Linking continuum and lattice quark mass functions via an effective charge

Khépani Raya Montaño

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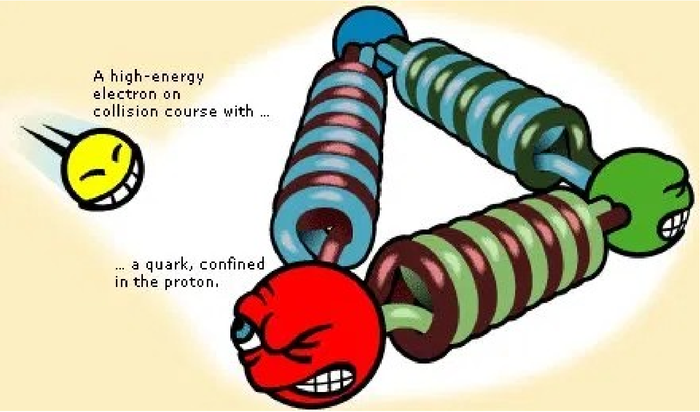
QCD: Basic Facts

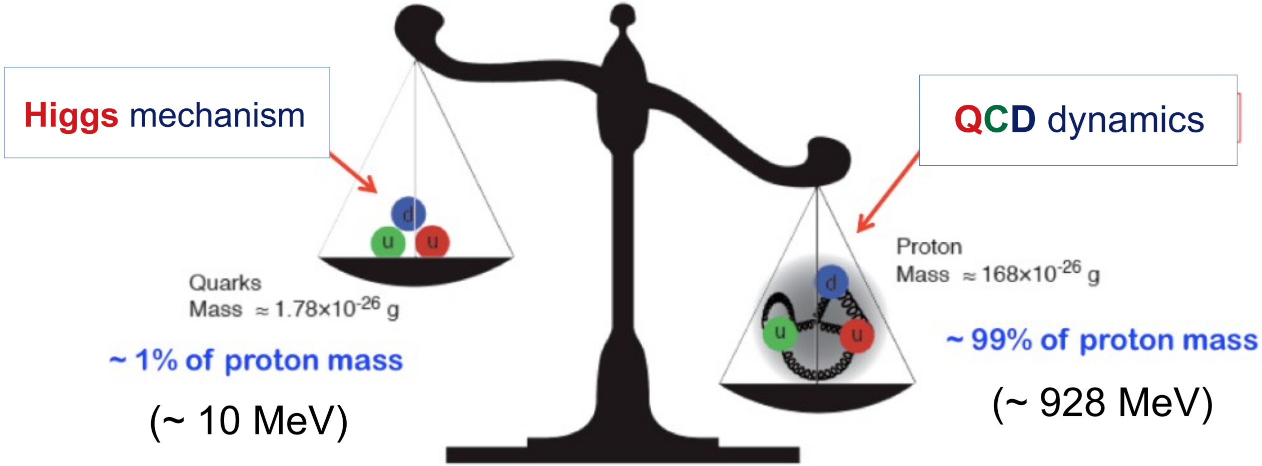
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons

- Emergence of hadron masses (EHM) from QCD **dynamics**





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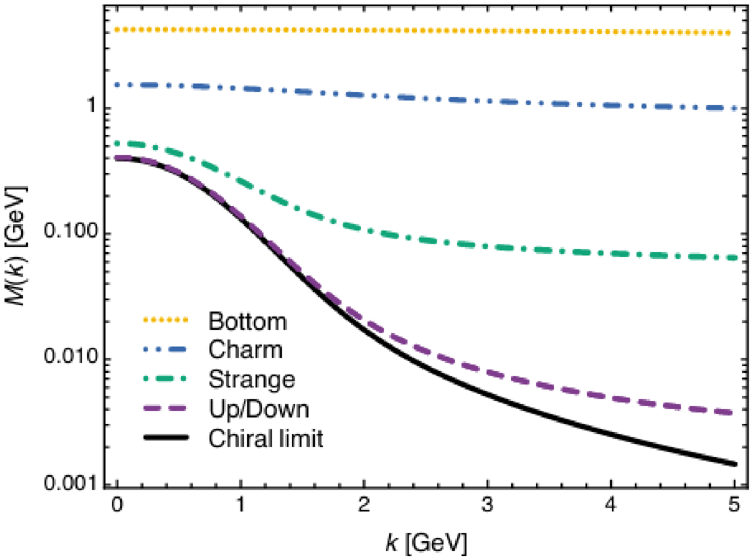
Can we trace them down to fundamental d.o.f?

◆ **In gauge and matter sectors?**

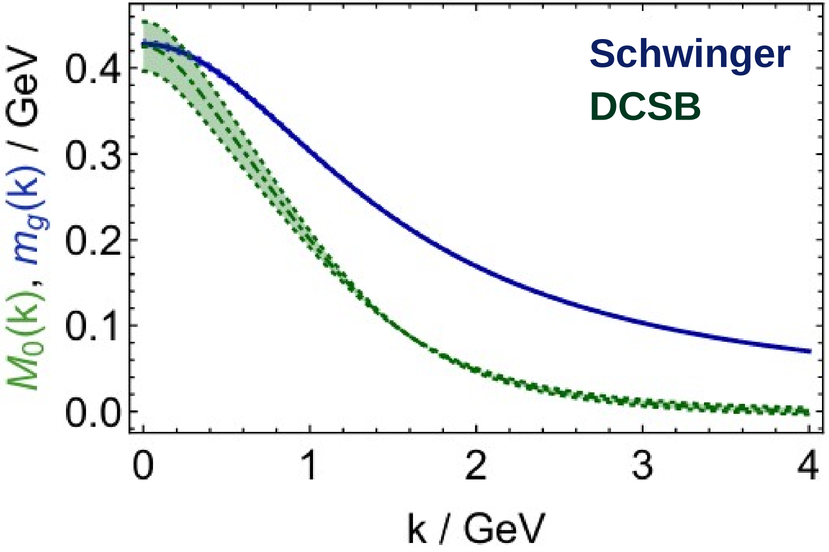
Dynamical masses

(Dynamical Chiral Symmetry Breaking)



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$



Gluon and quark *running masses*

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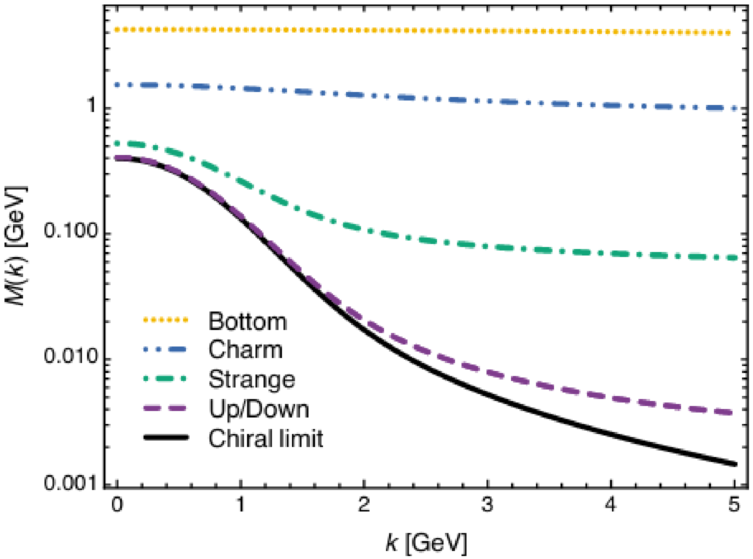
Dynamical masses

(**D**ynamical **C**hiral **S**ymmetry **B**reaking)



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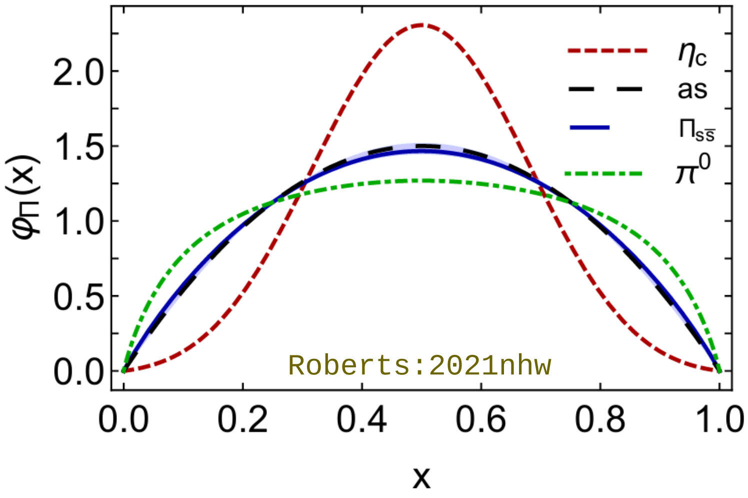
$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

- **What are the implications in the hadron spectrum and properties?**



e.g. Distribution amplitudes

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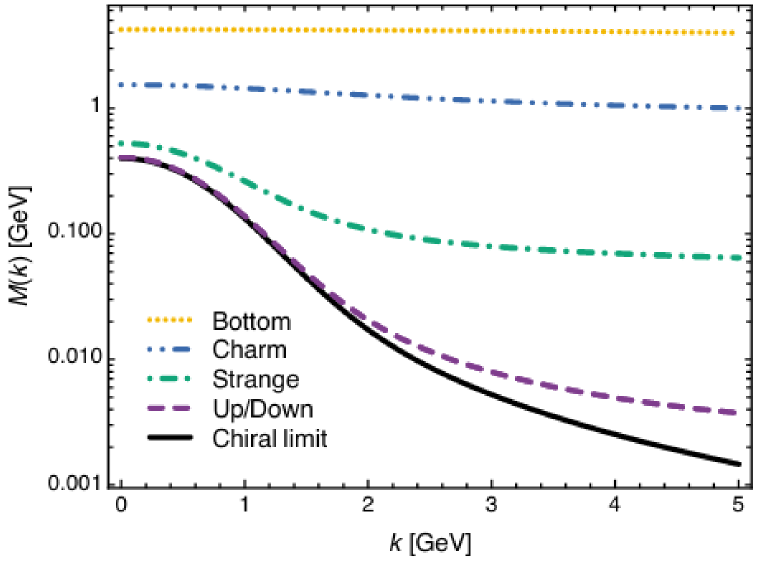

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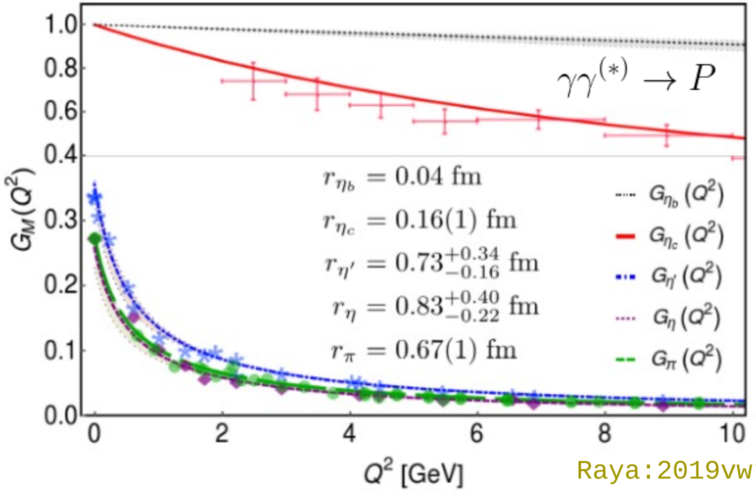

Raya:2019vwr

e.g. Electromagnetic FFs

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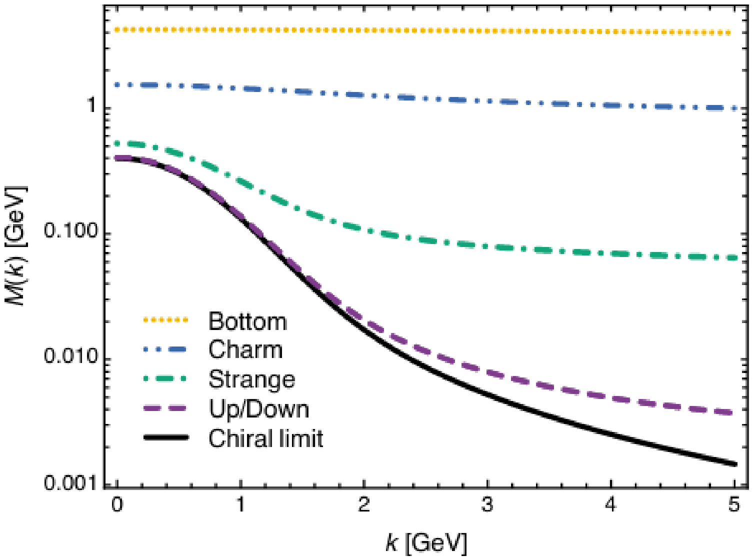


"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

◆ **Furthermore...**

Properties of the **massless pion** are a direct measure of the dressed-**quark mass function**:

$$f_\pi E_\pi(p;0) = M(p^2)/Z(p^2)$$

"Goldstone theorem"

Pions exist if, and only if, DCSB occurs

$$\Gamma_\pi(p;P) \propto iE_\pi(p;P)$$

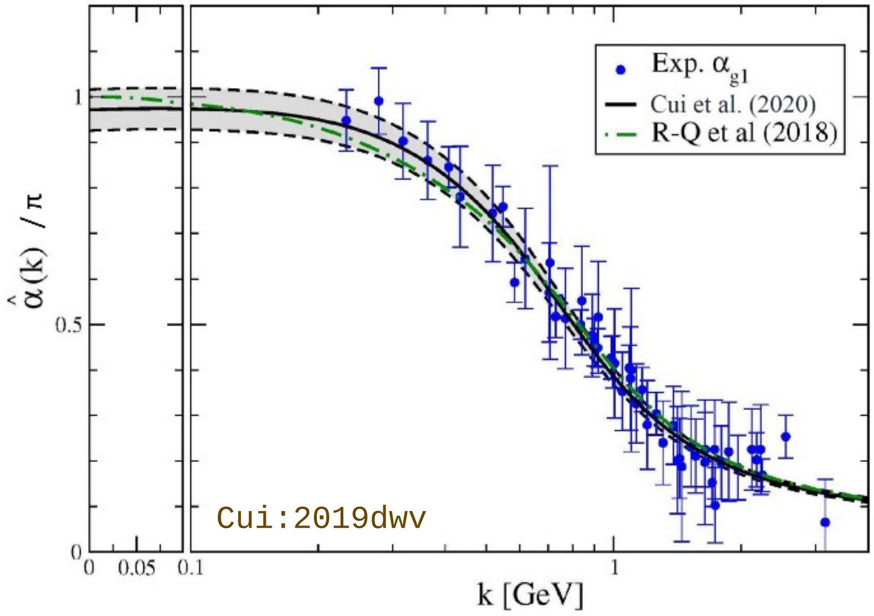
QCD: Basic Facts

J. Rodriguez-Quintero's talk

- **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.

Effective charge from lattice QCD*

Z.-F. Cui(崔著钫)[1,2] J.-L. Zhang(张金利)[3] D. Binosi[4] F. De Soto[5] C. Mezrag[6] J. Papavassiliou[7]
C. D. Roberts[1,2;1)] J. Rodríguez-Quintero[8;2)] J. Segovia[5,2] S. Zafeiropoulos[9]

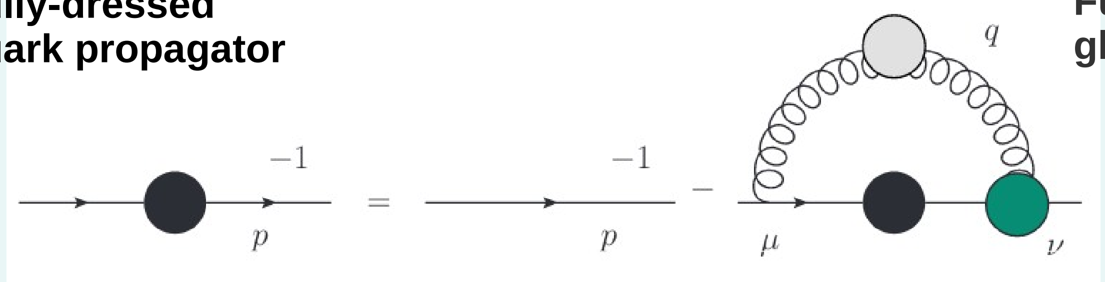


- Our **modern** picture, obtained from combined **lattice QCD** and **continuum** analysis:
  - Compares very well with world data for Bjorken-sume-rule charge.
  - **Cured** from **Landau pole**.
  - Instead, it **saturates** in the IR and exposes a mass scale, the **hadron scale**.
  - Process-independent, **parameter free** prediction.

Quark Mass Function in the Continuum

Fully-dressed
quark propagator

Fully-dressed
gluon propagator



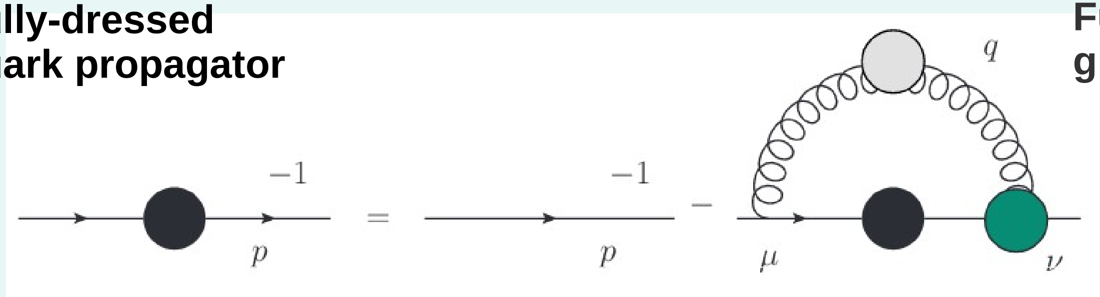
Fully-dressed
quark-gluon vertex

Gap equation: Quark propagator

- The starting point is the **Dyson-Schwinger** equation for the **quark propagator**:

Fully-dressed quark propagator

Fully-dressed gluon propagator



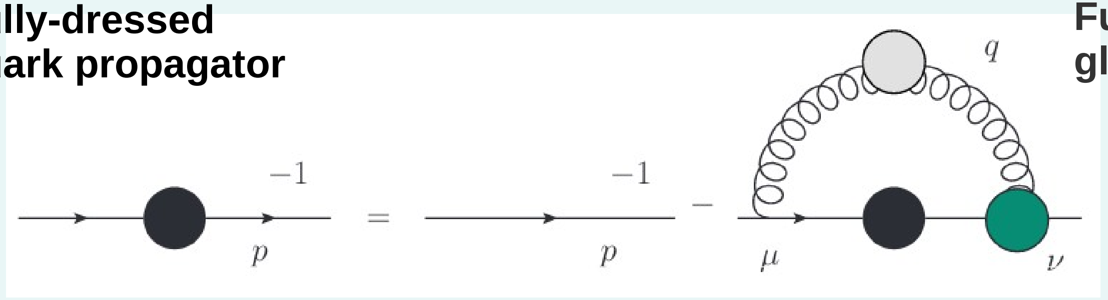
Fully-dressed quark-gluon vertex

- Each blob in the equation obeys its own **DSE**.
  - **Infinite** tower of coupled integral equations: must be systematically **truncated**.
- No assumptions on the strength of the **coupling**.
  - Perturbative and <u>non perturbative</u> facets of **QCD** can be properly **captured**.

Gap equation: Quark Propagator

➢ The starting point is the **Dyson-Schwinger** equation for the **quark propagator**:

Fully-dressed quark propagator



Fully-dressed gluon propagator

Fully-dressed quark-gluon vertex

The **fully-dressed** quark propagator may be written:

$$S(p;\zeta) = \frac{Z(p^2;\zeta^2)}{i\gamma\cdot p + M(p^2)}$$

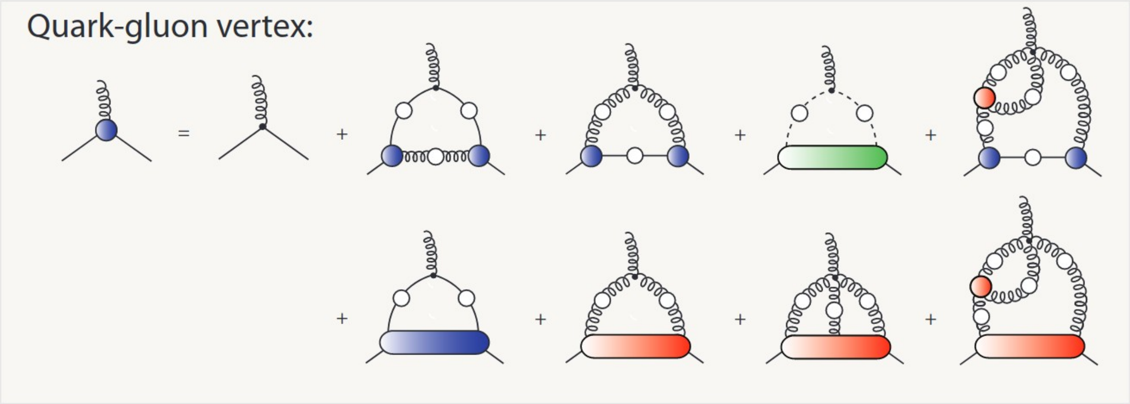
In analogy with its **tree-level** counterpart

$$S^{(0)}(p;\zeta) = \frac{1}{i\gamma\cdot p + m_{\mathrm{bm}}}$$

- The mass function encodes all the complex non-perturbative effects.

- Its shape impacts the hadron structural properties.

Quark-gluon vertex



$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$$

$$\Gamma^L_\mu(k,p) = \sum_{j=1}^{4} \lambda_j(k,p)\, L^j_\mu(k,p)$$

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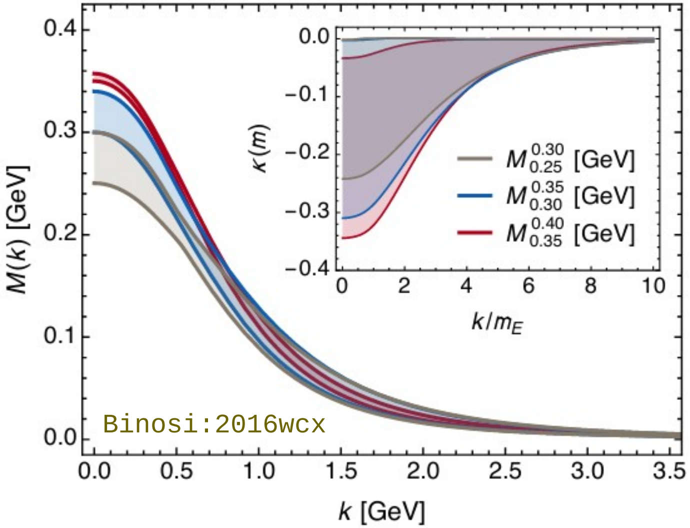


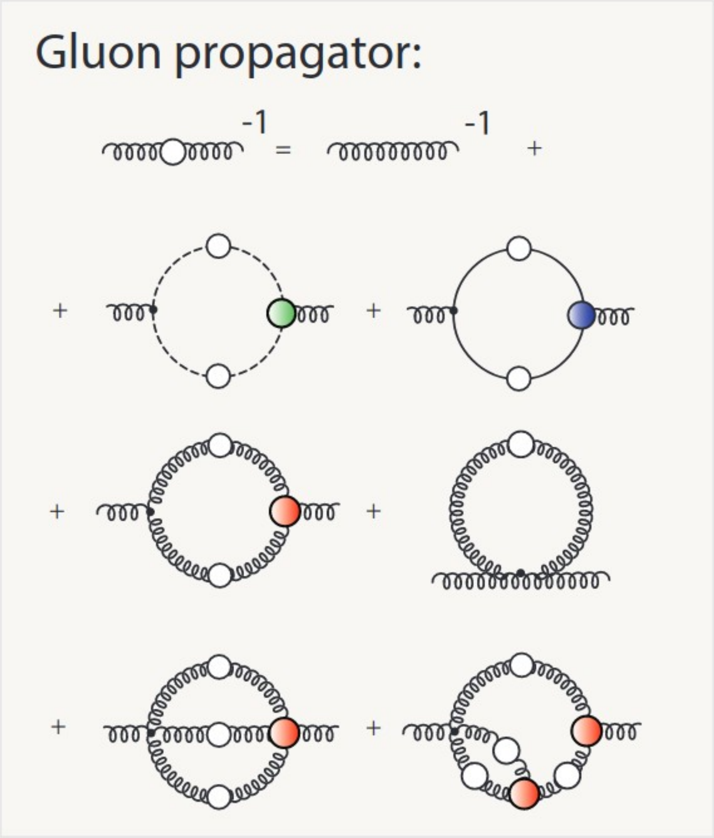
- **QGV**: characterized by 12 tensor structures.
- Mathematical **principles** provide valuable constraints:
  - ✓ Ward-Green-Takahashi / Slavnov-Taylor identities
  - ✓ Perturbation theory limits, etc.

Albino:2018ncl

- Phenomenological inputs may be useful as well.
  - ✓ For instance, the strength of quark anomalous chromo magnetic moment **(ACM)**.

Chang:2010hb

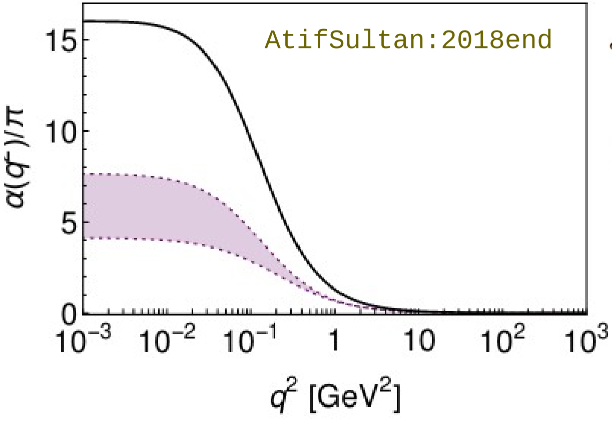
Gluon propagator

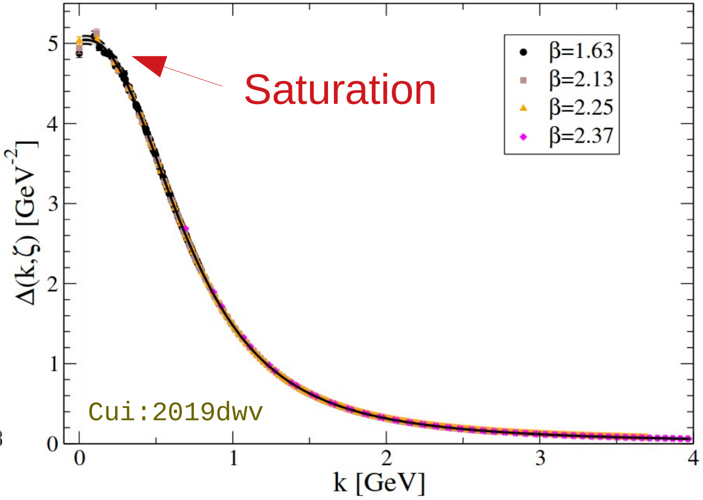


- **Gluon propagator**: characterized by a single dressing function: $$D_{\mu\nu}(k) = \Delta(k^2)T_{\mu\nu}(k)$$ $$(T_{\mu\nu}(k) = \delta_{\mu\nu} - k_\mu k_\nu/k^2)$$
- To get this piece, we often appeal to external inputs: **phenomenological** models and **lattice QCD**.

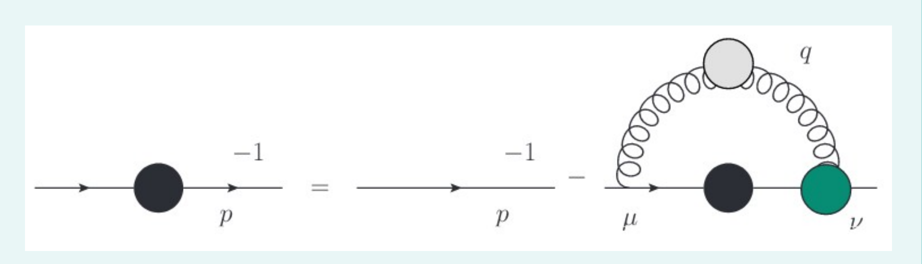
Aguilar:2009nf

Qin:2011dd





Gap equation: Effective Interaction



$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + M_f(p^2))$$
$$= Z_2 S_{f(0)}^{-1}(p) + \Sigma_f(p) \,,$$
$$\Sigma_f(p) = \frac{4}{3} Z_1 \int_{dq}^{\Lambda} \underbrace{g^2 D_{\mu\nu}(p-q)}_{\text{Gluon propagator}} \gamma_\mu S_f(q) \underbrace{\Gamma_\nu^f(p,q)}_{\text{QGV}} \,.$$

Z_1 : Quark-gluon vertex renormalization constant

Z_2 : Quark wavefunction renormalization constant

- Within the **PT-BFM**, a unique **QCD** running coupling can be defined from the gauge-field two-point Green's functions.

Aguilar:2009nf

- Then, in the **gauge** sector, one has:

$$g^2 D_{\mu\nu}(k) \to g^2 \widehat{D}_{\mu\nu}(k) = 4\pi \widehat{d}(k^2) T_{\mu\nu}(k)$$

Taylor coupling

$$k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}$$

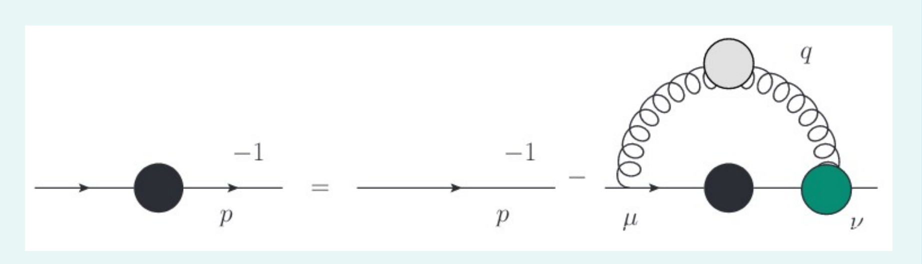
Ghost/Gluon **Ghost**

↓

Renormalization Group Invariant (**RGI**) interaction

Binosi:2016nme, Rodriguez-Quintero:2018wma, Cui:2019dwv

Gap equation: Effective Interaction



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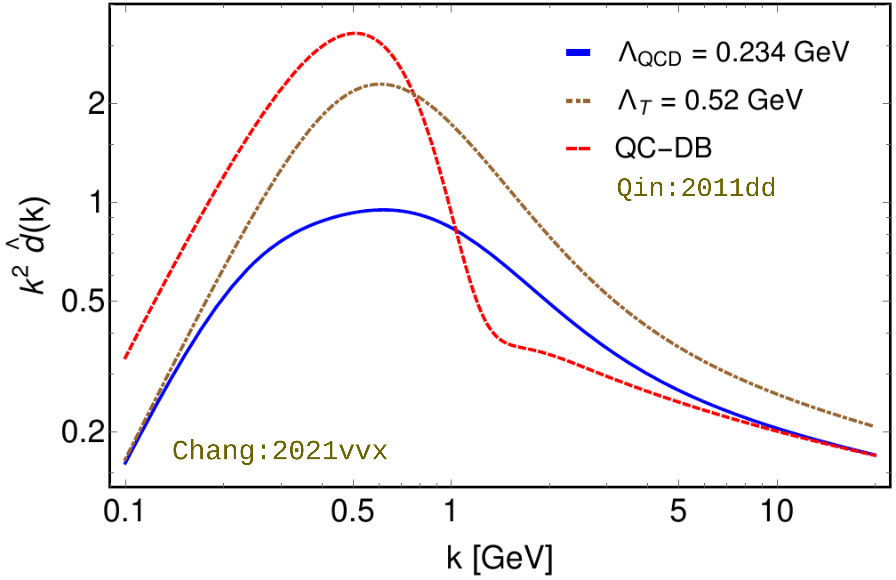
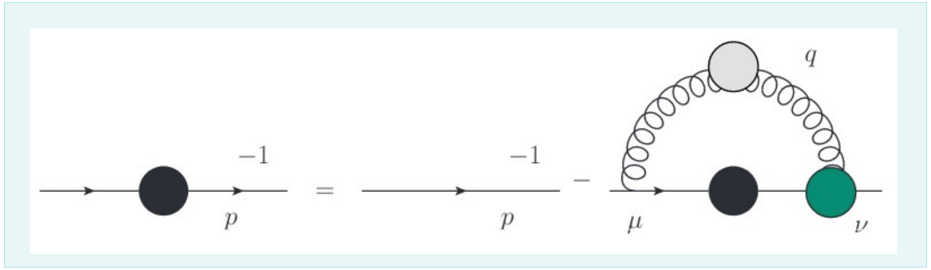


(we shall adopt the blue one, which defines the all orders DGLAP evolution)

Gap equation: The Vertex



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- Furthermore, within the **PT-BFM**, QCD Green's functions are rearranged in such a way that their DSEs satisfy **linear STIs**.

Aguilar:2009nf

- Then, in the **matter** sector, one has:

$$Z_1 \Gamma_\nu^f(p,q) \rightarrow Z_2 \hat{\Gamma}_\nu^f(p,q)$$

- Thus arriving at a *QED-like* structure.
  - **WGTI** might be employed to constrain the **longitudinal** part of the **QGV**.

$$i(k-p)_\nu \hat{\Gamma}_\nu^f = S_f^{-1}(k) - S_f^{-1}(p)$$

Ball:1980ay

  - The **transverse** part is far more richer and difficult to **constrain**.

Albino:2018ncl

Gap equation: The vertex

- A sensible, practical **Ansatz** is given by:

$$\hat{\Gamma}^f_\nu(p,q) = \Gamma^{f,\mathrm{BC}}_\nu(p,q) + \Gamma^{f,\mathrm{ACM}}_\nu(p,q)$$

$$i(k-p)_\nu \hat{\Gamma}^f_\nu = S_f^{-1}(k) - S_f^{-1}(p)$$

$$M(p^2) = B(p^2)/A(p^2)$$
$$Z(p^2) = 1/A(p^2)$$

  - **Ball-Chiu** basis, determines the longitudinal part of the vertex by the requirement of **gauge invariance**. Ball:1980ay

$$\Gamma^{f,\mathrm{BC}}_\nu(k,p) = \lambda^f_1(k^2,p^2)\,\gamma_\nu - i\lambda^f_2(k^2,p^2)\,t_\nu + \lambda^f_3(k^2,p^2)\,t_\mu \gamma\cdot t/2 \qquad t = k+p$$

$$\lambda^f_1(k^2,p^2) = \frac{A_f(k^2)+A_f(p^2)}{2} \qquad \lambda^f_2(k^2,p^2) = \frac{B_f(k^2)-B_f(p^2)}{k^2-p^2} \qquad \lambda^f_3(k^2,p^2) = \frac{A_f(k^2)-A_f(p^2)}{k^2-p^2}$$

Enhanced structures by the effects of
Dynamical Chiral Symmetry Breaking

Gap equation: The vertex

- **8 structures** characterize the **transverse** part of the **QGV**
- One might appeal to **transverse Takahashi** identities or **perturbation theory** requirements to **constraint it.**

  Albino:2018ncl, Bashir:2011dp, Chang:2010hb, Qin:2013mta, etc.

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$$

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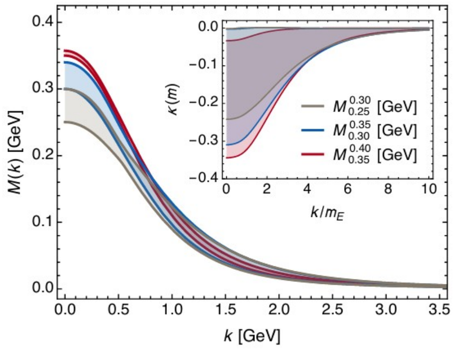
- *Mathematics* and *phenomenology* had highlighted the importance of **quark anomalous chromomagnetic** (ACM) moment term.

  (whose large size is a consequence of DCSB)

Binosi:2016wcx, Chang:2010hb, Qin2020:jig



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$$\hat{\Gamma}^f_\nu(p,q) = \Gamma^{f,\mathrm{BC}}_\nu(p,q) + \Gamma^{f,\mathrm{ACM}}_\nu(p,q)$$

$$\Gamma^{f,\mathrm{ACM}}_\nu(p,q) = \eta\, \sigma_{\nu\alpha} k_\alpha \frac{B_f(p^2) - B_f(q^2)}{p^2 - q^2} \mathcal{H}\left(k^2\right)$$

Where, again, explicit structures connected with **DCSB** appear

Gap equation: The vertex

- *Mathematics* and *phenomenology* had highlighted the importance of **quark anomalous chromomagnetic** (**ACM**) moment term.

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$$

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Where, again, explicit structures connected with **DCSB** appear

The **profile function**, which controls the **UV** convergence:

$$(s/m_0^2)\mathcal{H}(s) \;=\; (1 - e^{-s/m_0^2})$$

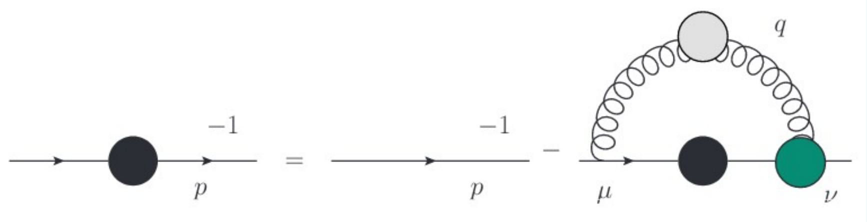
And the **flavor-independent** parameters:

$$m_0 = 2\ \mathrm{GeV} \qquad \eta = (1.27,\ 1.32)$$

Gap equation: Recap

The final **DSE** for the **quark propagator** is:

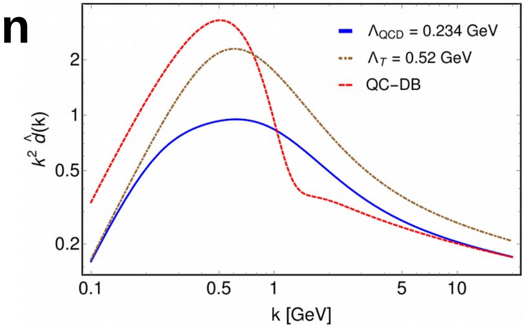
$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + M_f(p^2))$$
$$= Z_2 S_{f(0)}^{-1}(p) + \Sigma_f(p) \, ,$$



$$\Sigma_f(p) = \frac{4}{3} Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2) T_{\mu\nu}(k) \gamma_\mu S_f(q) \widehat{\Gamma}_\nu^f(p,q)$$

Chang:2021vvx

The effective interaction

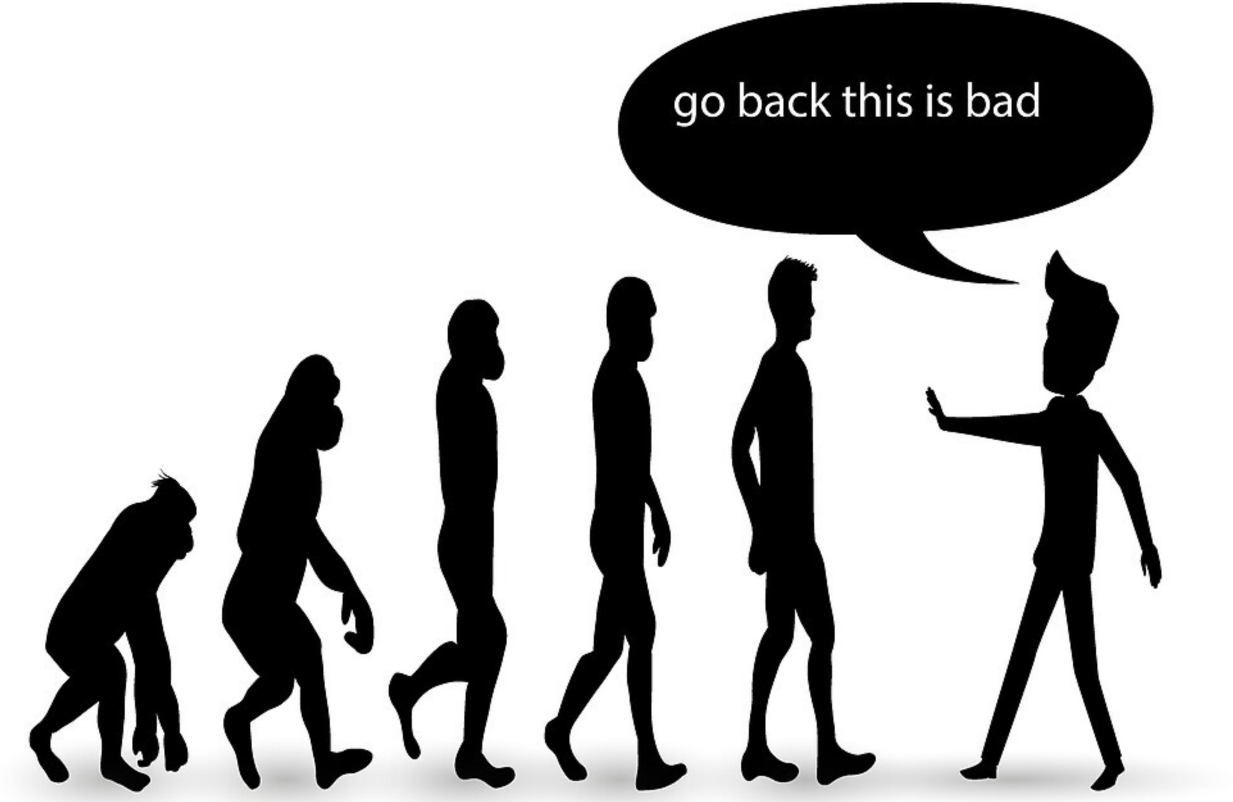


The quark-gluon vertex:

$$\hat{\Gamma}_\nu^f(p,q) = \Gamma_\nu^{f,\mathrm{BC}}(p,q) + \Gamma_\nu^{f,\mathrm{ACM}}(p,q)$$

$$\Gamma_\nu^{f,\mathrm{ACM}}(p,q) = \eta \, \sigma_{\nu\alpha} k_\alpha \frac{B_f(p^2) - B_f(q^2)}{p^2 - q^2} \mathcal{H}\left(k^2\right)$$

EFFECTIVE INTERACTION AND EVOLUTION



Effective Interaction and DGLAP

J. Rodriguez-Quintero's talk

Assume there is an **effective charge** that defines *all orders* DGLAP evolution.

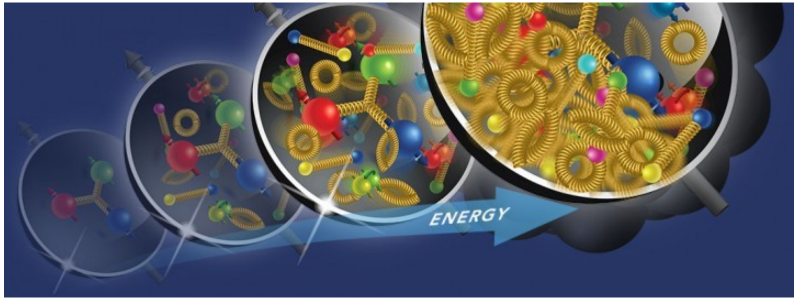
Starting from fully-dressed **quasiparticles**, at ζ_H

(at which **valence quarks** carry ***all*** meson's **properties**)

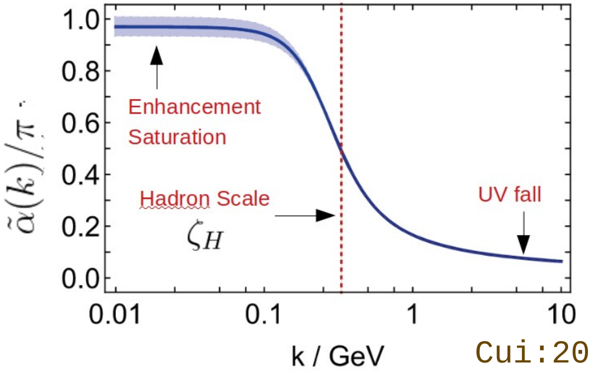
Sea and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) - \boxed{\frac{\tilde{\alpha}(\zeta^2)}{4\pi}}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{{\rm NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$

Exact equation → **"All orders scheme"**



QCD's effective charge could be our answer:



Cui:2020tdf

Effective Interaction and DGLAP

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_0,\zeta_f)\right)\langle x^n(\zeta_0)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)}^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$$q = u, \bar{d}$$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

This ratio encodes the information of the charge

Implication 2: glue and sea-quark DFs $(n_f = 4)$

Momentum sum rule:

$$\boxed{\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1}$$

$$\langle 2x(\zeta_f)\rangle_q = \cancel{\exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right)}, \qquad q = u, \bar{d};$$

$$\zeta_f/\zeta_H \to \infty$$

$$\langle x(\zeta_f)\rangle_{\rm sea} = \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}),$$

$$= \boxed{\frac{3}{7}} + \frac{4}{7}\cancel{\langle 2x(\zeta_f)\rangle_u^{7/4}} - \cancel{\langle 2x(\zeta_f)\rangle_u}$$

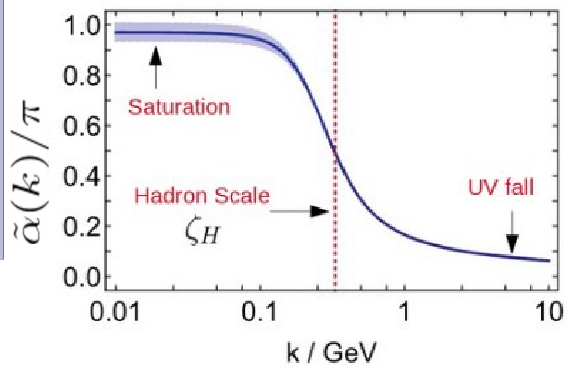
$$\langle x(\zeta_f)\rangle_g = \boxed{\frac{4}{7}}\left(1 - \cancel{\langle 2x(\zeta_f)\rangle_u^{7/4}}\right);$$

A textbook result:
G. Altarelli, Phys. Rep. 81, 1 (1982)

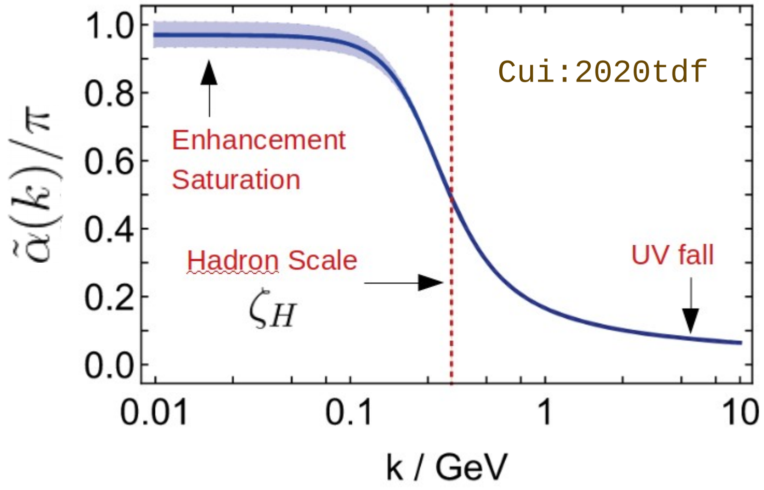
J. Rodriguez-Quintero's talk

- Closed **algebraic** relations between momentum fractions
- **Recovery** of sum rule and asymptotic limits
- Clear connection with the **hadron scale**.

→ Therefore, the scale is **unambiguously** defined (**not tuned**)



Effective Interaction and DGLAP



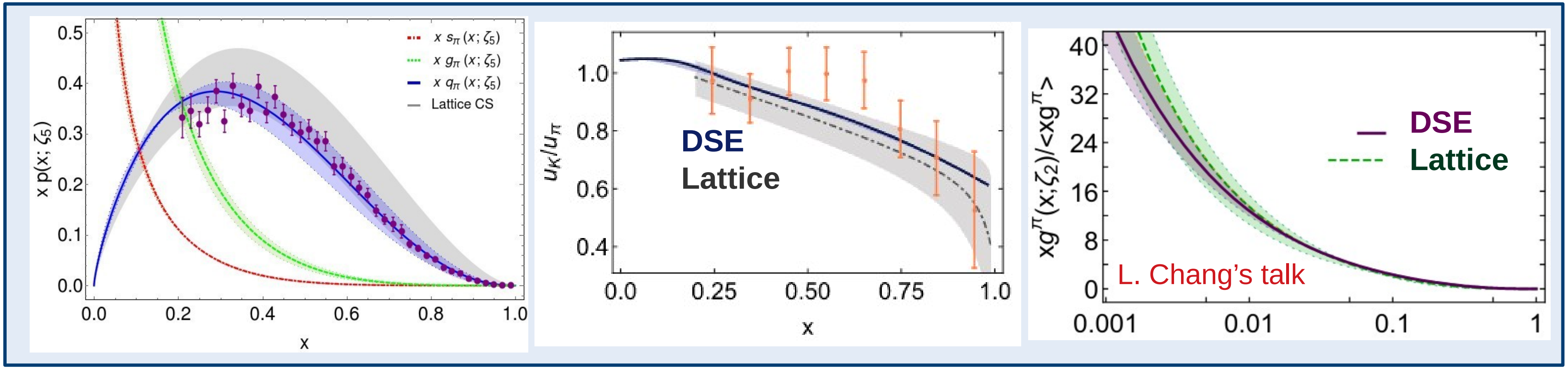
- This idea, supplemented by the effective coupling on the left, has yielded an array of predictions for **pion** and **Kaon** **distribution functions** and **GPDs.**

K. Raya et al. 2109.11686

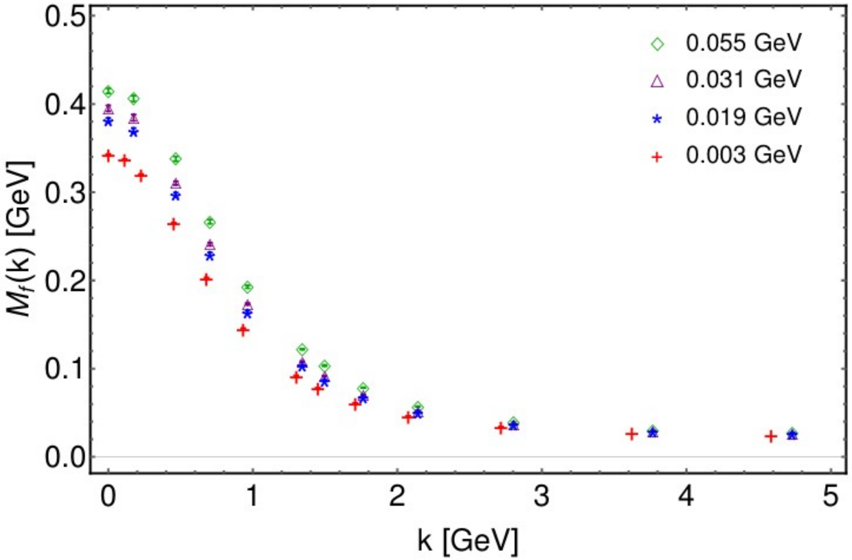
- Its phenomenological **success** inspires us to choose:

$$\widehat{d}(k^2) = \widetilde{\alpha}(k^2)\mathcal{D}(k^2)$$

as the **effective interaction** in the gap equation.

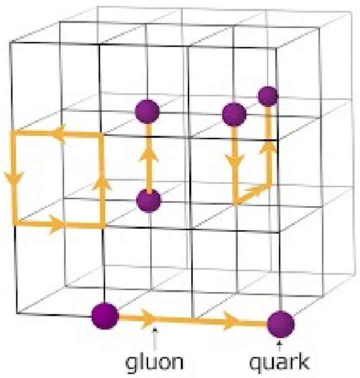


The Mass Function from Lattice QCD



Mass Function from Lattice

- **Some preliminary technical aspects:**
  - Overlap fermions for the valence quarks *(on five RBC/UKQCD 2+1 flavor Domain-Wall fermion ensambles).*
  - 5 different ensembles to analyze **discretization errors** and **sea-quark** mass dependence.



Symbol	$L^3 \times T$	a (fm)	m_π(MeV)	m_K(MeV)	N_{cfg}
64I	$64^3 \times 128$	0.0837(2)	139	508	40
48I	$48^3 \times 96$	0.1141(2)	139	499	40
24I	$24^3 \times 64$	0.1105(3)	340	593	203
24Ih	$24^3 \times 64$	0.1105(3)	432	626	143
24Ih2	$24^3 \times 64$	0.1105(3)	576	660	85

Physical point light-quark masses

Larger sea-quark masses

Mass Function from Lattice

- **Quark mass function** can be obtained from:

[1] $$M_f^{\mathrm{RI'}}(p^2) = \frac{1}{12}\mathrm{Tr}[S_{\mathrm{lat}}^{-1}(p)]/Z_2^{\mathrm{RI'}}(p^2)$$

[2] $$Z_2^{\mathrm{RI'}}(p^2) = \frac{1}{12}\mathrm{Tr}[\not{p}\, S_{\mathrm{lat}}^{-1}(p)]/p^2 \;;$$

$$S_{\mathrm{lat}}(p) \equiv \sum_{x,y} e^{-ipx} S(p,x)/V$$

$$S_{\mathrm{lat}}(p,w) = \langle \psi(w) \sum_y \bar{\psi}(y) e^{ipy} \rangle$$

- [2], however, is *not defined* at $p^2 = 0$
- **Alternatively**, one can define:

[3] $$Z_2^{\mathrm{Ver}}(p^2) = \frac{Z_V}{36}\mathrm{Tr}[\gamma_\nu \Lambda(p,\gamma_\mu) T_{\mu\nu}(p)]$$

$$\Lambda(p,\Gamma) = \frac{S_{\mathrm{lat}}^{-1}(p)\,\langle\Gamma\rangle\, S_{\mathrm{lat}}^{-1}(p)}{V} \,,$$

$$\langle\Gamma\rangle = \left\langle \sum_w \gamma_5 S_{\mathrm{lat}}^{\dagger}(p,w)\gamma_5 \Gamma S_{\mathrm{lat}}(p,w) \right\rangle$$

… and use it in Eq. [1] ([2,3] are the same under dimensional regularization).

Gracey:2003yr

Mass Function from Lattice

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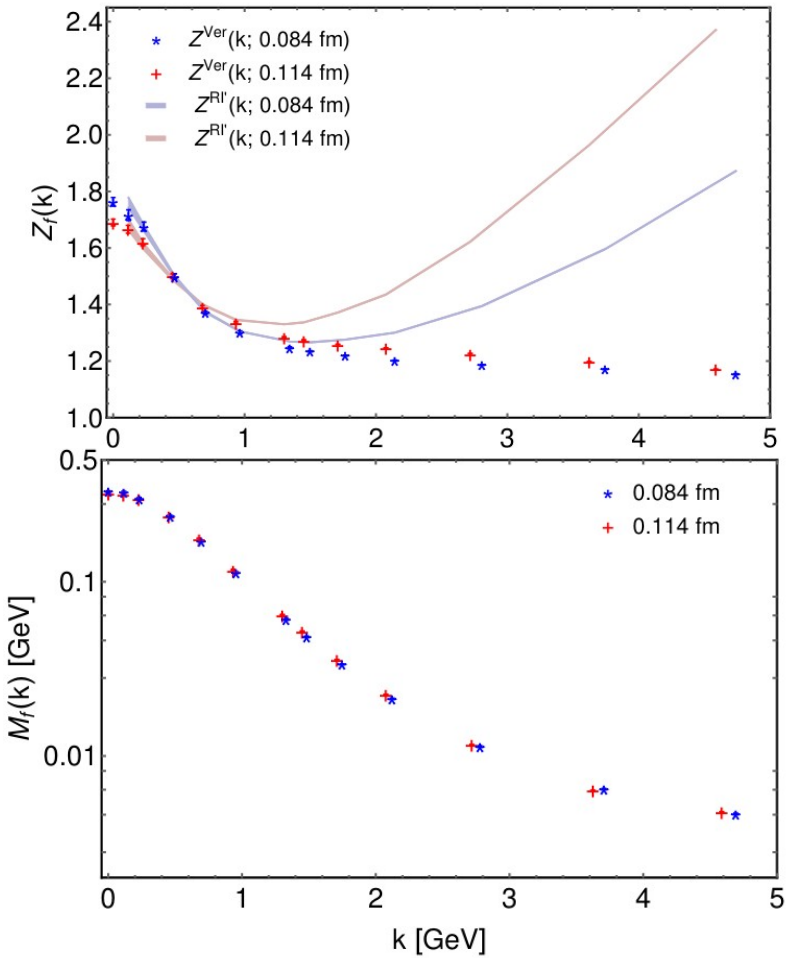

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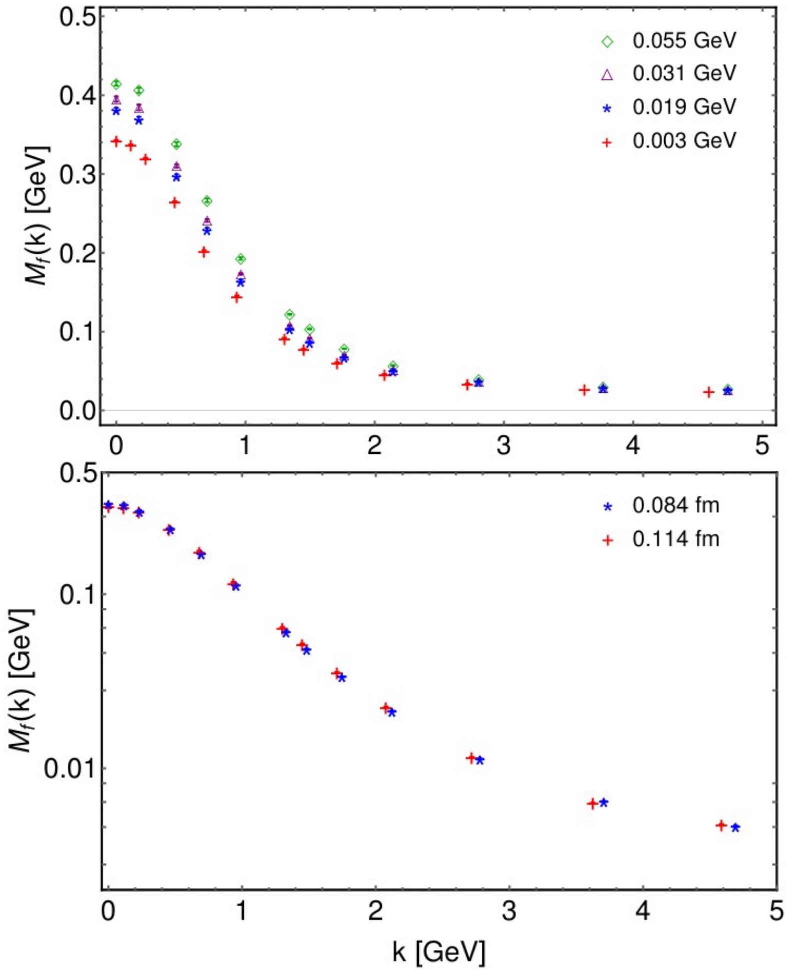
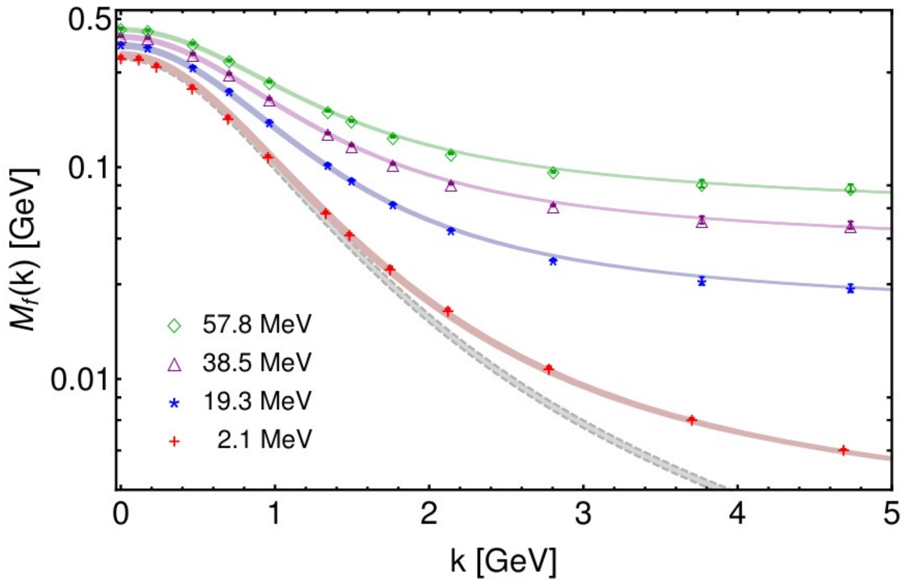
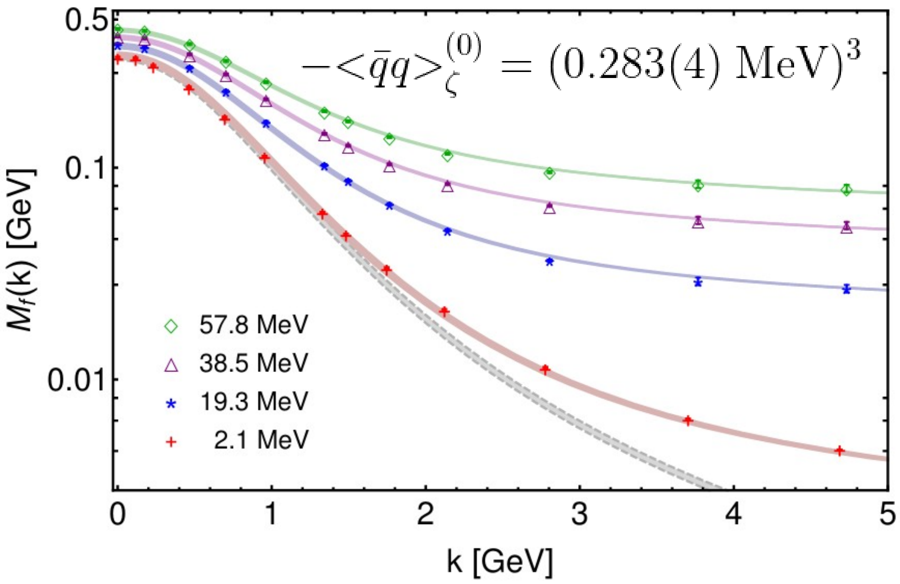


Continuum ↔ Lattice
Quark Mass Function



Continuum ↔ Lattice QMF

Chang:2021vvx



$f_\pi^{(0)} = 89.5(1.8)\ \mathrm{MeV}$ **[Continuum]**

$f_\pi^{(0)} = 86.2(5)\ \mathrm{MeV}$ **[Lattice]**

(FLAG review 2019)

Chiral limit pion decay constant

- Modern **lattice QCD** ensembles and methods enabled us to produced **reduced** error bands.
- In the **continuum**, the **matter** and **gauge-** sector is connected via the **PT-BFM**.
  - The **longitudinal** piece of the **QGV** is fixed by **symmetry principles** (**WGTI**).
  - The **transverse** one accounts for the **quark ACM**, with a reduced number of flavor-independent parameters.
  - The **effective interaction** is the same that defines the **all-orders evolution** of **PDFs** (and **GPDs**).

(driven by the PI effective charge)

Continuum ↔ Lattice QMF

Chang:2021vvx

- Modern **lattice QCD** ensembles and methods enabled us to produced **reduced** error bands.
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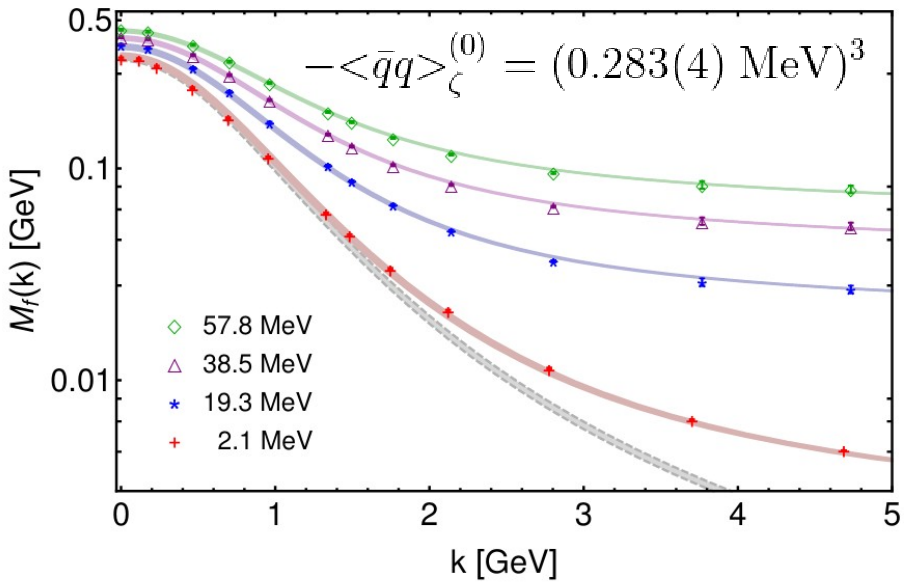


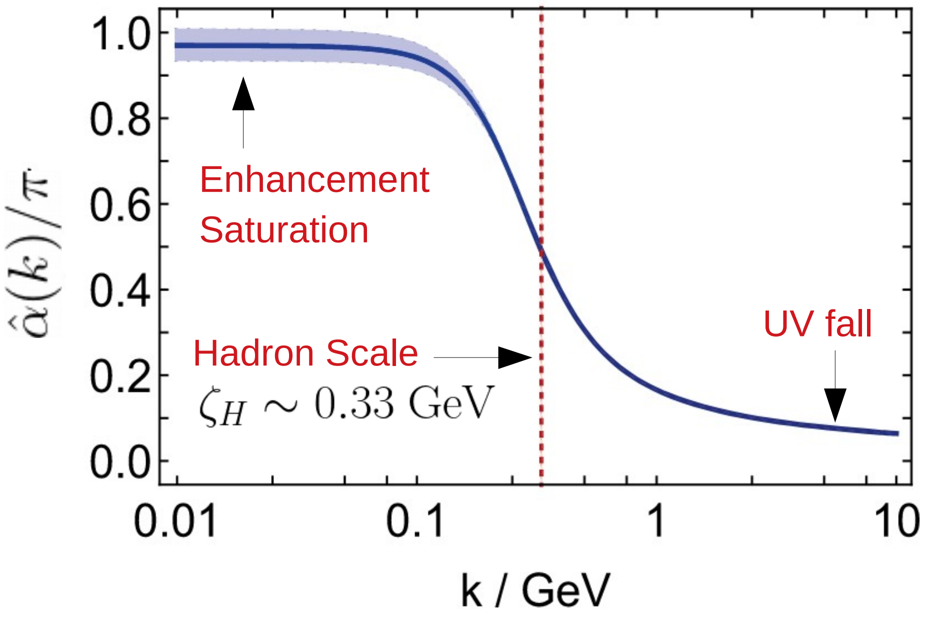
This **connects lattice** and **continuum** methods, producing a **very congruent picture** within the range of explored current quark masses.

Backup Slides

The effective interaction

J. Rodriguez-Quintero's talk

- **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.



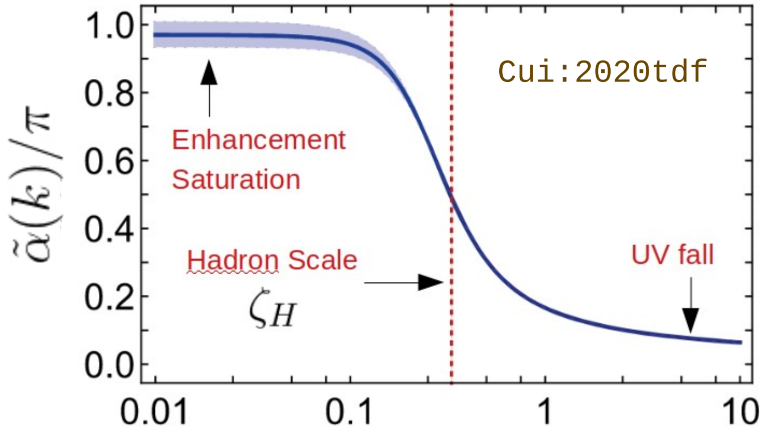
$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\text{QCD}}}\right]}, \quad \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

- Our **modern** picture, obtained from combined **lattice QCD** and **continuum** analysis:
  - Compares very well with world data for Bjorken-sume-rule charge.
  - **Cured** from **Landau pole**.
  - Instead, it **saturates** in the IR and exposes a mass scale, the **hadron scale**.
  - Process-independent, **parameter free** prediction.

ζ_H : The scale at which fully dress valence quarks express all hadron's properties

Employed in all-orders **DGLAP** evolution...

The effective interaction



$$\tilde{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]} \qquad \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

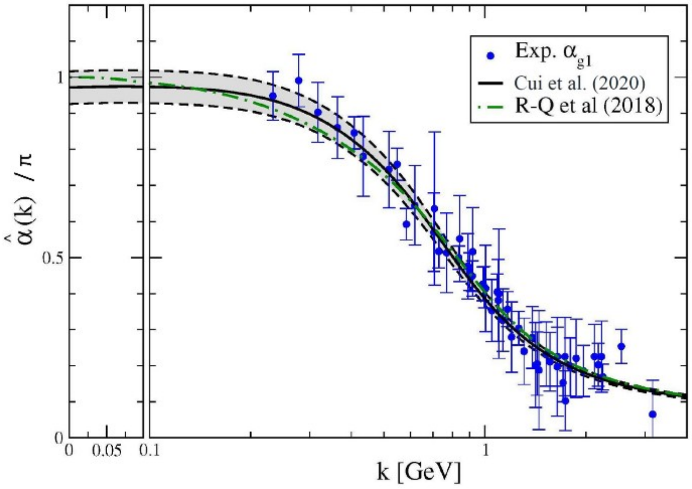
$$\Lambda_{\rm QCD} = 0.234\ \text{GeV},\ n_f = 4$$

$$\{a_0,\ a_1,\ b_0\} = \{0.104(1), 0.0975, 0.121(1)\}$$

(in appropriate powers of GeV)

$$\tilde{\alpha}(0) = \hat{\alpha}(0) \qquad \tilde{\alpha}(k^2) \neq \hat{\alpha}(k^2)$$

Matching at an infrared fixed point, but with different perturbative tails.



Obtained by replacing:

$$\Lambda_{\rm QCD} \to \Lambda_T = 0.52\ \text{GeV}$$

(this one being the MOM-scheme value for the QCD Λ parameter)

and: $\{a_0, a_1, b_1\} \to \{a_0, a_1, b_0\} \times (\Lambda_{\rm QCD}/\Lambda_T)^2$