Higher-Twist Effect in Pion Parton Distribution

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Perceiving the Emergence of Hadron Mass through

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outline

- Motivation of this work
- A modified-DGLAP equation (ZRS equation)
- Comparing with data and some other works
- Conclusion

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1、Motivation of this work

- Pion is the simplest quark component hadron in the world, while its parton distributions are still not very clear, especially the distributions of sea quark and gluon.

- A natural and wonderful idea is that all the sea quarks and gluons come from the radiation of valence quarks, while according to the evolution of the standard QCD equation-DGLAP equation, the situation seems not so simple. The DGLAP equation can not naturally connect the valence quark distribution at very low momentum scale to the experiment data at high Q^2.

- The DGLAP equation can well describe the parton evolution in high Q^2 region, but in low Q^2 and small x region, some complicated interactions should be considered. A modified-DGLAP equation (ZRS) which contained twist-4 effects may extend the application range of DGLAP equation.

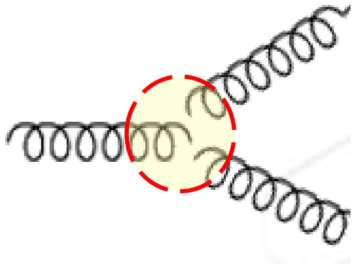
- Starting with a well defined valence quark distribution, with the evolution of ZRS equation, the pion parton distributions may be well described.

2、A modified-DGLAP equation

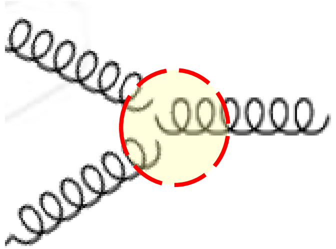
(1) DGLAP equation

$$\frac{dxG(x,Q^2)}{d\ln Q^2} = P_{gg}^{\mathrm{AP}} \otimes G(x,Q^2) + P_{gq}^{\mathrm{AP}} \otimes S(x,Q^2)$$

$$\frac{dxS(x,Q^2)}{d\ln Q^2} = P_{qg}^{\mathrm{AP}} \otimes G(x,Q^2) + P_{qq}^{\mathrm{AP}} \otimes S(x,Q^2)$$



splitting

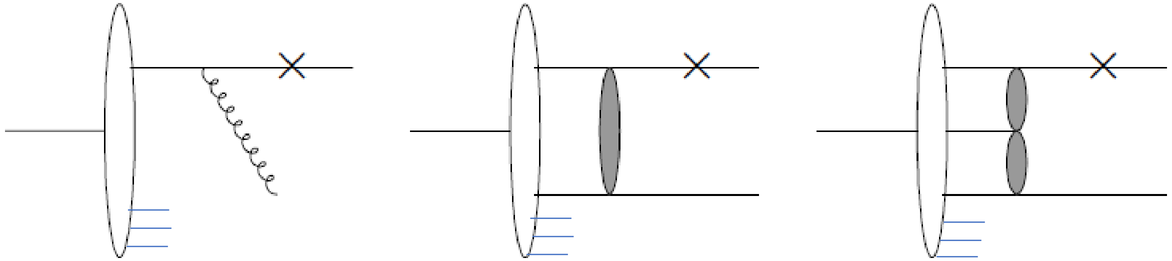


recombination

The necessity to modify the DGLAP equation:

- In small x region, the rapid increase of parton distributions with decreasing x will cause the violation of unitarity bound.
- The increase of the number of partons cannot go on unlimited. When the density of partons becomes too large, they can no longer be considered as independent free particles, they begin to interact with each other.

(2) GLR-MQ equation (Phys. Rep. 100 (1983) 1, Nucl. Phys. B 268 (1986)427)



$$Q^2\frac{\partial}{\partial Q^2}xG(x,Q^2)=\frac{\alpha C_A}{\pi}\int_x^1\frac{\mathrm{d}x'}{x'}\frac{x}{x'}\gamma^{GG}\left(\frac{x}{x'}\right)x'G(x',Q^2)$$

$$-\frac{4\pi^3}{N^2-1}\left(\frac{\alpha C_A}{\pi}\right)^2\frac{1}{Q^2}\int_x^1\frac{\mathrm{d}x'}{x'}(x')^2G^{(2)}(x',Q^2).$$

- GLR-MQ equation violates the momentum conservation

(3) Modified-DGLAP (ZRS) equation

（Nucl. Phys. B 559 (1999) 378， Phys. Rev. D 68 (2003) 094015
Int. J. Mod. Phys. E 23 (2014) 1450057）

$$
\begin{aligned}
\frac{dxG(x,Q^2)}{d\ln Q^2} &= P_{gg}^{\mathrm{AP}} \otimes G(x,Q^2) + P_{gq}^{\mathrm{AP}} \otimes S(x,Q^2) \\
&+ \frac{\alpha_s^2}{4\pi R^2 Q^2} \int_{x/2}^{x} dx_1 x x_1 G^2(x_1,Q^2) \\
&\cdot \sum_i P_i^{gg\to g}(x_1,x) \\
&- \frac{\alpha_s^2}{4\pi R^2 Q^2} \int_{x}^{1/2} dx_1 x x_1 G^2(x_1,Q^2) \\
&\cdot \sum_i P_i^{gg\to g}(x_1,x), \qquad (1a)
\end{aligned}
$$

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Small-x analysis on the effect of gluon recombinations inside hadrons in light of the GLR-MQ-ZRS equation

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Modified DGLAP = DGLAP

+ antishadowing (+)

+ shadowing (-)

An India research group studied this equation in detail

the gluon momentum lost due to shadowing is believed to be compensated in terms of new gluons with comparatively larger *x*.

3、Comparing with data and some other works

(A) The input pion parton distributions

BLFQ :（Phys. Rev. D 101 (2020) 034024, Phys. Rev. Lett. 122 (2019) 172001 ）

valence quark input distribution

$$Q_0^2 = 0.12GeV^2$$

$$f(x) = B^{-1}(1+\alpha, 1+\beta)x^{\alpha}(1-x)^{\beta}$$

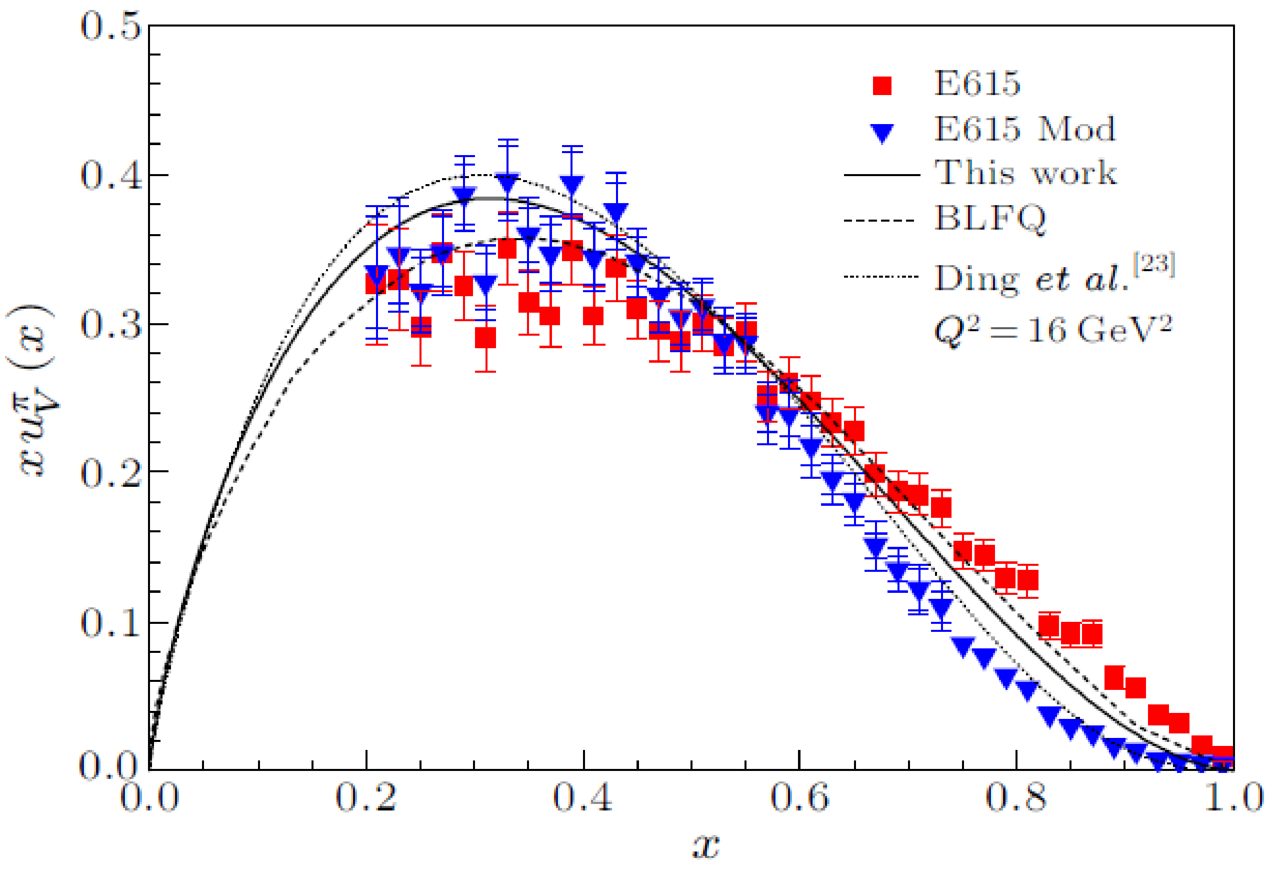
$$\alpha = \beta = 0.5961$$

Our model : valence quark input distribution

$$Q_0^2 = 0.09GeV^2$$

$f(x)$: the same as BLFQ

(B) Parton distributions comparison



valence quark distributions

BLFQ: $Q_0^2 = 0.12 GeV^2$

DGLAP evolution equation

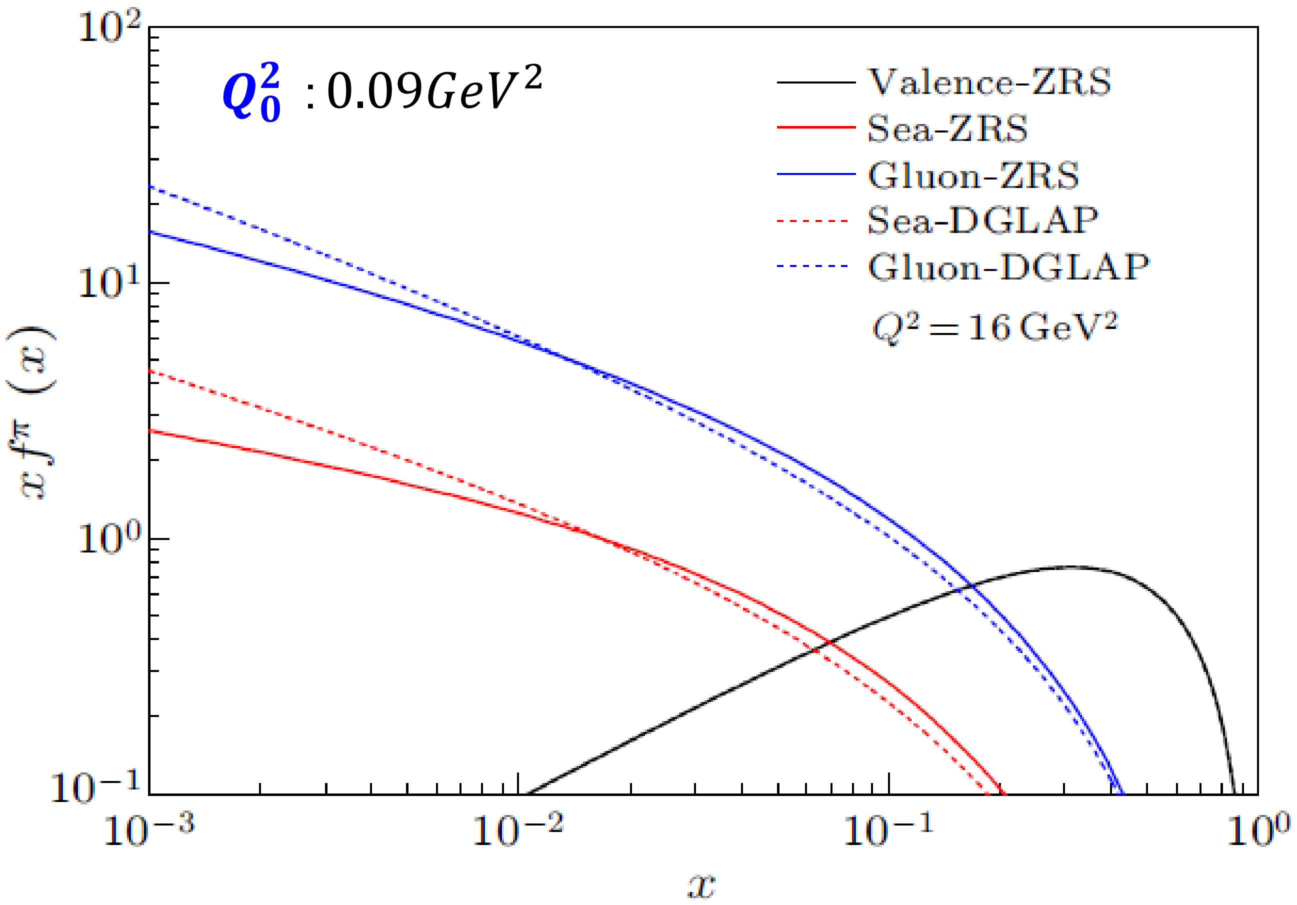
Our model : $Q_0^2 = 0.09 GeV^2$

ZRS evolution equation

Ding et al: $Q_0^2 = 0.09 GeV^2$

$$f(x) = 213.32\, x^2 (1-x)^2 \times [1 - 2.9342 \sqrt{x(1-x)} + 2.2911\, x(1-x)].$$

（Phys. Rev. D 101 (2020) 054014)
Chin. Phys. C 44 (2020) 031002）



valence quark, sea quark and gluon distributions

input distribution :

BLFQ

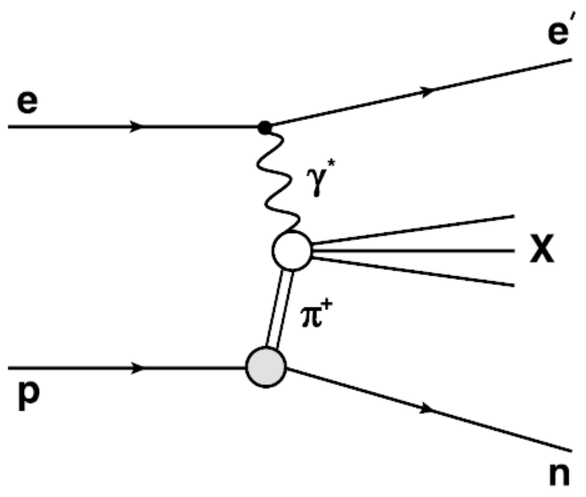
Evolution equation:

DGLAP equation

ZRS equation

- This fig directly show the effect of shadowing and anti-shadowing . If the equation contains only the shadowing part, without the anti-shadowing part, the solid lines will always be lower than the dashed lines, and momentum conservation will not be maintained.

(C) Structure functions



$$\mathrm{d}\sigma\left(ep \to e'nX\right) = f_{\pi^+/p}(x_L, t) \cdot \mathrm{d}\sigma\left(e\pi^+ \to e'X\right),$$

$f_{\pi^+/p}(x_L, t)$ the pion flux associated with the splitting of a proton into a $\pi+n$ system

$d\sigma(e\pi+ \to e'X)$: the cross section of the e-pion interaction

H1: one pion exchange model

$x_L = 0.73 \quad 0.68 < x_L < 0.77$

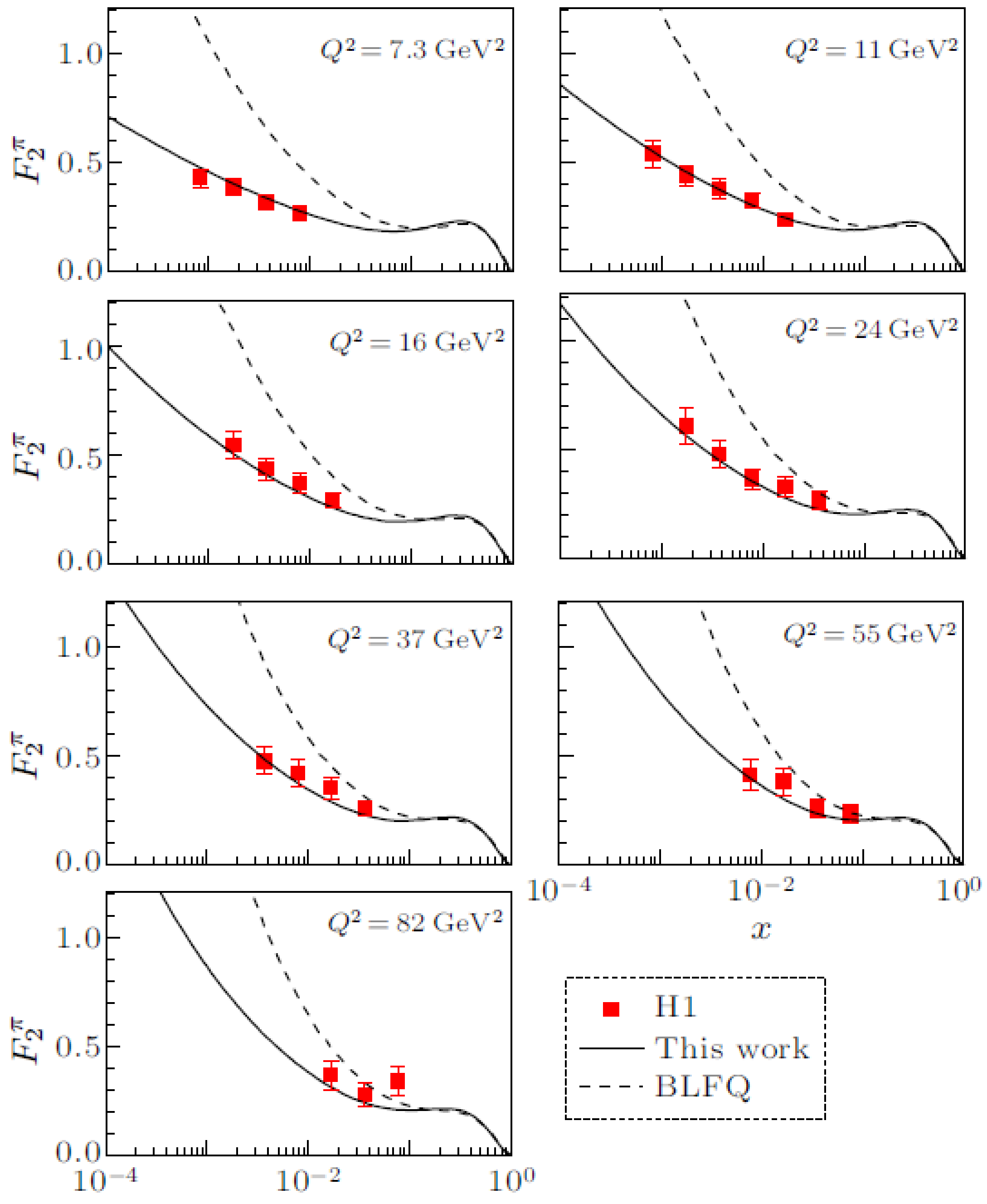
$p_T < 0.2\ GeV$

ZEUS: effective one-pion-exchange model

additive quark model

$x_L = 0.73 \quad 0.64 < x_L < 0.82$

$p_T < 0.656 x_L$



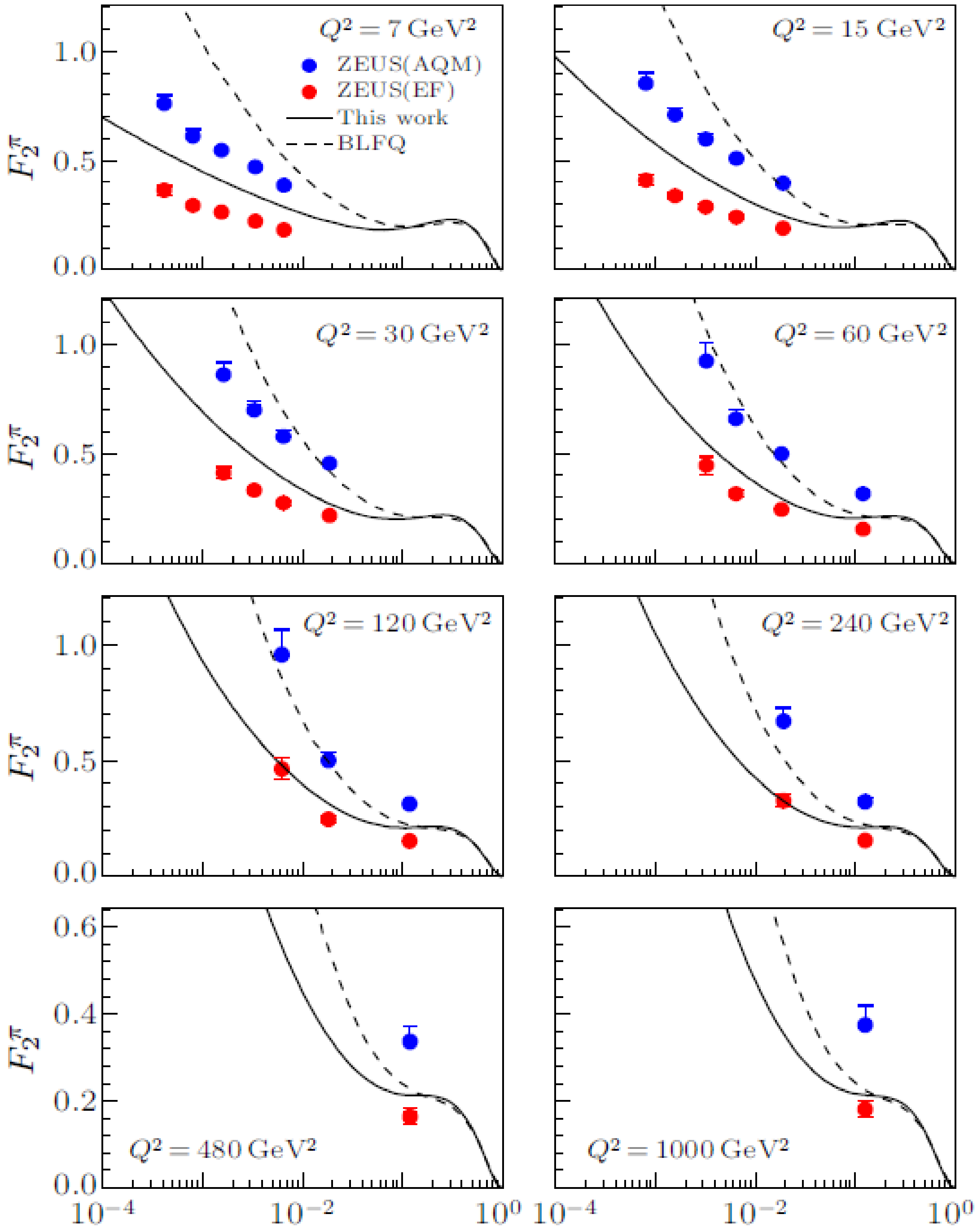
Comparing with H1 data

The dashed lines obtained via the evolution of the DGLAP equation are clearly higher than those for the experimental data.

The solid lines are obtained via the evolution of the ZRS equation The higher-twist correction played an important role in the small x region.

In the leading-order approximation

$$F_2^\pi(x, Q^2) = \sum_q e_q^2 x[f_q^\pi(x, Q^2) + f_{\bar{q}}^\pi(x, Q^2)]$$

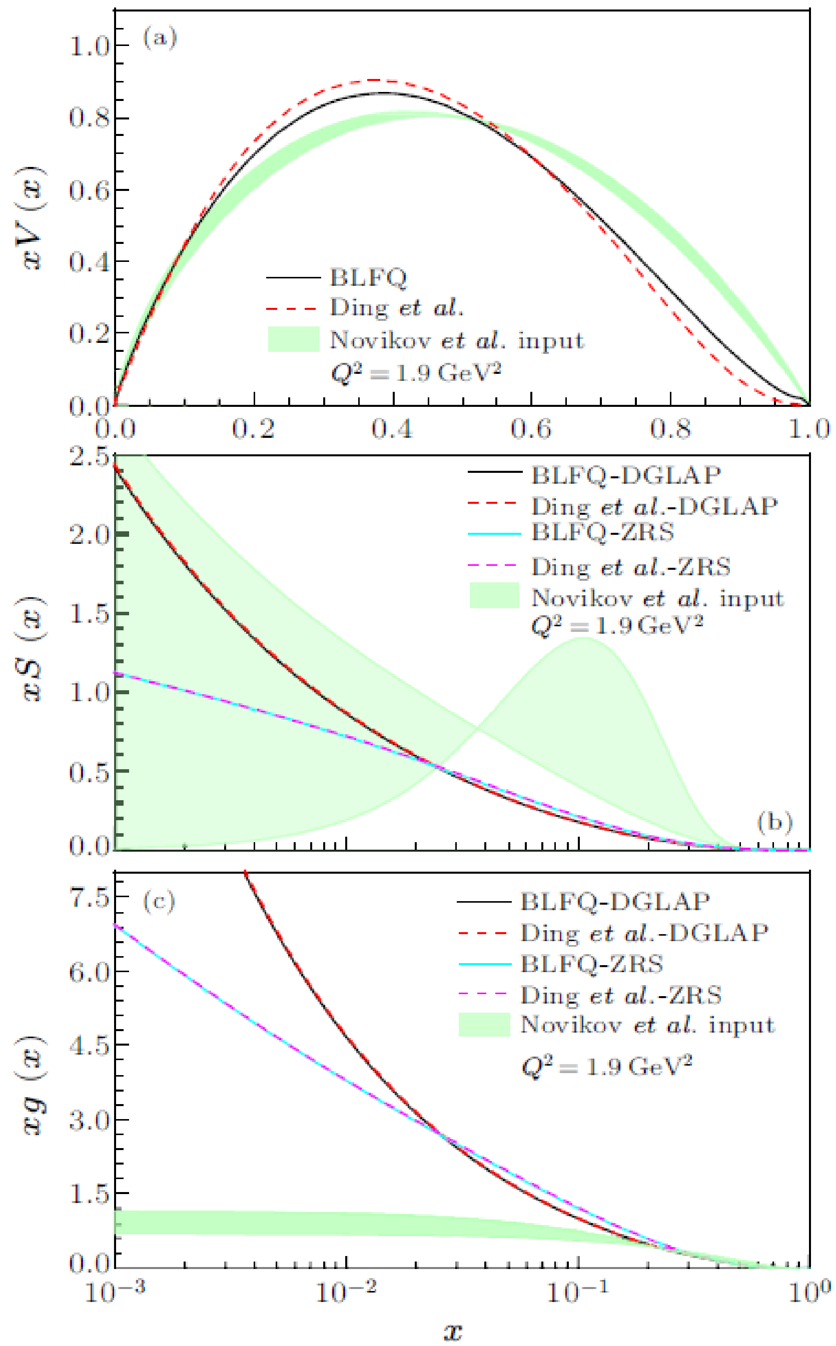


Comparing with ZEUS data

There are two groups of data of ZEUS， our lines are in-between the two sets of data。

The results of BLFQ are much closer to the data of the additive quark model, their line slope is obviously larger than that of the data, particularly in the small x region.

It seems that higher-twist corrections cannot be neglected, even in comparison with the additive quark model.



Comparision of BLFQ、Ding et al., and Novikov at al.

$$Q^2 = 1.9GeV^2$$

Valence quark: BLFQ is similar to Ding

Novikov: obviously higher in large x region

Sea quark: BLFQ 、Ding almost the same

Novikov: a very large uncertainty region

obviously higher in large x region

gluon: BLFQ 、Ding almost the same

Novikov much smaller in small x region

Novikov at al: Phys. Rev. D 102(2020) 014040

(D) momentum comparison

Momentum fractions of pion, carried by valence quark, sea quark and gluon at $Q^2 = 1.9\ GeV^2$

	$\langle xV \rangle$	$\langle xS \rangle$	$\langle xg \rangle$
BLFQ (DGLAP)	0.532	0.068	0.400
BLFQ (ZRS)	0.532	0.068	0.400
Ding *et al.* (DGLAP)	0.530	0.069	0.401
Ding *et al.* (ZRS)	0.530	0.069	0.401
Novikov *et al.* input	0.56	0.21	0.23

- The momentum fraction of BLFQ and Ding are almost the same
- The sea quark momentum of Novikov is much larger than the other two models, and the gluon momentum is much smaller.
- The evolution of ZRS equation does not influence the momentum fraction, since it is momentum conservation, it will only change the shape of sea quark and gluon distributions

4、Conclusion:

- ZRS equation is an evolution equation considering twist-4 corrections, which satisfies the conservation of momentum. Using the input distribution function of BLFQ and the evolution of ZRS equation, the experimental data of valence quarks distribution and structure functions can be well explained.

- The experimental data of structure function show that the higher-twist effect should not be neglected in small x region.

- Through comparison, it is found that BLFQ model is very close to Ding's model, but they are quite different from Novikov model. It is expected to be further determined by experiments in the future.

Thank you for your attention!