Synchrotron Radiation improvements in MAD-X for FCC-ee simulation studies

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Abstract

MAD-X is being used for FCCee modelling and optimization. We present a review of the implementation of Synchrotron Radiation (SR) effects in MAD-X in the TWISS, TRACK and EMIT calculations modules. We present few examples in which radiation calculation shows issues by comparing results from different modules and analytical calculation, and show how to address them in the MAD-X code. The main objective is to improve and update MAD-X in order to model the Interaction Region (IR) of the FCC-ee.

Objectives

FCCee goals	Code Results
Find field quality specifications for the FCCee IR magnets with focus on improving software tools	Improve physics of thin multipoles, including solenoids, in TWISS, TRACK, EMIT
Contribute to vertical equilibrium emittance studies	Improve physics of machine imperfection (misalignments and field errors) in TWISS, TRACK, EMIT

- Validate the multipole and solenoid analytical physics in MADX.
- Obtain a realistic electromagnetic field map.
- Solenoid modeling : Expand and slice in multipoles to check field residuals in order to validate the physical model.
- Tracking for dynamic aperture, momentum acceptance and equilibrium emittance.
- Scan of individual multipoles to identify most critical components.
- Find acceptable range from each multipoles.

SR Theory

The power loss by an accelerated relativistic particle is given by the Larmor formula :

$$-\frac{dE}{dt} = \frac{q^2}{6\pi\varepsilon_0 m^2 c^3}\frac{dP^\mu}{d\tau}\frac{dP_\mu}{d\tau}$$

In presence of a magnetic field orthogonal to the direction of motion we have :

$$\frac{dP^\mu}{d\tau}\frac{dP_\mu}{d\tau} = (q\gamma\beta c B_\perp)^2 = (\frac{qPB_\perp}{m})^2 = \left(\frac{PP_0 b}{m}\right)^2$$

Where $b = \frac{q}{B_0}B_\perp$ is the scaled magnet B-field.

We then have : $-\frac{dE}{dt} = \frac{q^2 P_0^2 P^2}{6\pi\varepsilon_0 m^4 c^3} b^2$

In a multipole, assuming kick-drift splitting, one can lump the effect of the radiation loss at the entrance and the exit by assuming that the power loss is constant over the path length inside the multipole.

And with $\frac{dt}{ds} = \frac{1+hx}{\beta c}$

We have: $\Delta E = \int_0^L \frac{dE}{dt}\frac{dt}{ds}ds = \frac{q^2P^2P_0^2}{6\pi\varepsilon_0 m^4 c^3}\left(\frac{\Delta p}{\Delta s}\right)^2 \frac{1+hx}{\beta c}\Delta s$

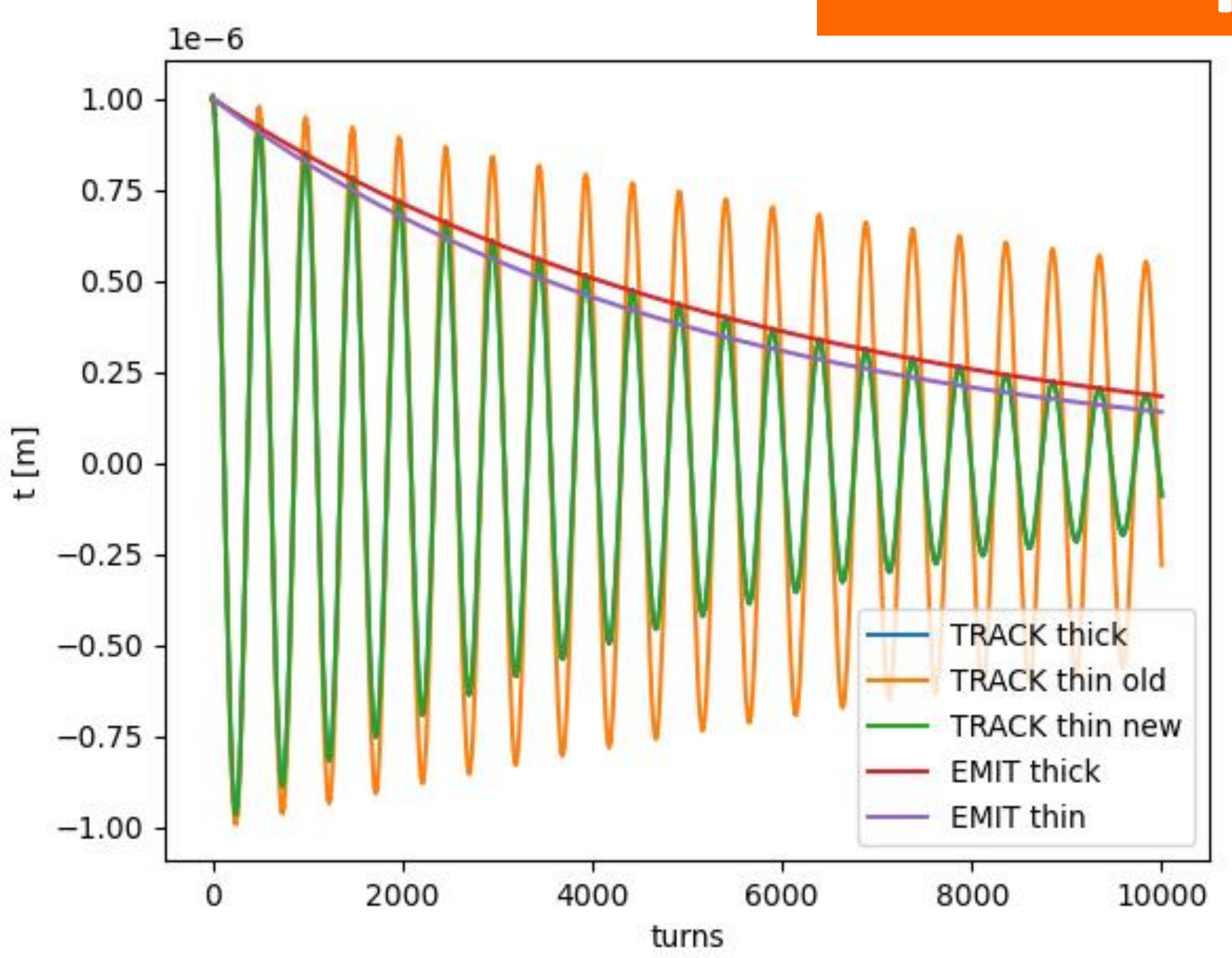
The integrated transverse momentum change the multipole field:

$\langle b \rangle = \frac{\Delta p}{\Delta s}$ and $\Delta p = \sqrt{\Delta p_x^2 + \Delta p_y^2}$

Using :$E = \beta P c, q = q_e e, mc^2 = m_e v e$

We get: $-\frac{\Delta E}{E} = \frac{e}{6\pi\varepsilon_0}\frac{q_e^2\beta_0^3\gamma_0^3}{m_e v}\left(\frac{\Delta p}{\Delta s}\right)^2(1+\delta)(1+hx)\Delta s$

SR comparison with simplified lattice



Review of the damping decay with the TRACK and EMIT module for thick and thin lattices with an electron of 2.4 GeV with a lattice from the Elettra synchrotron.

Methods of simulation	Damping constant [1/s]
EMIT Thick	196.3
EMIT Thin	227.4
TRACK Thick	196.3
TRACK Thin (before fix)	70.12
TRACK Thin (after fix)	198.4
Twiss thin using $D = \frac{\oint k_0 D_x(k_1 + k_0^2)\,ds}{\oint k_0^2\,ds}$ $\alpha_t = \frac{W_0}{2E_0T_0}(2+D)$	198.2

```
1261 1268          do jtrk = 1,ktrack
     1269  +          x = track(1,jtrk)
     1270  +          pt = track(6,jtrk)
1262 1271             curv = sqrt((dipr + dxt(jtrk))**2 + (dipi + dyt(jtrk))**2) / elrad
1263 1272             if (quantum) then
1264 1273                call trphot(elrad,curv,rfac,pt)
1265 1274             else
1266       -             rfac = const * curv**2 * elrad
     1275  +             delta_plus_1 = sqrt(pt*pt + two*pt*beti + one);
     1276  +             rfac = const * curv**2 * delta_plus_1 * elrad * (one + dipr/elrad * x)
1267 1277             endif
```

Addition of the (1+δ)(1+hx) term in the code to correct the damping time

The results shown above were conclusive for the EMIT and TRACK modules for thick lattices, whereas the module EMIT for thin lattices gave a result more than 10 % higher than expected. For the TRACK module in thin configuration, a missing term in (1+δ)(1+hx) was found and then added, correcting the wrong damping constant obtained.
Similar tests will be conducted with the FCCee lattices in order to monitor the errors already present in the code and to apply the needed corrections before going further in the optimization of SR in TWISS, EMIT and TRACK modules and the implementation of the solenoids physics for the IR in MAD-X.

Perspectives

- **Code :**
  - Fixing the issues linked to the multipoles in the EMIT module.
  - Test random fluctuations with multipoles and equilibrium emittances.
  - Addressing tapering test with multipoles.
- **Implementation :**
  - Solenoid tests
  - Collect field map and implement sequence with slicing.

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