Quark propagation and hadronization

Andrea Signori

University of Pavia and INFN

Perceiving the emergence of hadron mass through AMBER@CERN

September 29, 2021

Outline

Quark hadronization, propagation, mass generation

Inclusive jets

Semi-inclusive processes

Some references:

A selection of references related to the topics discussed in this talk:

- Collinear factorization for deep inelastic scattering structure functions at large Bjorken x_B

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- On the connection between quark propagation and hadronization

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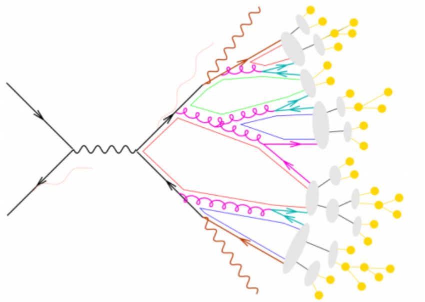
  A. Accardi, A. Signori - 2005.11310 - EPJC
- Pion parton distribution and fragmentation functions beyond the leading twist in a confining Nambu–Jona-Lasinio model
  I. Cloet, A. Signori - in preparation

Fragmentation functions

Hadronization: dynamical generation of hadronic properties from quarks/gluons
→ fundamental topic

It follows any QCD hard scattering event and populates the final states with hadrons.

Maps of hadronization in momentum space: fragmentation functions (FFs)



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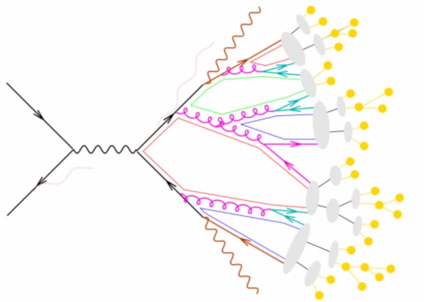

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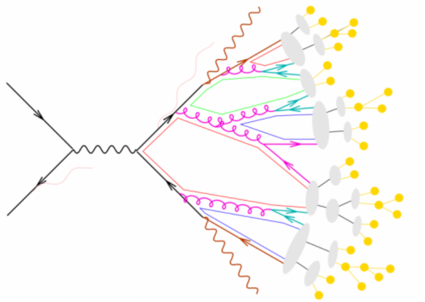

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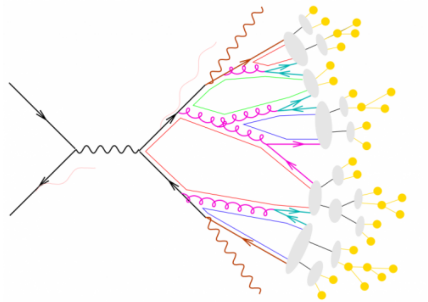

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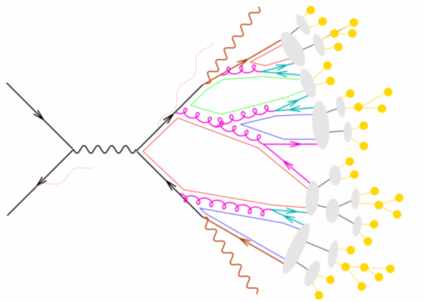

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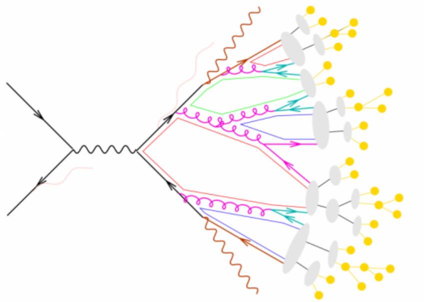

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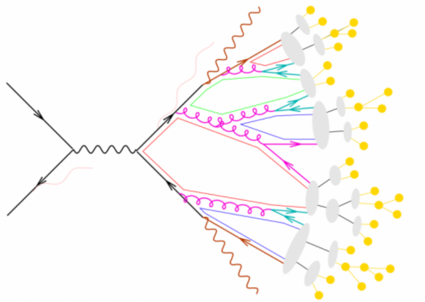

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The generation of mass

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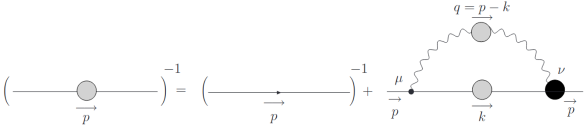
The dynamical generation of mass in QCD can be addressed in different ways:

- gap equation
  e.g. in the NJL model of QCD: $M_q = m_q - 4G_\pi \langle \overline{q} q \rangle \gg m_q$



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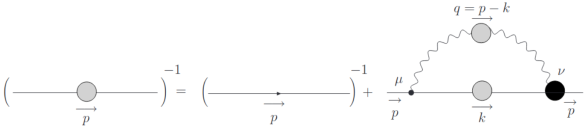

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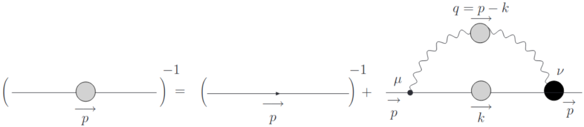
- Energy Momentum Tensor → hadron mass decomposition
- "mass sum rule" for fragmentation functions - new and observable!



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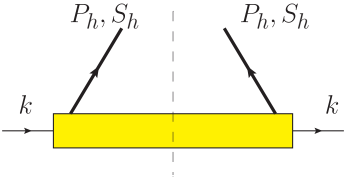
Quark 1h-FFs

$$\Delta_{ij}(k, P_h, S_h) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathsf{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi)\psi_i(\xi)\, a^\dagger a\, \overline{\psi}_j(0) W_2(0,\infty)|\Omega\rangle$$



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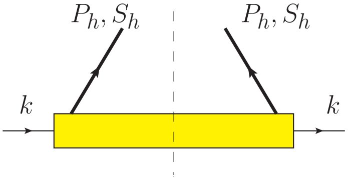

quark pol.

hadron pol.

	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

8 (TMD) fragmentation functions at leading twist

Quark higher twist 1h-FFs

Twist 3 transverse momentum dependent FFs $\mathcal{D}_{\dots}^{a\to h}(z, P_{h\perp}^2)$
for a quark hadronizing into a spin $1/2$ hadron

quark pol.

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	U	L	T
U	$D^\perp$	$G^\perp$	E, H
L	$D_L^\perp$	$G_L^\perp$	H_L, E_L
T	D_T, $D_T^\perp$	G_T, $G_T^\perp$	H_T, $H_T^\perp$, E_T, $E_T^\perp$

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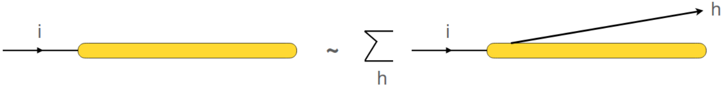
	T	D_T, $D_T^{\perp}$	G_T, $G_T^{\perp}$	H_T, $H_T^{\perp}$, E_T, $E_T^{\perp}$

Black and magenta: survive transverse momentum integration
Red and magenta: T-odd
Blue: T-even, w/o collinear counterpart

Inclusive jets and 1h-FFs

Accardi, Signori 1903.04458 - PLB
Accardi, Signori 2005.11310 - EPJC

"Fully" inclusive jet *correlator* (quark propagator)
$\equiv$ inclusive limit of 1h-fragmentation *correlator*



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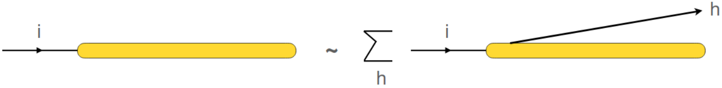

$$k^\mu\, \Xi^i(k) = \sum_{h,S_h} \int \frac{d^4 P_h}{(2\pi)^3}\, \delta(P_h^2 - M_h^2)\, P_h^\mu\, \Delta^{i\to h}(k, P_h, S_h)$$

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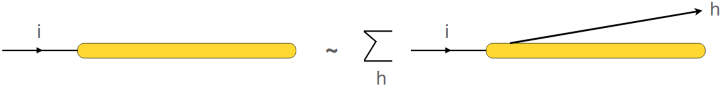

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Dirac projections:

momentum sum rules for FFs in terms of quark propagator

Quark propagator

Källen-Lehman representation in terms of spectral functions $\rho_{1,3}$:

$$\boxed{\Xi(k) \to S_F(k) = \int \frac{d\mu^2}{(2\pi)^4} \left\{ \not{k}\, \rho_3(\mu^2) + \sqrt{\mu^2}\, \rho_1(\mu^2)\, \mathbb{I} \right\} \frac{\theta(\mu^2)}{k^2 - \mu^2 + i\epsilon}}$$

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The non-perturbative structure of the jet is trivial at twist 2, but not at twist 3

Quark/jet mass

"Mass sum rule" for twist 3 E fragmentation function:

$$\boxed{\sum_h \int dz M_h E^h(z) = M_j}$$

quark/jet dynamical mass M_j as the "average" of produced hadron masses weighted by chiral-odd E FF

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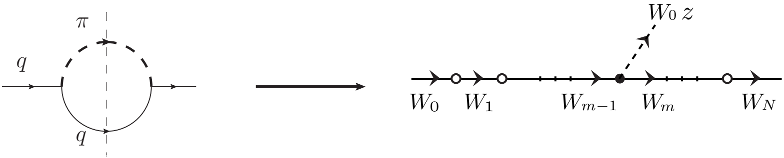
Full QCD: $M_j = m_q + m_q^{corr}$ (current and dynamical components), where

$$\boxed{\sum_h \int dz M_h \tilde{E}^h(z) = M_j - m_q = m_q^{corr}}$$

$\tilde{E}$ *and* m_q^{corr} *probe* *quark-gluon-quark* $\sim \langle 0|\overline{\psi} A \psi|0\rangle$ *dynamical correlations*

The NJL-jet model for FFs



- Within the NJL it is possible to calculate PDFs and FFs by calculating and regularizing the associated Feynman diagrams
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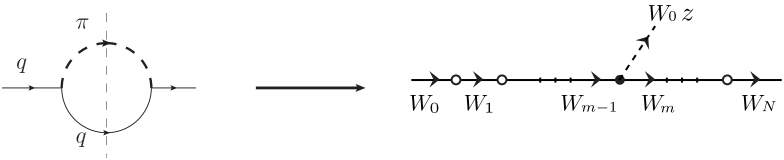

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$$D_q^\pi(z) = \sum_{m=1}^{N} \int_0^1 d\eta_1 \cdots \int_0^1 d\eta_N \, 6^N \sum_{Q_N} d_q^{Q_1}(\eta_1) \cdots d_{Q_{m-1}}^{\pi}(z) \cdots d_{Q_{N-1}}^{Q_N}(\eta_N)$$

The physical FF D_q^π can be calculated from the *elementary* d_q^π solving two integral Volterra equations

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estimate of M_j in NJL model

evolution: large-N_c and LO in α_s

(A. Belitsky - hep-ph/9703432)

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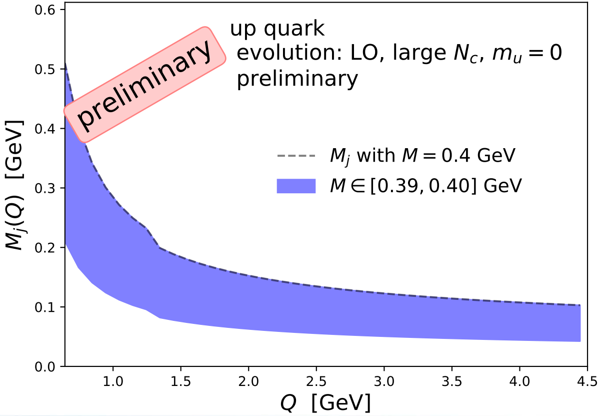

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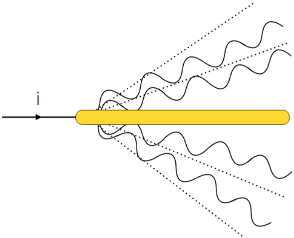

Inclusive jet function $J_i(s)$:
sensitive to the jet virtuality s

(within a defined cone)

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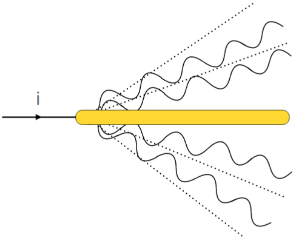

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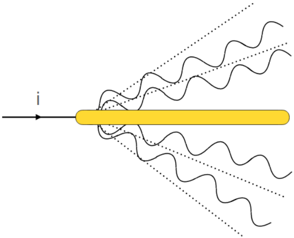

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- non-perturbative radiation (low s, dashed lines)

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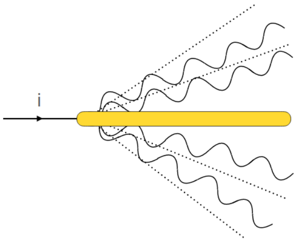

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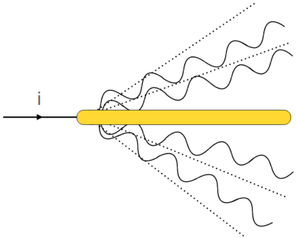

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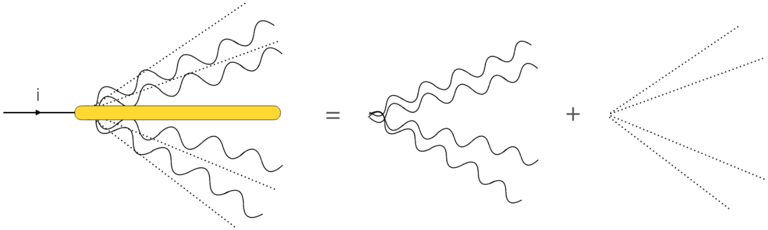
- non-perturbative radiation (low s, dashed lines)
- non-perturbative quark propagation (yellow blob) ← relevance?

Twist two jets

Procura, Stewart 0911.4980 - PRD
Jain, Procura, Waalewijn 1101.4953 - JHEP



Twist two jets

Procura, Stewart 0911.4980 - PRD
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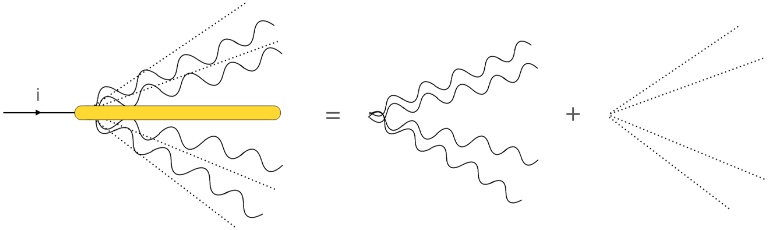

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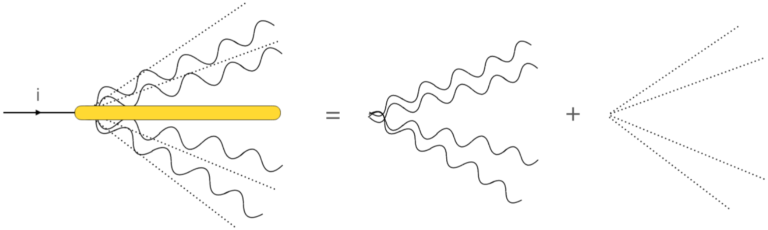

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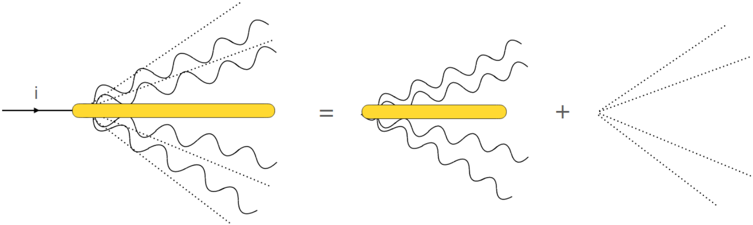
At twist 2 the jet function $J_i(s)$ "decouples" from the 1h-FF $D_1(z)$
and the non-perturbative structure gets simplified

Twist three jets



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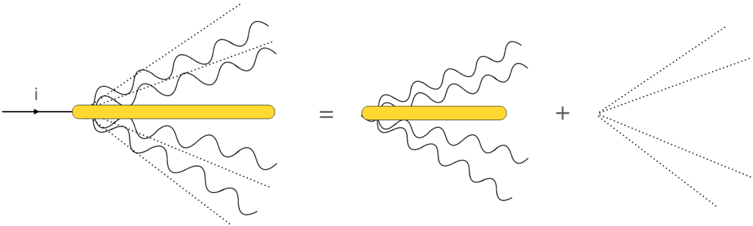

$$\tilde{J}_i(s) \sim M_j \otimes \tilde{J} + \mathcal{O}(\Lambda_{qcd}^2\, s^{-1})$$

(because of the “mass sum rule” for E)

More complex non-perturbative structure:
normalization of the associated quark spectral function (ρ_1 in this case)

Just a speculation?

Just a speculation? **NO**

Accardi, Signori 1903.04458 - PLB
Accardi, Signori 2005.11310 - EPJC

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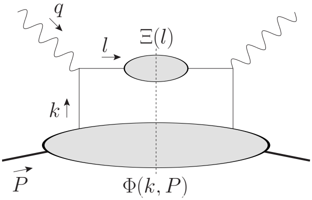

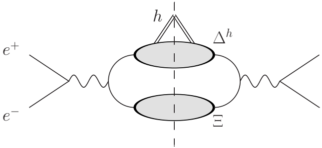

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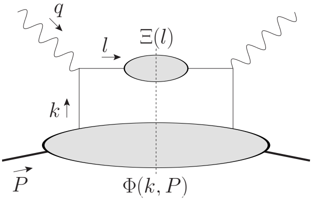

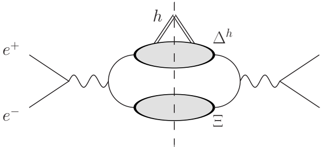

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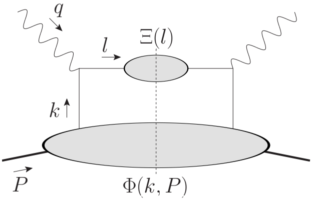

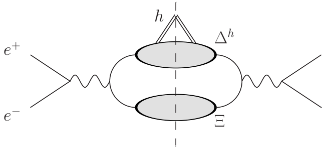

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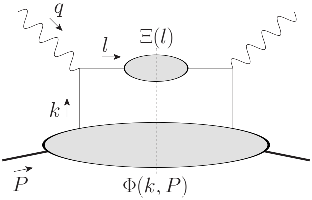
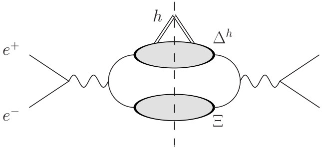
- calculable: quark E FFs $[M_j = \sum_h \int dz \, M_h \, E^h(z)]$ $\leftarrow$ this work!





Outline

Quark hadronization, propagation, mass generation

Inclusive jets

Semi-inclusive processes

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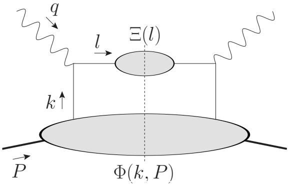

dynamical mass coupled to the transversity PDF

A. Accardi, A. Bacchetta - 1706.02000 - PLB

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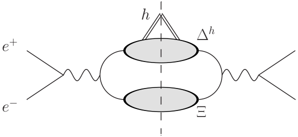

(Accardi, Signori et al. - in progress)

$$\frac{d\sigma^L(e^+e^- \to h^\uparrow X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2}\,\lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2}\lambda_h G_{1L}(z) \right.$$

$$\left. + D(y)\, |\boldsymbol{S}_T|\, \cos(\phi_S) \frac{2M_h}{Q} \left(\frac{G_T(z)}{z} + \frac{m_q^{corr}}{M_h} H_1(z) \right) \right\}$$

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$$H\, p^{\uparrow} \to h\, j\, X \;\; : \;\; f_1^H(x_1) \otimes h_1^{p^{\uparrow}}(x_2) \otimes D_1^h(z) \otimes m_q^{corr}$$

(fixed-target configuration: AMBER, AFTER, Compass, etc.)

Conclusions

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- this mass is gauge-invariant, and the dynamical component can be measured at twist three in scattering experiments

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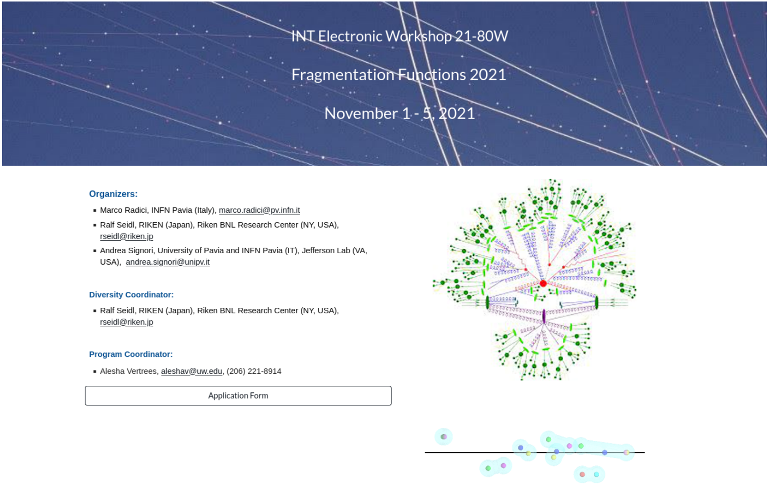

INT Electronic Workshop 21-80W

Fragmentation Functions 2021

November 1 - 5, 2021

Organizers:

- Marco Radici, INFN Pavia (Italy), marco.radici@pv.infn.it
- Ralf Seidl, RIKEN (Japan), Riken BNL Research Center (NY, USA), rseidl@riken.jp
- Andrea Signori, University of Pavia and INFN Pavia (IT), Jefferson Lab (VA, USA), andrea.signori@unipv.it

Diversity Coordinator:

- Ralf Seidl, RIKEN (Japan), Riken BNL Research Center (NY, USA), rseidl@riken.jp

Program Coordinator:

- Alesha Vertrees, aleshav@uw.edu, (206) 221-8914

https://sites.google.com/uw.edu/int/programs/21-80w

Backup

Useful references/1:

A selection of useful references related to inclusive jets and dynamical mass effects:

- Fully unintegrated parton correlation functions and factorization in lowest order hard scattering

  J.C. Collins, T.C. Rogers, A.M. Stasto - 0708.2833

- Collinear factorization for deep inelastic scattering structure functions at large Bjorken x_B

  A. Accardi, J.W. Qiu - 0805.1496

- Quark fragmentation as a probe of dynamical mass generation

  A. Accardi, A. Signori - 1903.04458

- On the connection between quark propagation and hadronization

  A. Accardi, A. Signori - 2005.11310

- Accessing the nucleon transverse structure in deep-inelastic scattering

  A. Accardi, A. Bacchetta - 1706.02000

Useful references/2:

A selection of useful references dealing with fragmentation functions, inclusive jets in pQCD, e^+e^- annihilation:

- Parton fragmentation functions (review)

  A. Metz, A. Vossen - 1607.02521

- Quark fragmentation within an identified jet

  M. Procura, I. Stewart - 0911.4980

- Parton fragmentation within an identified jet at NNLL

  A. Jain, M. Procura, W. Waalewijn - 1101.4953

- Asymmetries in polarized hadron production in e^+e^- annihilation up to order $1/Q$

  D. Boer, R. Jakob, P.J. Mulders - hep-ph/9702281

- Angular dependences in inclusive two-hadron production at `Belle`

  D. Boer - 0804.2408

The NJL model of QCD

The Nambu–Jona-Lasinio (NJL) model of QCD is a chiral effective theory which is useful to help understand non-perturbative phenomena in low energy QCD. In particular:

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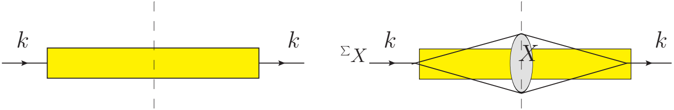
The NJL model has been used to describe:

- hadrons as bound states of quarks
- nuclear matter and nuclei in terms of quarks (medium modifications)
- phases of strongly interacting matter at high densities (e.g. neutron stars, etc.)

(Klevansky - Rev.Mod.Phys. 64 (1992) 649-708)

The cut quark propagator

$$\Xi_{ij}(k;v) = \mathsf{Disc}\int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathsf{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi;v)\psi_i(\xi)\overline{\psi}_j(0)W_2(0,\infty;v)|\Omega\rangle$$



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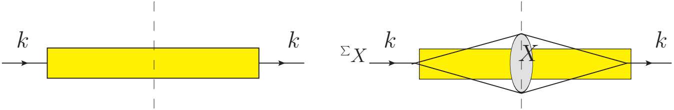

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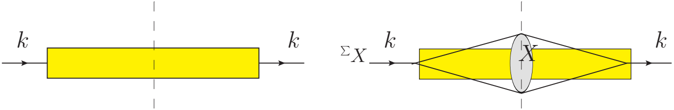

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- insights into dynamical generation of mass and momentum and chiral symmetry breaking

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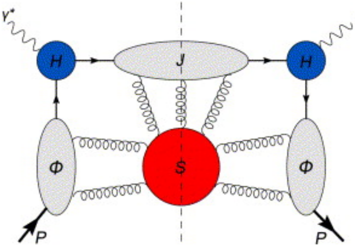


See Sterman NPB 281 ('87) 310, Chen et al. NPB 763 ('07) 183, Accardi et al. - 0805.1496, Collins et al. - 0708.2833 (and refs. therein)

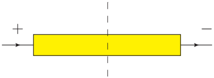
(figure from Chen et al.)

- Ξ emerges in the factorization theorem for DIS at *large x*, where a new semi-hard scale appears
- Ξ captures the physics at $Q^2(1-x) \sim Q\Lambda_{QCD}$, which becomes increasingly non-perturbative at low energy and large x
- the end-point factorization should be extend to different processes (e.g. e^+e^-)
- here we study the properties of Ξ and Δ regardless of processes

The quark/jet mass

$$\boxed{M_j(k^-) \sim \int dk^+ \mathrm{Tr}_D \left[\Xi \, \mathbb{I}\right]}$$

$\sim$



Mass associated with the scalar term (chiral-odd) of the cut quark propagator:

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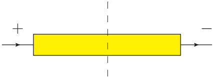

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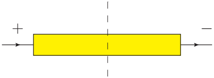

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This mass term:

- gauge-invariant
- renormalization scale dependent
- calculable via the spectral functions of the cut quark propagator
- accessible via momentum sum rules for twist-3 FFs