Light Meson Structure from Basis Light-front Quantization

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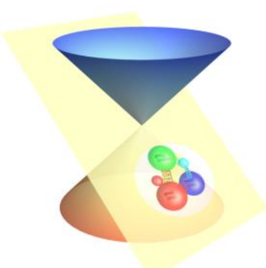

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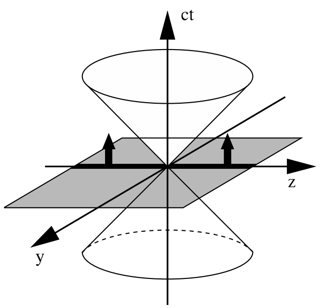
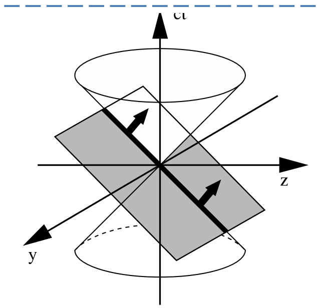
Outline

- Basis Light-front Quantization approach
- Application to π and K mesons
  - Preliminary results on π/K GPDs and TMDs
- Conclusion and outlook

Light-Front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\phi(t)\rangle = H\lvert\phi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\phi(x^+)\rangle = \frac{1}{2}P^-\lvert\phi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Advantages:

- Frame-independent wave functions
- Direct access to parton distributions
- Simple vacuum structure
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i(2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

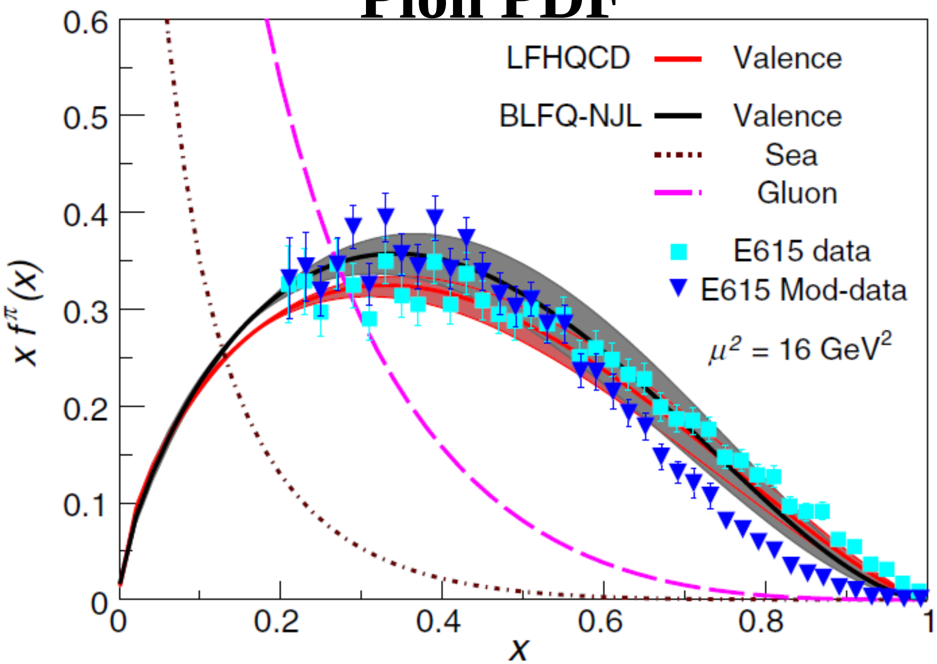
Large N_{max} and K : High UV cutoff & low IR cutoff

PDF from BLFQ and QCD Evolution for Light Mesons

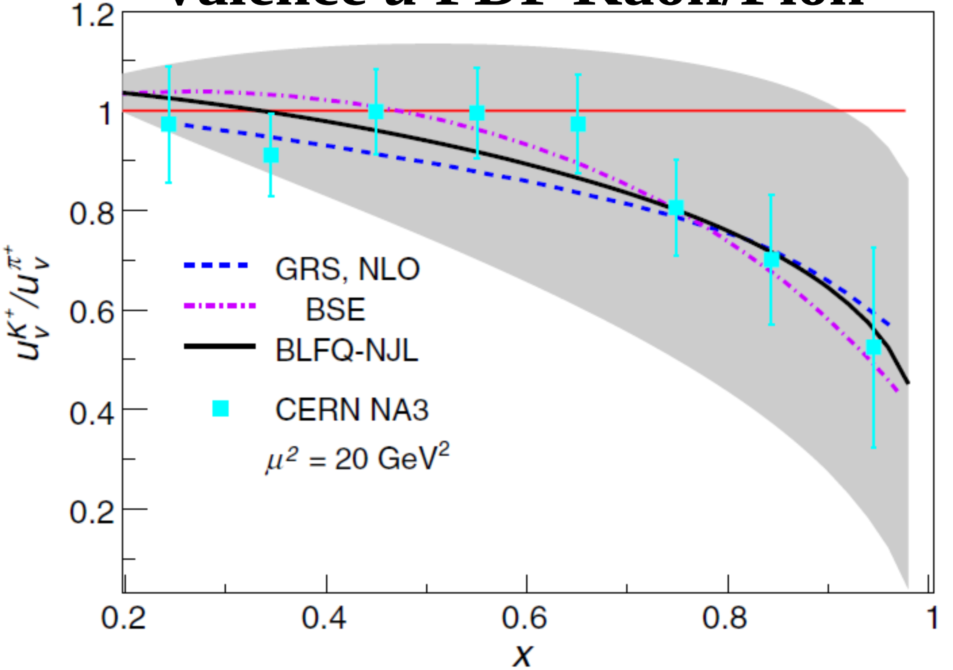
$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

PDF for the valence quark result from the light-front wave functions obtain by diagonalizing the effective Hamiltonian.

Pion PDF



Valence *u* PDF Kaon/Pion



[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

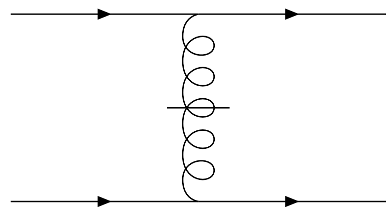
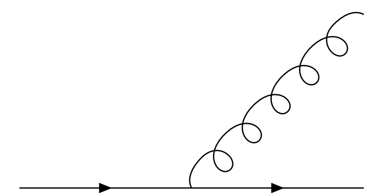
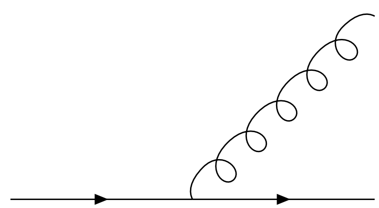
Agree with experimental results

$$|\pi\rangle = |q\bar{q}\rangle + \cdots$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

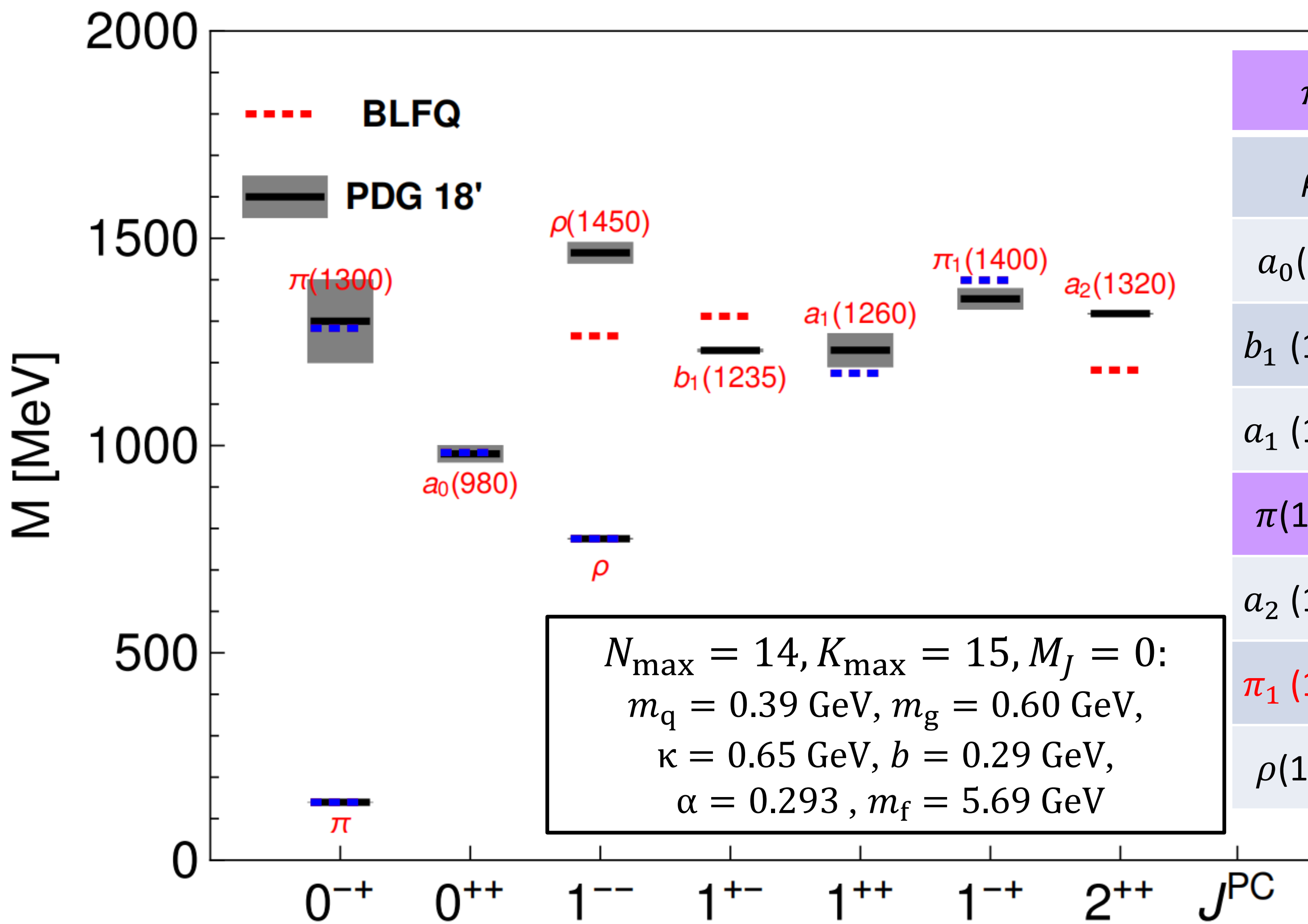
Interaction Part of Hamiltonian

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

H_{int}	$\|q\bar{q}\rangle$	$\|q\bar{q}g\rangle$
$\langle q\bar{q}\|$		
$\langle q\bar{q}g\|$		0

$$P^- = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + H_{\text{int}}$$

Mass Spectrum



$$N_{\max} = 14, K_{\max} = 15, M_J = 0:$$
$$m_q = 0.39 \text{ GeV}, m_g = 0.60 \text{ GeV},$$
$$\kappa = 0.65 \text{ GeV}, b = 0.29 \text{ GeV},$$
$$\alpha = 0.293, m_f = 5.69 \text{ GeV}$$

norm $q\bar{q}$ **DC[MeV]**

π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

Fix the parameters by fitting six blue states

[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion Mass, *DC*, *Radii*

$$\langle r_c^2\rangle = -6\frac{\partial}{\partial Q^2}F(Q^2)|_{Q^2\to 0}$$

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

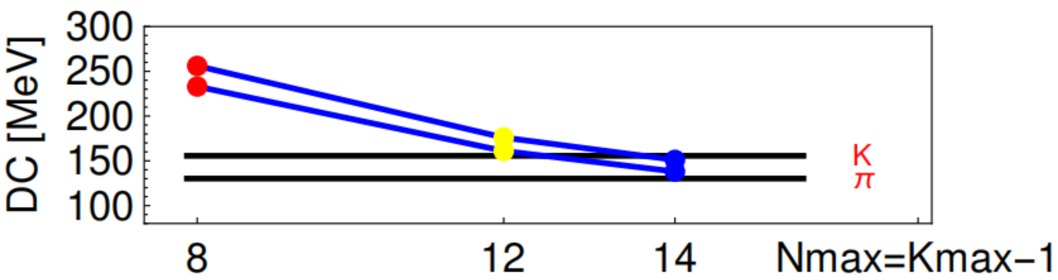
$$\langle 0|\bar{\psi}(0)\gamma^+\gamma_5\psi(0)|P(p)\rangle = ip^+ f_P,$$

$$\langle 0|\bar{\psi}(0)\gamma^+\psi(0)|V(p,\lambda)\rangle = e_\lambda^+ M_V f_V.$$

	m_{π^+} [MeV]	m_{ρ^+} [MeV]	f_{π^+} [MeV]	f_{ρ^+} [MeV]	$\sqrt{\langle r_c^2\rangle}\|_{\pi^+}$[fm]	norm $q\bar{q}$
BLFQ	139.57	775.26	138.2	129.0	0.516~1.456	0.492
PDG [Tanabashi, et al, PRD(2018)]	***139.57***	***775.26±0.25***	***130.2±1.7***	***221±2***	***0.672±0.008***	
BLFQ-NJL [Jia, Vary, PRC(2018)]	*139.57*	*775.23±0.04*	*202.10/$\sqrt{2}$*	*100.12/$\sqrt{2}$*	*0.68±0.05*	

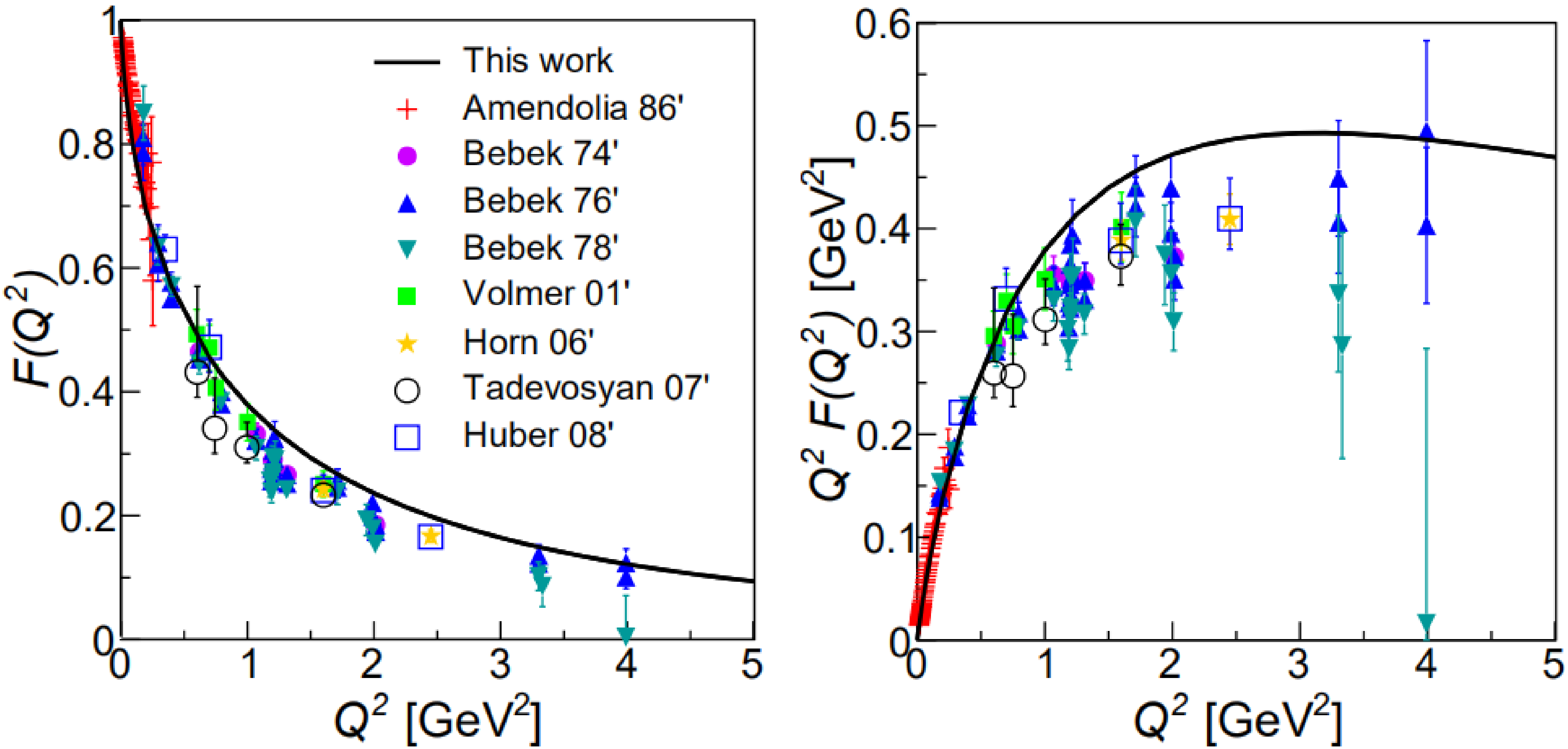
BLFQ

$N_{\max} = 14, K_{\max} = 15, M_J = 0$
$m_q = 0.39$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV



[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

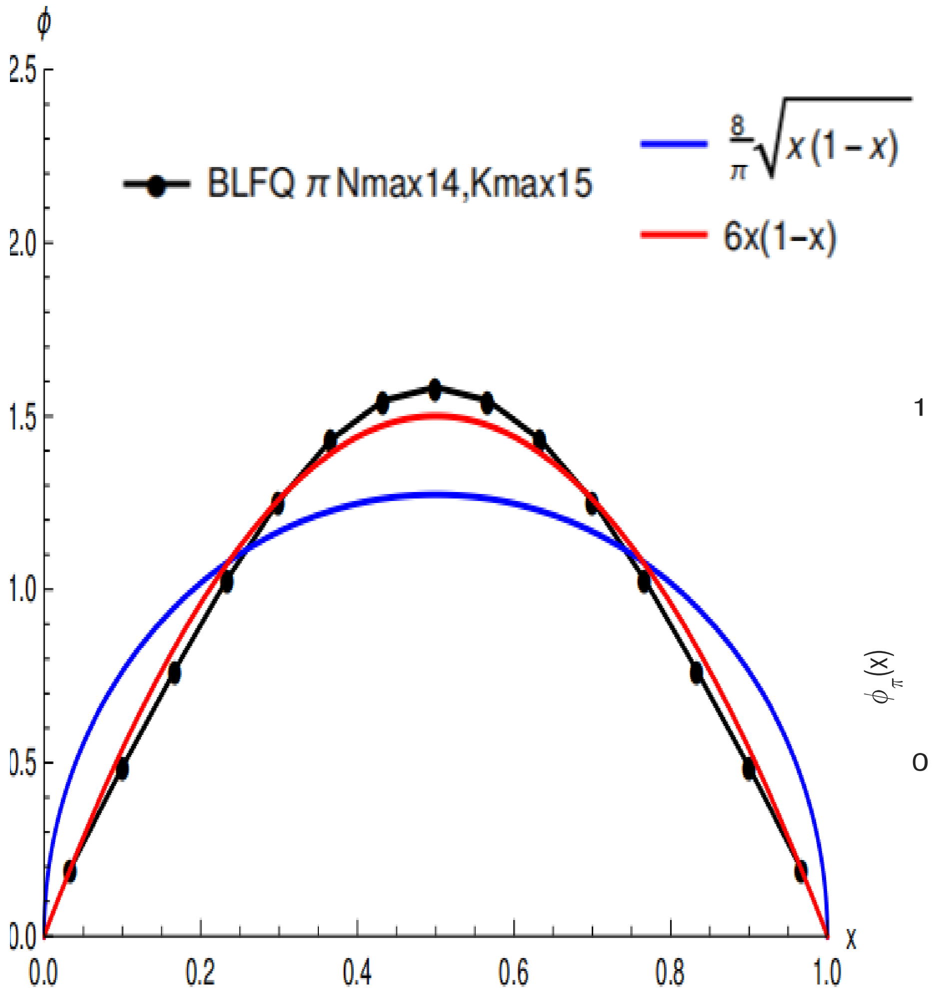
Pion Electromagnetic Form Factor



- Our FF is output .vs. the FF of NLFQ-NJL model obtain by fitting
- $F(Q^2) \propto 1/Q^2$ for large Q^2, consistent with pQCD

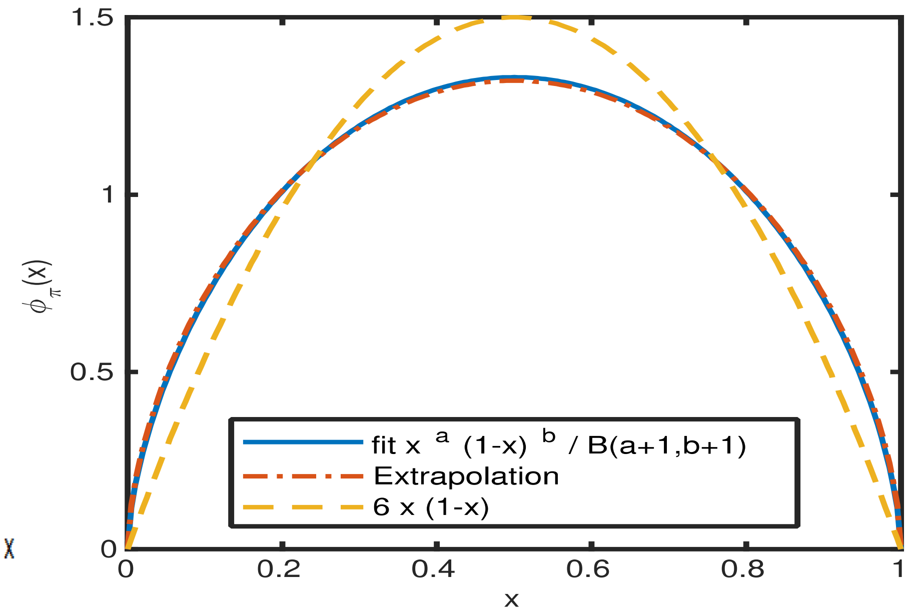
[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion PDA



Ads/QCD

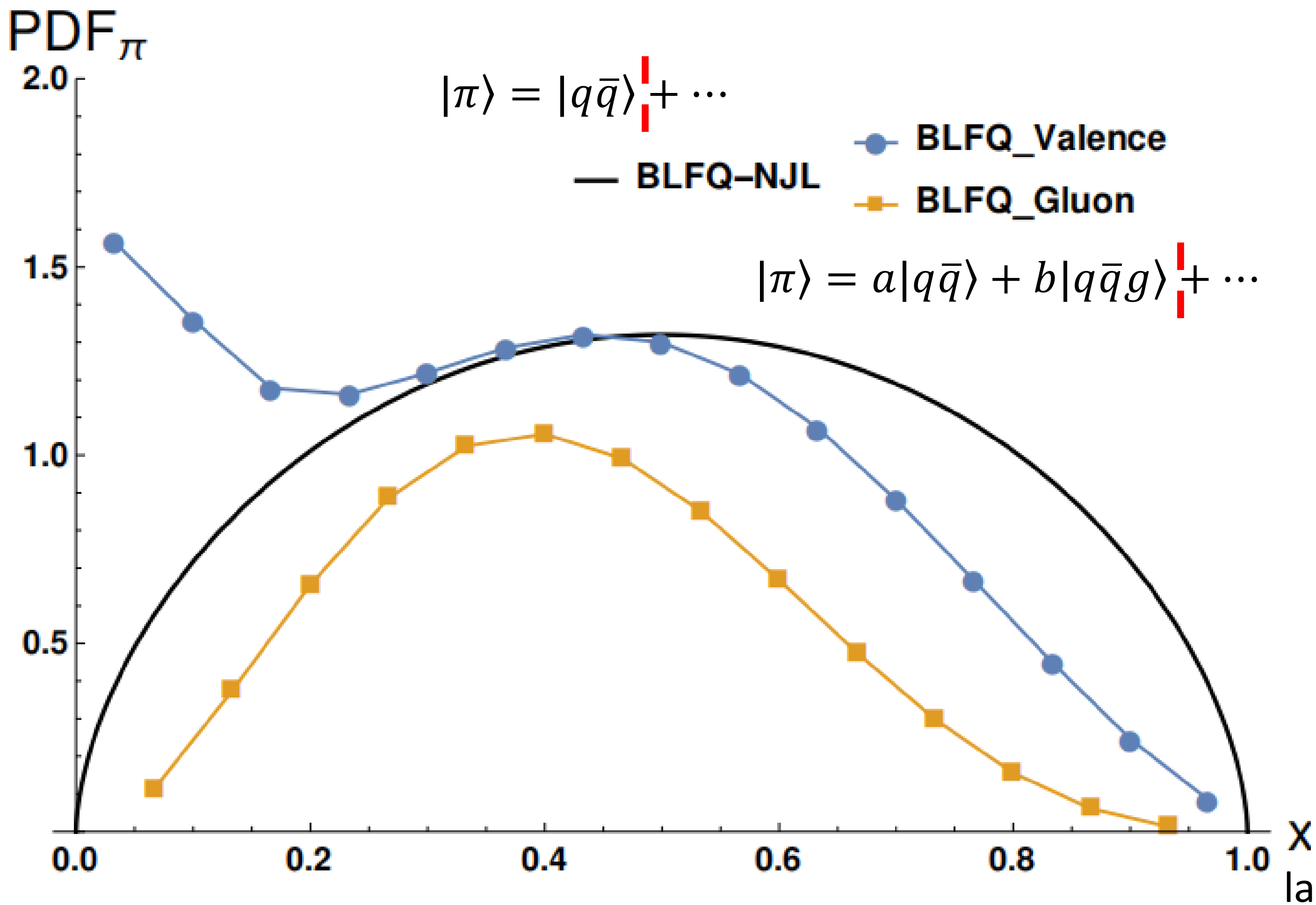
pQCD



[Jia and Vary, PRC 99, 035206 (2019)]

- Endpoint behavior agrees with pQCD

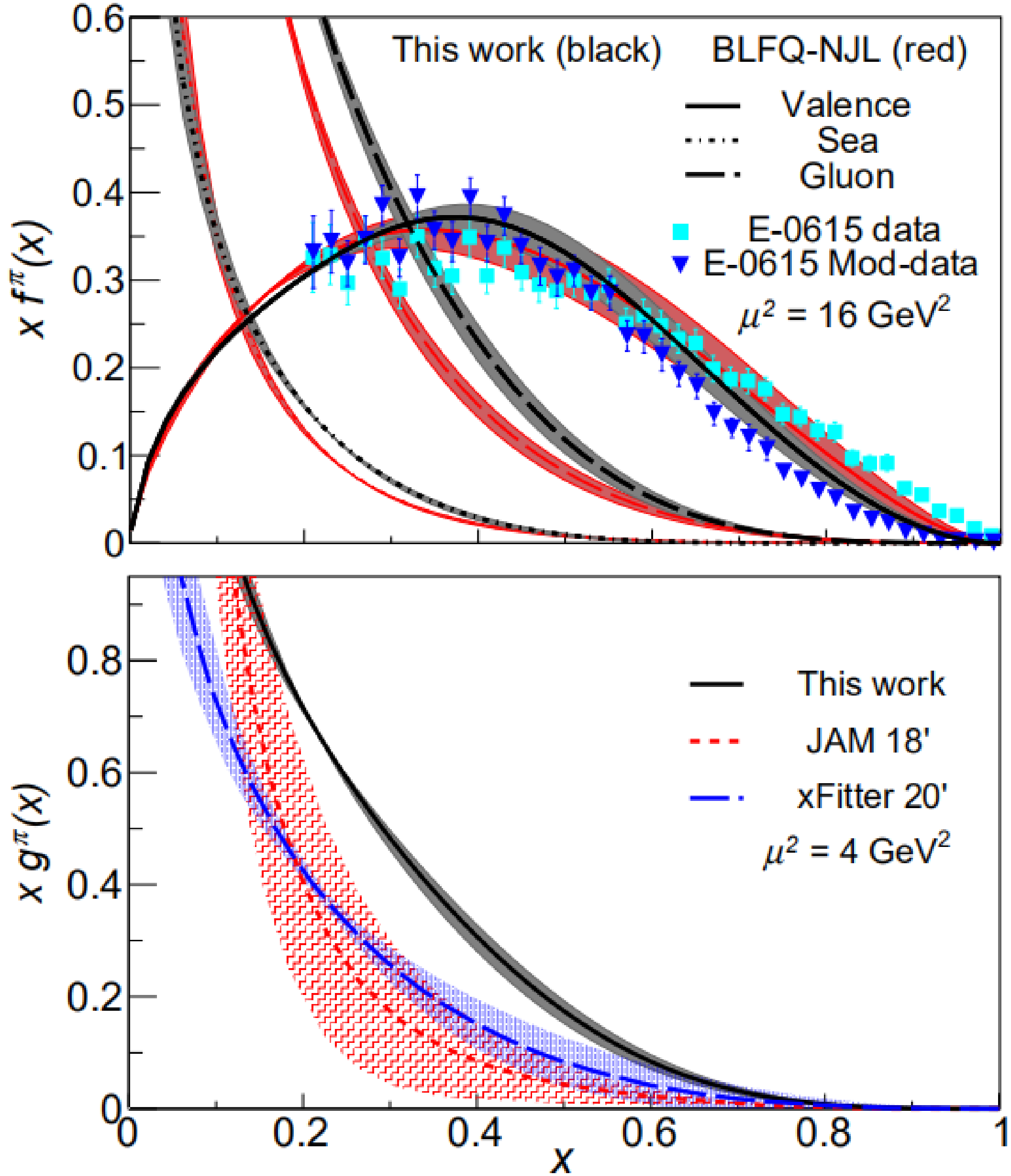
Pion initial PDF



$\mu^2_{0\mathrm{BLFQ-NJL}} = 0.240\ \mathrm{GeV}^2$	$\langle x \rangle_{\mathbf{gluon}} = \mathbf{0}$;	$\langle x \rangle_{\mathbf{valence}\ u} = \mathbf{0.5}$	$(1-x)^{0.596}$
$\mu^2_{0\mathrm{BLFQ}} = 0.34\ \mathrm{GeV}^2$	$\langle x \rangle_{\mathbf{gluon}} = \mathbf{0.216}$;	$\langle x \rangle_{\mathbf{valence}\ u} = \mathbf{0.392}$	$(1-x)^{1.4}$

[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion PDF



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

- Large-x behavior $(1-x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ	**0.483**	**0.421**	**0.096**
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019']	0.48(3)	0.41(2)	0.11(2)

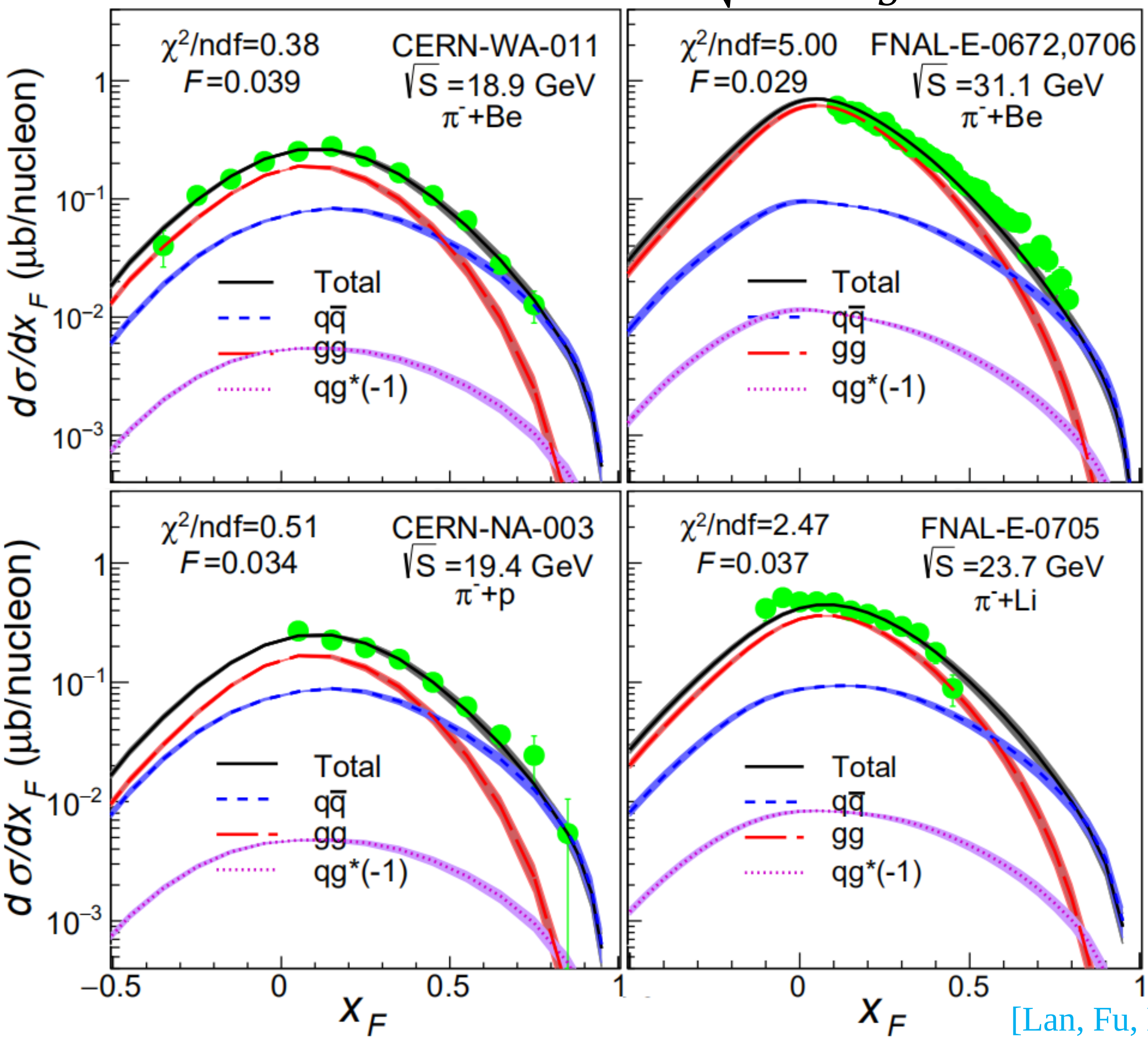
[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

J/ψ production cross section

$\pi^{\pm} N \rightarrow J/\psi\ X$

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

Agree with experimental data (**FNAL E672, E706, E705, CERN NA3,WA11**).

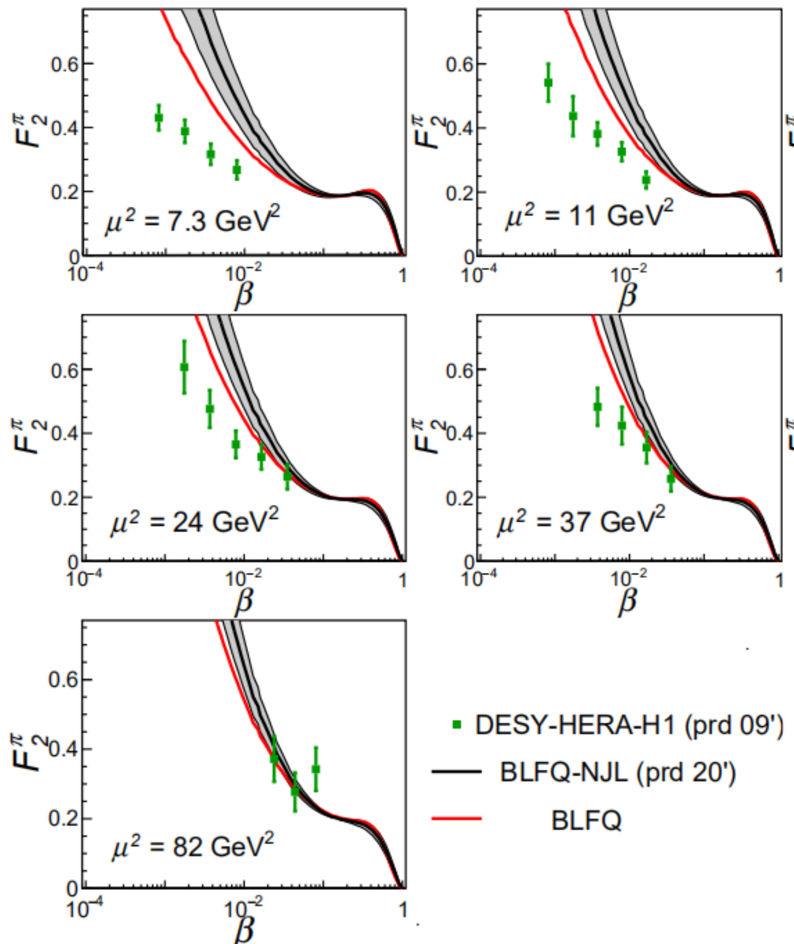
[Lan, Fu, Mondal, Zhao, Vary, arXiv 2106.04954]

Pion Structure function

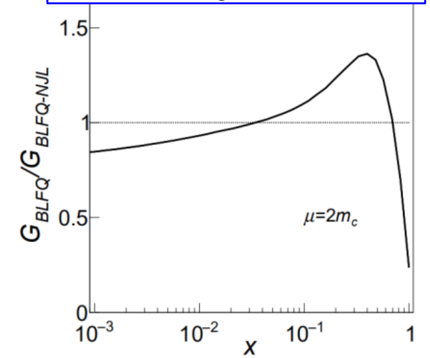
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$F_2^{\pi}(\beta,\mu^2) = \sum_{q,g} e_q^2 \beta\{f_q^{\pi}(\beta,\mu^2) + f_{\bar{q}}^{\pi}(\beta,\mu^2) + \frac{\alpha_s(\mu^2)}{2\pi}[C_{q,2}\otimes(f_q^{\pi} + f_{\bar{q}}^{\pi}) + 2C_{g,2}\otimes f_g^{\pi}]\}$$



$\mu = 2m_c = 3$ GeV



better than BLFQ-NJL results

- Higher initial scale
- less gluon/sea quark at small x

in future $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \cdots$

Pion 3D Structure

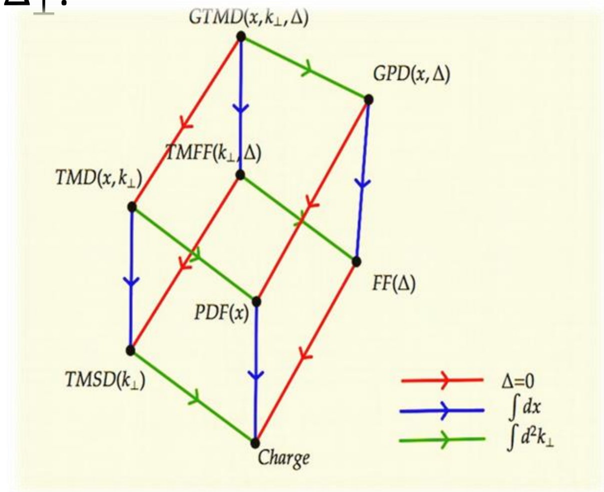
GPD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

$$H(x,0,t) = \sum_{n,s_i} \int \prod_{i=1}^{n} \frac{dx_i d^2\vec{p}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_j x_j\right) \delta^2\left(\sum_{j=1}^{n} \vec{p}_{\perp j}\right) \delta(x - x_1)$$

$$\Psi_n\left(x_i', \vec{p}_{\perp i}', s_i\right) \Psi_n\left(x_i, \vec{p}_{\perp i}, s_i\right),$$

where n=2 for only $|q\bar{q}\rangle$, n=3 for $a|q\bar{q}\rangle + b|q\bar{q}g\rangle$

$x_1' = x_1,\ \vec{p}_{\perp 1}' = \vec{p}_{\perp 1} - (1 - x_1)\Delta_\perp, x_i^\perp = x_i, \vec{p}_{\perp i}' = \vec{p}_{\perp i} + x_i\Delta_\perp.$



Preliminary

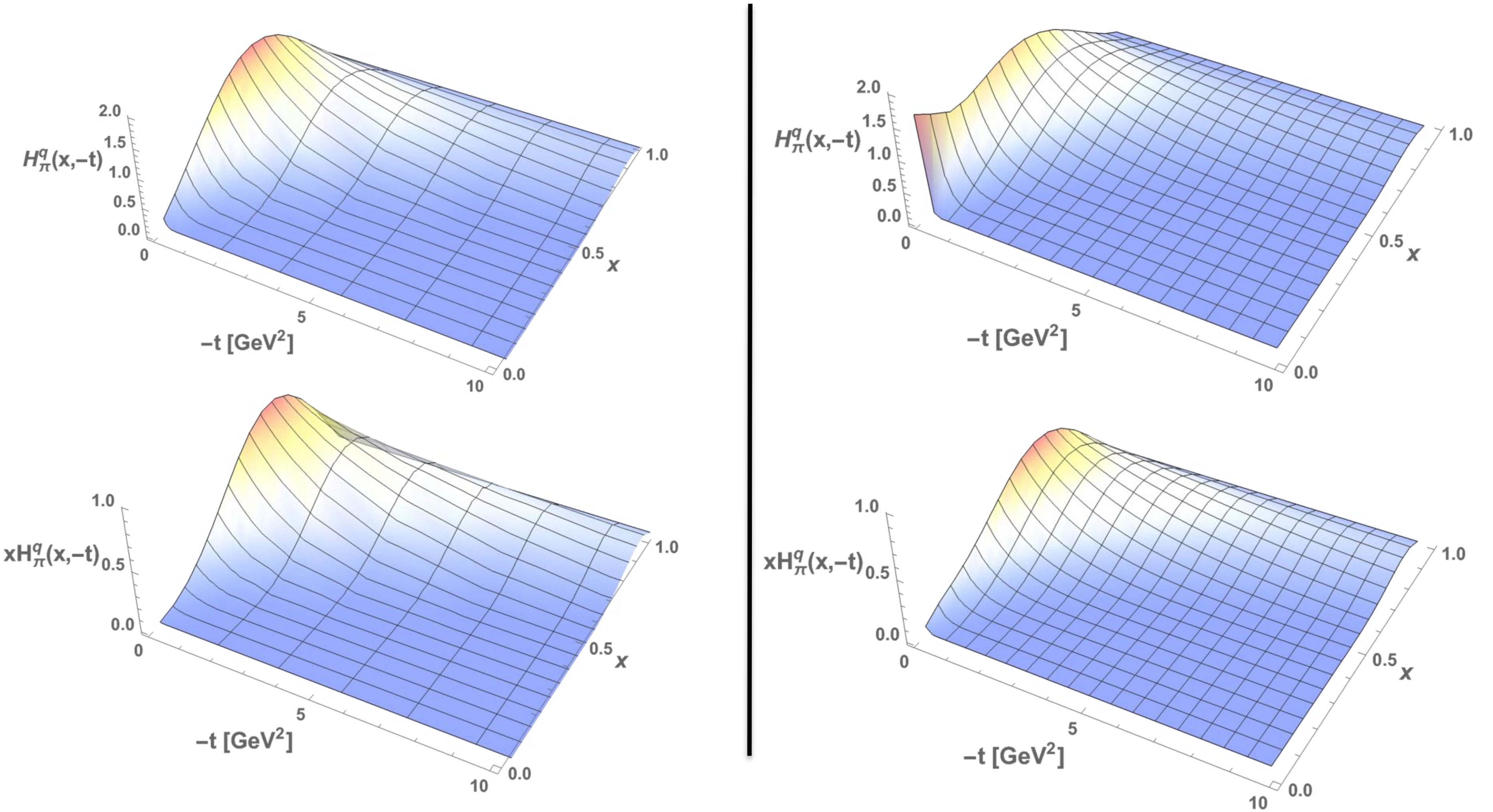
Pion GPD

Preliminary

GPD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

Zero skewness

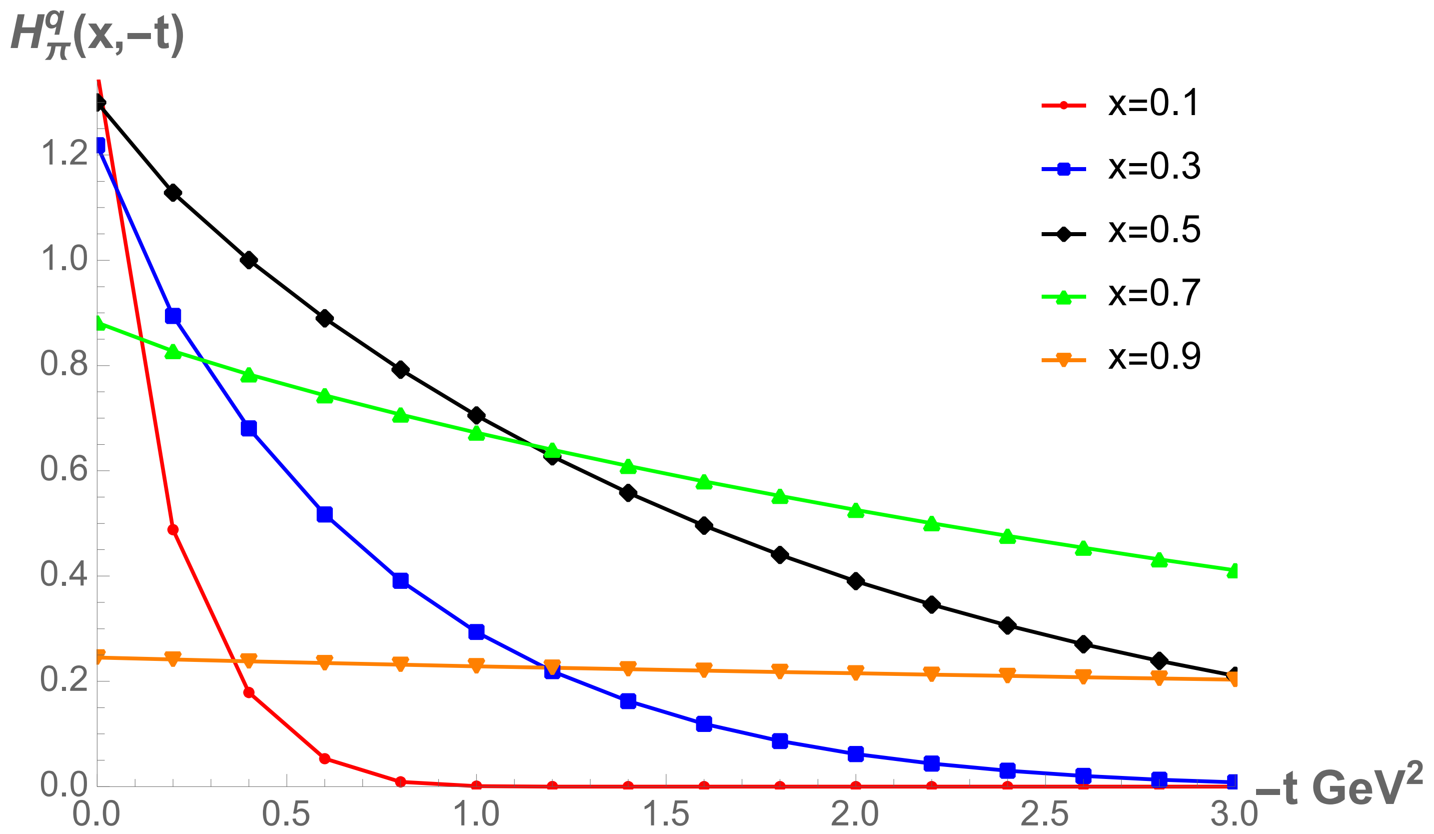
Only $|q\bar{q}\rangle$ With $|q\bar{q}g\rangle$



- Quark content enhanced at small x with $|q\bar{q}g\rangle$

Pion GPD

GPD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

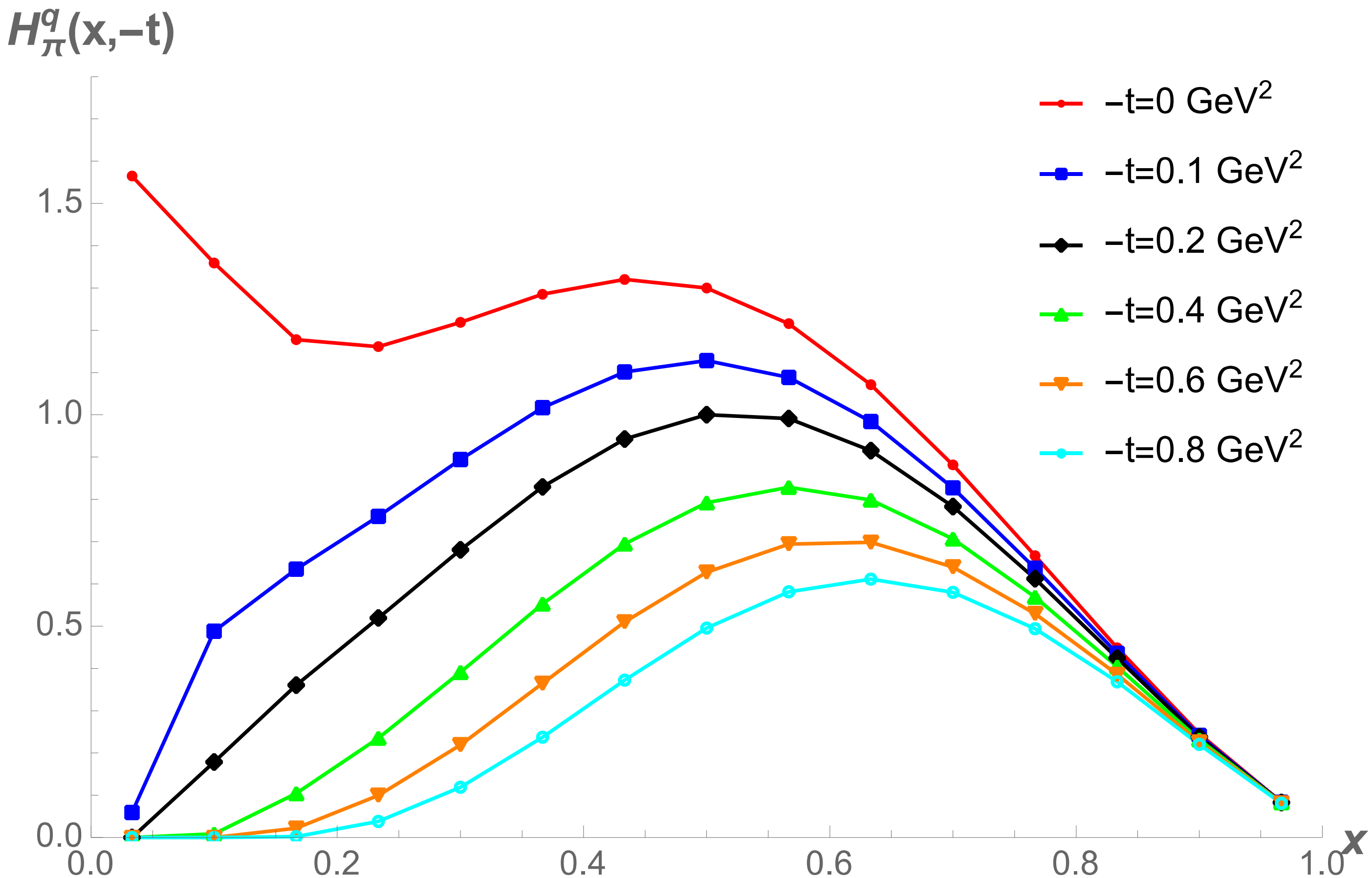


- Distribution is broader at larger x

Preliminary

Pion GPD

GPD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

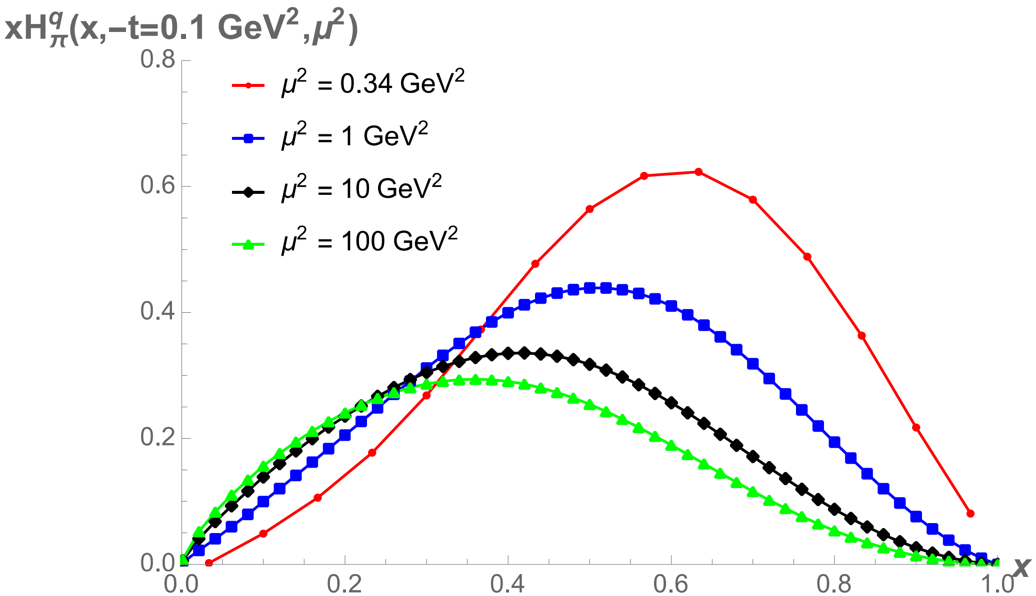


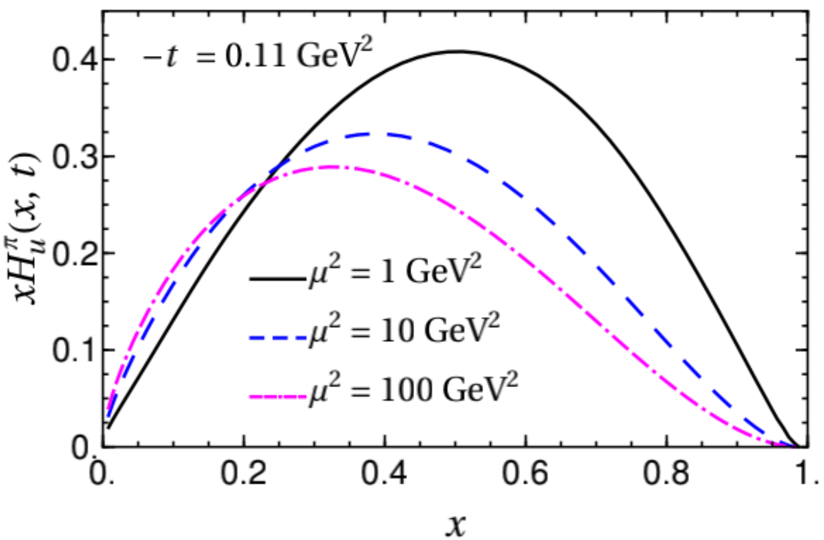
Preliminary

Pion GPD with Scale Evolution

GPD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

BLFQ-NJL $\mu_0^2 = 0.24\ \text{GeV}^2$





- Include valence quark only
- Scale evolution performed with DGLAP (HOPPET)

Preliminary

Pion TMD

TMD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

$$f_1^{q/\bar{q}} = \Phi^{[\gamma^+]} = \Phi_{++}^{q/\bar{q}} + \Phi_{--}^{q/\bar{q}}$$

$$f_1^{g} = \delta_{ij}\Phi^{g[i+;j+]} = \Phi_{++}^{g} + \Phi_{--}^{g}$$

Only $|q\bar{q}\rangle$

$$\Phi^{q}_{\lambda_1'\lambda_1}(P;x,p^{\perp}) = \sum_{\lambda_2} N_{all}\psi^*_{\lambda_1'\lambda_2}(p,p_2)\psi_{\lambda_1\lambda_2}(p,p_2)$$

$$\Phi^{\bar{q}}_{\lambda_2'\lambda_2}(P;x,p^{\perp}) = \sum_{\lambda_1} N_{all}\psi^*_{\lambda_1\lambda_2'}(p_1,p)\psi_{\lambda_1\lambda_2}(p_1,p)$$

With $|q\bar{q}g\rangle$

$$\Phi^{q}_{\lambda_1'\lambda_1}(P;x,p^{\perp}) = \sum_{\lambda_2\lambda_3}\int dx_2 d^2p_2^{\perp} N_{all}\psi^*_{\lambda_1'\lambda_2\lambda_3}(p,p_2)\psi_{\lambda_1\lambda_2\lambda_3}(p,p_2)$$

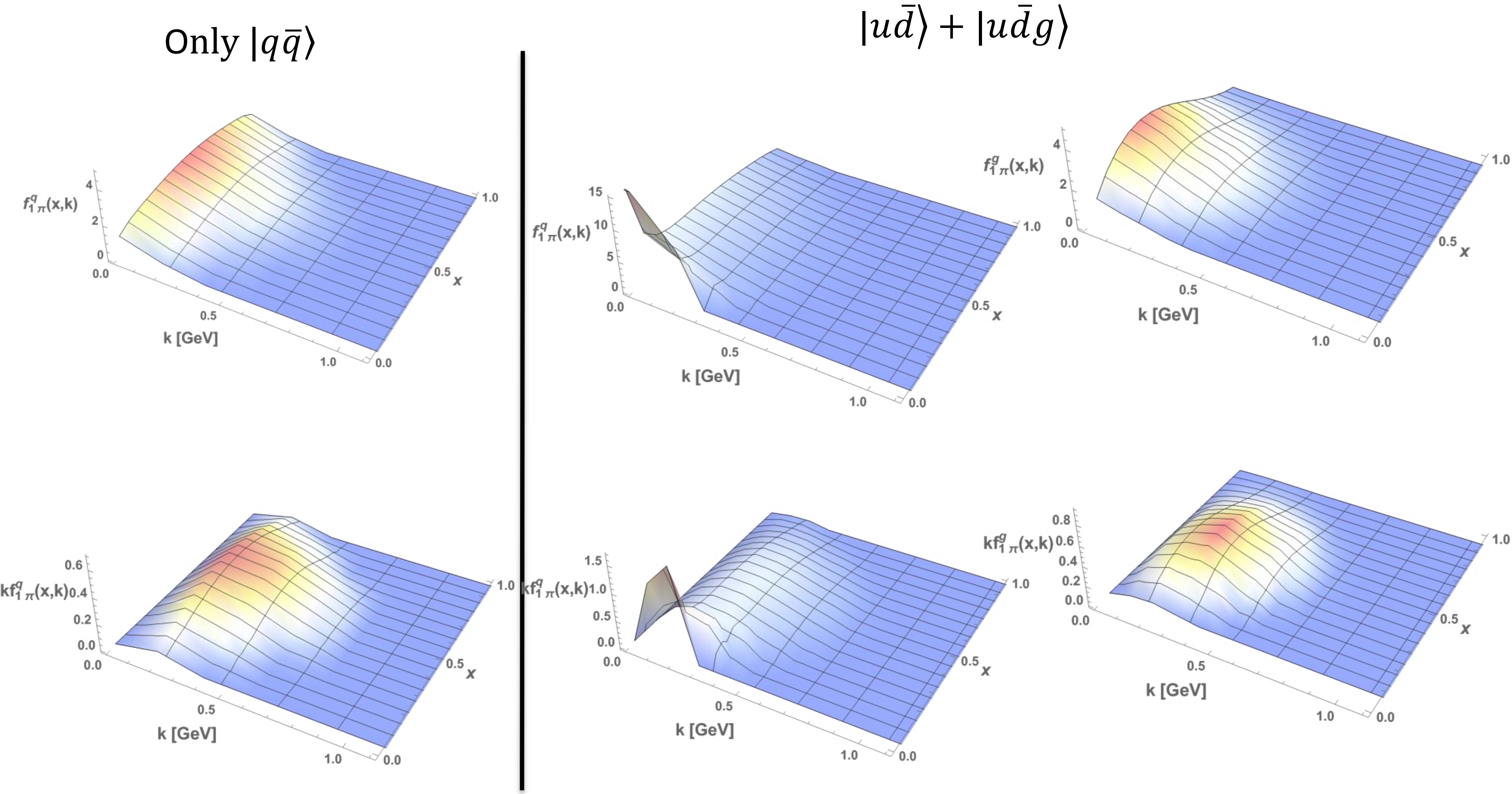
$$\Phi^{\bar{q}}_{\lambda_2'\lambda_2}(P;x,p^{\perp}) = \sum_{\lambda_1\lambda_3}\int dx_1 d^2p_1^{\perp} N_{all}\psi^*_{\lambda_1\lambda_2'\lambda_3}(p_1,p)\psi_{\lambda_1\lambda_2\lambda_3}(p_1,p)$$

$$\Phi^{g}_{\lambda_3'\lambda_3}(P;x,p^{\perp}) = \sum_{\lambda_1\lambda_2}\int dx_2 d^2p_2^{\perp} N_{all}\psi^*_{\lambda_1\lambda_2\lambda_3'}(p_2,p)\psi_{\lambda_1\lambda_2\lambda_3}(p_2,p)$$

Preliminary

Pion 3D Structure

TMD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$



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Pion 3D Structure

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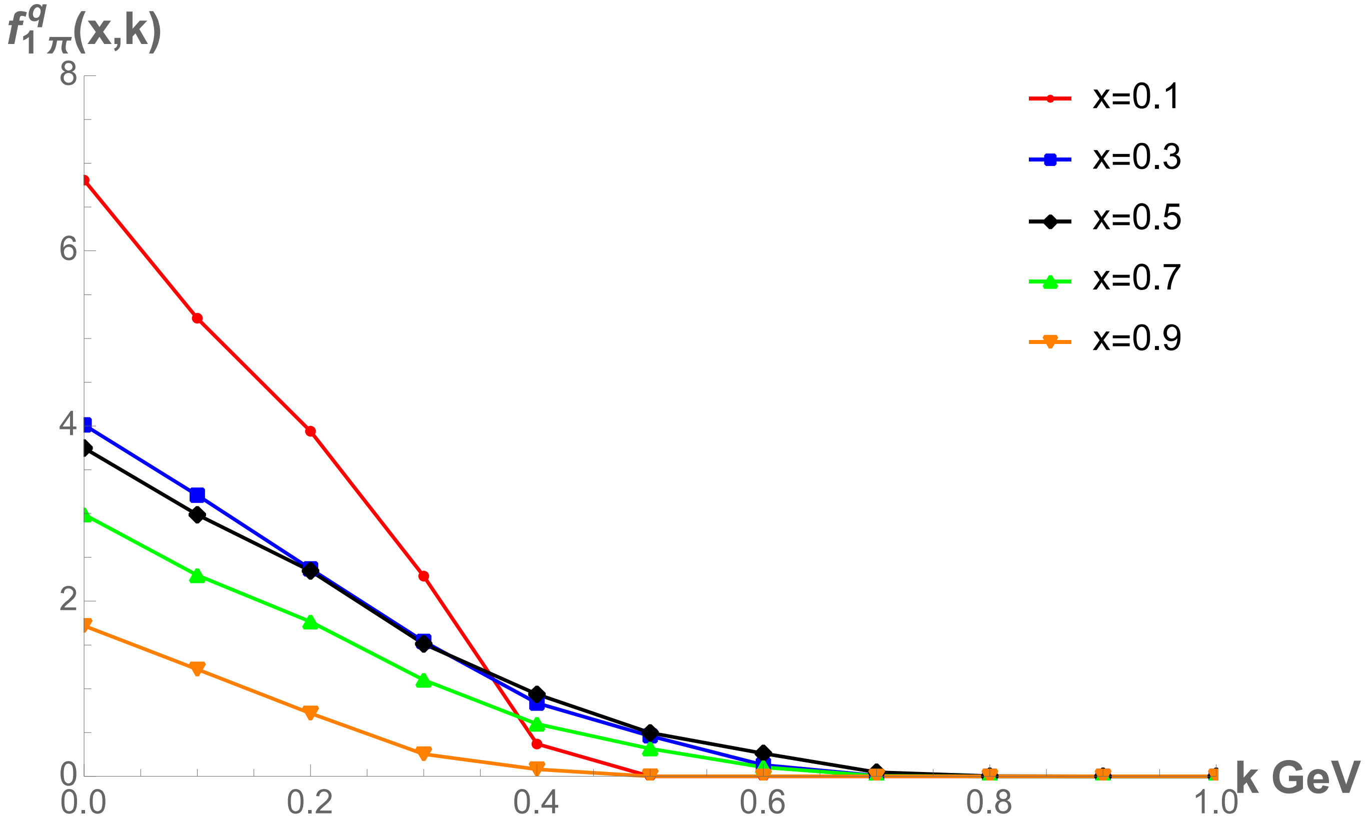

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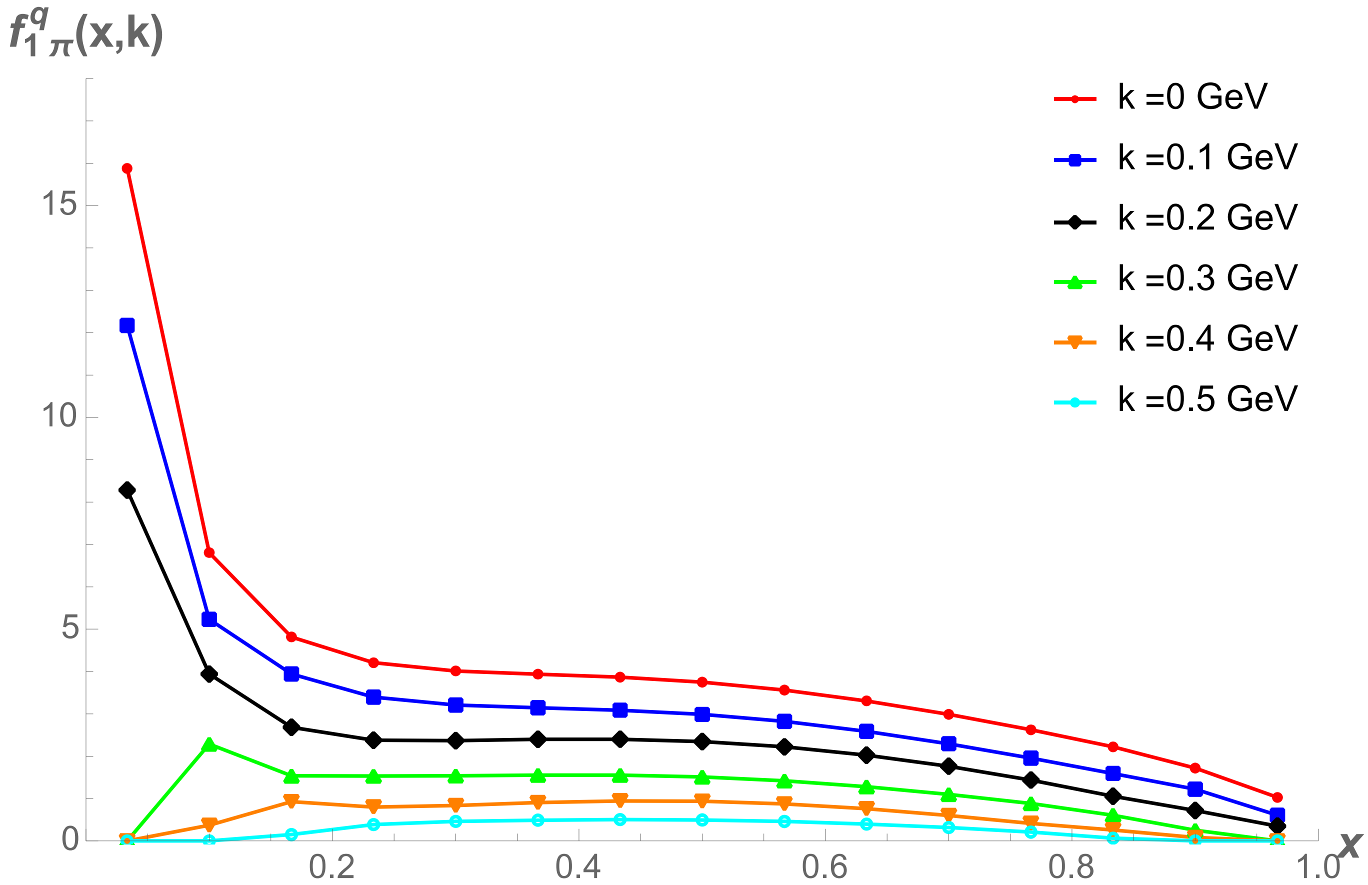
Pion 3D Structure

TMD $|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

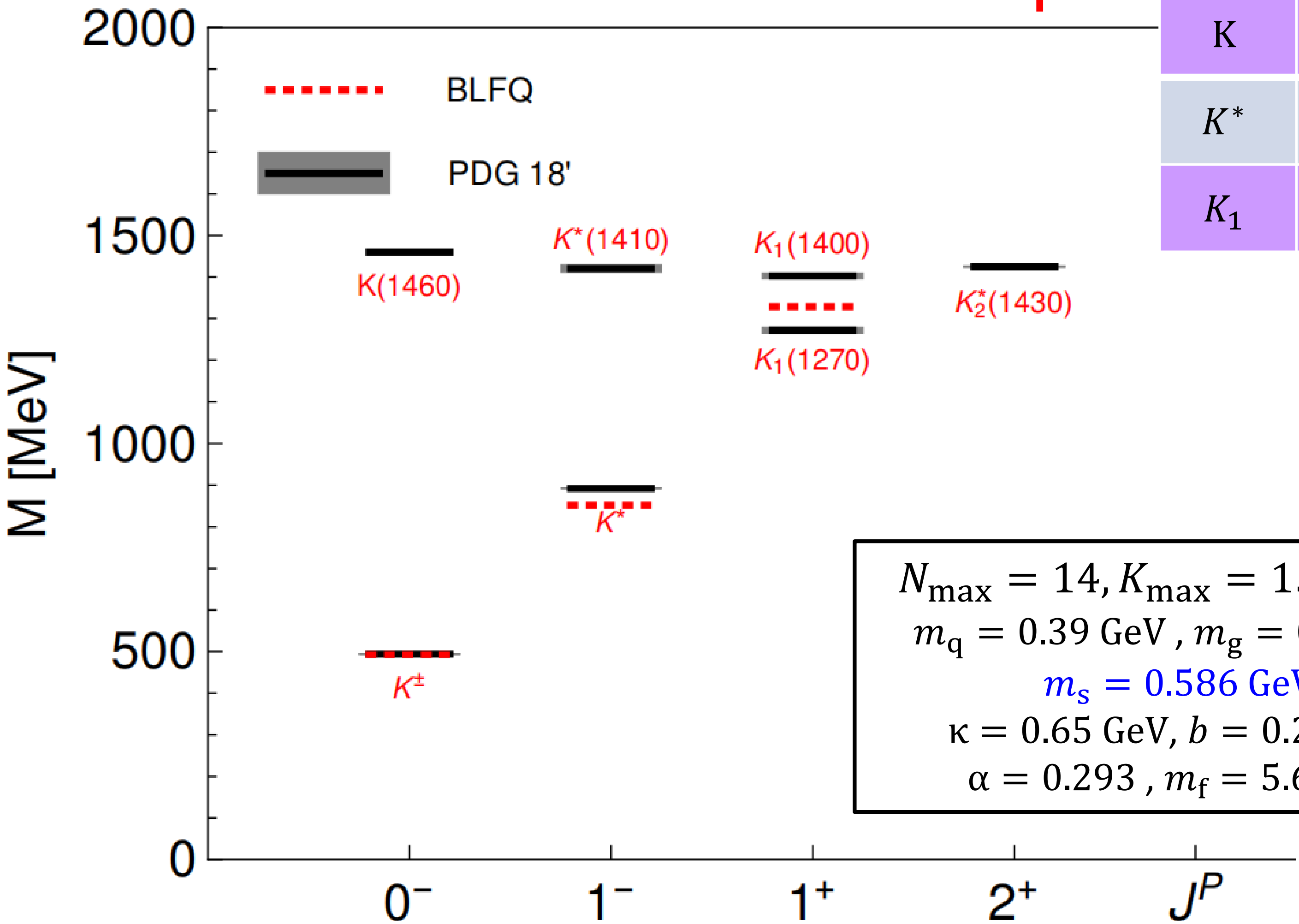


Preliminary

Kaon Spectrum

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$

	Norm1	DC[MeV]
K	0.588	151
K^*	0.602	142
K_1	0.419	13



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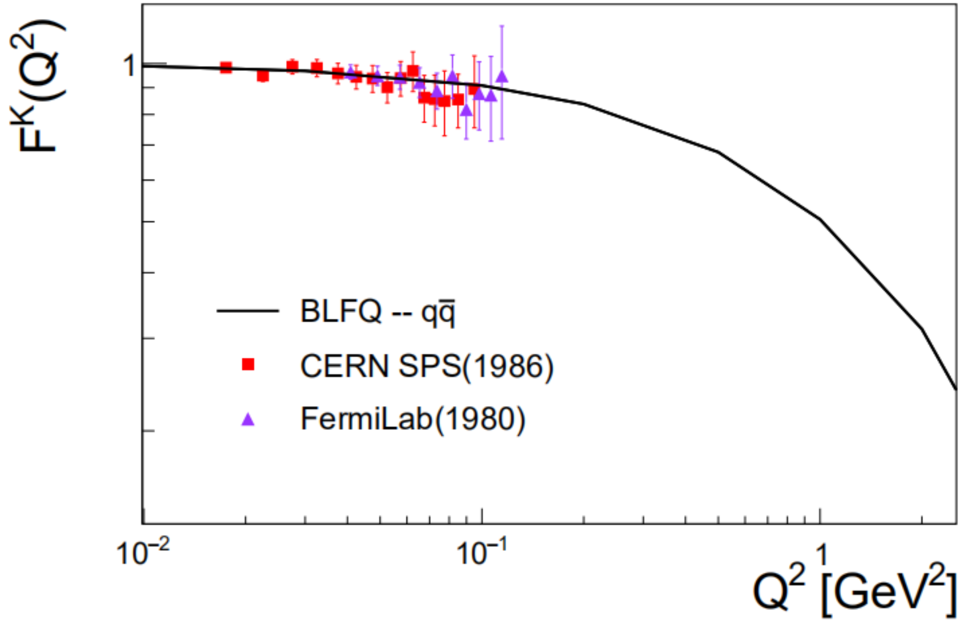
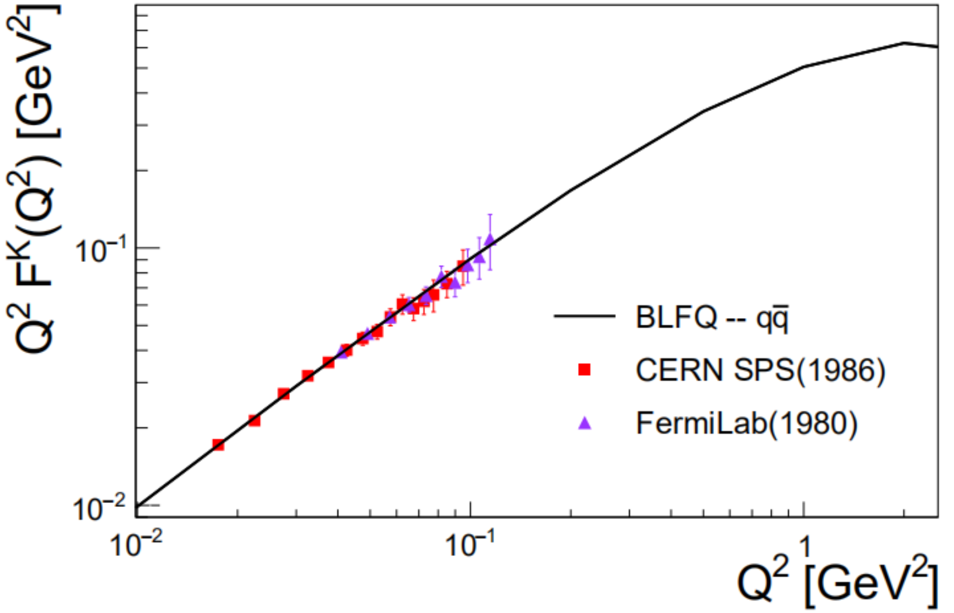
$m_q = 0.39$ GeV , $m_g = 0.60$ GeV,
$m_s = 0.586$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

Preliminary

Kaon Form Factor

$$F(Q^2) = \sum_i e_i \int dx_i H(x_i, 0, Q^2)$$

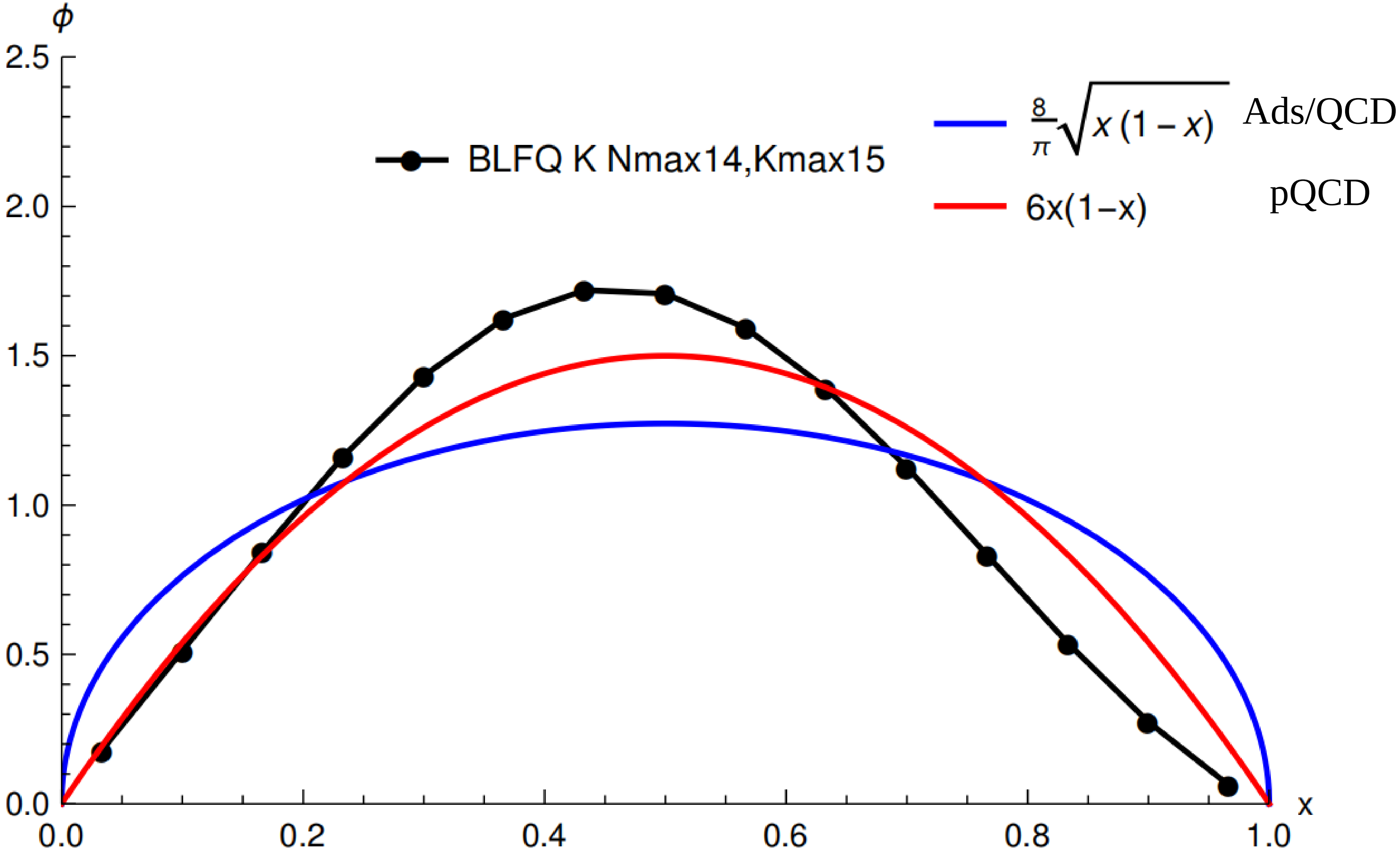




Preliminary: based on leading Fock Sector WF

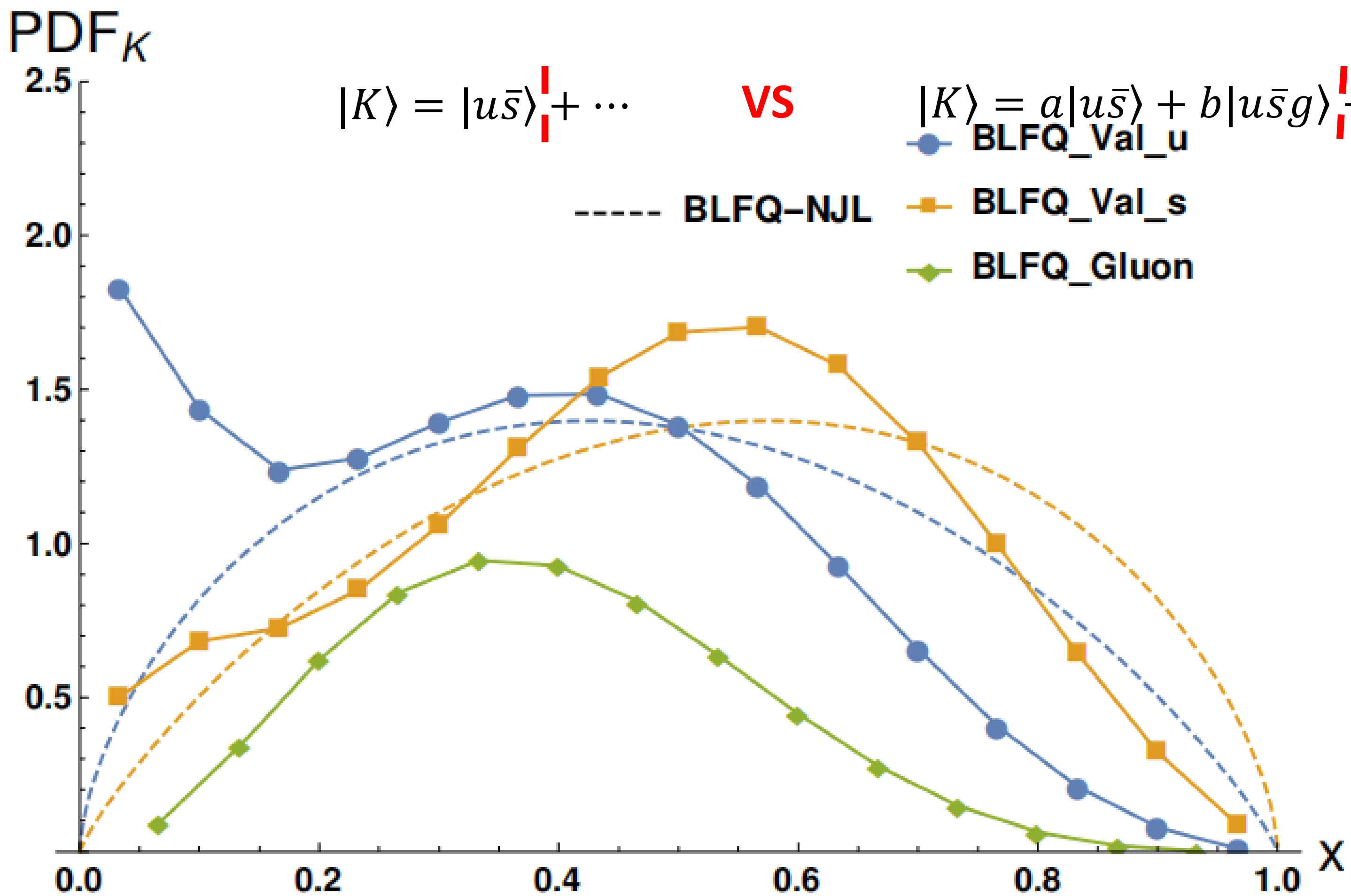
[Lan, et al, in progress]

Kaon PDA



Preliminary

Kaon initial PDF



u large x

$\mu^2_{0\text{BLFQ-NJL}} = 0.247\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = \mathbf{0}$; $\langle x\rangle_{\text{valence } u} = \mathbf{0.468}$; $\langle x\rangle_{\text{valence } s} = \mathbf{0.532}$ $(1-x)^{0.8546}$

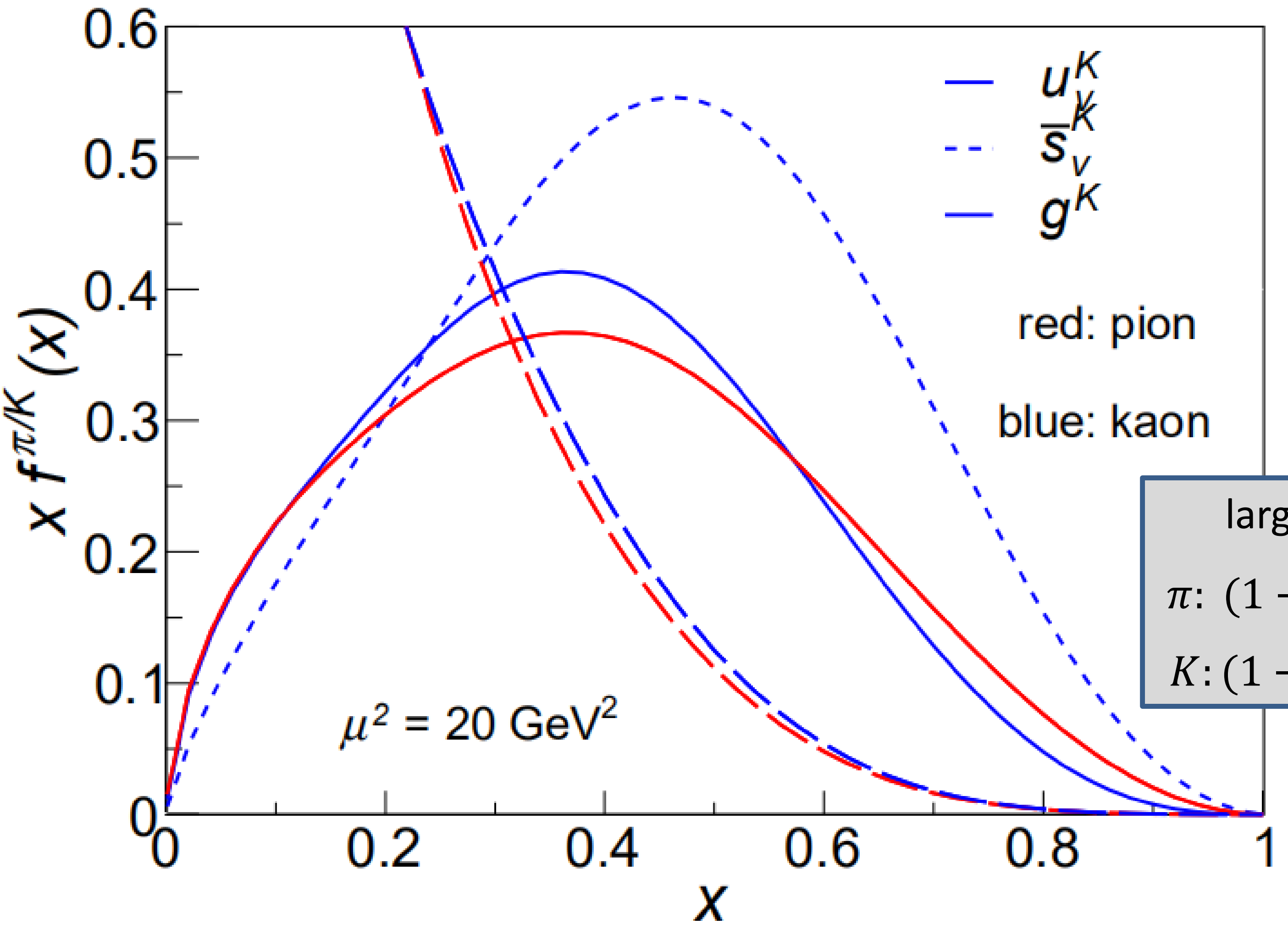
$\mu^2_{0\text{BLFQ}} = 0.47\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = \mathbf{0.162}$; $\langle x\rangle_{\text{valence } u} = \mathbf{0.353}$; $\langle x\rangle_{\text{valence } s} = \mathbf{0.485}$ $(1-x)^{1.92}$

[Lan, et al, in progress]

Preliminary

Kaon PDF

$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



large x

π: $(1-x)^{1.80}$

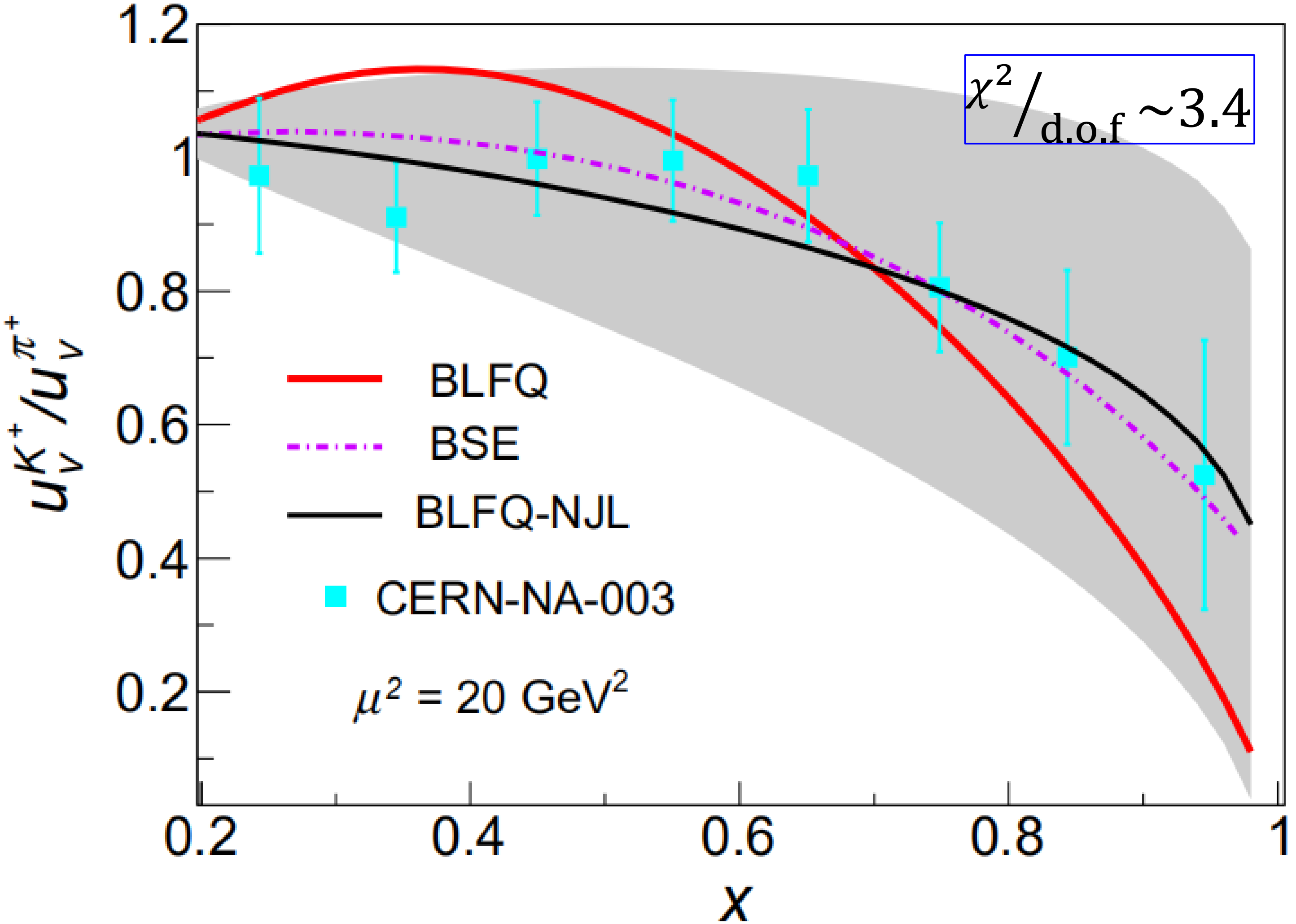
K: $(1-x)^{2.46}$

Preliminary

[Lan, et al, in progress]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **vs** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$

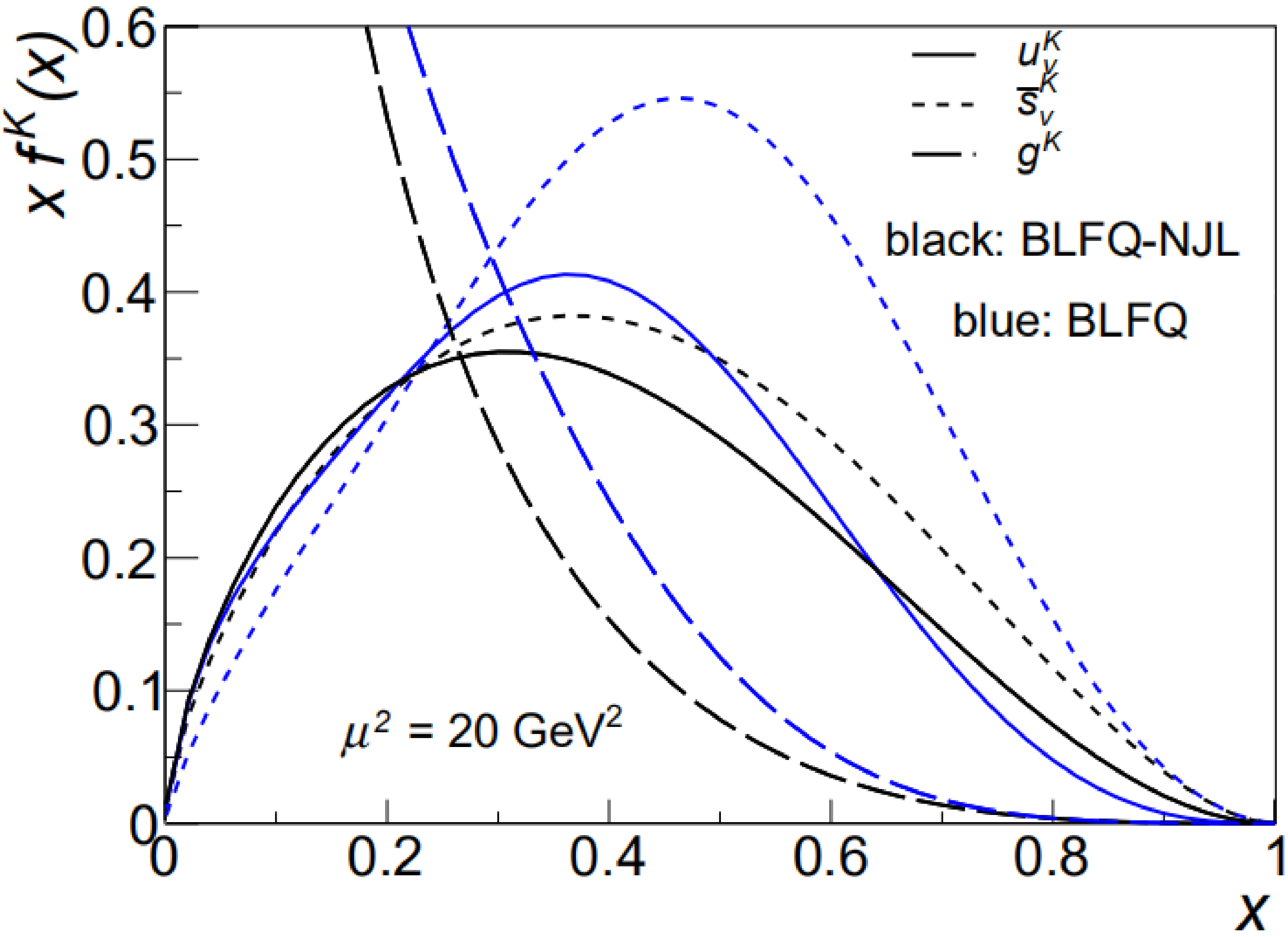


Preliminary

[Lan, et al, in progress]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$

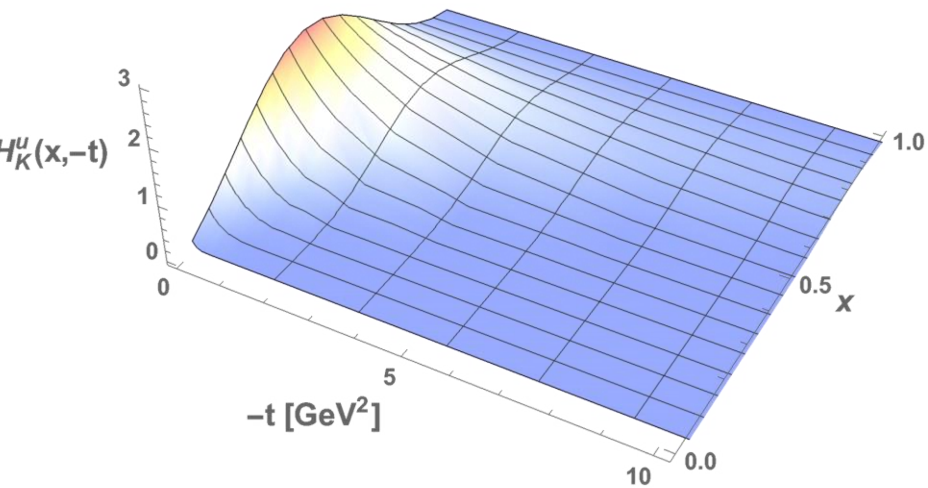


[Lan, et al, in progress]

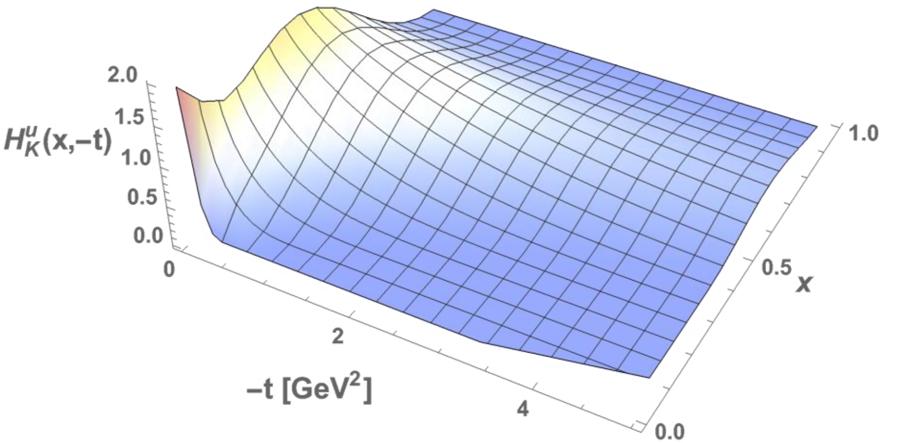
Kaon 3D Structure

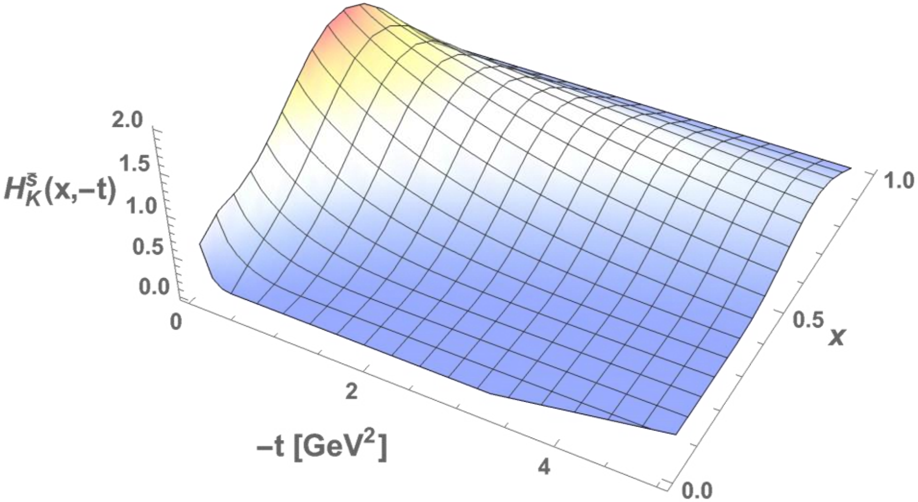
GPD $|\boldsymbol{K}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

Only $|q\bar{q}\rangle$



$|u\bar{s}\rangle + |u\bar{s}g\rangle$

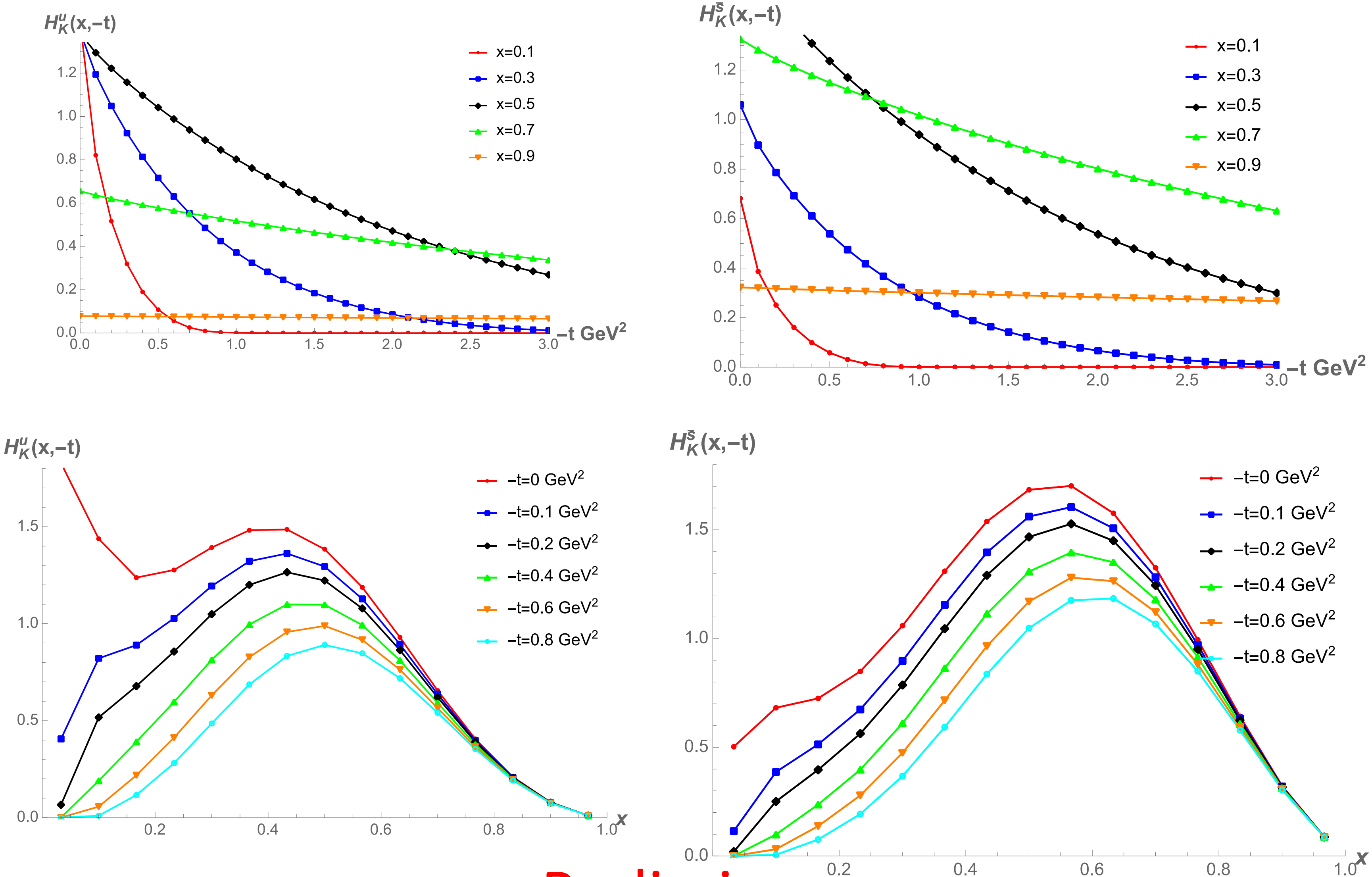




Preliminary

Kaon 3D Structure

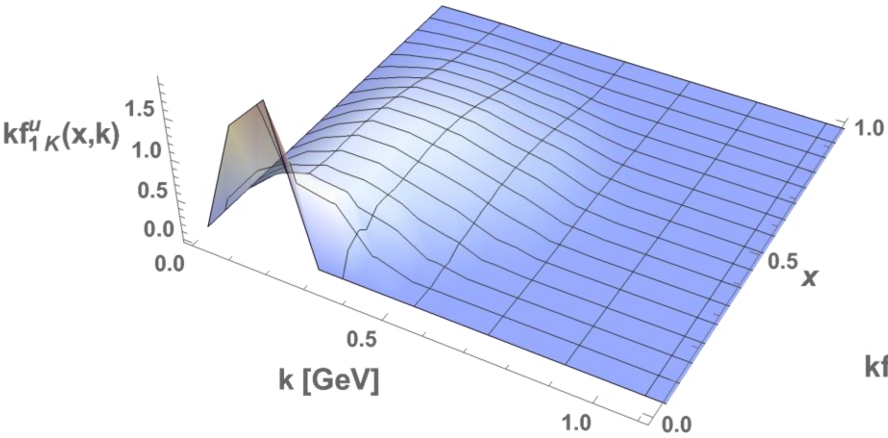
GPD $|\boldsymbol{K}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

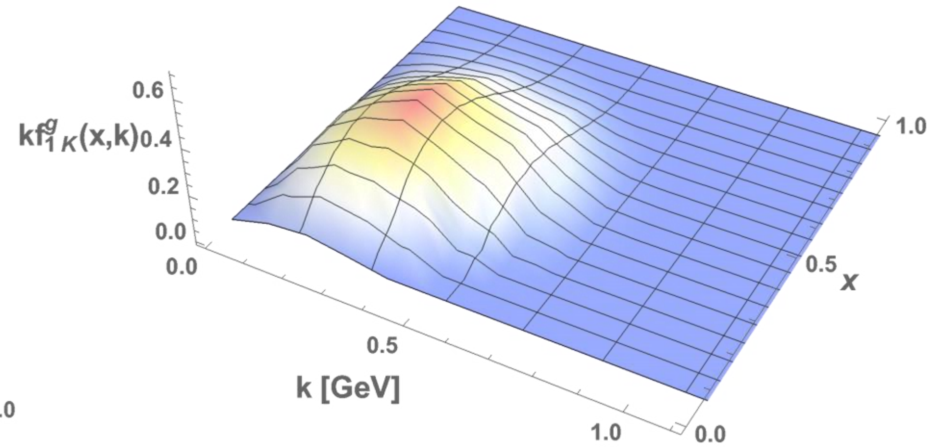


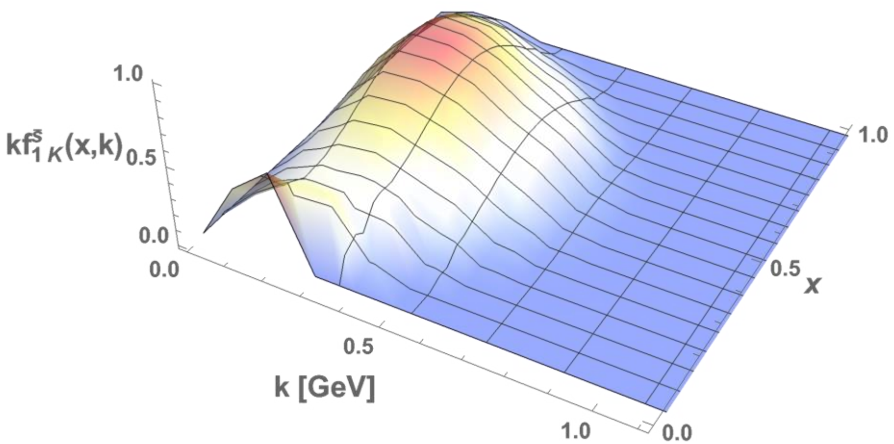
Preliminary

Kaon 3D Structure

TMD $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



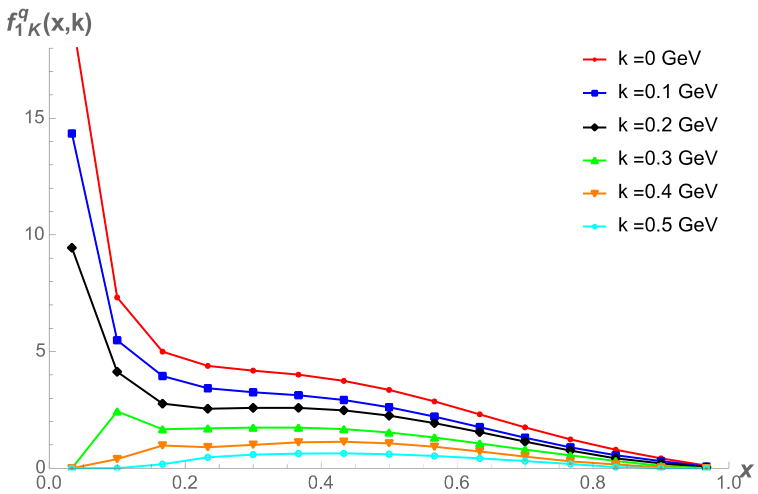


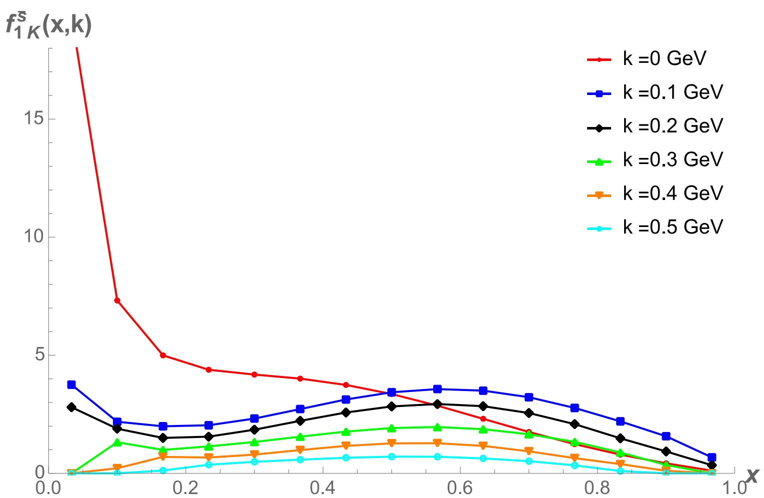


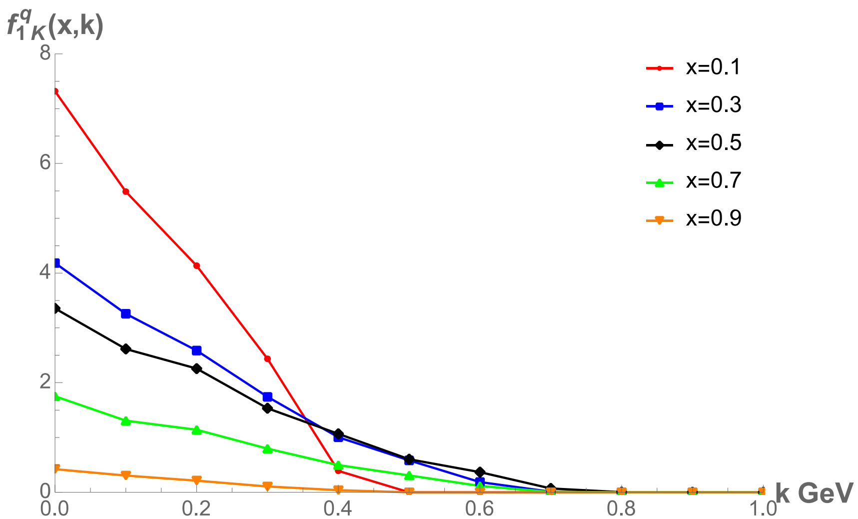
Preliminary

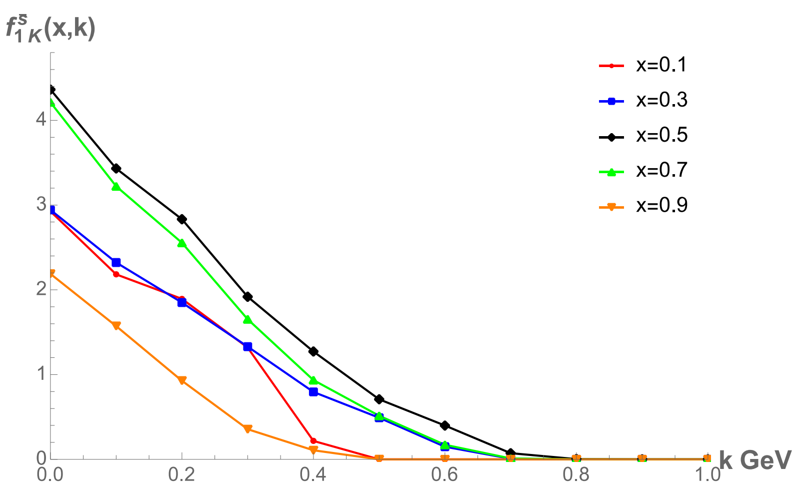
Kaon 3D Structure

TMD $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$









Preliminary

Conclusion and outlook

- Light-front Hamiltonian framework:
  - Relativistic approach
  - Wave functions are available
  - Systematically expandable in Fock space
- Preliminary results on GPDs and TMDs of light mesons
- Next:
  - $|q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle$
  - More observables
  - Mesons of other quantum numbers

Thank you!