High-precision QCD at FCC-ee

FCC week 2022

Paris, 31st May 2022

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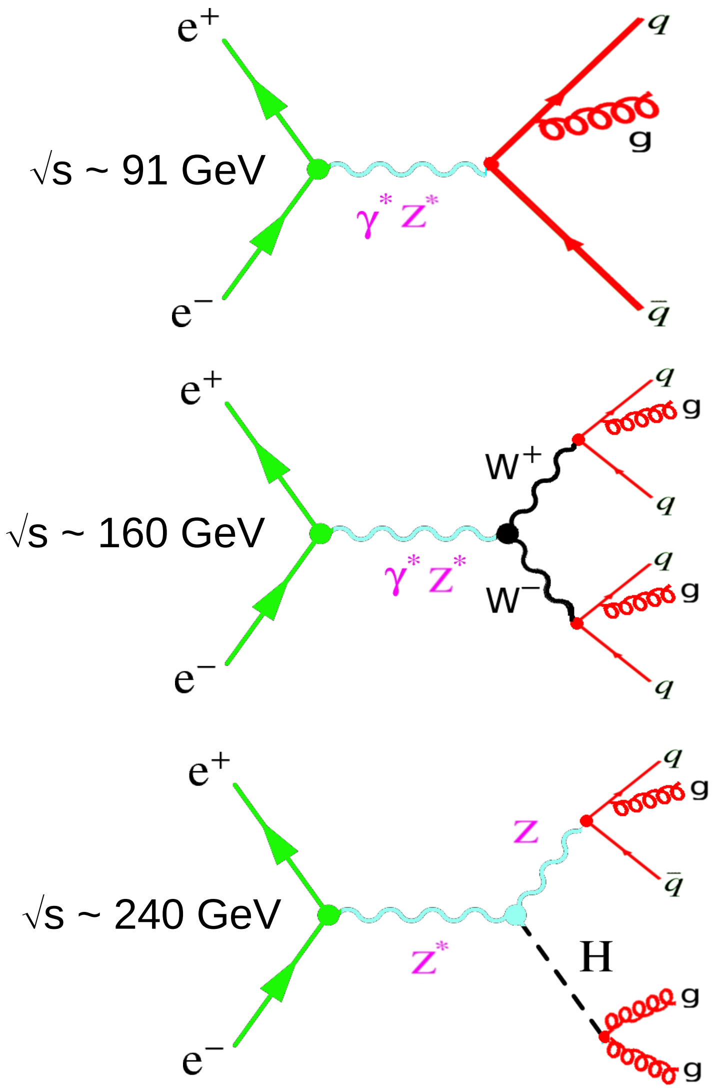
Pier Monni (CERN)

QCD = Key ingredient of future ee,pp colliders

- Though QCD is *not per se* the main driving force behind future colliders, QCD is crucial for many pp, ee measurements (signals & backgrounds):
  - High-precision α_s: Affects all x-sections & decays (esp. Higgs, top, EWPOs).
  - N^nLO corrs., N^nLL resummations: Affects all pQCD x-sections & decays.
  - High-precision PDFs: Affects all precision W,Z,H (mid-x) measurements & all searches (high-x) in pp collisions.
  - Heavy-Quark/Quark/Gluon separation (jet substructure, boosted topologies..): Needed for all precision SM measurements & BSM searches with final jets.
  - Semihard QCD (low-x gluon saturation, multiple hard parton interactions,…): Leading x-sections at FCC-pp (Note: Q_0~10 GeV at 100 TeV).
  - Non-perturbative QCD: Affects final-states with jets: Colour reconnection, $e^+e^-\to$WW, ttbar$\to$4j,6j... (m_W,m_{top} extractions). Parton hadronization,...

Precision QCD in e^+e^- collisions

- e^+e^- collisions provide an extremely clean environment with fully-controlled initial-state to probe very precisely q,g dynamics:



Advantages compared to p-p collisions:

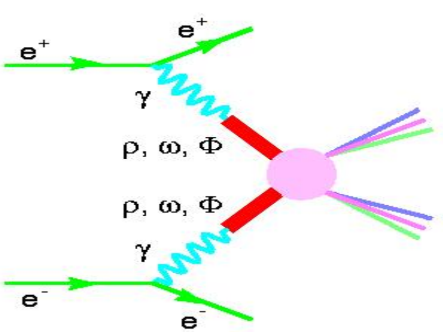
– QED initial-state with known kinematics

– Controlled QCD radiation (only in final-state)

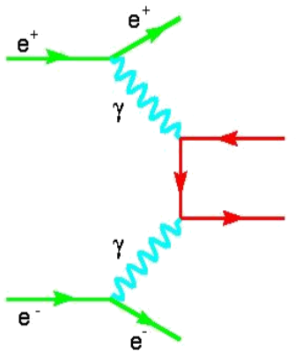
– Well-defined heavy-Q, quark, gluon jets

– Smaller non-pQCD uncertainties:
no PDFs, no QCD "underlying event",…
Direct clean parton fragmentation & hadroniz.

- Plus QCD physics in γγ (EPA) collisions:



(soft, VDM)



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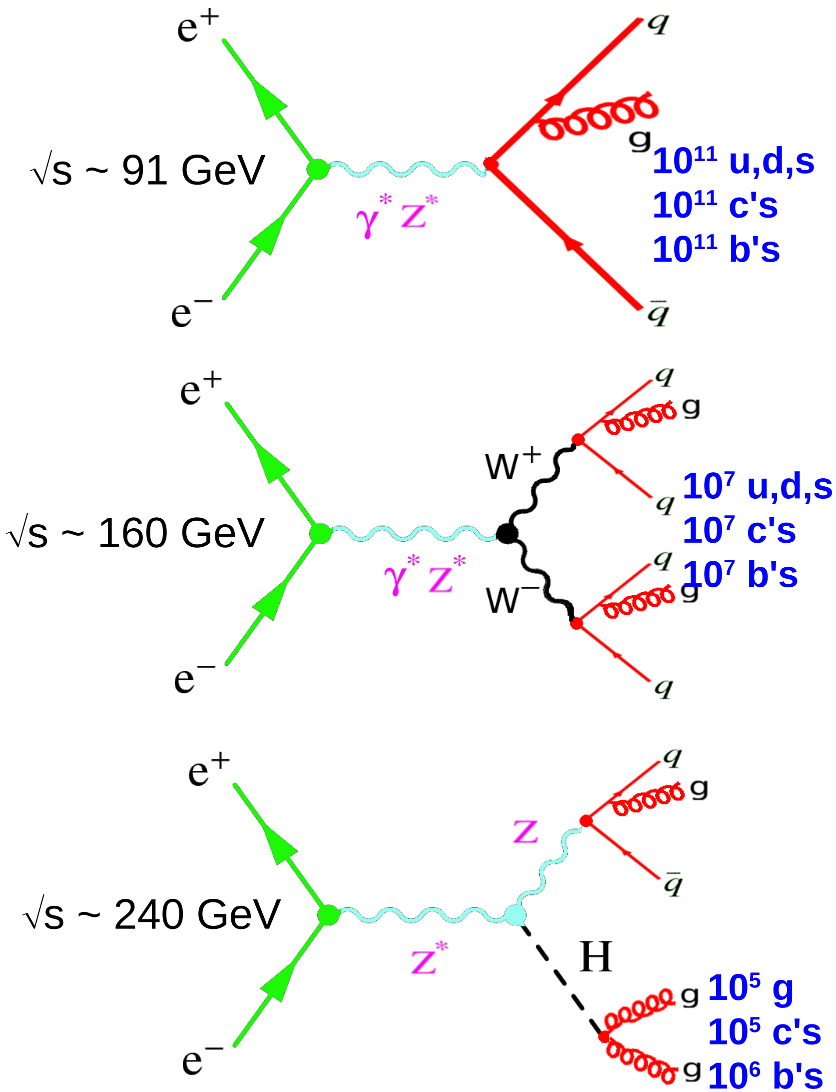

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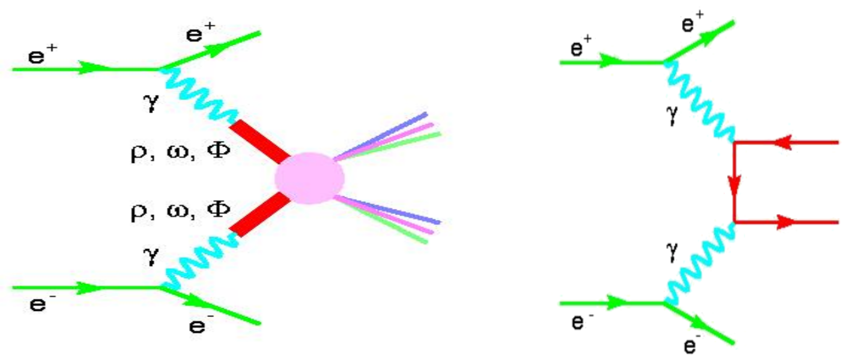


(soft, VDM) (direct)

QCD physics at FCC-ee

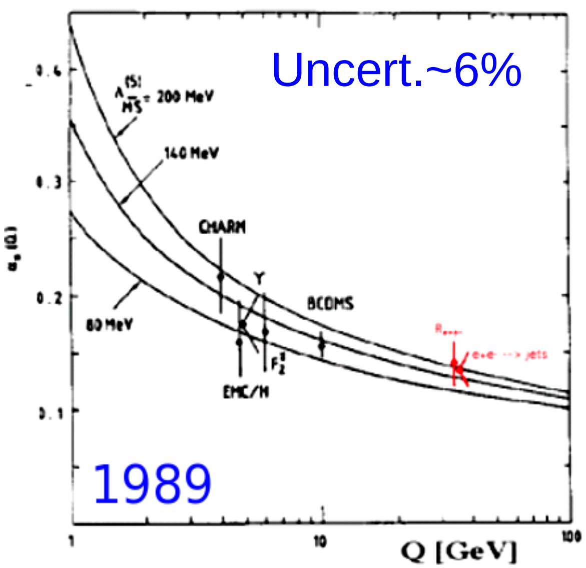
(1) QCD coupling

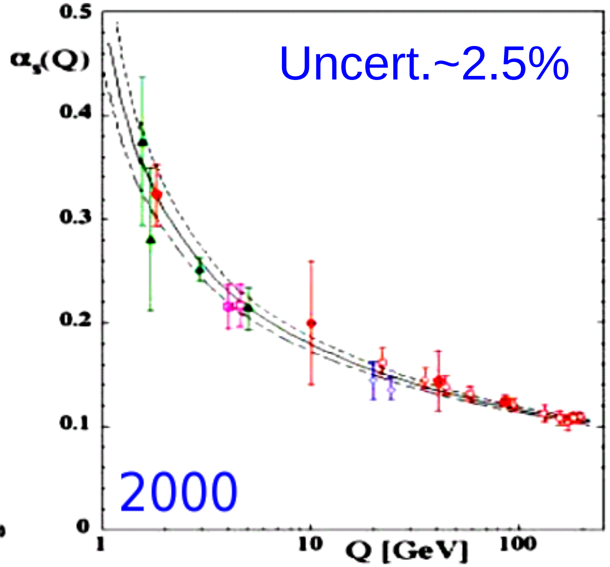
(2) Jet substructure & flavour tagging

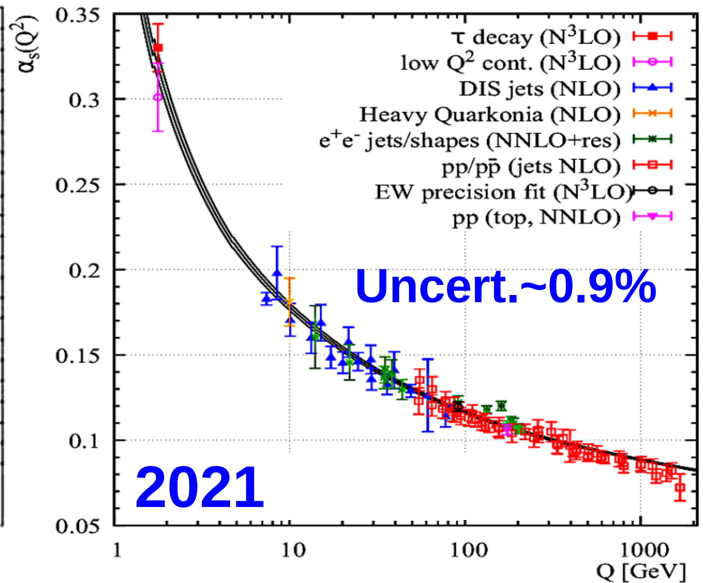
(3) Parton shower & Non-perturbative QCD

QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q \to 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$; $\Lambda \sim 0.2$ GeV







$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO) → $\alpha_s(M_Z) = 0.1184 \pm 0.0031$ (NNLO) → $\alpha_s(M_Z^2) = 0.1179 \pm 0.0010$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989

S. B., J. Phys. G 26, 2000

- Least precisely known of all interaction couplings !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

Importance of the QCD coupling α_s

- Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	σ (pb)	$\delta\alpha_s(\%)$	PDF $+\alpha_s(\%)$	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	± 3.0	$\pm$ 8.9	-9.3 +5.9

Channel	M_H[GeV]	$\delta\alpha_s(\%)$	Δm_b	Δm_c
$H \to c\bar{c}$	126	± 7.1	$\pm$ 0.1%	$\pm$ 2.3 %
$H \to gg$	126	± 4.1	$\pm$ 0.1%	$\pm$ 0 %

Msbar mass error budget (from threshold scan)

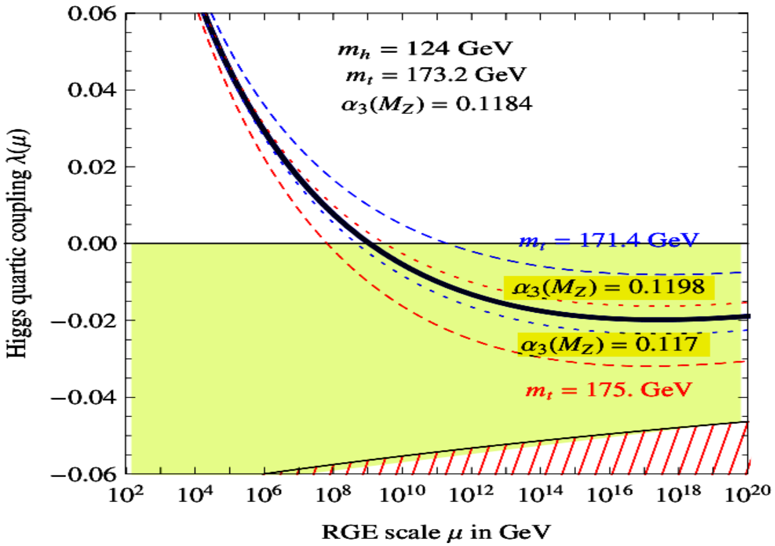
$(\delta M_t^{SD-low})^{exp}$	$(\delta M_t^{SD-low})^{theo}$	$(\delta\overline{m}_t(\overline{m}_t))^{conversion}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

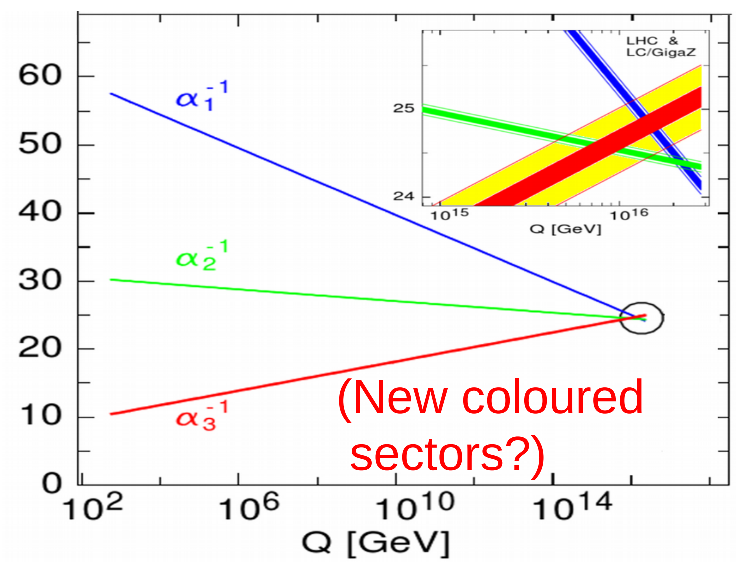
$\Rightarrow$ improvement in α_s crucial $\delta\alpha_s(M_Z) = 0.001$

Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3	$\delta\alpha_s$

Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

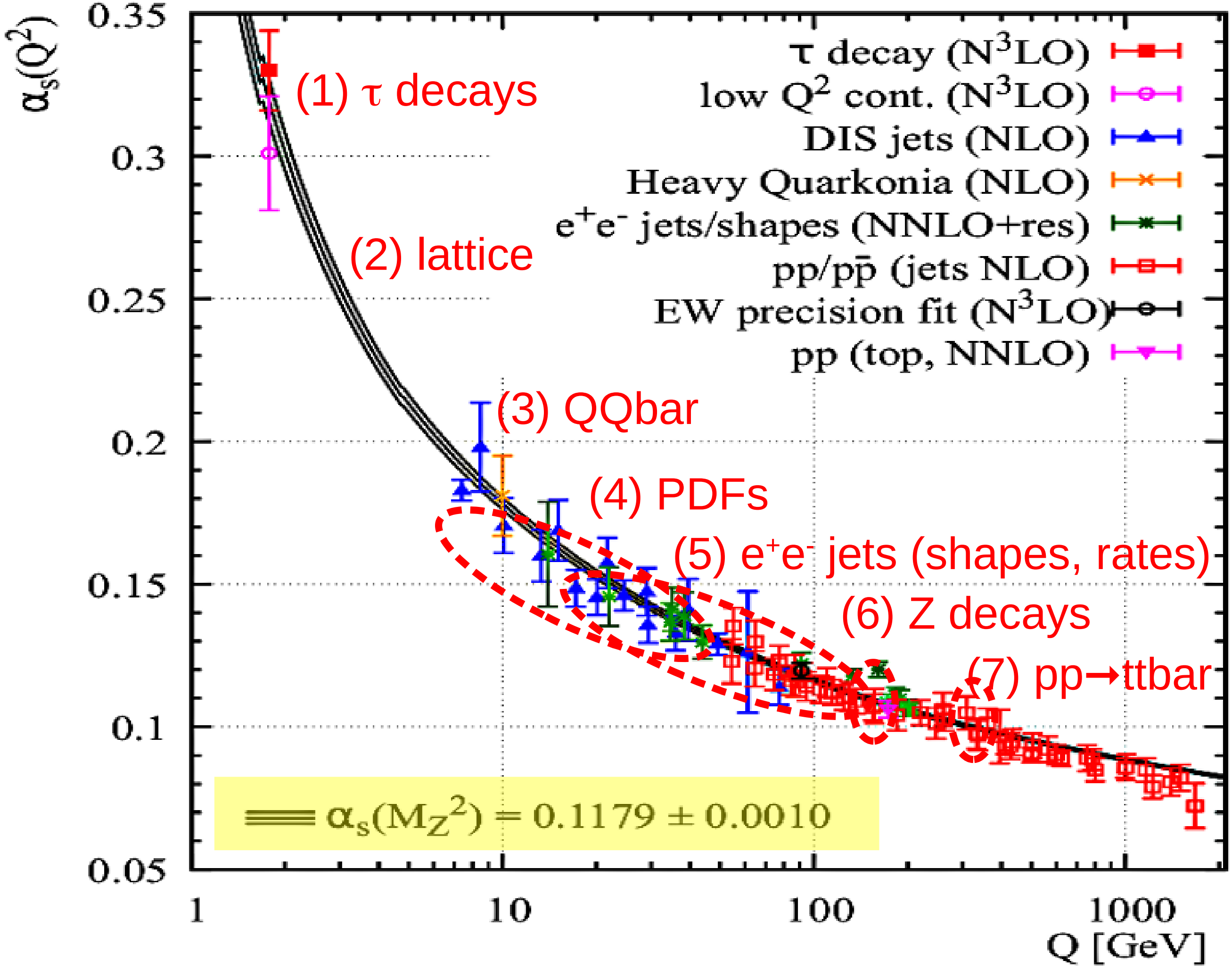
- Impacts physics approaching Planck scale: EW vacuum stability, GUT





World α_s determination (PDG today)

- Determined today by comparing 7 experimental observables to pQCD NNLO,N³LO predictions, plus global average at the Z pole scale:



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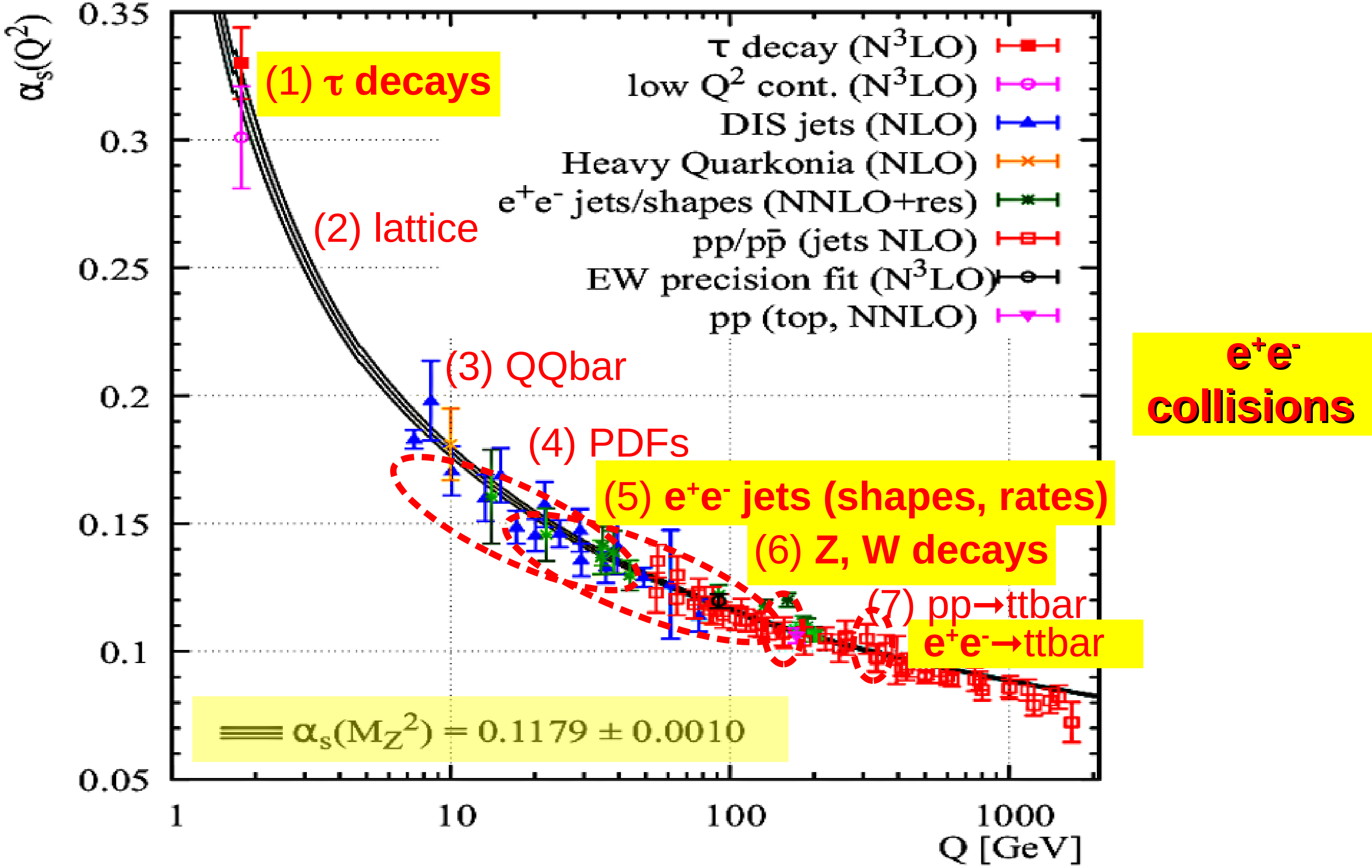
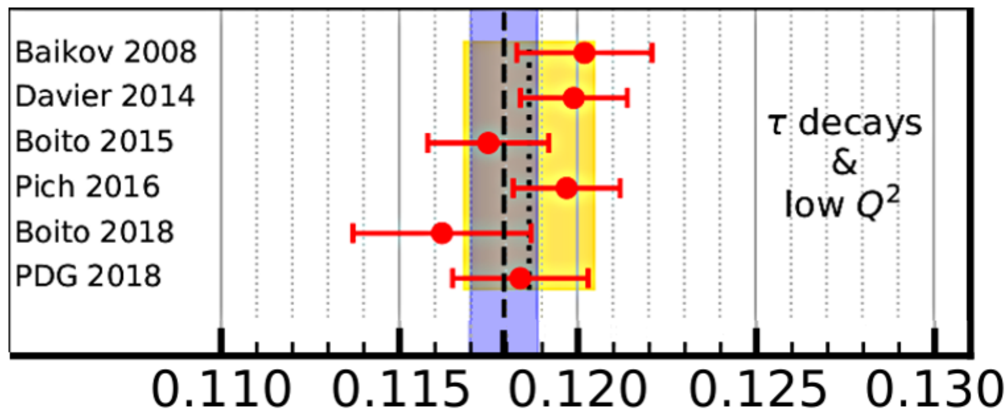


α_s from hadronic τ-lepton decays

- Computed at N³LO: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np})$
- Experimentally: $R_{\tau,\rm exp} = 3.4697 \pm 0.0080$ ($\pm 0.23\%$)
- Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections $(\Lambda/m_\tau)^2 \sim 2\%$, yield different results.



$\alpha_s(M_Z^2) = 0.1187 \pm 0.0018$ **(±1.5%)** $\alpha_s(M_Z^2)$

- <u>FCC-ee</u>:
  – Huge stats: $\mathcal{O}(10^{11})$ from $Z\to\tau\tau$ at FCC-ee(90)
  – Much better τ spectral functions than LEP (stats & syst.).
  – New extraction of Γ width from ultra-precise τ lifetime
  – TH: Better understanding of FOPT vs CIPT diffs.
  Calculation of N⁴LO corrections.

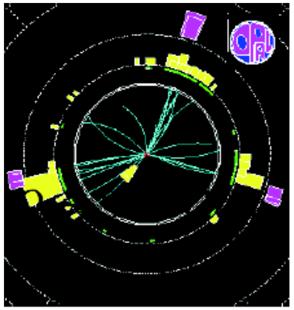
$\delta\alpha_s/\alpha_s$ << 1%

α_s from e⁺e⁻ event shapes & jet rates (today)

- Computed at N^{2,3}LO+N^{(2,3)}LL accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections

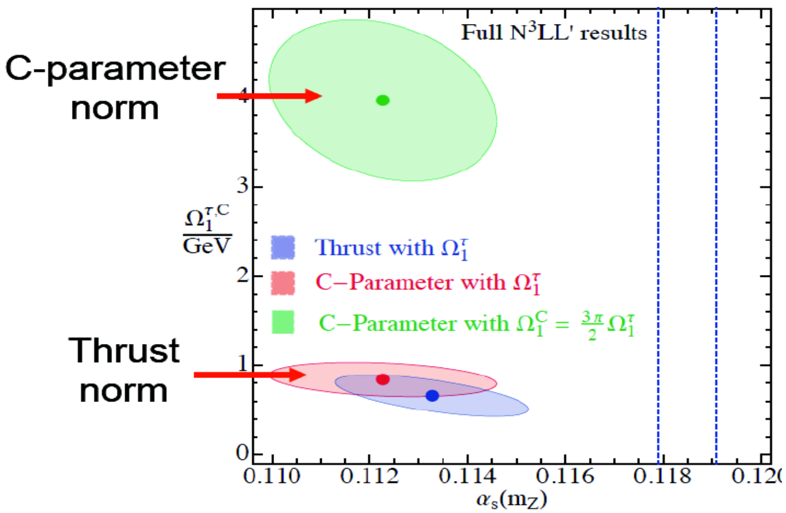
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

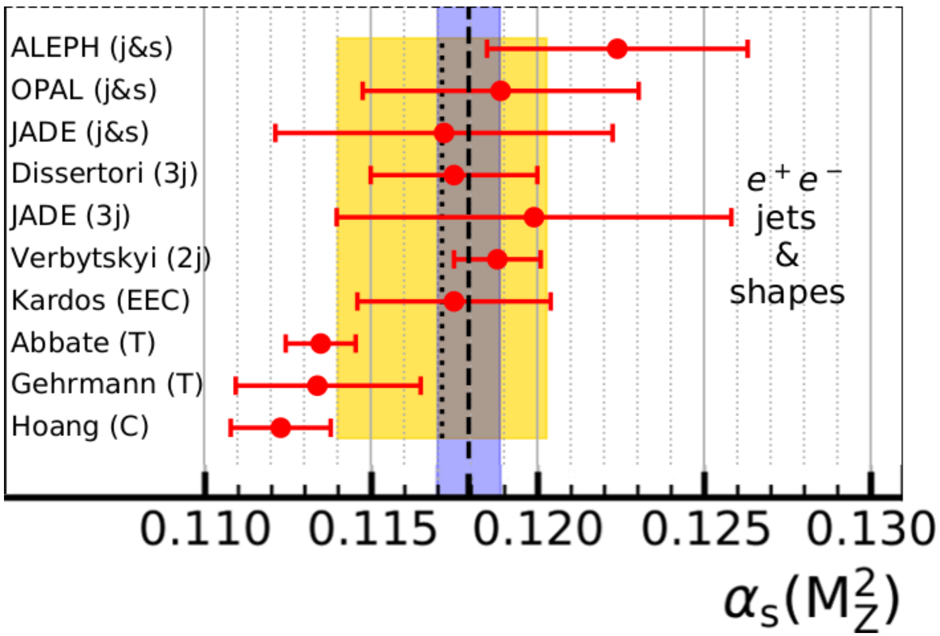
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$



OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:





- Wide span of TH extractions...

$$\alpha_s(M_Z^2) = 0.1171 \pm 0.0031$$ **(±2.6%)**

α_s from e^+e^- event shapes & jet rates (FCC-ee)

- Computed at N^{2,3}LO+N$^{(2,3)}$LL accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
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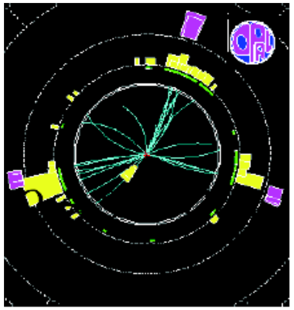

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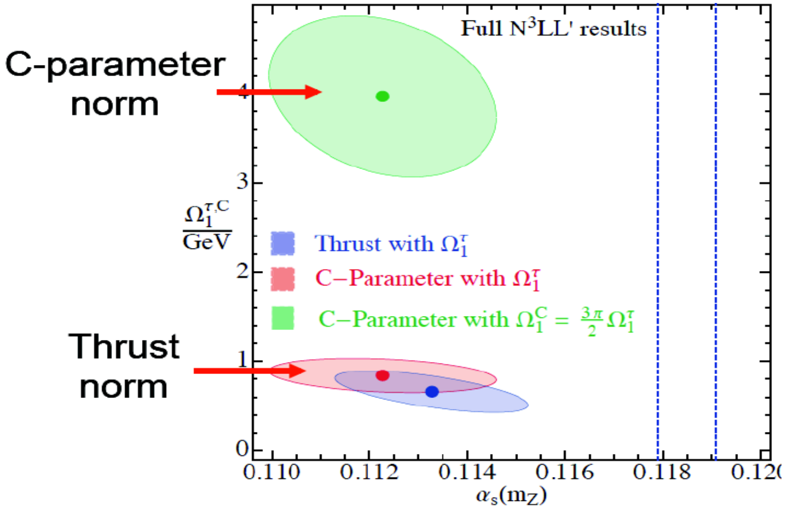
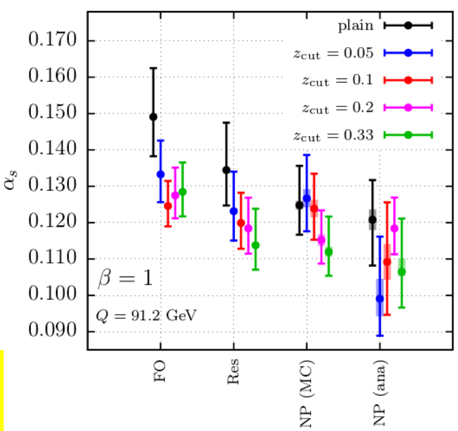
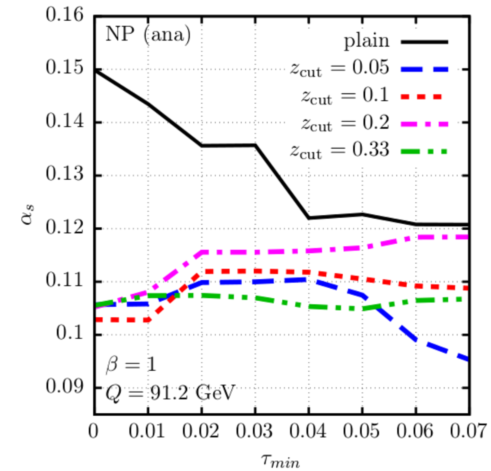


- Modern jet substructure techniques:
  "Soft drop" to reduce non-pQCD corrections for groomed shapes:





- FCC-ee: **$\delta\alpha_s/\alpha_s$ << 1%**

– Huge data sets at lower-√s (ISR) for shapes, higher-√s for jet rates
– TH: Improved (N^{2,3}LL) resummation for rates, hadronization for shapes

α_s from hadronic Z decays (FCC-ee)

- α_s extracted at N^{3}LO from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

- FCC-ee:

– Huge Z pole stats. ($\times 10^5$ LEP)
– Exquisite systematic/parametric precision (stat. uncert. negligible):

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$
$$\Delta \Gamma_Z^{tot} = 0.1 \text{ MeV}, \quad \Gamma_Z^{tot} = 2495.2 \pm 0.1 \text{ MeV}$$
$$\Delta \sigma_Z^{had} = 4.0 \text{ pb}, \quad \sigma_Z^{had} = 41\,494 \pm 4 \text{ pb}$$
$$\Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV}$$
$$\Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta\alpha_{had}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

– TH uncert. to be reduced by $\times 4$ computing missing α_s^5, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

- 20! times better precision than today: $\delta\alpha_s/\alpha_s \sim \pm 0.2\%$ (tot), $\pm 0.1\%$ (exp)
Strong (B)SM consistency test.

- The W and Z hadronic widths :

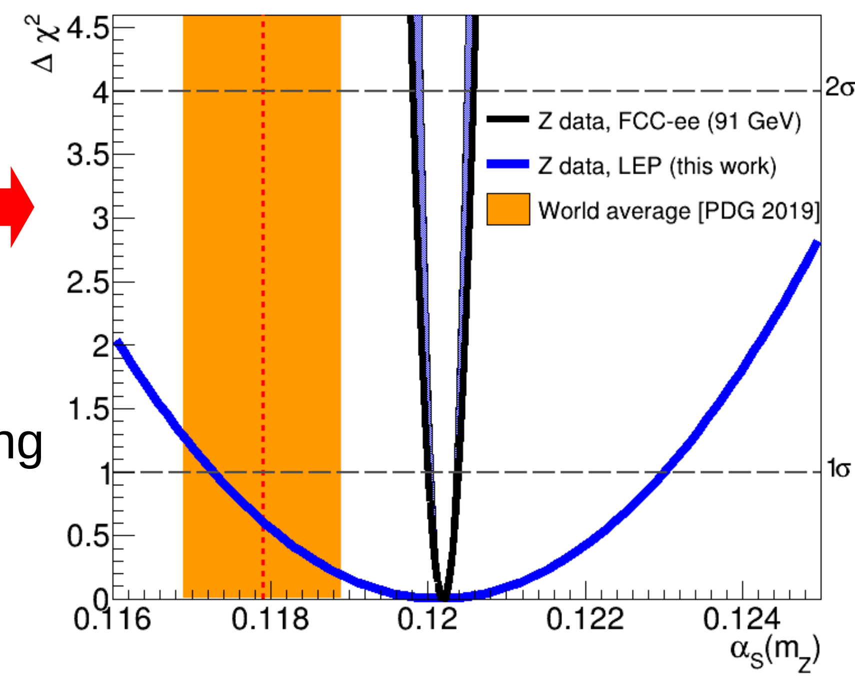
$$\Gamma_{W,Z}^{had}(Q) = \Gamma_{W,Z}^{Born}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{EW} + \delta_{mix} + \delta_{np}\right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{had}(Q)}{\Gamma_{W,Z}^{lep}(Q)} = R_{W,Z}^{EW}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{mix} + \delta_{np}\right)$$

- In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma_Z^{had} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{had}}{(\Gamma_Z^{tot})^2}$$



$\alpha_s(m_z)$ = 0.12030 ± 0.00014 (±0.1%)

α_s from hadronic W decays (FCC-ee)

- Q extracted from N^{3}LO fit of combined Γ_W, R_W W boson pseudo-observ.:

- The W and Z hadronic widths :

$$\Gamma^{\rm had}_{\rm W,Z}(Q) = \Gamma^{\rm Born}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm EW} + \delta_{\rm mix} + \delta_{\rm np}\right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$\mathrm{R}_{\rm W,Z}(Q) = \frac{\Gamma^{\rm had}_{\rm W,Z}(Q)}{\Gamma^{\rm lep}_{\rm W,Z}(Q)} = \mathrm{R}^{\rm EW}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm mix} + \delta_{\rm np}\right)$$

- FCC-ee:

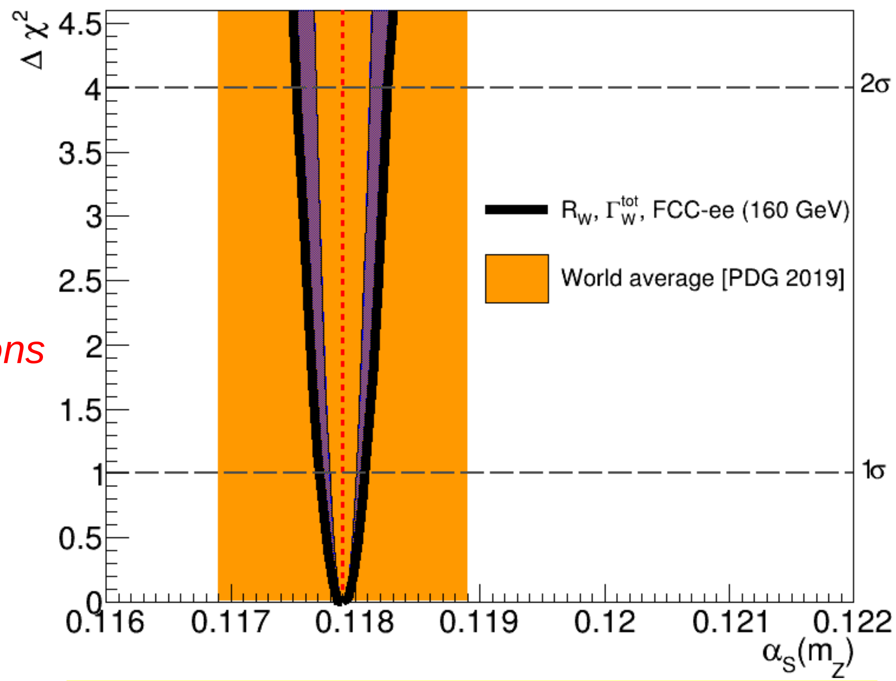
– Huge W pole stats. ($\times 10^4$ LEP-2).

– Exquisite syst./parametric precision:

$$\Gamma_{\rm W}^{\rm tot} = 2088.0 \pm 1.2 \text{ MeV}$$
$$\mathrm{R_W} = 2.08000 \pm 0.00008$$
$$m_{\rm W} = 80.3800 \pm 0.0005 \text{ GeV}$$
$$|V_{cs}| = 0.97359 \pm 0.00010$$
← O(10^{12}) D mesons

– TH uncertainty to be reduced by $\times 10$ by computing missing α_s^5, α^2, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



- 150! times better precision than today:

$\alpha_s(m_z) = 0.101 \pm 0.027$ (±27%) → **$\alpha_s(m_z) = 0.11790 \pm 0.00023$ (±0.2%)**

QCD physics at FCC-ee

(1) QCD coupling

(2) Jet substructure & flavour tagging

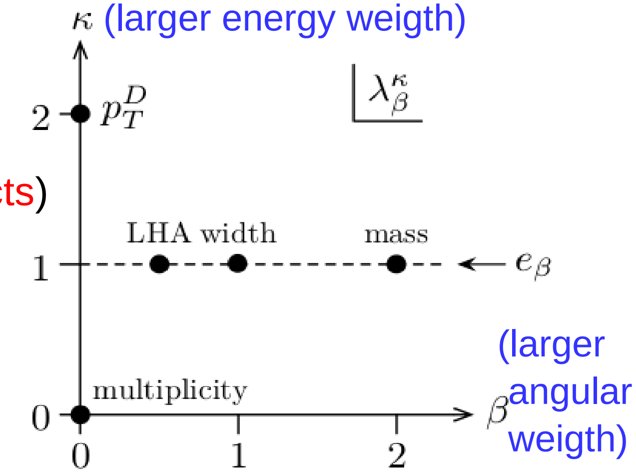
(3) Parton shower & Non-perturbative QCD

Jet substructure & flavour tagging (today)

- State-of-the-art jet substr. studies based on Lund Plane & angularities:

$$\lambda^\kappa_\beta = \sum_{i\in \text{jet}} z_i^\kappa \theta_i^\beta ,$$

(normalized $E^n \times \theta^n$ products)

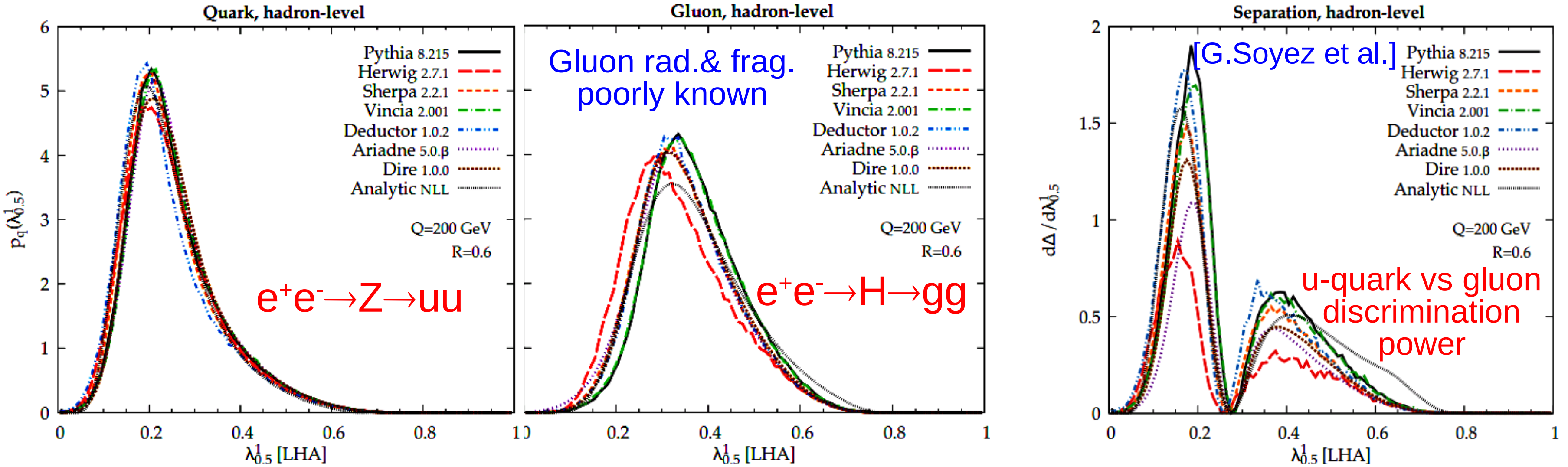


[Larkoski,Salam,Thaler,13]
[Larkoski,Thaler,Waalewijn,14]

- "Sudakov"-safe variables of jet constituents: multiplicity, LHA, width/broadening, mass/thrust, C-parameter,...

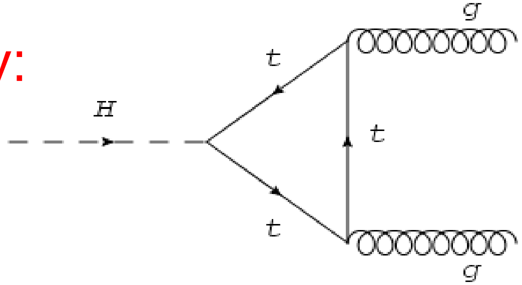
- k=1: IRC-safe computable (N^nLO+N^nLL) via SCET (but uncertainties from non-pQCD effects)

- MC parton showers differ on gluon (less so quark) radiation patterns:

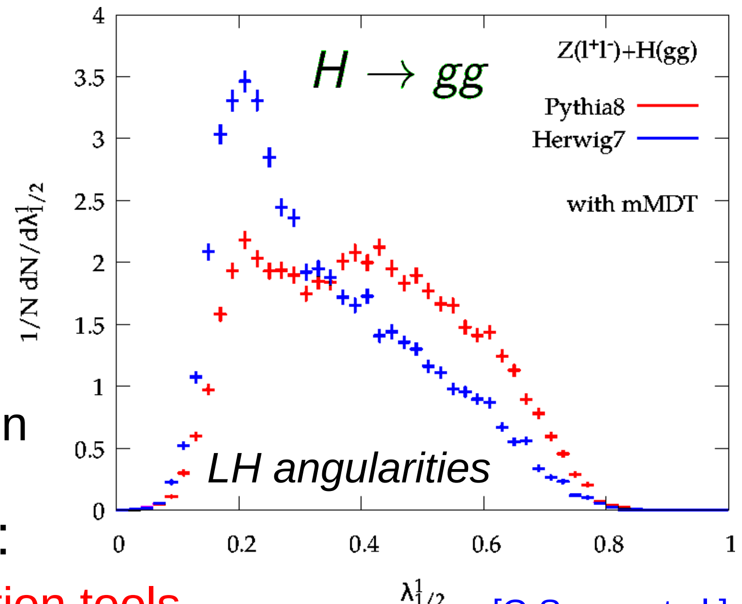


High-precision g & q jet studies (FCC-ee)

- Exploit FCC-ee H(gg) as a ”pure gluon” factory: H→gg (BR~8% accurately known) provides 100.000 extra-clean digluon events.



- Multiple handles to study gluon radiation & g-jet properties:
  - Gluon vs. quark via H→gg vs. Z→qq (Profit from excellent g,b separation)
  - Gluon vs. quark via Z→bbg vs. Z→qq(g) (g in one hemisphere recoiling against 2-b-jets in the other).
  - Vary E_{jet} range via ISR: $e^+e^-\to Z^*,\gamma^*\to jj(\gamma)$
  - Vary jet radius: small-R down to calo resolution



[G.Soyez et al.]

- Multiple high-precision analyses at hand:
  – Higgs/BSM/flavour: Improve q/g/Q discrimination tools
  – pQCD: Check N^nLO antenna functions. High-precision QCD coupling.
  – non-pQCD: Gluon fragmentation: Octet neutralization? (zero-charge gluon jet with rap gaps). Colour reconnection? Glueballs ? Leading η's,baryons?

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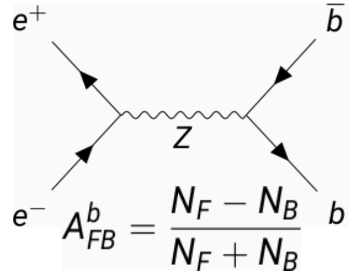
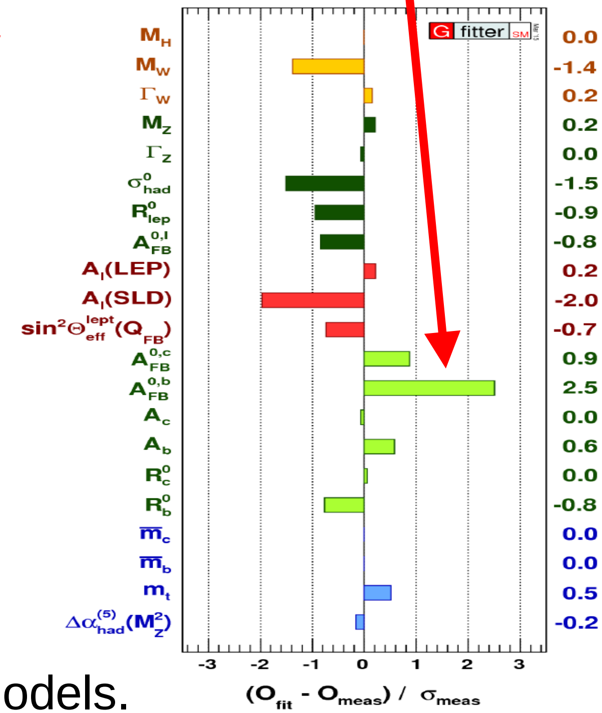
QCD uncertainties on EWK observables



- With $\times 10^5$ more Z's than LEP, some EWPO uncerts at FCC-ee will be dominated by QCD syst. Example: $e^+e^- \to bb$ forward–backward asymmetry
  - – 8 measurements at LEP: 4 lepton-, 4 jet-charge-based
  - – Largest EWPO discrepancy today wrt. the SM: 2.8σ



- Exp. uncertainties at LEP: ~1.6%
  - – Statistical: $\pm$1.5% (~0.05% at FCC-ee)
  - – Systematics: $\pm$0.6% (QCD-related: $\pm$0.4%)

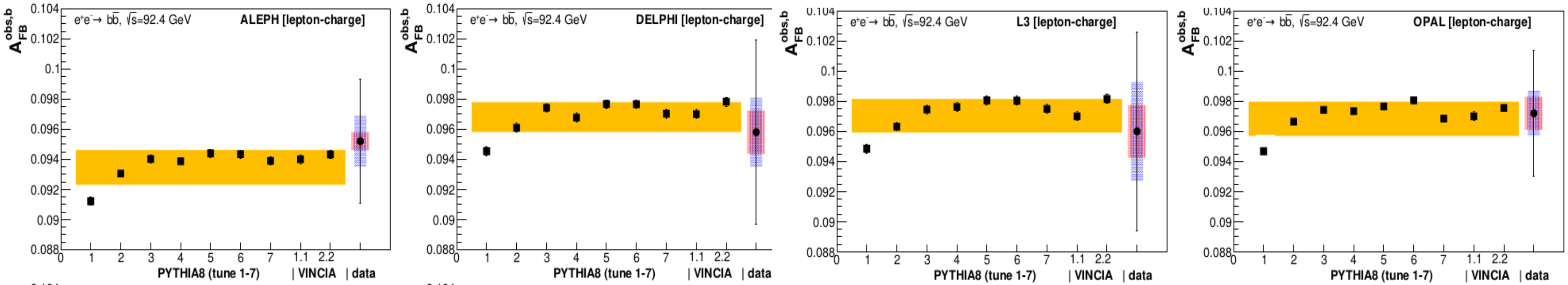
- QCD effects on $A^{0,b}_{FB}$ (depending strongly on exp. selection procedure):
  - – Gluon splitting (TH control: α_s^2 corrections)
  - – Smearing of b-jet/thrust axis
  - – b and c radiation & fragmentation. B and D decay models.

  [Uncertanties estimated by Abbaneo et al., EPJC 4 (1998)]

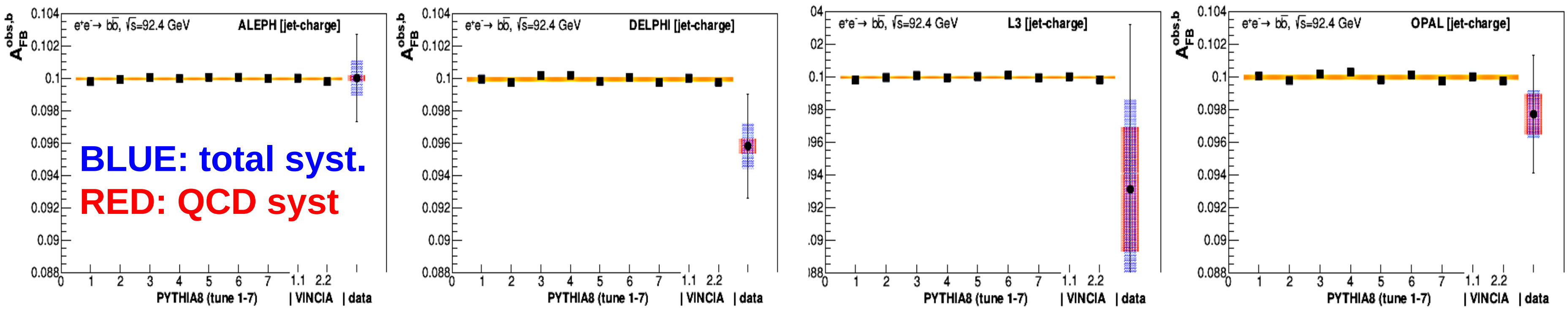
- Impact of QCD effects on $A^{0,b}_{FB}$ revisited by implementing original analyses in up-to-date retuned parton-shower+hadronization MCs

Reduced QCD uncertainties on A_{FB} at Z pole

- QCD uncertainties recomputed from PYTHIA8.226 (7 tunes) & VINCIA2.2
- $e^+e^- \rightarrow b\bar{b}$ forward–backward asymmetry for lepton-based analyses:



- $e^+e^- \rightarrow b\bar{b}$ forward–backward asymmetry for jet-charge-based analyses:



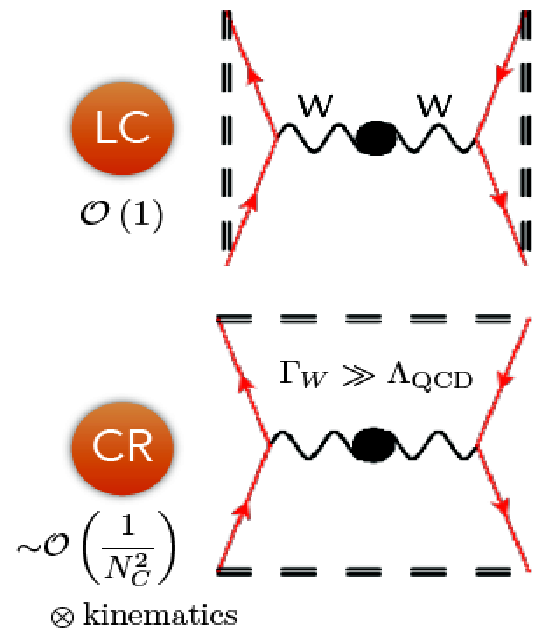
- 2020 vs. 1998 parton shower+hadronization uncertainties halved:
  – Lepton-based analyses: ~1.4% → ~0.7%
  – Jet-charge-based analyses: ~0.7% → ~0.3%

[DdE & Yan, 2011.00530 [hep-ph]]

- FCC-ee data needed to much further improve PS & non-pQCD syst. uncert.

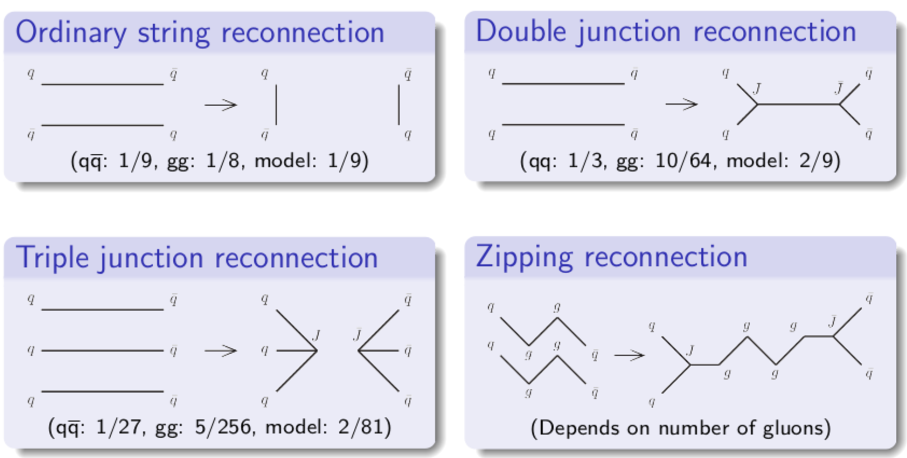
Non-pQCD: Colour reconnection

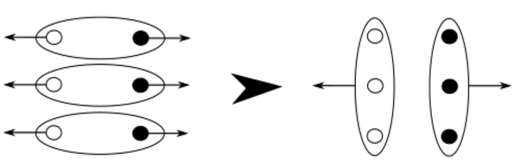
- Colour reconnection among partons is source of uncertainty in m_W, m_{top}, aGC extractions in multijet final-states. Especially in pp (MPI cross-talk).
- CR "string drag" effect impacts all FCC-ee multi-jet final-states: $e^+e^- \to WW(4j)$, H(2j,4j), ttbar,…
  - – Shifted masses & angular correlations (CP studies).
  - – Combined LEP $e^+e^- \to WW(4j)$ data best described with 49% CR, 2.2σ away from no-CR.



- Exploit huge W stats ($\times 10^4$ LEP) to measure m_W leptonically & hadronically and constrain CR:

"Recent" PYTHIA option: QCD-inspired CR (QCDCR) (1505.01681):

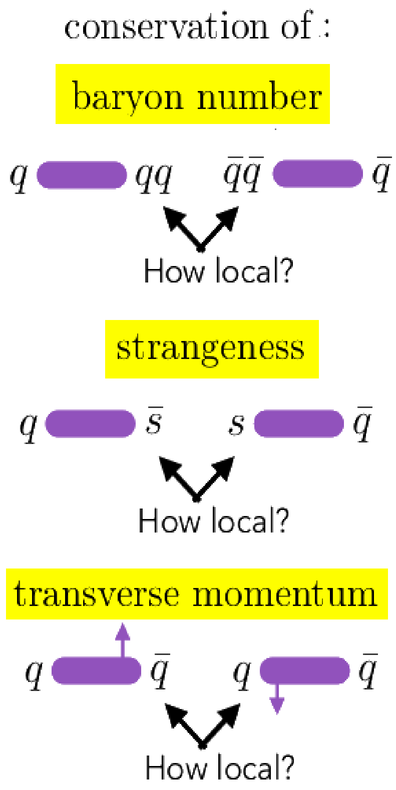


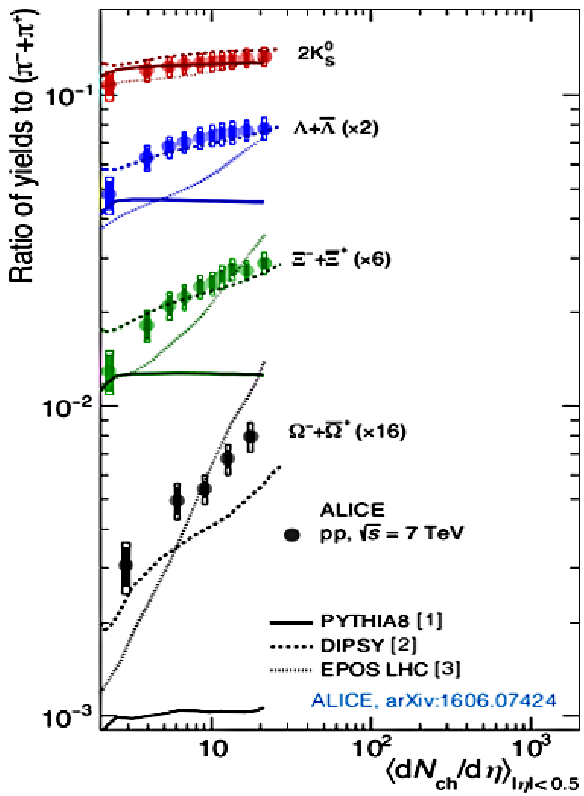


Triple-junction also in HERWIG cluster model. (1710.10906)

Non-pQCD: Detailed hadronization studies

- High-precision low-p_T PID hadrons in e^+e^- required for detailed studies:
  - Baryon & strangeness production. Colour string dynamics.
  - Final-state correlations (spin: Bose-Einstein, Fermi-Dirac; momenta; space)
  - Bound state formation: Onia, multi-quark states, glueballs, ...





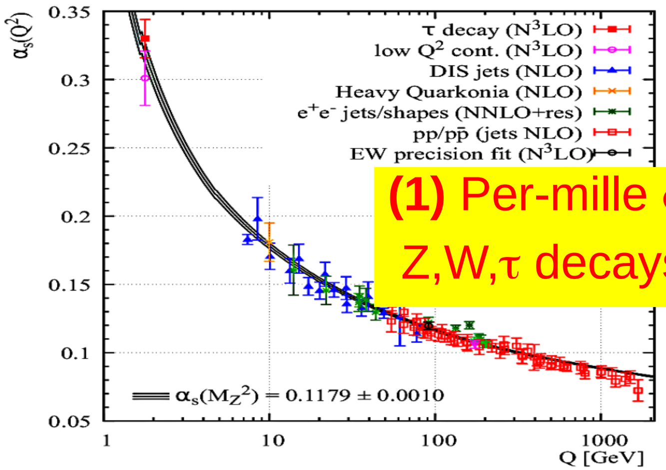
- ▶ Understand breakdown of universality of parton hadronization with system size observed at LHC.

- Baseline vacuum e^+e^- studies for high-density QCD in small & large systems.

Also impact e.g. ultra-high-energy cosmic-ray MCs (muon puzzle)

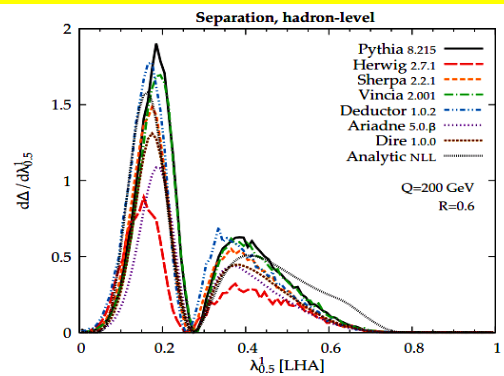
Summary: High-precision QCD at FCC-e⁺e⁻

- The precision needed to fully exploit all future ee/pp/ep/eA/AA SM & BSM programs requires exquisite control of pQCD & non-pQCD physics.
- Unique QCD precision studies accessible at FCC-ee:

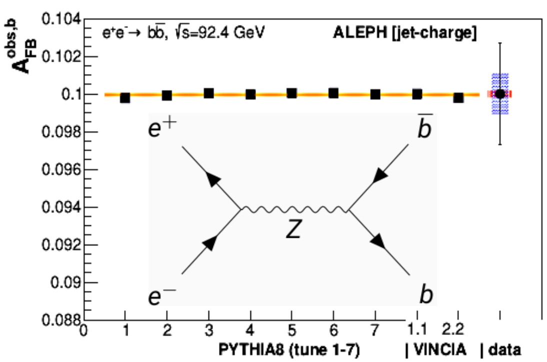


(1) Per-mille α_s via hadronic Z,W,τ decays, evt shapes...

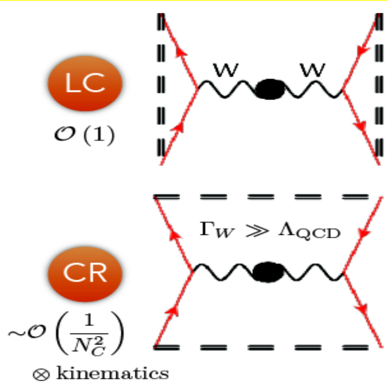
(2) NⁿLO+NⁿLL jet structure
Ultimate g/q/Q discrimination



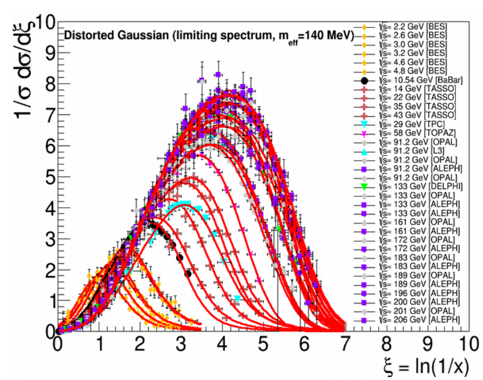
(3) Reduced PS+hadroniz. uncert. of EWPOs

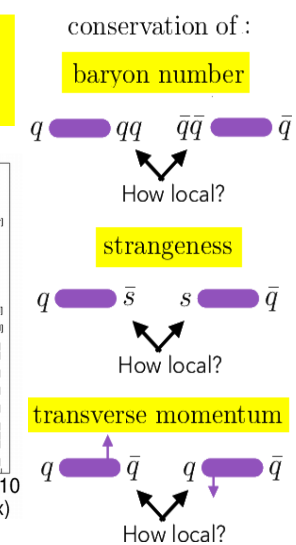


(4) <<1% control of colour reconnection



(5) High-precision hadronization:





Backup slides

Unparalleled Z, W, H, t data samples at FCC-ee

- Unparalleled Z, W, jets, τ,… data sets: Negligible stat. uncerts
- State-of-the-art detectors + exquisite E_{beam} control: Tiny syst. uncerts. (10^{-5})

Working point	Z, years 1-2	Z, later	WW	HZ	$t\bar{t}$		(s-channel H)
$\sqrt{s}$ (GeV)	88, 91, 94		157, 163	240	340-350	365	m_H
Lumi/IP (10^{34} cm^{-2}s^{-1})	115	230	28	8.5	0.95	1.55	(30)
Lumi/year (ab^{-1}, 2 IP)	24	48	6	1.7	0.2	0.34	(7)
Physics Goal (ab^{-1})	150		10	5	0.2	1.5	(20)
Run time (year)	2	2	2	3	1	4	(3)
Number of events	5×10^{12} Z		10^8 WW	10^6 HZ + 25k WW $\to$ H	10^6 $t\bar{t}$ +200k HZ +50k WW $\to$ H		(6000)

# of light-q jets/year:	**$\mathcal{O}(10^{12})$**	$\mathcal{O}(10^{7})$	$\mathcal{O}(10^{5})$	–	$\mathcal{O}(10^{8})$
# of gluon-jets/year:	$\mathcal{O}(10^{11})$	$\mathcal{O}(10^{6})$	**$\mathcal{O}(10^{4})$**	–	$\mathcal{O}(10^{6})$
# of heavy-Q jets/yr:	$\mathcal{O}(10^{12})$	$\mathcal{O}(10^{7})$	$\mathcal{O}(10^{5})$	**$\mathcal{O}(10^{6})$**	$\mathcal{O}(10^{8})$

α_s from hadronic Z, W decays

- Z & W observables theoretically known at N³LO accuracy:

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

• The W and Z hadronic widths :

$$\Gamma^{\rm had}_{\rm W,Z}(Q) = \Gamma^{\rm Born}_{\rm W,Z}\left(1+\sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm EW} + \delta_{\rm mix} + \delta_{\rm np}\right)$$

• The ratio of W, Z hadronic-to-leptonic widths :

$$\mathrm{R}_{\rm W,Z}(Q) = \frac{\Gamma^{\rm had}_{\rm W,Z}(Q)}{\Gamma^{\rm lep}_{\rm W,Z}(Q)} = \mathrm{R}^{\rm EW}_{\rm W,Z}\left(1+\sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm mix} + \delta_{\rm np}\right)$$

• In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma^{\rm had}_{\rm Z} = \frac{12\pi}{m_{\rm Z}} \cdot \frac{\Gamma^e_{\rm Z}\Gamma^{\rm had}_{\rm Z}}{(\Gamma^{\rm tot}_{\rm Z})^2}$$

TH uncertainties:
(α^2,α^3 included for Z):
±0.015–0.03% (Z)
±0.015–0.04% (W)

Param. uncerts.:
($m_{Z,W}$, α, $V_{cs,ud}$):
±0.01–0.03% (Z)
±1.1–1.7% (W)
±0.03% (W, CKM unit)

- Measured at LEP with ±0.1–0.3% (Z), ±0.9–2% (W) exp. uncertainties:

	theory			experiment		
	previous	new (this work)	change	previous [6]	new [20, 21]	change
$\Gamma^{\rm tot}_{\rm Z}$ (MeV)	$2494.2 \pm 0.8_{\rm th}$	$2495.2 \pm 0.6_{\rm par} \pm 0.4_{\rm th}$	+0.04%	2495.2 ± 2.3	2495.5 ± 2.3	+0.012%
$\mathrm{R_Z}$	$20.733 \pm 0.007_{\rm th}$	$20.750 \pm 0.006_{\rm par} \pm 0.006_{\rm th}$	+0.08%	20.767 ± 0.025	20.7666 ± 0.0247	−0.040%
$\sigma^{\rm had}_{\rm Z}$ (pb)	$41\,490 \pm 6_{\rm th}$	$41\,494 \pm 5_{\rm par} \pm 6_{\rm th}$	+0.01%	$41\,540 \pm 37$	$41\,480.2 \pm 32.5$	−0.144%

Recent update of LEP luminosity bias(*) change the Z values by few permil

W boson observables	GFITTER 2.2 (NNLO)	this work (N³LO) (exp. CKM)	this work (N³LO) (CKM unit.)	experiment
$\Gamma^{\rm had}_{\rm W}$ (MeV)	–	$1440.3 \pm 23.9_{\rm par} \pm 0.2_{\rm th}$	$1410.2 \pm 0.8_{\rm par} \pm 0.2_{\rm th}$	1405 ± 29
$\Gamma^{\rm tot}_{\rm W}$ (MeV)	$2091.8 \pm 1.0_{\rm par}$	$2117.9 \pm 23.9_{\rm par} \pm 0.7_{\rm th}$	$2087.9 \pm 1.0_{\rm par} \pm 0.7_{\rm th}$	2085 ± 42
$\mathrm{R_W}$	–	$2.1256 \pm 0.0353_{\rm par} \pm 0.0008_{\rm th}$	$2.0812 \pm 0.0007_{\rm par} \pm 0.0008_{\rm th}$	2.069 ± 0.019

(*) Voutsinas et al. arXiv:1908.01704, Janot et al. arXiv:1912.02067

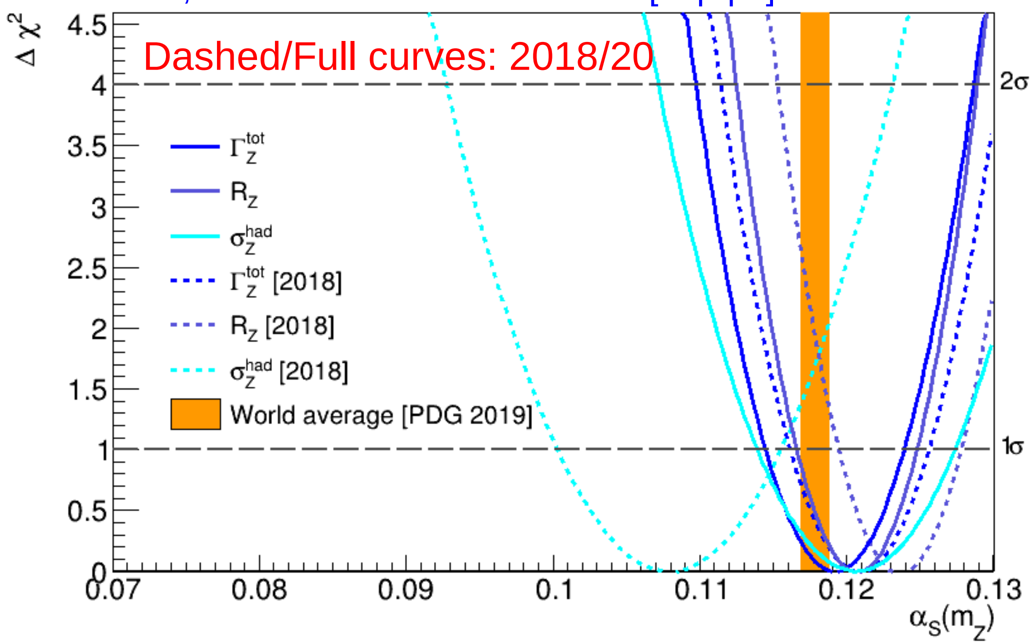
α_s from hadronic Z decays (today)

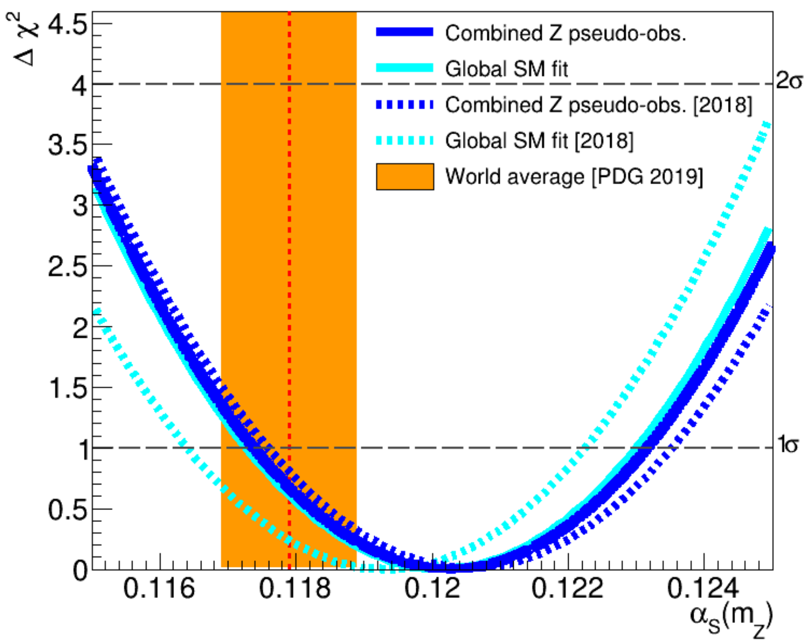
- QCD coupling extracted from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

Z boson observable	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_Z^{tot}	0.1192 ± 0.0047	± 0.0046	± 0.0005	± 0.0008
R_Z	0.1207 ± 0.0041	± 0.0041	± 0.0001	± 0.0009
σ_Z^{had}	0.1206 ± 0.0068	± 0.0067	± 0.0004	± 0.0012
All combined	0.1203 ± 0.0029	± 0.0029	± 0.0002	± 0.0008
Global SM fit	0.1202 ± 0.0028	± 0.0028	± 0.0002	± 0.0008

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]





- LEP lumi-bias updates lead to much better agreement among Γ_Z, R_Z, σ_0 extractions:
- Improved **$\alpha_s(m_z) = 0.1203 \pm 0.0028$ (±2.3%)**

PDG'19: $\alpha_s(m_z) = 0.1205 \pm 0.0030$ (±2.5%)

- EXP/TH updates lead to better agreement with full SM fit:
- **$\alpha_s(m_z) = 0.1202 \pm 0.0028$**

PDG'19: $\alpha_s(m_z) = 0.1194 \pm 0.0029$

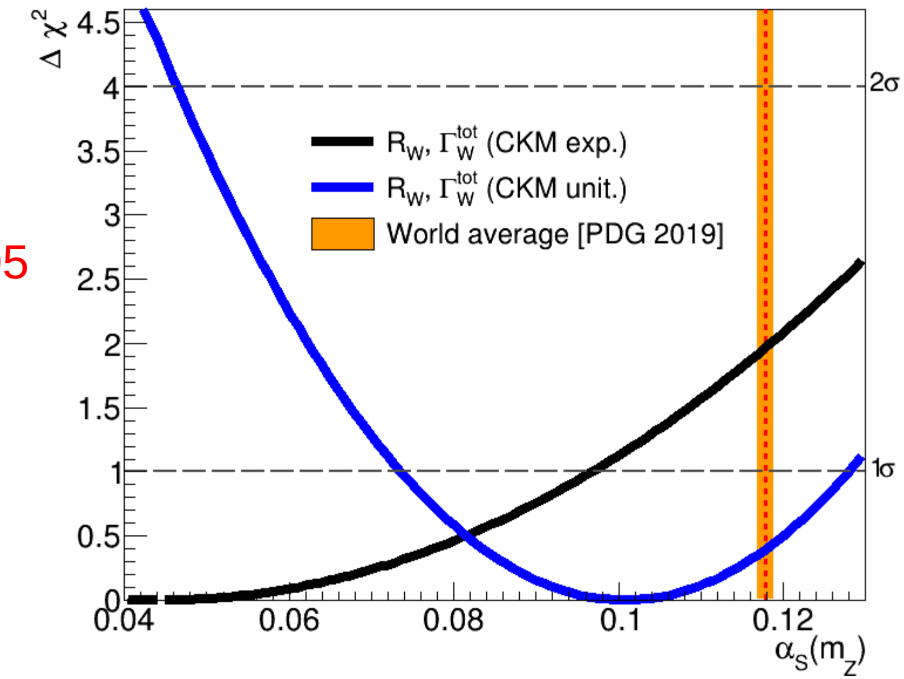
α_s from hadronic W decays (today)

- QCD coupling extracted from new N³LO fit of combined Γ_W, R_W pseudo-observ.:

W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_W^{tot}, R_W (exp. CKM)	0.044 ± 0.052	±0.024	±0.047	(±0.0014)
Γ_W^{tot}, R_W (CKM unit.)	0.101 ± 0.027	±0.027	(±0.0002)	(±0.0016)
Γ_W^{tot}, R_W (FCC-ee, CKM unit.)	0.11790 ± 0.00023	±0.00012	±0.00004	±0.00019

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

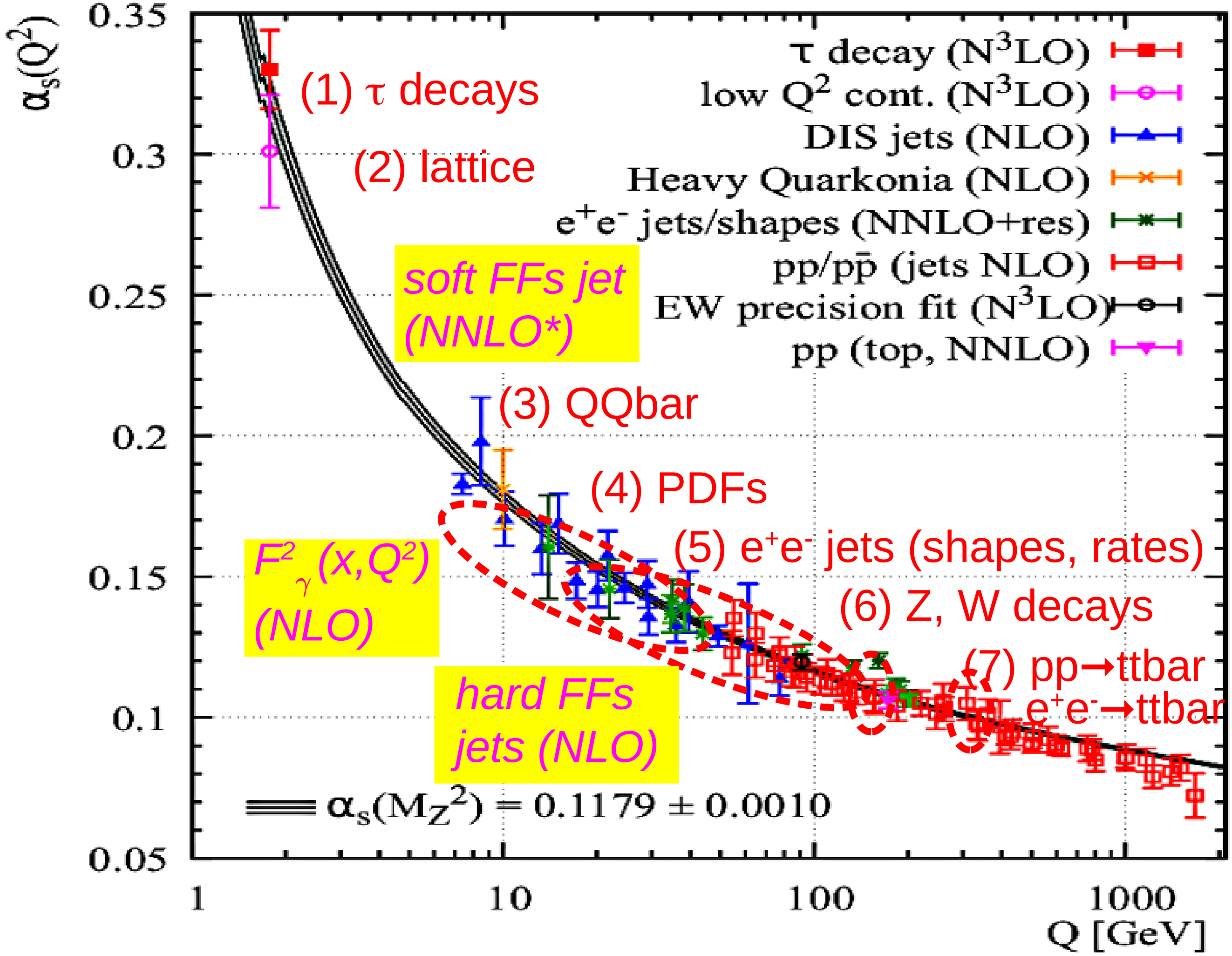
- Very imprecise extraction:
  - Large propagated parametric uncert. from poor V_{cs} exp. precision (±2%): QCD coupling unconstrained: 0.04±0.05
  - Imposing CKM unitarity: large exp. uncertainties from Γ_W, R_W (0.9–2%): QCD extracted with ~27% precision
  - Propagated TH uncertainty much smaller today: ~1.5%



$\alpha_s(m_Z)$ = 0.101 ± 0.027 (±27%)

Other α_s extractions (not *yet* in world average)

- There are few other classes of e^+e^- observables, computed today at lower accuracy (NLO, NNLO*), that can be used to extract the QCD coupling:



α_s from photon QCD structure function (NLO)

- Computed at NNLO: $$\int_0^1 dx F_2^{\gamma}(x,Q^2,P^2) = \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{\frac{4\pi}{\alpha_s(Q^2)}c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi}c_{NNLO} + \mathcal{O}(\alpha_s^2)\right\}$$

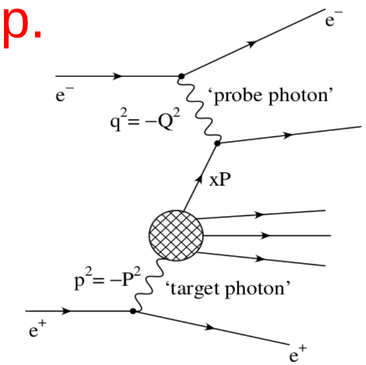
- Poor $F^2_{\gamma}(x,Q^2)$ experimental measurements:

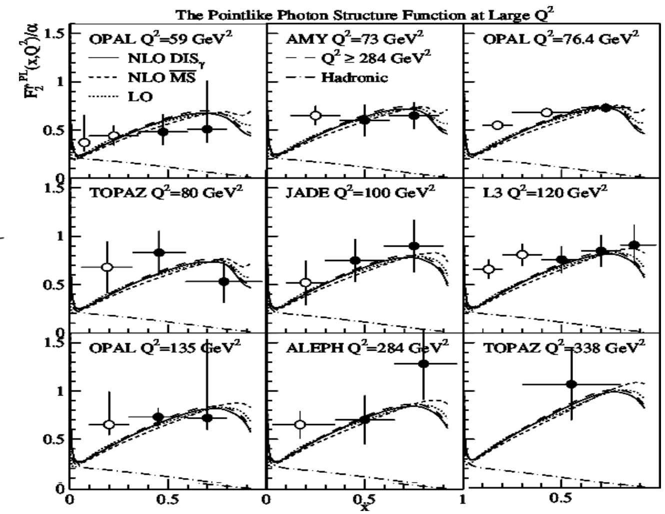
- Extraction (NLO) with large exp. uncertainties today:

α_s (m$_z$) = 0.1198 ± 0.0054

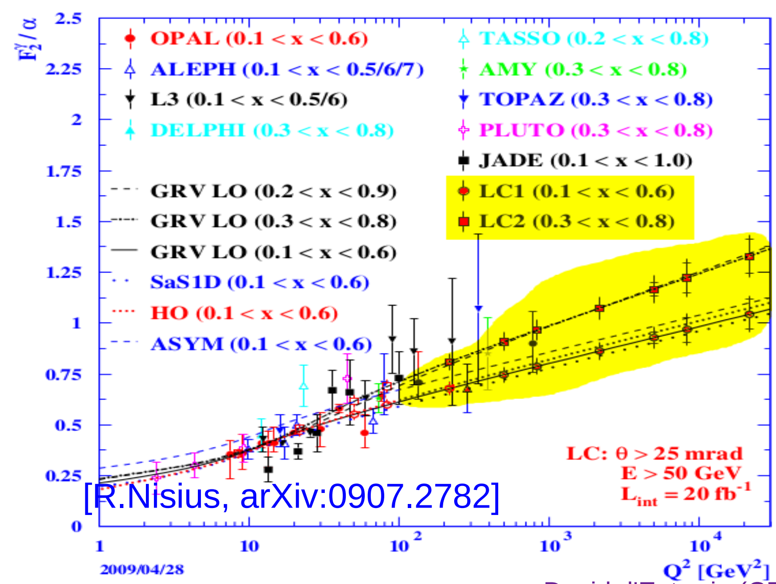
(±4.5%)

[M.Klasen et al. PRL89 (2002)122004]





- Future prospects:
  - Fit with NNLO F^2_{γ} evolution (ongoing)
  - Better data badly needed: Belle-II ?
  - Dedicated simul. studies at ILC exist:
  - Huge $\gamma\gamma$ (EPA) stats at FCC-ee will lead to: $\delta\alpha_s/\alpha_s$ < 1%

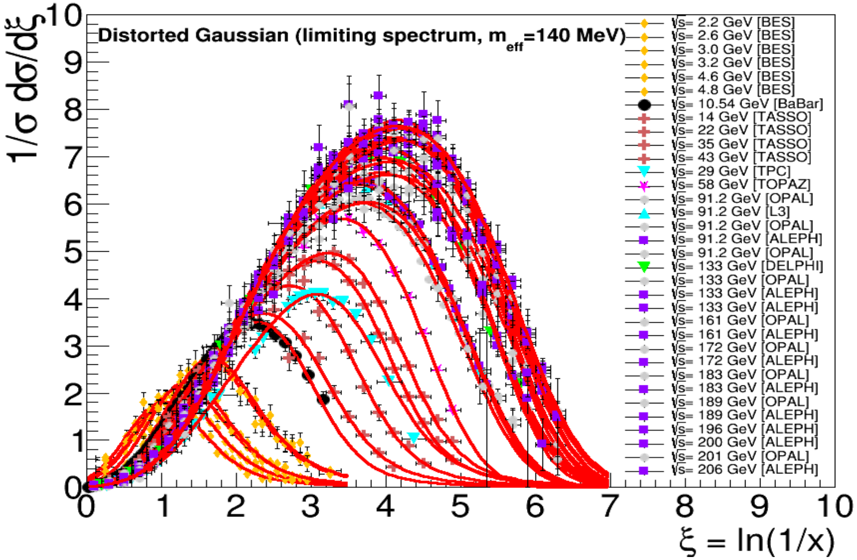


[R.Nisius, arXiv:0907.2782]

α_s extractions from jet fragmentation (NLO,NNLO*)

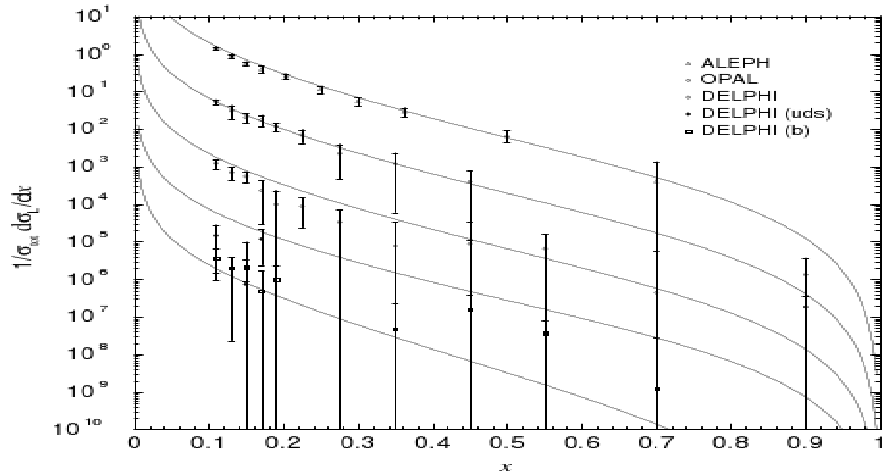
- Soft parton-to-hadron FFs (NNLO*+NNLL):

[D.d'E.,R.Perez-Ramos, arXiv:1505.02624]



- Hard parton-to-hadron FFs (NLO):

$\alpha_s(m_Z) = 0.1176 \pm 0.0055$ (±4.7%)

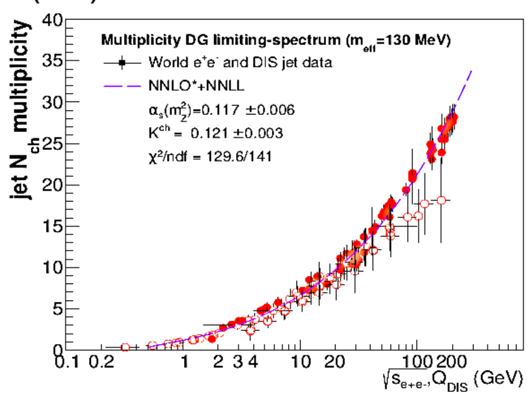


[AKK, B. Kniehl et al.,NPB 803(2008)42]

Combined fit of the jet-energy evolution of the FF moments (multiplicity, peak, width,…) with α_s as single free parameter:

$\alpha_s(m_Z) = 0.1205 \pm 0.0022$ (±2%)

(full-NNLO corrections missing)



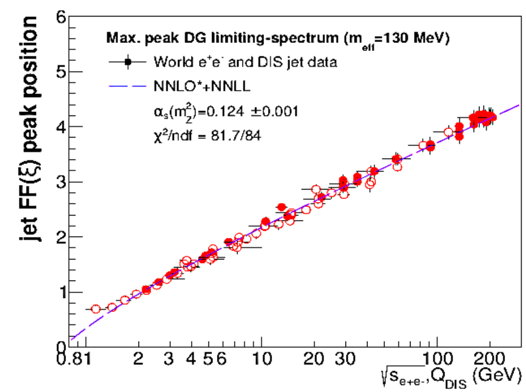
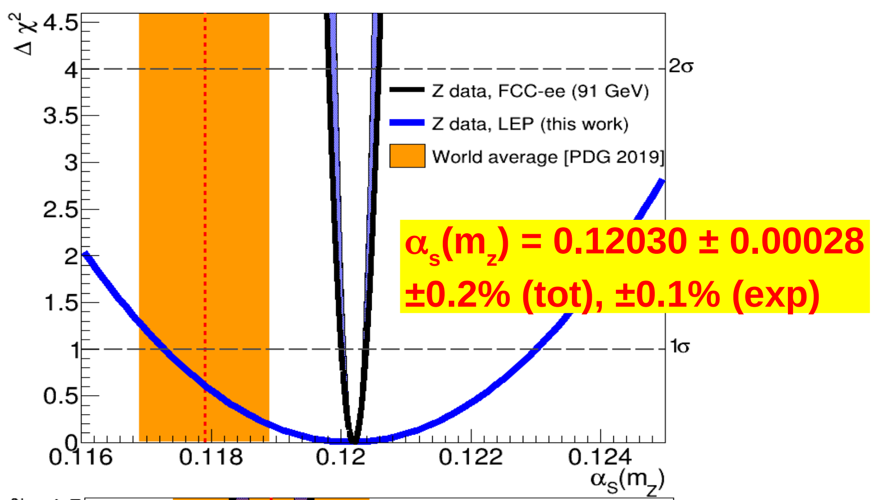


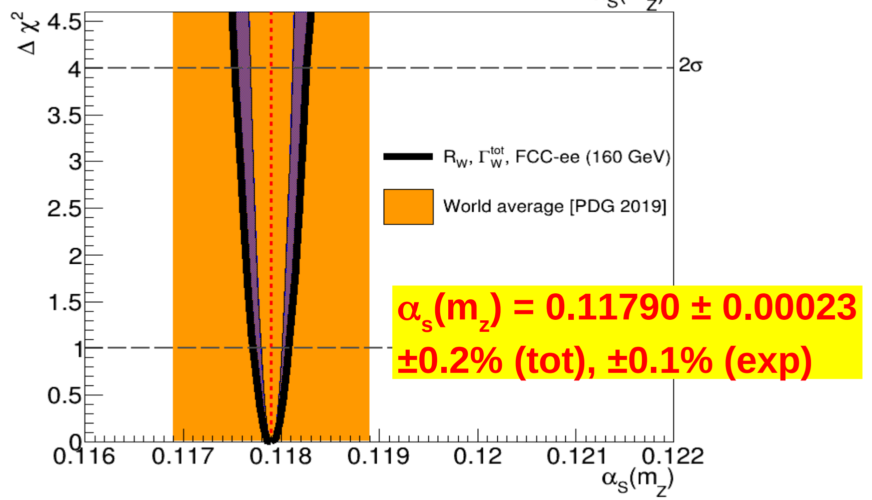
Figure 3: Energy evolution of the charged-hadron multiplicity (left) and of the FF peak position (right) measured in e^+e^- and DIS data fitted to the NNLO*+NNLL predictions. The obtained $\mathcal{K}_{ch}$ normalization constant, individual NNLO* $\alpha_s(m_Z)$ values, and the goodness-of-fit per degree-of-freedom χ^2/ndf.

α_s at future e^+e^- colliders (summary)

- World-average QCD coupling at $N^{2,3}LO$ today:
  - *Determined from 7 observables with combined 0.85% uncertainty (least well-known gauge coupling).*
  - *Impacts all LHC QCD x-sections & decays.*
  - *Role beyond SM: GUT, EWK vacuum stability, New colored sectors?*
- e^+e^- extractions:
  - Hadronic tau decays: ±1% TH
  - Event shapes, jet rates: ±1% TH
  - Z&W pseudo-observ.: ±0.1% TH
- State-of-the-art Z, W extractions:
  - Z boson: New fit with high-order EW corrections + updated LEP data: ~2.3% (exp.) uncertainty today.
  - W boson: New N^3LO fit to Γ_W, R_W ~27% (exp.) uncertainty today.

- Permil uncertainty only possible with a machine like FCC-e^+e^-



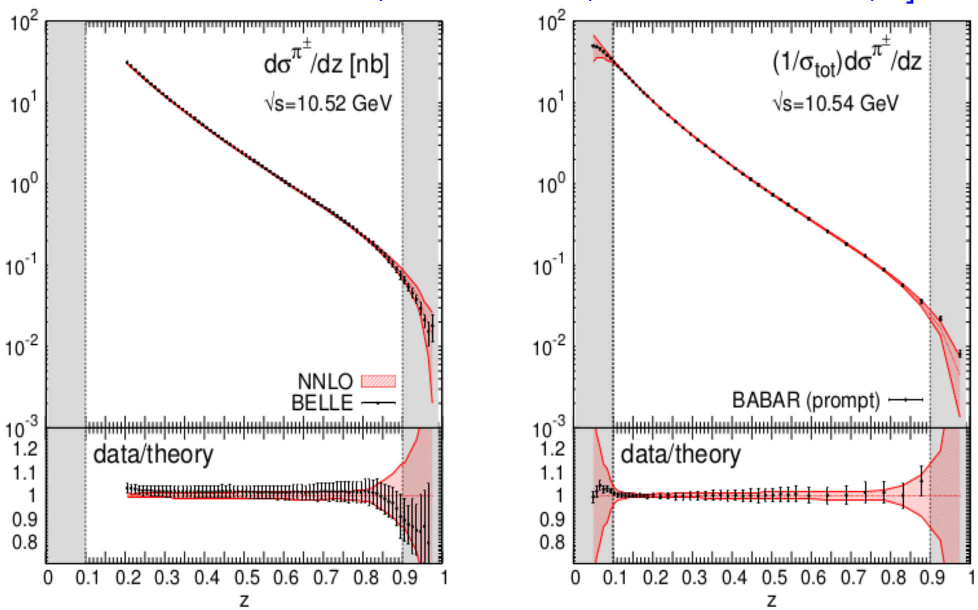


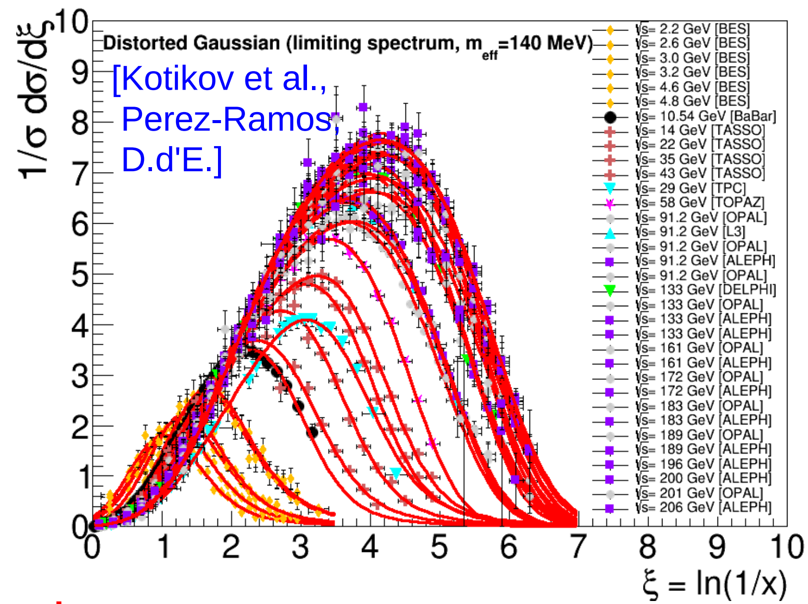
What are the detector design improvements needed to bring propagated syst. uncert. on W,Z pseudo-observ. below 0.1% ?

High-precision parton FFs

- Parton-to-hadron fragment. functions evolution known at NNLO at high-z & at NNLO*+NNLL at low-z:

[D.Anderle et al., A.Vossen et al., B.Kniehl et al., V.Bertone et al., N.Sato et al., D.deFlorian et al.,...]





provide additional QCD coupling extractions:

Method	Current $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2)$ uncertainty (theory & experiment state-of-the-art)	Future $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2)$ uncertainty (theory & experiment progress)
soft FFs	$1.8\%_{th} \oplus 0.7\%_{exp} \approx 2\%$ (NNLO* only (+NNLL), npQCD small)	$0.7\%_{th} \oplus 0.7\%_{exp} \approx 1\%$ ($\sim$2 yrs), $<1\%$ (FCC-ee) (NNLO+NNLL. More precise e^+e^- data: 90–350 GeV)
hard FFs	$1\%_{th} \oplus 5\%_{exp} \approx 5\%$ (NLO only. LEP data only)	$0.7\%_{th} \oplus 2\%_{exp} \approx 2\%$ (+B-factories), $<1\%$ (FCC-ee) (NNLO. More precise e^+e^- data)

- FCC-ee (much broader z range) allows for α_s extraction with **$\delta\alpha_s < 1\%$**