Mellin moments of spin dependent and independent PDFs of the ρ and π

[2108.07544]

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for RQCD collaboration

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RQCD

Motivation

Experiment

- Pion measurement potential EIC candidate [1907.08218]
- Deuteron measurements at HERMES and EIC [1903.01119]

COMPASS++/AMBER (proto-)collaboration at CERN

Drell-Yan reactions with (secondary) pion beams, π-PDFs for $x \gtrsim 0.15$

Theory

Computation of the energy momentum tensor

Lattice Studies

- Update previous studies [hep-lat/9703014]
- Stochastic threepoint functions allow studies for a large set of spin structures, momenta, flavors, ...
- Taking disconnected contributions fully into account
- Controlled chiral- and continuum extrapolation to the physical limit using CLS data

Outline

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Generals properties of PDFs

Parton distribution functions from DIS [hep-ph/9204208]

- Factorize matrix element of DIS cross-section in $l_{\mu\nu}W^{\mu\nu}$
- Further decompose the hadronic tensor

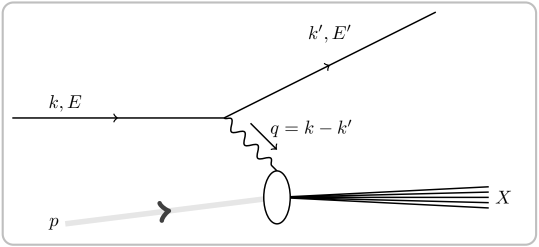
$$W^{\mu\nu} = \int \mathrm{d}x\, f_q(x)\, C^{\mu\nu}_{q,\text{partonic}}$$

$f_q(x)$ is the PDF for flavor q and $C_q^{\mu\nu}$ the corresponding coefficient

Definitions:

$$Q^2 = -q^2$$
$$x = \frac{Q^2}{2p\cdot q}$$

- c.f. talk about GPDs by Cédric Mezrag



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- Parton model density functions can be expressed in terms of matrix elements of bilocal twist-2 operators

$$f_q(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z^-}{4\pi} \, e^{ip^+ z^-} \left\langle \mathbf{p}, \lambda \middle| \bar{q}\left(\frac{-z}{2}\right) \Gamma \, q\left(\frac{-z}{2}\right) \middle| \mathbf{p}, \lambda \right\rangle \Big|_{z^+=0, \mathbf{z}=0}$$

with light-cone coordinates p^+ and z^- and quark spinors q

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Moments of PDFs

- Wick rotation on the lattice $\rightarrow$ imaginary distance $z^- = z^0 - z^3$
- Compute moments of PDFs using local matrix elements

$$M_n(f) = \int_0^1 \mathrm{d}x\, x^{n-1} f(x) = \langle x^{n-1} \rangle_f$$

$$\langle x^{n-1} \rangle_{f_q} \sim \frac{1}{2p_+^n} \langle \mathbf{p}| \mathcal{O}^{\mu_1 \cdots \mu_n} |\mathbf{p}\rangle$$

Generals properties of PDFs

Structure functions from DIS

Simplest case decomposition of the hadronic tensor

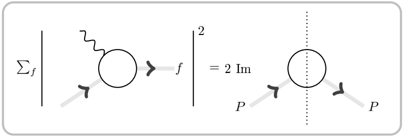
$$W^{\mu\nu} = F_1(x,Q^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{q^2}\right) + F_2(x,Q^2)\left(\frac{p^\mu}{p\cdot q} + \frac{p^\nu}{q^2}\right)\left(\frac{p^\nu}{p\cdot q} + \frac{p^\mu}{q^2}\right)$$

Structure functions are related to PDFs

$$F_2(x,Q^2) = \sum_q e_q^2\, x\, f_q(x) \qquad F_1(x,Q^2) = \frac{1}{2x} F_2(x,Q^2)$$

with electrical charge e_q for flavor q

Generals properties of PDFs



Structure functions on the lattice

Interpretation of structure functions

via sum rules and operator product expansion (OPE)

Use optical theorem to relate forward Compton scattering amplitude and hadronic tensor

$$2\pi\, W^{\mu\nu} = \frac{i}{4\pi} \int \mathrm{d}^4 x e^{iq\cdot x} \left\langle p, \lambda' \right| T\left[J^\mu(x) J^\nu(0)\right] \left| p, \lambda \right\rangle$$

Compute Compton scattering structure functions

$$\mathrm{Im}\, \tilde{F}_1(\omega + i\epsilon) = 2\pi F_1(\omega), \ldots$$

Operator product expansion (OPE)

Expand the product of two currents into a series local operators using the OPE

$$\lim_{z\to 0} \mathcal{O}_a(z)\,\mathcal{O}_b(0) = \sum_k c_{abk}(z)\,\mathcal{O}_k(0)$$

The most dominant twist $t = 2$ operators are given by

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \frac{1}{2^{n-1}}\,\mathcal{S}\,\bar{q}\,\gamma^{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n}\,q\,, \quad \ldots$$

where the covariant, symmetrized derivatives are defined as $\overleftrightarrow{D}^{\mu} = \overrightarrow{D}^{\mu} - \overleftarrow{D}^{\mu}$ and $\mathcal{S}$ projects out the completely symmetrized and traceless components of the r.h.s. tensor.

Reduced matrix elements

Now compute matrix elements of the operators and decompose for pseudoscalar

$$\langle \mathbf{p} | \, \mathcal{O}^{\mu_1 \cdots \mu_n} \, | \mathbf{p} \rangle = 2 \, \mathcal{S} \left[v_n^q p^{\mu_1} \cdots p^{\mu_n} \right]$$

and vector mesons

$$\langle \mathbf{p}, \lambda | \mathcal{O}^{\mu_1 \cdots \mu_n} \, | \mathbf{p}, \lambda \rangle = 2 \, \mathcal{S} \Bigg[a_n^q p^{\mu_1} \cdots p^{\mu_n} + d_n^q \left(m^2 \epsilon^{* \mu_1}(\mathbf{p}, \lambda) \, \epsilon^{\mu_2}(\mathbf{p}, \lambda) - \frac{1}{3} p^{\mu_1} p^{\mu_2} \right) p^{\mu_3} \cdots p^{\mu_n} \Bigg]$$

to obtain the reduced matrix elements v_n^q, a_n^q and d_n^q.

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Structure function sum rules

Evaluate $T\left[J^{\mu}(x)J^{\nu}(0)\right]$ to relate Compton structure functions to reduced matrix elements

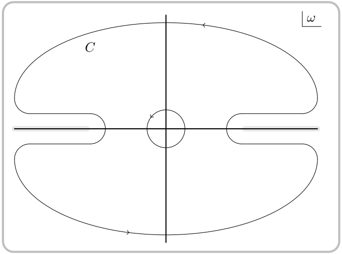
$$\tilde{F}_1(\omega) = \sum_{n=2,4,\ldots}^{\infty} 2\,C_n^{(1)}\,a_n\,\omega^n$$

$$\tilde{F}_2(\omega) = \sum_{n=2,4,\ldots}^{\infty} 4\,C_n^{(2)}\,a_n\,\omega^{n-1}$$

$$\tilde{b}_1(\omega) = \sum_{n=2,4,\ldots}^{\infty} 2\,C_n^{(1)}\,d_n\,\omega^n$$

$$\tilde{b}_2(\omega) = \sum_{n=2,4,\ldots}^{\infty} 4\,C_n^{(2)}\,d_n\,\omega^{n-1}$$

via sum-rules using $\omega = 1/x$, with $|\omega| = 1$, and Wilson coefficients C_n.



Relation to physical region

Extract coefficient functions by contour integral

$$2C_n^{(1)} a_n = \frac{1}{2\pi i} \oint_C \frac{\mathrm{d}\omega}{\omega^{n+1}} \tilde{F}_1(\omega)$$

Using the opitical theorem + symmetries we find

$$2M_n(F_1) = 2\int_0^1 \mathrm{d}x\, x^{n-1} F_1(x) = C_n^{(1)}\, a_n \qquad \text{for } n \text{ even}$$

General Properties of PDFs (summary)

Hadronic tensor (DIS)

$$W^{\mu\nu}(\mathbf{p},\lambda) = \int \frac{\mathrm{d}^4 z}{4\pi}\; e^{iq\cdot z} \langle \mathbf{p},\lambda|\, [j^\mu(z), j^\nu(0)]\, |\mathbf{p},\lambda\rangle$$

Structure functions from hadronic tensor decomposition

Spin-0 target:

coupling to spin-0 operator $\rightarrow$ 2 structure functions

Spin-1 target:

coupling to spin-(0,1,2) operator $\rightarrow$ 8 structure functions

Computation of the 2nd $(n = 2)$ moments

$$2M_n(F_1^\pi) = C_n^{(1)} v_n$$

$$2M_n(F_1^\rho) = C_n^{(1)} a_n$$

$$2M_n(b_1^\rho) = C_n^{(1)} d_n$$

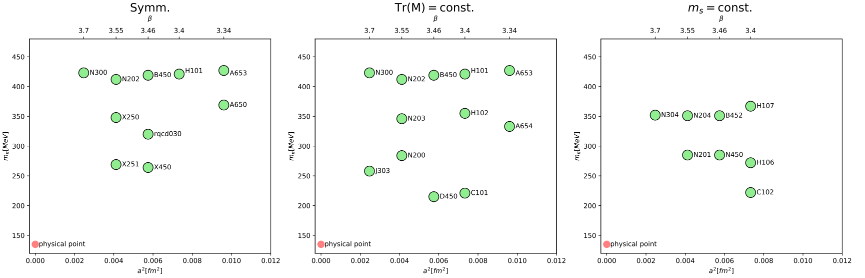
Operator

$$\mathcal{O}_{\mathrm{v2a}} = \mathcal{O}^{4i} \qquad \mathcal{O}_{\mathrm{v2b}} = \mathcal{O}^{44} - \frac{1}{3}\left(\mathcal{O}^{11} + \mathcal{O}^{22} + \mathcal{O}^{33}\right)$$

Lattice computation

Gauge ensembles generated by CLS [1411.3982]

- Tree-level Symanzik improved gauge action
- $N_f = 2 + 1$ flavors of non-perturbatively order a improved Wilson fermions
- Stable Monte Carlo sampling by twisted mass determinant reweighting
- Wuppertal smearing to improve overlap of interpolating currents + APE smoothed gauge links



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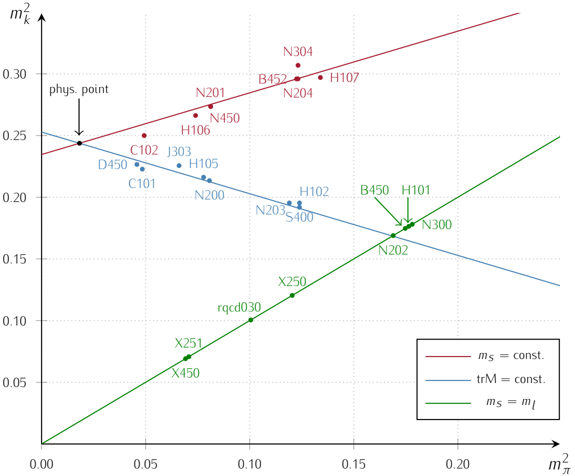


Renormalization [2012.06284]

- Two step procedure
  1. Non-pert. calculation in RI'/SMOM
  2. Conversion to $\overline{\text{MS}}$
- Final results in $\overline{\text{MS}}$ at 2 GeV

Operator mixing

Isosinglet operators $(\bar{u}u + \bar{d}d + \bar{s}s)$ mix under renormalization with gluonic operators

$$\mathcal{O}^{\text{ren}} = Z^{qq}\mathcal{O} + Z^{qg}\mathcal{O}_g$$

- Neglect mixing within statistical accuracy for the moment
- Approximate isosinglet renormalization by nonsinglet renormalization

2π contributions

Possible 2π contributions for ρ mesons (resonance nature) [1512.08678]

Ratios

Extract matrix element using direct fits to ratios $C_{3,\mathbf{p},t,\tau}/C_{2,\mathbf{p},t}$

Pseudoscalar

$$R_{\mathbf{p}} = \frac{C_{3,\mathbf{p},t,\tau}}{C_{2,\mathbf{p},t}} \xrightarrow{t \gg \tau \gg 0} \frac{\langle \mathbf{p}| \mathcal{O} |\mathbf{p}\rangle}{2E^{\pi}_{\mathbf{p}}}$$

Vector

$$R^i_{\mathbf{p}} = \frac{C^{ii}_{3,\mathbf{p},t,\tau}}{C^{ii}_{2,\mathbf{p},t}} \xrightarrow{t \gg \tau \gg 0} \frac{m^2}{E^2} \sum_{\lambda,\lambda'} \epsilon^i(\mathbf{p},\lambda')\epsilon^{i*}(\mathbf{p},\lambda) J^{\mathbf{p}}_{\lambda'\lambda}$$

$\underline{\hat{\mathbf{p}} = \pm \mathbf{e}_i :}$

$$J^{\mathbf{p}}_{00} = R^i_{\mathbf{p}} \qquad J^{\mathbf{p}}_{++} + J^{\mathbf{p}}_{--} = \sum_{j \neq i} R^j_{\mathbf{p}}$$ using $J^{\mathbf{p}}_{\lambda'\lambda} \equiv \langle \mathbf{p}, \lambda' | \mathcal{O} | \mathbf{p}, \lambda \rangle / (2E^{\rho}_{\mathbf{p}})$

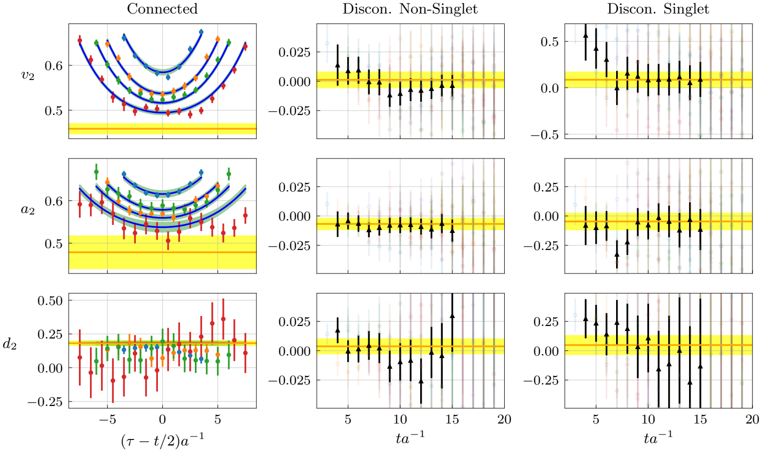
Reduced matrix elements

- Include disconnected contributions (large statistical error)
- Use stochastic estimators for connected and disconnected computation [1711.02384], [0910.3970]
- Extract reduced matrix elements separately for con. and discon.

Relation to lattice ratios (for $\hat{\mathbf{p}} = \pm \mathbf{e}_i$)

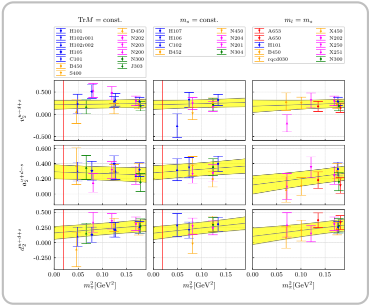
$\mathcal{O}^i_{\mathrm{v2a}} = \mathcal{O}^{0i}$	$\mathcal{O}_{\mathrm{v2b}} = \frac{4}{3}\mathcal{O}^{00}$
$v_2 = \frac{1}{p^i} R_{\mathbf{p}}\big(\mathcal{O}^i_{\mathrm{v2a}}\big)$	$v_2 = \frac{3E}{4E^2-m^2} R_{\mathbf{p}}\big(\mathcal{O}_{\mathrm{v2b}}\big)$
$a_2 = \frac{1}{3p^i} \sum_j R^j_{\mathbf{p}}\big(\mathcal{O}^i_{\mathrm{v2a}}\big)$	$a_2 = \frac{E}{4E^2-m^2} \sum_j R^j_{\mathbf{p}}\big(\mathcal{O}_{\mathrm{v2b}}\big)$
$d_2 = \frac{3}{4p^i}\big(2R^i_{\mathbf{p}}\big(\mathcal{O}^i_{\mathrm{v2a}}\big) - \sum_{j\neq i} R^j_{\mathbf{p}}\big(\mathcal{O}^i_{\mathrm{v2a}}\big)\big)$	$d_2 = \frac{3E}{8(E^2-m^2)}\big(2R^i_{\mathbf{p}}\big(\mathcal{O}_{\mathrm{v2b}}\big) - \sum_{j\neq i} R^j_{\mathbf{p}}\big(\mathcal{O}_{\mathrm{v2b}}\big)\big)$

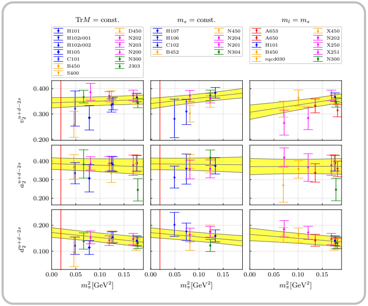
Extraction of ground state matrix elements: N204, $\mathbf{p}^2 = 1, \mathcal{O}_{\mathrm{v2b}}$



Extrapolation to physical and continuum limit

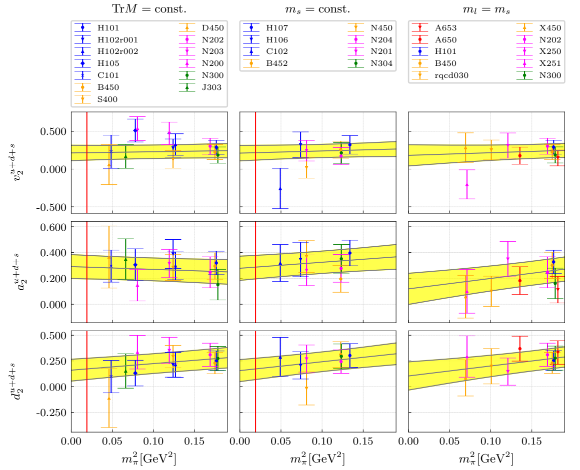
- Parametrization: $f\left(a, m_\pi^2\right) = c_0 + c_1 a + c_2 m_\pi^2 + c_3 m_K^2$
- So far 25 ensembles analyzed
- Data points are corrected for mass / latt. spacing effects





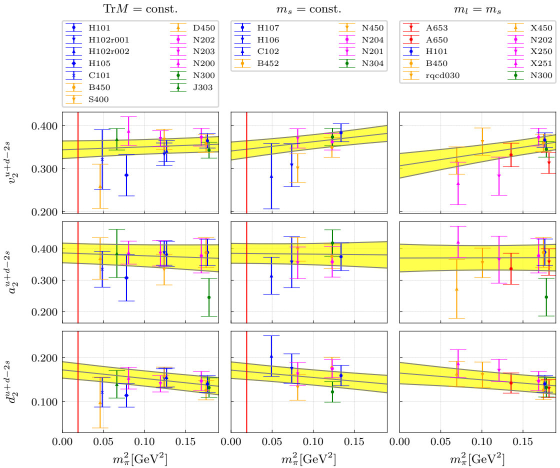
Extrapolation to physical and continuum limit

Flavor singlet contribution $(u + d + s)$



Extrapolation to physical and continuum limit

Flavor non-singlet contribution $(u + d - 2s)$



Summary

- First computation including disconnected contributions
- Considerable error reduction for connected contribution compared to previous studies [hep-lat/9703014]
- Controlled chiral- and continuum extrapolation to the physical limit

Possible extensions

- Analysis of additional spin structures
- Computation of non-forward limit matrix elements
- Renormalization of singlet combinations
- Further investigation of possible 2π states

Thank you for your attention

Extrapolation to physical and continuum limit

Weighted average for a-dependence