}_1[L_s] = \int_s^\infty \frac{\rho_1(\sigma)}{\sigma}\, d\sigma \overset{1l}{=} \frac{1}{\pi} \arccos \frac{L_s}{\sqrt{\pi^2 + L_s^2}}$$

Inequality :

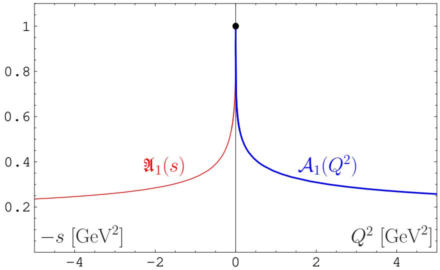
$$a_s^n[L] > (\mathcal{A}_n[L], \mathfrak{A}_n[L]) \xrightarrow{L\to\infty} a_s^n[L]$$

Generalization of $(\mathcal{A}_n, \mathfrak{A}_n)$: $\mathcal{I}_n$ **[Ayala et al. PRD98 2018, 096017]** :

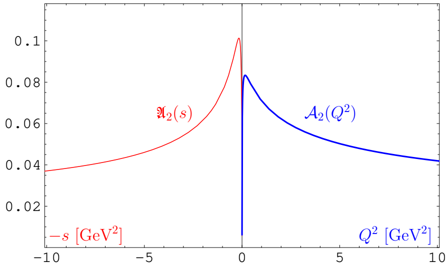
$$\mathcal{I}_n(s, Q^2) = \int_s^\infty \frac{\rho_n(\sigma)}{\sigma + Q^2}\, d\sigma \; .$$

APT: Distorting mirror **[Shirkov&Solovtsov1997-2007]**

Coupling images: $\mathfrak{A}_1(s)$ & $\mathcal{A}_1(Q^2)$

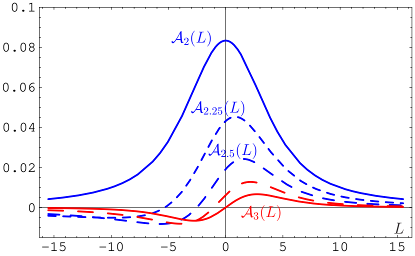


Square-images: $\mathfrak{A}_2(s)$ & $\mathcal{A}_2(Q^2)$

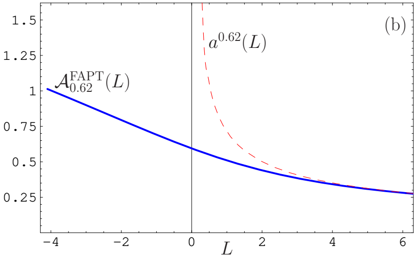


$$\mathcal{A}_\nu[L] = \frac{1}{L^\nu} - \frac{\mathrm{Li}_{1-\nu}(e^{-L})}{\Gamma(\nu)}$$

Fractional $\nu \in [2, 3]$:



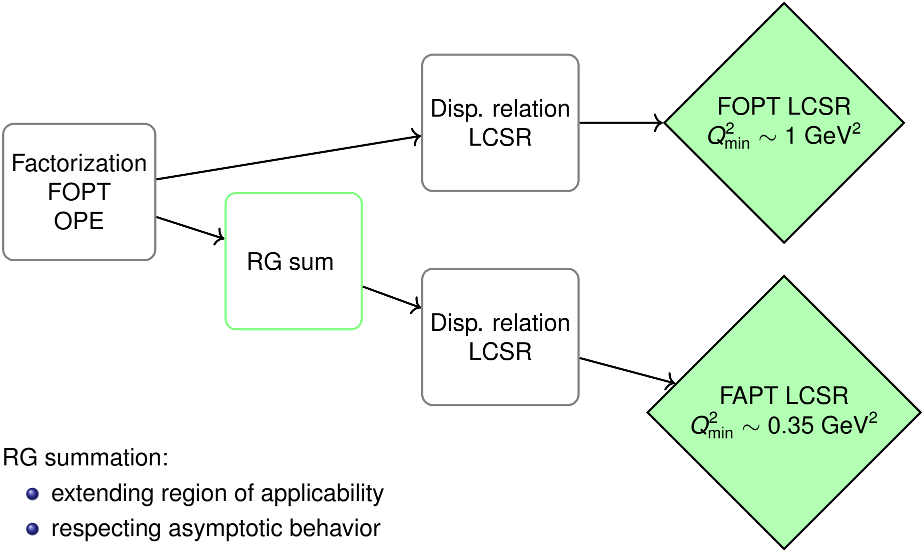
Comparison with $\bar{a}_s^\nu[L]$:



where $\nu = 0.62 = \gamma_2/2\beta_0$

$$\bar{a}_s^{1+\nu}[L] \gg \mathcal{A}_{1+\nu}[L] \gg \mathcal{A}_{2+\nu}[L] \text{ at } L \sim 1$$

Improvements, Bird's eye view



RG summation:

- extending region of applicability
- respecting asymptotic behavior
- model independent
- parameter free

$$\gamma(q^2 \simeq 0)\gamma^*(Q^2) \to \pi^0$$

Light Cone Sum Rules with FAPT,

New prediction for the pion TFF

Pimikov A., Ayala C. &M.S. &Stefanis N.

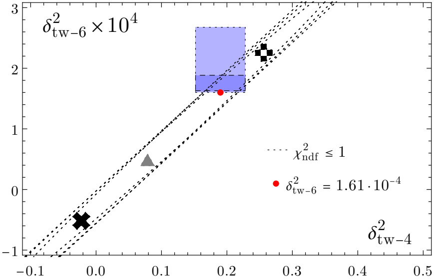
PRD103 (2021) 096003, PRD98 (2018) 096017,
EPJ Web Conf. 222 (2019) 0301

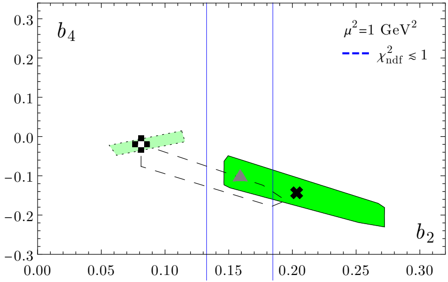
From BESIII, CELLO, Cleo data in window $0.35 \leq Q^2 \leq 3.1$ GeV^2, we extract and reconcile the hadronic characteristics presented in TFF: twist-2 DA, the scales of twist-4,6 **[PRD103 (2021) 096003]**.

$$F^{\gamma\pi}_{\mathrm{LCSR}}(Q^2) = F^{\gamma\pi}_{\mathrm{LCSR};0}(Q^2) + \sum_{n=2,4} b_n(\mu^2)\, F^{\gamma\pi}_{\mathrm{LCSR};n}(Q^2) + \delta^2_{\text{tw-4}}(\mu^2) F^{\gamma\pi}_{\text{tw-4}}(Q^2) + \delta^2_{\text{tw-6}} F^{\gamma\pi}_{\text{tw-6}}(Q^2)$$

First, we consider three models of twist-2 DA given by b_2 , b_4:

- DA from QCD SR with NLC ✖ **[BMS2001]**
- platykurtic DA ✜ **[Stefanis PLB738, 2014]**
- ▲: b_2 from lattice (vert. blue lines) **[Bali et al. JHEP08,(2019]**, b_4 from BMS domain



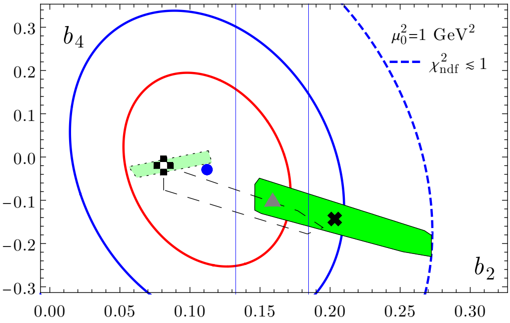


- Twist-4 from **[Bakulev et al. PRD67,2003]** $\delta^2_{\text{tw-4}}(\mu_0^2) = 0.19(4)$ GeV$^2 \sim \langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle$
- Twist-6 is extracted from data $\delta^2_{\text{tw-6}}(\mu_0^2) = 1.61(26) \times 10^{-4}$ GeV$^6 \sim \alpha_S\langle\bar{q}q\rangle^2$

The processing of the experimental data on $\mathrm{TFF}_{\mathrm{LCSR}}$ up to 3.1 GeV^2 (2)

Second, we fit twist-2 DA given by b_2 , b_4: using obtained twist-4,-6 parameters:

- Twist-4 from **[Bakulev et al. PRD67,2003]** $\delta^2_{\text{tw-4}}(\mu_0^2) = 0.19(4)\ \text{GeV}^2 \sim \langle \bar{q} D^2 q\rangle / \langle \bar{q} q\rangle$
- Twist-6 is extracted from data $\delta^2_{\text{tw-6}}(\mu_0^2) = 1.61(26) \times 10^{-4}\ \text{GeV}^6 \sim \langle \bar{q} q\rangle^2$



- best fit DA $\chi^2_{\text{ndf}} = 0.38$

red line - 1σ-region

blue solid line - 2σ-region

blue dashed line - $\chi^2_{\text{ndf}} \leq 1$-region

✖ NLC QCD SR **[BMS2001]** $\in 2\sigma$-region

✜ platykurtic DA **[Stefanis PLB738, 2014]**

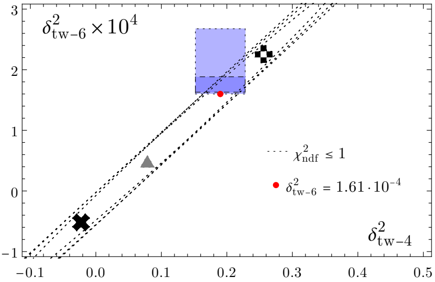
▲ b_2 from lattice (vert. blue linef) b_4 from BMS domain

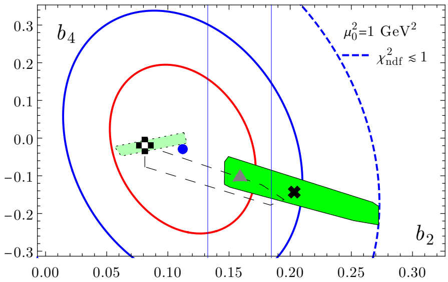
- Pion DA model from BMS domain marked as ▲ is within 1σ-region and is suggested for postdictions of pion TFF.
- Low sensitivity of TFF to pion DA at low momenta
- Considered models are in a good agreement with data

The processing of the experimental data on TFF_{LCSR} up to 3.1 GeV^2 (3)

Processing BESIII, CELLO, Cleo data in window $0.35 \leq Q^2 \leq 3.1$ GeV^2 .
We extract twist-2 DA, the scales of twist-4,6. **[PRD103 (2021) 096003]**

$$F^{\gamma\pi}_{\text{LCSR}}(Q^2) = F^{\gamma\pi}_{\text{LCSR};0}(Q^2) + \sum_{n=2,4} b_n(\mu^2)\, F^{\gamma\pi}_{\text{LCSR};n}(Q^2) + \delta^2_{\text{tw-4}}(\mu^2) F^{\gamma\pi}_{\text{tw-4}}(Q^2) + \delta^2_{\text{tw-6}} F^{\gamma\pi}_{\text{tw-6}}(Q^2)$$

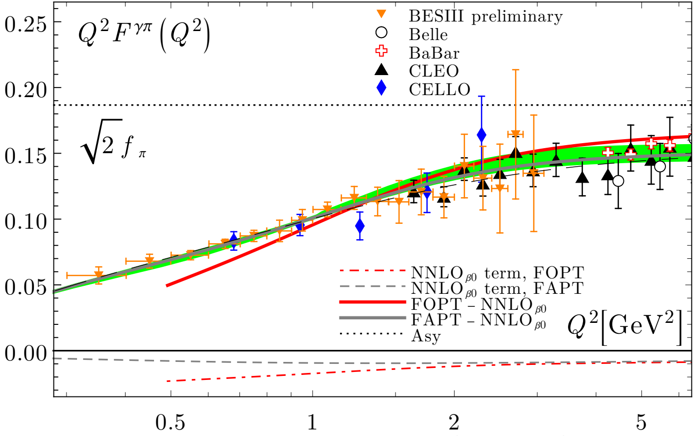




- Twist-4 $\delta^2_{\text{tw-4}}(\mu_0^2)$ from QCD SR **[Bakulev et al. PRD67,2003]**
- Twist-6 is extracted from data and found to be in good agreement with estimations **[here, Cheng et al.,PRD102,2020]** based on Gell-Mann-Oakes-Renner relation
- twist-2 DA chosen in intersection of BMS domain ∩ lattice result is in 1σ-region: $\boldsymbol{(b_2(\mu_0^2) = 0.159, b_4(\mu_0^2) = -0.098)}$ ▲

Predictions of $\mathrm{TFF}_{\mathrm{LCSR}}$ in FAPT vs the experimental data

$$F^{\gamma\pi}_{\mathrm{LCSR}}(Q^2) = F^{\gamma\pi}_{\mathrm{LCSR};0}(Q^2) + \sum_{n=2,4} b_n(\mu^2)\, F^{\gamma\pi}_{\mathrm{LCSR};n}(Q^2) + \delta^2_{\text{tw-4}}(\mu^2) F^{\gamma\pi}_{\text{tw-4}}(Q^2) + \delta^2_{\text{tw-6}} F^{\gamma\pi}_{\text{tw-6}}(Q^2)$$



Gray line&green strip around - FAPT predictions to $\boldsymbol{Q^2 F^{\gamma\pi}_{\mathrm{LCSR}}}$, $\chi^2_{ndf} = \mathbf{0.57}$

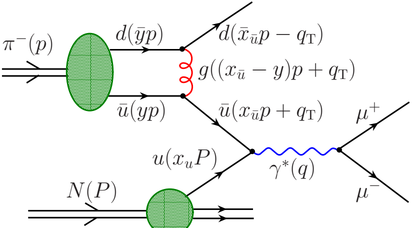
Red line - FOPT prediction at N^{2}LO to $\boldsymbol{Q^2 F^{\gamma\pi}_{\mathrm{LCSR}}}$ – fall down

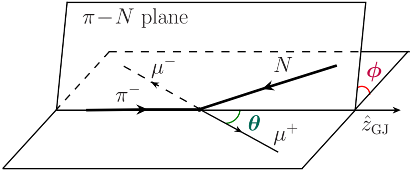
The fitted parameter is the scale of twist-6 $\langle\bar{q}q\rangle^2$ with the certain pattern of pion DA from BMS bunch and QCD SR based estimation for twist-4 parameter.

Pion induced Drell-Yan process $\pi^- + N \to \mu^+ + \mu^- + X$

Angular distribution of μ^+ in the pair rest frame **[Brandenburg et. al. PRL73(1994)]**

$$\frac{d^5\sigma}{dQ^2 dQ_T^2 dx_L\, d\cos\theta d\phi} \propto N(\tilde{x}, \rho)\left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right),$$

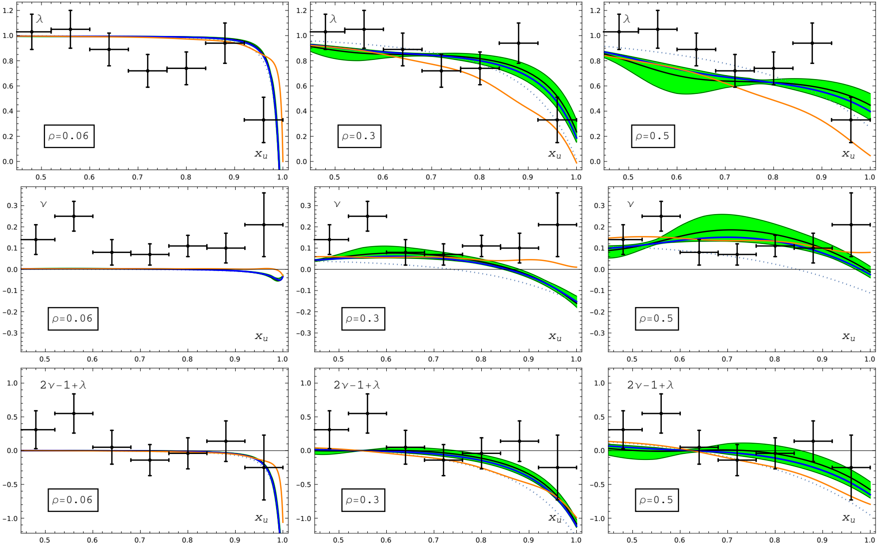




Gottfried-Jackson frame

Detailed analysis of E615@Fermilab data **[Bakulev et. al. PRD76(2007)]**

Pion induced Drell-Yan process, E615@Fermilab



Blue line - [here], orange - DSE-DB DA [Chang et al„ PRL 2013], Data from [Conway et al., PRD39 (1989)]

We look forward to new AMBER and COMPASS data that can provide detailed information on pion DA.

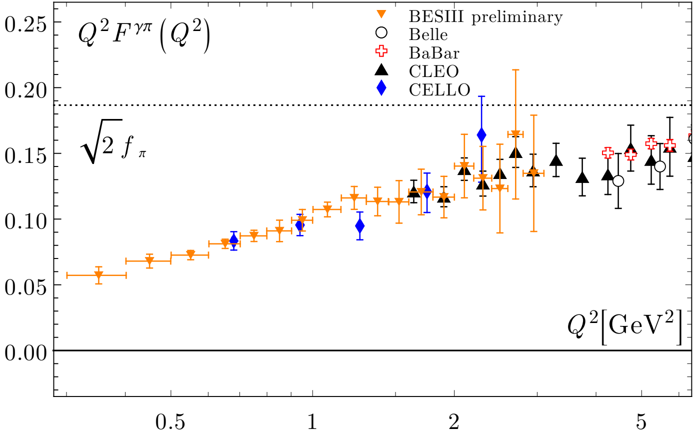
CONCLUSIONS

1. LCSRs and **RG summation** applied for radiative corrections of transition FF yield improvement of Q^2 behavior and extension of the domain QCD applicability well below 1 GeV^2.

2. This composition of **RG sum** and LCSRs naturally leads to a generalization of Fractional APT that improves perturbative corrections to amplitudes.

3. The first time processing of low energy experimental data $[\mathbf{0.35} \leqslant \boldsymbol{Q^2} \leqslant \mathbf{3.1}]$ GeV^2 is performed.
   We have reconciled all twist-2, -4, -6 hadronic characteristics and the lattice result.

Experimental Data on $F_{\gamma\gamma^*\pi}$: BESIII (2019), CELLO, CLEO, BaBar



Uncertainties of BESIII preliminary data are compatible to BaBar data errorbars. Unfortunately, its low energy tail is unreachable for standard QCD.

Theoretical status of pion TFF in QCD FOPT

Hard process at $-Q^2, -q^2 \gg m_\rho^2 \Rightarrow$ collinear factorization

$$F^{(\text{tw=2})}_{\text{FOPT}}(Q^2, q^2) = N_{\text{T}} \left(T_{\text{LO}} + a_s T_{\text{NLO}} + a_s^2 T_{\text{NNLO}} + \ldots \right) \otimes \varphi_\pi^{(2)}$$

$$\begin{aligned}
\boldsymbol{T_{\text{LO}}} &= a_s^0(\mu_F^2)\; T_0(y) \equiv 1/\left(q^2 \bar{y} + Q^2 y\right) \\
\boldsymbol{a_s T_{\text{NLO}}} &= a_s^1(\mu_F^2)\; T_0(y) \otimes \left[\mathcal{T}^{(1)} + \underline{L\; V_0}\right](y,x)\,, \\
\boldsymbol{a_s^2 T_{\text{NNLO}}} &= a_s^2(\mu_F^2)\; T_0(y) \otimes \Big[\mathcal{T}^{(2)} - \underline{L\,\mathcal{T}^{(1)}\beta_0} + \underline{L\,\mathcal{T}^{(1)} \otimes V_0} - \underline{\frac{L^2}{2}\,\beta_0 V_0} \\
&\qquad + \underline{\frac{L^2}{2}\; V_0 \otimes V_0} + \underline{\underline{L\; V_1}}\Big](y,x)\,,
\end{aligned}$$

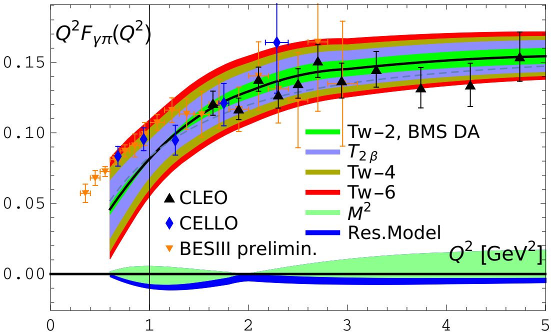
where $L = \ln\left[(q^2 \bar{y} + Q^2 y)/\mu_F^2\right]$.

Plain terms $\mathcal{T}^{(1)}, \mathcal{T}^{(2)}(\mathcal{T}_\beta^{(2)})$ - corrections to parton subprocess;

Underlined terms due to $\bar{a}_s(y)$ and ERBL, V_0 - kernel; $\underline{\underline{\text{underlined}}}$ term - two loops ERBL, V_1 - kernel.

Pion TFF in LCSR in QCD FOPT vs exp. data

Challenge for low energy discription
Total rad. corrections -18 % at 3 GeV^2 .



Mikhailov&AP&Stefanis, PRD93(2016)114018

Source	Uncertainty (%) at 3 GeV^2
Unknown NNLO term $\mathcal{T}_c^{(2)}$	∓ 5
Range of Tw-2 BMS DAs	$-3.4 \div 4.1$
Tw-4 $\delta^2 = [0.152 - 0.228]$ GeV^2	± 3.0
Tw-6 $\langle \bar{q}q \rangle^2 = (0.24 \pm 0.01)^6$ GeV^6	$-2.4 \div 3.0$
Total	$-13.6 \div 14.9$