FCC-ee: Physics of **Precision** and **Discoveries**

Janusz Gluza

FCC Week 2022

31 May 2022, Paris (Talk on-line)

`‘La victoire appartient au plus persévérant’`

– Roland Garros

NATIONAL SCIENCE CENTRE POLAND

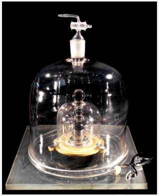
SOLIDARNI Z UKRAINĄ

Paris, world center for over 200 years of precision studies

Today: quantum mass measurement

BIPM

Pavillon de Breteuil



Precision: 10^{-8} $(10\ \mu\text{g/kg})$

2020's result from the Paris lab on $\alpha_{QED}(0)$

REPORT

Measurement of the fine-structure constant as a test of the Standard Model

Richard H. Parker[1,*], Chenghui Yu[1,*], Weicheng Zhong[1], Brian Estey[1], Holger Müller[1,2,†]
+ See all authors and affiliations

Science 13 Apr 2018:
Vol. 360, Issue 6385, pp. 191-195
DOI: 10.1126/science.aap7706

Article | Published: 02 December 2020

Determination of the fine-structure constant with an accuracy of 81 parts per trillion

Léo Morel, Zhibin Yao, Pierre Cladé & Saïda Guellati-Khélifa

Nature 588, 61–65(2020) | Cite this article

$\alpha^{-1}(Cs)$= 137.035 999 046(27)

$\alpha^{-1}(Rb)$= 137.035 999 206(11)

$\alpha^{-1}(a_e)$= 137.035 999 139(31)

Remarks:
(i) new result - deviation from SM in the same direction as in $(g-2)_\mu$,
(ii) substantial disagreement with Cs ($\sim 5.4\sigma$).

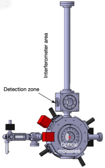
Over 2 decades of improvements
https://www.nature.com/articles/s41586-020-2964-7 [02 December 2020]

$\alpha_{QED}(0)$ and BSM

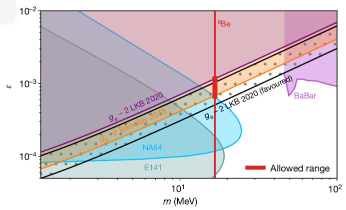
Table 1 | Error budget on α

Source	Correction ($\times 10^{-11}$)	Relative uncertainty ($\times 10^{-11}$)
Gravity gradient	−0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman light shift	2.3	2.3
Index of refraction	0	<0.1
Internal interaction	0	<0.1
Light shift (two-photon transition)	−11.0	2.3
Second-order Zeeman effect		0.1
Phase shifts in Raman phase-lock loop	−39.8	0.6
Global systematic effects	**64.2**	**6.8**
Statistical uncertainty		**2.4**
Relative mass of ^{87}Rb[a]: 86.9091805310(60)		3.5
Relative mass of the electron[b]: $5.48579909065(16)\times 10^{-4}$		1.5
Rydberg constant[b]: 10,973,731.568160(21) m^{-1}		0.1
Total: α^{-1} = 137.035999206(11)		**8.1**

For each systematic effect, more discussion can be found in Methods.
[a]From ref. [13].
[b]From https://pml.nist.gov/cuu/Constants/.



Morel, L., Yao, Z., Cladé, P. et al. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. Nature 588, 61–65 (2020).



Substructure: $\alpha_{QED}(0) \longrightarrow$ modification of $\delta a_e \simeq m_e/m^*$
Excluded (light, states, weakly coupled):

$$m^* < 520 \text{ GeV}.$$

Future δa_e improvement by an order of magnitude in next years, sensitivity similar as for $(g-2)_\mu$.

FCC-ee discovery strategy

From a bird's eye perspective, the physics plan includes

1. searching for new elusive particles that could interact extremely weakly;
2. unveiling the existence of new heavy particles by their indirect (virtual loops) effect on ultra-precise measurements.

Two ways for discoveries (in both cases precision is crucial):

1. within the known theory (anomalies[1])
2. new processes and (rare) phenomena;

See the overview talk by Christophe Grojean (pdf) and talks by Matthew Mccullough, Christoph Paus and David d'Enterria on physics and theoretical aspects of FCC-ee feasibility studies.

[1] 'I have always suspected that, one day, (...) they [JG: experimentalists] would like to see what would happen, just for the fun of it, if they falsely report that there exists a certain bump, or an oscillation in a certain curve, and see how the theorists predict it. I know these men so well that the moment I thought of that possibility I have honestly always been concerned that some day they will do just that. Then you can imagine how absurd the theoretical physicists would sound, making all these complicated calculations to demonstrate the existence of such a bump, while these fellows are laughing up their sleeves.' – R.P. Feynman

In quest of new elusive particles and interactions

The FCC-ee physics covers an entire spectrum of problems presented in particle physics

- Higgs scalar potential, scalar particles;
- Flavor mixings, mass hierarchies, types of neutrinos;
- CP (a)symmetry (quarks, neutrinos, scalars);
  Note the 100^{th} Birthday Anniversary of Prof. Chen Ning Yang,
  G. t'Hooft 'Projecting local and global symmetries to the Planck scale', 2202.05367
- Astro and cosmological problems (DM, BAU).

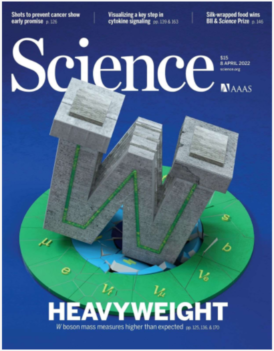
link: ECFA 1st Workshop of the WG1-SRCH group (searches for new scalars, last week)

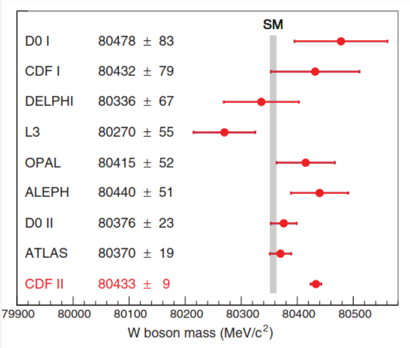
link: FCC Higgs group
(+EW and Precision, Top, Flavours (quark and leptons), BSM)

Which BSM model in case of the anomaly?

I have chosen to discuss:
(i) the M_W measurement problem,
(ii) recent progress in Feynman integrals evaluation methods

SM TESTS: M_W





Science 376 (2022) 6589, 170-176

$$
\begin{aligned}
\text{SM}: M_W &= 80.357 \pm 6 \text{ MeV}, \ (\text{PDG2020}) \\
\text{Global}: M_W &= 80.379 \pm 12 \text{ MeV}, \ (\text{PDG2020}) \\
\text{CDFII}: M_W &= 80433.5 \pm 9.4 \text{ MeV}
\end{aligned}
$$

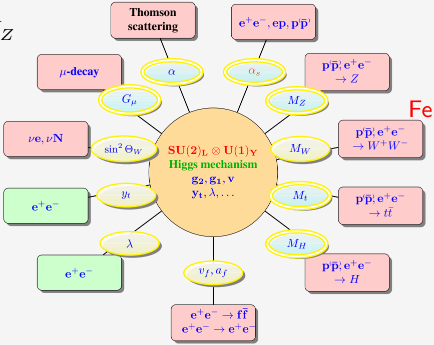
$$
\texttt{FCC-ee} \text{ forecast}: M_W = X \pm \mathbf{0.4} \text{ MeV!}
$$

Conclusion?

Input and calculated/measured parameters

Schemes: G_μ vs M_W,...

G_μ, $\sin^2\theta^\ell_{eff}$, M_Z



Fermilab!

Fig. from the FCC-ee report 'α_{QED}' by F. Jegerlehner in 1905.05078

Introduction to Precision Electroweak Analysis by J. Welss, 0512342

Input and calculated/measured parameters

Experimental values:

$$\begin{aligned}
\hat{\alpha} &= 1/137.0359895(61),\ \gamma^* \to e^+e^- \\
\hat{G}_F &= 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}\ \text{muon decay} \\
\hat{m}_Z &= 91.1875 \pm 0.0021\,\mathrm{GeV} \\
\hat{m}_W &= 80.426 \pm 0.034\,\mathrm{GeV} \\
\hat{s}^2_{\text{eff}} &= 0.23150 \pm 0.00016, \text{effective} \sin^2\theta_{\mathrm{W}},\ A_{LR} \equiv \frac{(1/2-\hat{s}^2_{\text{eff}})^2 - \hat{s}^4_{\text{eff}}}{(1/2-\hat{s}^2_{\text{eff}})^2 + \hat{s}^4_{\text{eff}}} \\
\hat{\Gamma}_{l^+l^-} &= 83.984 \pm 0.086\,\mathrm{MeV}
\end{aligned}$$

$$\left\{ \begin{array}{l} \mathbf{g}(=e/s_W)\ SU(2) \\ \mathbf{g}'(e/c_W)\ U(1)_Y \\ \mathbf{v}\ \text{VEV}, \end{array} \right. \longrightarrow \left\{ \begin{array}{l} \hat{\alpha} = \frac{e^2}{4\pi} \\ \hat{G}_F = \frac{1}{\sqrt{2}v^2} \\ \hat{m}_Z^2 = \frac{e^2v^2}{4s^2c^2} \\ \hat{m}_W^2 = \frac{e^2v^2}{4s^2} \\ \hat{s}^2_{\text{eff}} = s^2 \\ \hat{\Gamma}_{l^+l^-} = \frac{v}{96\pi}\frac{e^3}{s^3c^3}\left[\left(-\frac{1}{2}+2s^2\right)^2 + \frac{1}{4}\right] \end{array} \right.$$

Shaping the SM, tree level estimates

In terms of $\hat{\alpha}, \hat{G}_F$ and $\hat{m}_Z$

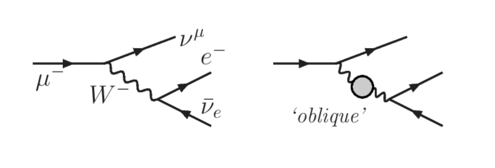
$$\hat{m}_W^2 = \pi\sqrt{2}\hat{G}_F^{-1}\hat{\alpha}\left(1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^{-1}$$

$$\hat{s}_{\text{eff}}^2\hat{c}_{\text{eff}}^2 = \frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2} \quad \equiv \quad \hat{s}_{\text{eff}}^2 = \frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}$$

$$\hat{\Gamma}_{l^+l^-} = \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{12\pi}\left\{\left(\frac{1}{2}-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^2+\frac{1}{4}\right\}$$

$$\begin{aligned} Prediction : \hat{m}_W &= 80.939 \pm 0.003\,\text{GeV}\ 15\sigma\ \text{away} \\ Prediction : \hat{s}_{\text{eff}}^2 &= 0.21215 \pm 0.00003\ 120\sigma\ \text{away} \\ Prediction : \hat{\Gamma}_{l^+l^-} &= 84.843 \pm 0.012\,\text{MeV}\ 10\sigma\ \text{away} \end{aligned}$$

Shaping SM, oblique corrections also not sufficient



$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

$$\begin{aligned} \frac{(\hat{G}_F)^{\rm th}}{\sqrt{2}} &= \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0} \\ &= \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right]. \end{aligned}$$

Primary role of SM radiative corrections, F. Jegerlehner, in 1905.05078

$$\sin^2\Theta_i\,\cos^2\Theta_i\,=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\,\frac{1}{1-\Delta r_i}\quad \Delta r_i=\Delta r_i(\alpha,G_\mu,M_Z,m_H,m_{f\neq t},m_t)\,,$$

$$\Delta r_i=-\frac{c_W^2}{s_W^2}\,\Delta\rho+\Delta r_{i\,\mathrm{reminder}}\,,$$

$$\Delta\rho=\frac{3\,m_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}$$

$$\hat{\alpha}(m_Z)=\frac{\hat{\alpha}}{1-\Delta\alpha(m_Z)}=\frac{e^2}{4\pi}\left[1+\frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2}\right]\sim 128\ (137\ \text{at the Thomson limit})$$

Still, well visible disagreement between SM prediction and experiment for EWPOs without subleading SM corrections, and only with the leading corrections $\Delta\alpha(m_Z)$ and $\Delta\rho$.

$r_{i\,\mathrm{reminder}}$ **matters!**

Input, theoretical and parametric errors,

A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee", https://arxiv.org/abs/1906.05379

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error (at start of FCC-ee)
M_W [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b $[10^{-5}]$	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l $[10^{-3}]$	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

‡The pure experimental precision on M_W is $\sim 0.5\,\mathrm{MeV}$.

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$0.5 - 1$	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_Z [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3 (0.7)	$\delta\alpha_s$

Important input parameter errors are $\delta(\Delta\alpha) = 3 \cdot 10^{-5}$, $\delta\alpha_s = 0.00015$.
$\alpha_s \longrightarrow$ see the talk by David d'Enterria.

Input and renormalization schemes

E.g. the bosonic 2-loop corrections shift the value of Γ_Z by 0.51 MeV when using M_W as input and 0.34 MeV when using G_μ as input.

Reminder: $\delta\Gamma_{Z,\mathrm{FCC-ee}} = 0.1$ MeV

Dubovyk et al, https://doi.org/10.1016/j.physletb.2018.06.037

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\mathrm{bos}})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

* Fixed values of M_W

A. Blondel, P. Janot, 2106.13885

Table 3 Measurement of selected precision measurements at FCC-ee, compared with present precision. Statistical errors are indicated in boed phase. The systematic uncertainties are initial estimates, aim is to improve down to statistical errors. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale Λ of 70 TeV in a description with dim 6 operators, and possibly much higher in specific new physics (non-decoupling) models

Observable	Present value ± error	FCC-ee stat.	FCC-ee syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan
				Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan
				Beam energy calibration
$\sin^2\theta_W^{eff}$ ($\times 10^6$)	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak
				Beam energy calibration
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	Small	From $A_{FB}^{\mu\mu}$ off peak
				QED&EW errors dominate
R_ℓ^Z ($\times 10^3$)	20767 ± 25	**0.06**	0.2–1	Ratio of hadrons to leptons
				Acceptance for leptons
$\alpha_s(m_Z^2)$ ($\times 10^4$)	1196 ± 30	**0.1**	0.4–1.6	From R_ℓ^Z above
σ_{had}^0 ($\times 10^3$) (nb)	41541 ± 37	**0.1**	4	Peak hadronic cross section
				Luminosity measurement
$N_\nu(\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections
				Luminosity measurement
R_b ($\times 10^6$)	216290 ± 660	**0.3**	$<$ 60	Ratio of $b\bar{b}$ to hadrons

Gluza

Future: W, t, H

- $e^+e^- \to W^+W^-$ at 161 GeV: $\delta m_W^{exp} = 0.5 \div 1$ MeV.
  Challenge to get the same TH error:
  NNLO $e^+e^- \to 4f$.

- $e^+e^- \to t\bar{t}$ at 350 GeV: $\delta {m_t}^{exp} =$ 17 MeV
  Big challenge for theory, today > 100 MeV, future projection $\leq$ 50 MeV:
  $\sim$ 10 MeV unc. from mass def.;
  $\sim$ 15 MeV from α_s unc. to threshold mass def.;
  ~ 30 MeV - h. orders resummation

- $e^+e^- \to HZ$ at 240 GeV: Kinematic constraint fits with $Z \to ll$ and $H \to bb$, ...,
  $m_H = 125.35$ GeV ± 150 MeV [link CMS], $\Gamma_H = 4.1^{5.1}_{4.0}$ MeV, $\Gamma_H <$ 13 MeV at 95 % C.L., 1901.00174
  $\delta m_H^{exp} =$ 10 MeV; Theory errors subdominant.

Monte Carlo generators (**not discussed!**) 'QED challenges at FCC-ee precision measurements', S. Jadach and M. Skrzypek, Eur.Phys.J.C 79 (2019) 9, 756 1903.09895

TOOLS

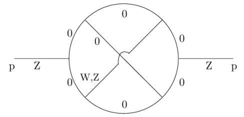
Direct numerical approach[2]

- Sector decomposition (SD) method:
  - FIESTA [2016], [A.V.Smirnov]
  - pySecDec [2022], Expansion by regions with pySecDec],
- The Mellin-Barnes (MB) method:
  - MB [M.Czakon, 2006]
  - MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] – Minkowskian kinematics
- Differential equations (DEs) method:
  - `AMFlow` [X. Liu, Y.-Q. Ma, 2022] `AMFlow`,
  - `SeaSyde` [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vi, 2022]

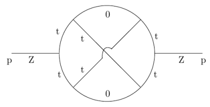
[2] All programs are public

MIs with high accuracy, results*

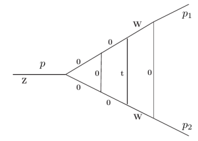
*Results for 3-loop EWPOs at the e^+e^- Z-resonance peak,
I. Dubovyk, A. Freitas, JG, K. Grzanka, M. Hidding, J. Usovitsch, ‘Evaluation of multi-loop multi-scale Feynman integrals for precision physics’, 2201.02576



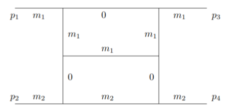
lhNp1



taNPl1



vtwPl



box2l

$$
\begin{aligned}
I_{\text{box2l}}[2,1,1,1,1,1,1,0,0,s,t,m_1^2,m_2^2] \quad &= \quad +0.000328707579/\epsilon^2 \\
&- \quad (0.0014129475 - 0.0020653306\ i)/\epsilon \\
&- \quad (0.005702737 - 0.000485980\ i) + \mathcal{O}(\epsilon), \\
& \quad\ \ 55\ \text{MIs},\ s = 2,\ t = 5,\ m_1^2 = 4,\ m_2^2 = 16.
\end{aligned}
$$

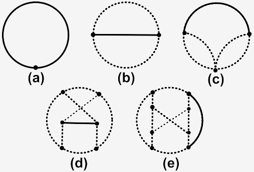
AMFlow method, $\eta = \infty \longrightarrow \eta = 0^+$ analytic continuation (auxiliary mass flow)

2. A set of Jan 27 2022 papers by Zhi-Feng Liu, Yan-Qin Ma and Xiao Liu:
   https://inspirehep.net/literature/2020677, https://inspirehep.net/literature/2020676, https://inspirehep.net/literature/2020880 and 1711.09572
   https://inspirehep.net/literature/1639025.

$$
\begin{aligned}
\widetilde{I}_{\vec{\nu}}(\eta) &= \int \left(\prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{i\pi^{D/2}} \right) \frac{\widetilde{\mathcal{D}}_{K+1}^{-\nu_{K+1}} \cdots \widetilde{\mathcal{D}}_N^{-\nu_N}}{\widetilde{\mathcal{D}}_1^{\nu_1} \cdots \widetilde{\mathcal{D}}_K^{\nu_K}}. \\
\widetilde{\mathcal{D}}_1 &= \ell_1^2 - m^2 + \mathrm{i}\eta \\
I_{\vec{\nu}} &= \lim_{\eta \to 0^+} \widetilde{I}_{\vec{\nu}}(\eta) \\
\mathrm{i}\frac{\partial}{\partial \eta} \overrightarrow{\widetilde{J}}(\eta) &= A(\eta) \widetilde{J}(\eta)
\end{aligned}
$$

Key point: boundary conditions at $\eta \to \infty$ are single mass scale bubble integrals, solved iteratively.

MIs with high accuracy by `AMFlow`, results



$$
\begin{aligned}
I[(e)] = & -2.073855510286740\epsilon^{-2} - 7.812755312590133\epsilon^{-1} \\
& -17.25882864945875 + 717.6808845492140\epsilon \\
& + 8190.876448160049\epsilon^{2} + 78840.29598046500\epsilon^{3} \\
& + 566649.1116484678\epsilon^{4} + 3901713.802716081\epsilon^{5} \\
& + 23702384.71086095\epsilon^{6} + 14214293.68205112\epsilon^{7},
\end{aligned}
$$

10 orders in ϵ, 16-digit precision.

Summary and Outlook[3,*]

1. Challenges at Z-pole:
   1.1 3-loop EW and mixed EW-QCD, leading 4-loop corrections for $Z \to 2f$ vertices
   1.2 QED interference effects, non-factorizable corrections
   1.3 Adjusting MC generators at NNLO and beyond (Bhabha (!), exclusive NNLO $e^+e^- \to f\bar{f}$).
2. Challenge to improve input parameters ($\alpha, \alpha_{\rm s}$, physics at ZH, WW, tt)
3. Challenge to optimize/understand paths towards BSM discovery (RHNs, DM, CP effects,...)
4. Challenge: SM(BSM)EFT, precision physics for concrete BSM models
5. Challenge: Tools (MC generators, ~~multiloop numerical~~, analytical programs)

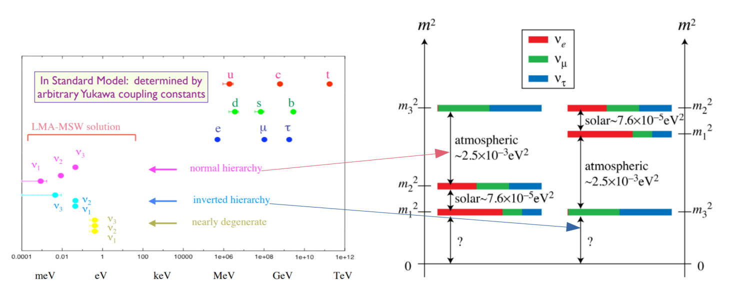
[*] 'FCC-ee: the challenge for theory', talk at 4th FCC Physics and Experiments Workshop, link

[3] 'At each meeting it always seems to me that very little progress is made. Nevertheless, if you look ever any reasonable length of time, a few years say, you find a fantastic progress and it is hard to understand how that can happen at the same time that nothing is happening in anyone moment (zeno's paradox).' - R.P. Feynman

BACKUP

NEUTRINOS

Neutrino parameters and the known unknowns

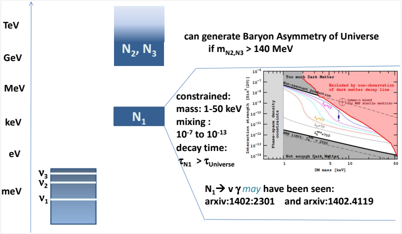


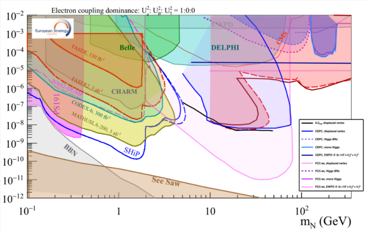
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\mathrm{CP}}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

BSM and RHNs, FCC-ee CDR vol.1

LFV Z-decays: $(10^{-6} \div 10^{-5})$. FCC-ee $\longrightarrow \sim 10^{-9}$ branching fractions.

A. Blondel et al. 1411.5230 ESPPU Briefieng Book 1910.11775





Low-scale leptogenesis with flavour and CP symmetries, M. Drewes et al, 2203.08538

Discrete Flavor Symmetries and Lepton Masses and Mixings, G. Chauhan, et al, 2203.08538 (Snowmass contribution)

Resonant Leptogenesis, Collider Signals and Neutrinoless Double Beta Decay from Flavor and CP Symmetries, G. Chauhan, B. Dev, 2203.08538

Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

$\Downarrow$

(Prior to 2012)

$s_{23} = 1/\sqrt{2}$ ($\theta_{23} = 45^\circ$) and $\theta_{13} = 0$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ Bimaximal Mixing	$\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ Tribimaximal Mixing	$\theta_{12} = 31.7^\circ$ Golden Ratio Mixing	$\theta_{12} = 30^\circ (s_{12} = 1/2)$ Hexagonal Mixing
$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$
Fukugita, Tanimoto, Yanagida PRD98;	Harrison Perkins, Scott PLB02;	Dutta,Ramond NPB03;	Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1 + \sqrt{5})/2$)

Flavor symmetries, why?

- Using the diagonalization relation

$$m_\nu = U_0^\star \mathrm{diag}(m_1, m_2, m_3) U_0^\dagger,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ *i.e.*, $\theta_{12} = 35.26^\circ$

Non-zero θ_{13}

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
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$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
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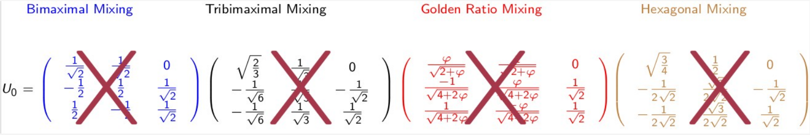
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$



Decendents of fixed pattern mixing schemes

- The CP-violating out-of-equilibrium decay of RH neutrinos into lepton and Higgs doublets in the early universe produces a net lepton asymmetry Fukugita, Yanagida, 1986; Covi, Roulet, Vissani 9605319
- The CP asymmetry parameter :

$$\epsilon_i^\alpha = \frac{\Gamma(N_i \to \ell_\alpha H) - \Gamma(N_i \to \overline{\ell}_\alpha \bar{H})}{\Gamma(N_i \to \ell_\alpha H) + \Gamma(N_i \to \overline{\ell}_\alpha \bar{H})} = \frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im}\left[\left((\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij}\right)^2\right]}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii}} f\left(\frac{m_i^2}{m_j^2}\right),$$

- Flavor symmetry dictates the structure of Y_ν and M_R, hence leaves its imprint on leptogenesis

'Probing Leptogenesis at Future Colliders', Antusch et al, JHEP 09 (2018) 124

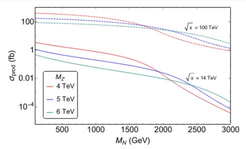
'CP Violating Effects in Heavy Neutrino Oscillations: Implications for Colliders and Leptogenesis', B. Dev et al, JHEP 11 (2019) 137

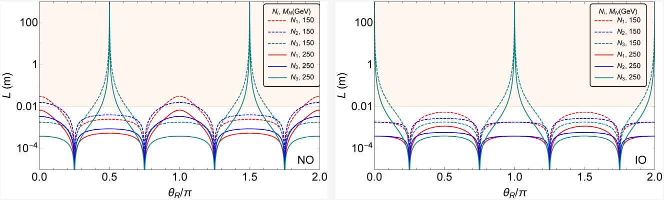
'Theories and Experiments for Testable Baryogenesis Mechanisms: A Snowmass White Paper', J.L. Barrow et al, Snowmass 2022, 2203.07059

'Searches for Long-Lived Particles at the Future FCC-ee', J.Alimena et al, Snowmass 2022, 2203.05502

Flavor Symmetries in Various Frontiers: Collider Physics

- CP phases present in Y_D can be related to the low-energy CP phases in $U_{\mathtt{PMNS}}$.
- PMNS mixing matrix depends on a single free parameter $\longrightarrow$ constrains and predictions for both low- and high-energy CP phases as well as the lepton mixing angles
- Example : G. Chauhan, B. Dev, 2203.08538 $\Delta(6n^2) \times CP \to Z_2 \times CP$

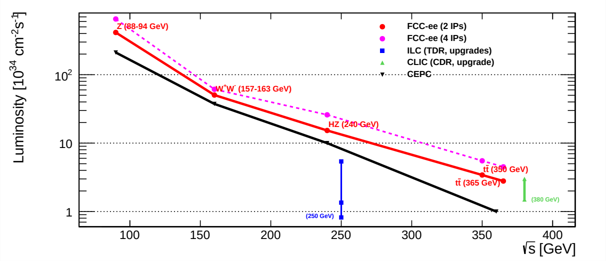




FCC-LLP 2203.0550

FCC-ee: SM EWK FACTORY

FCC-ee: Z,W,H,t and flavour electroweak factories



https://arxiv.org/abs/2203.06520 [The Future Circular Collider: a Summary for the US 2021 Snowmass Process]

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})	Event Statistics
FCC-ee-Z	4	88-94	150	$5 \cdot 10^{12}$ **Z decays**
FCC-ee-W	2	157-163	10	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events 25k WW $\to H$
FCC-ee-tt	5	340-365	$0.2 \div 1.5$	10^6 $t\bar{t}$ even ts 200k ZH 50k WW $\to H$

Jorgen D'Hondt, "Strategies and plans for particle physics in Europe",

Epiphany 2021, https://indico.cern.ch/event/934666

---

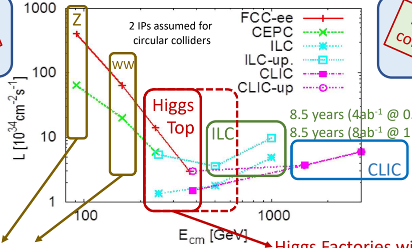
e^+e^- Higgs Factories (incl. B/c/τ/EW/top factories)

precision frontier — circular colliders

synchrotron radiation

for the same power, less luminosity at higher E_{cm}

(Energy Recovery Linac technology might mitigate this & allow to go to higher E_{cm})



8.5 years ($4ab^{-1}$ @ 0.5 TeV)
8.5 years ($8ab^{-1}$ @ 1 TeV)

7 years ($2.5ab^{-1}$ @ 1.5 TeV)
8 years ($5ab^{-1}$ @ 3 TeV)

linear colliders — energy frontier

B/c/τ/EW Factories

per detector in e^+e^-	# Z	# B	# τ	# charm	# WW
LEP	4×10^6	1×10^6	3×10^5	1×10^6	2×10^4
SuperKEKB	-	10^{11}	10^{11}	10^{11}	-
FCC-ee	2.5×10^{12}	7.5×10^{11}	2×10^{11}	6×10^{11}	1.5×10^8

Higgs Factories with complementarity

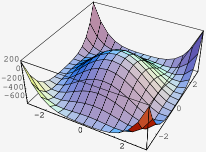
- g_{HZZ} (250GeV) versus g_{HWW} (380GeV)
- top quark physics
- beam polarization for EW precision tests

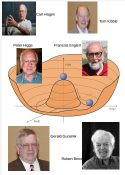
(transverse polarization in circular e^+e^- colliders only at lower E_{cm} while longitudinal polarization at linear colliders)

SCALARS

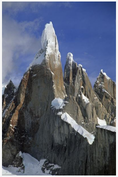
What Is a Particle Physics scalars landscape?

Mount Mayon (Renowned as the "perfect cone" because of its almost symmetric conical shape)

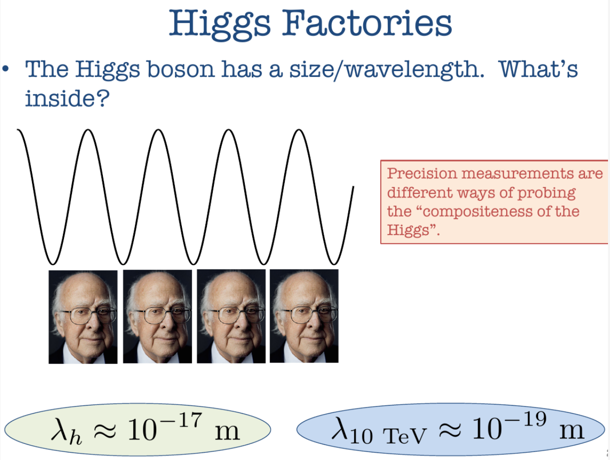




$$
\begin{aligned}
V_{SM} &= -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \\
V_{HTM} &= -m_\Phi^2\left(\Phi^\dagger\Phi\right) + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 + M_\Delta^2 \mathrm{Tr}\left(\Delta^\dagger\Delta\right) \\
&+ \left[\mu\left(\Phi^T i\sigma_2 \Delta^\dagger\Phi\right) + \mathrm{h.c.}\right] \\
&\quad +\lambda_1\left(\Phi^\dagger\Phi\right)\mathrm{Tr}\left(\Delta^\dagger\Delta\right) + \lambda_2\left[\mathrm{Tr}\left(\Delta^\dagger\Delta\right)\right]^2 \\
&+ \lambda_3 \mathrm{Tr}\left[\left(\Delta^\dagger\Delta\right)^2\right] + \lambda_4\Phi^\dagger\Delta\Delta^\dagger\Phi\,.
\end{aligned}
$$



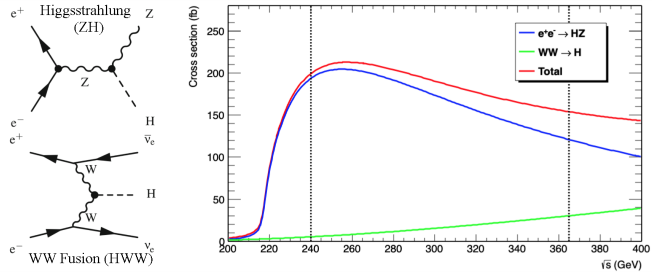
Compositeness - testing substructures of elementary particles



Matthew Philip Mccullough, Oxford 2019,

https://indico.cern.ch/event/783429/contributions/3305140/attachments/1829729/2996092/CEPC.pdf

Higgs Physics in 'FCC Physics Opportunities', EPJCC 79 (2019) 6, 474

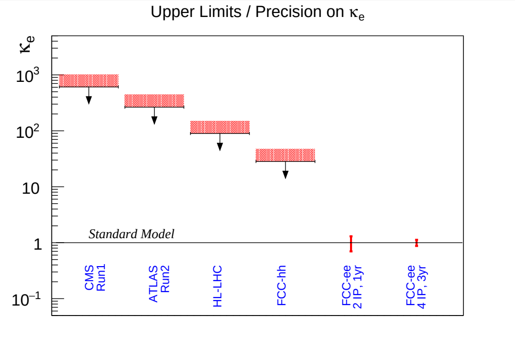


$$\sigma_{\mathrm{ZH}} \times \mathcal{B}(\mathrm{H} \to X\bar{X}) \propto \frac{g^2_{\mathrm{HZZ}} \times g^2_{\mathrm{HX\bar{X}}}}{\Gamma_{\mathrm{H}}}$$

$$\sigma_{\mathrm{H}\nu_e\bar{\nu}_e} \times \mathcal{B}(\mathrm{H} \to X\bar{X}) \propto \frac{g^2_{\mathrm{HWW}} \times g^2_{\mathrm{HX\bar{X}}}}{\Gamma_{\mathrm{H}}} .$$

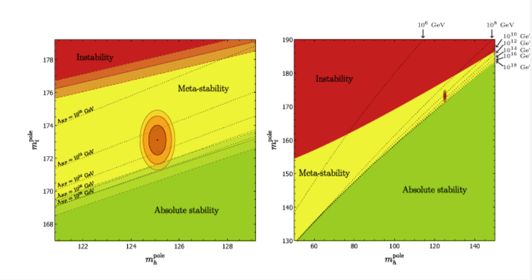
ZH cross section measurement gives a model-independent measurement of $g_{\mathrm{HZZ}} \longrightarrow$ normalization for the measurements of other Higgs boson couplings, a unique feature of e^+e^- colliders.

Sensitivity of FCC-ee, comparisons, Blondel & Janot 1912.11871



Current upper limits on the Higgs boson coupling modifier to electrons, κ_e, from and ATLAS; projected κ_e upper limits at HL-LHC and FCC-hh; and projected κ_e precisions at FCC-ee in two different running configurations (one year with 2 IPs, or three years with 4 IPs).

The ‘universe’ stability fate phase diagram, https://arxiv.org/abs/1707.08124



Dotted lines indicating the scale at which the addition of higher-dimension could stabilize the SM (one of possible BSM scenarios). Is BSM needed there?

‘The Standard Model of Particle Physics as a Conspiracy Theory and the Possible Role of the Higgs Boson in the Evolution of the Early Universe’, F. Jegerlehner, 2106.00862

F. Jegerlehner, in 1905.05078

Example: the W and Z mass from $\alpha(M_Z)$, G_μ and $\sin^2\Theta_{\ell\,\mathrm{eff}}$:
(i) $\sin^2\Theta_W = 1 - M_W^2/M_Z^2$,
$\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) = \left(1 + \frac{\cos^2\Theta_W}{\sin^2\Theta_W}\,\Delta\rho\right)\sin^2\Theta_W$,

$$\Delta\rho = \frac{3\,M_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}\ ;\quad M_t = 173 \pm 0.4\ GeV$$

The iterative solution with input $\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) \equiv (1 - v_\ell/a_\ell)/4 = 0.23148$ (EXP!) is $\sin^2\Theta_W = 0.22426$.
(ii) $M_W^{\mathrm{exp}} = 80.379 \pm 0.012\ \ ;\ \ M_Z^{\mathrm{exp}} = 91.1876 \pm 0.0021\ \mathrm{GeV}$,
$\longrightarrow 1 - \mathrm{M_W^2/M_Z^2} = 0.22263$.
Predicting then the masses we have

$$M_W = \frac{A_0}{\sin^2\Theta_W}\ ;\ \ A_0 = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_\mu}}\ ;\ \ M_Z = \frac{M_W}{\cos\Theta_W}$$

where, including photon VP correction $\alpha^{-1}(M_Z) = 128.953 \pm 0.016$. For the W, Z mass we then get

$$M_W^{\mathrm{the}} = 81.1636 \pm 0.0346\ \ ;\ \ M_Z^{\mathrm{the}} = 92.1484 \pm 0.0264\ \ .$$

Deviavions (errors added in quadrature): $W:\ \ 23\,\sigma\ ;\ \ Z:\ \ 36\,\sigma$

E. Torrente-Lujan, 1209.0474v2

If

$$\rho_t = \frac{m_Z m_t}{m_H^2},$$

then (for ATLAS, CMS combined $m_H = 125.6 \pm 0.4 \pm 0.5$)

$$\rho_t^{(exp)} = 1.0022 \pm 0.007 \pm 0.009$$

Separately,

$$\rho_t^{(exp)} = 1.0077 \pm 0.007 \pm 0.009 \qquad (m_{h,ATLAS}),$$
$$\rho_t^{(exp)} = 0.9965 \pm 0.007 \pm 0.007 \qquad (m_{h,CMS})$$

Observable	present value ± error	FCC-ee **Stat.**	FCC-ee Syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan Beam energy calibration
$\sin^2\theta_W^{eff}(\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	small	from $A_{FB}^{\mu\mu}$ off peak QED&EW errors dominate
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	0.2-1	ratio of hadrons to leptons **acceptance for leptons**
$\alpha_s(m_Z^2)$ $(\times 10^4)$	1196 ± 30	**0.1**	0.4-1.6	from R_ℓ^Z above
σ_{had}^0 $(\times 10^3)$ (nb)	41541 ± 37	**0.1**	4	peak hadronic cross section luminosity measurement
$N_\nu(\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections Luminosity measurement
R_b $(\times 10^6)$	216290 ± 660	**0.3**	< 60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD
$A_{FB}^b, 0$ $(\times 10^4)$	992 ± 16	**0.02**	1-3	b-quark asymmetry at Z pole from jet charge
$A_{FB}^{pol,\tau}$ $(\times 10^4)$	1498 ± 49	**0.15**	<2	τ polarization asymmetry τ decay physics
τ lifetime (fs)	290.3 ± 0.5	**0.001**	0.04	radial alignment
τ mass (MeV)	1776.86 ± 0.12	**0.004**	0.04	momentum scale
τ leptonic $(\mu\nu_\mu\nu_\tau)$ B.R. (%)	17.38 ± 0.04	**0.0001**	0.003	e/μ/hadron separation
m_W (MeV)	80350 ± 15	**0.25**	0.3	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2085 ± 42	1.2	0.3	From WW threshold scan Beam energy calibration
$\alpha_s(m_W^2)(\times 10^4)$	1170 ± 420	**3**	small	from R_ℓ^W
$N_\nu(\times 10^3)$	2920 ± 50	**0.8**	small	ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV/c^2)	172740 ± 500	**17**	small	From $t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV/c^2)	1410 ± 190	45	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	1.2 ± 0.3	**0.10**	small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	$\pm$ 30%	0.5 – 1.5%	small	From $\sqrt{s} = 365\,\mathrm{GeV}$ run

Higgs boson decays: theoretical status, M. Spira, pdf

Estimated theoretical uncertainties from missing higher orders and the perturbative orders (QCD/elw.) of the results included in the analysis.

Partial Width	**QCD**	**Electroweak**	**Total**	**on-shell Higgs**
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	N^{4}LO / NLO
$H \to \tau^+\tau^-/\mu^+\mu^-$	—	$\sim 0.5\%$	$\sim 0.5\%$	— / NLO
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	N^{3}LO / NLO
$H \to \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \to Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	LO / LO
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	NLO/NLO

Higgs boson decays: theoretical status

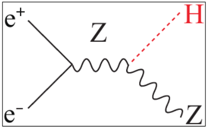
Projected intrinsic and parametric uncertainties for the partial and total Higgs-boson decay width predictions. The last column: the target of FCC-ee precisions.

decay	intrinsic	para. m_q	para. $\alpha_{\rm s}$	para. $M_{\rm H}$	FCC-ee prec. on g^2_{HXX}
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 0.8\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.4\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.1\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 12\%$
$H \to gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.0\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \to ZZ$	$\lesssim 0.3\%^\dagger$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
$\Gamma_{\rm tot}$	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

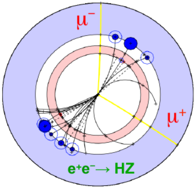
† From $e^+e^- \to HZ$ production

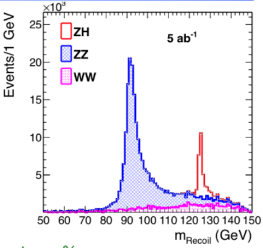
Absolute coupling and width measurement

- Higgs tagged by a Z, Higgs mass from Z recoil



$$m_H^2 = s + m_Z^2 - 2\sqrt{s}(E_+ + E_-)$$





- Total rate $\propto g_{HZZ}^2$ → measure g_{HZZ} to 0.2%
- ZH → ZZZ final state $\propto g_{HZZ}^4 / \Gamma_H$ → measure Γ_H to a couple %
- ZH → ZXX final state $\propto g_{HXX}^2\, g_{HZZ}^2 / \Gamma_H$ → measure g_{HXX} to a few per-mil / per-cent
- Empty recoil = invisible Higgs width; Funny recoil = exotic Higgs decays

- Note: The HL-LHC is a great Higgs factory (10^9 Higgs produced) but …
  - $\sigma_{i\to f}^{(observed)} \propto \sigma_{prod}\,(g_{Hi})^2\,(g_{Hf})^2 / \Gamma_H$
    - Difficult to extract the couplings : σ_{prod} is uncertain and Γ_H is largely unknown
      - Must do physics with ratios or with additional assumptions.

E.g. effective weak mixing angle

The weak mixing angle $s_W^2 \equiv \sin^2\theta_W$ has three potential different meanings or functions in the model-building:

(i) It describes the ratio of the two gauge couplings,

$$g'/g = c_W/s_W,$$

usually in the $\overline{\text{MS}}$ scheme.

(ii) It describes the ratio of two gauge boson (on-shell) masses,

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}.$$

(iii) It describes the ratio of the vector and axial-vector couplings of an (on-shell) Z boson to fermions,

$$\frac{v_f}{a_f} = 1 - 4|Q_f|s_W^2.$$

This definition is called the effective weak mixing angle, denoted as $\sin^2\theta_W^{f,\text{eff}}$.

Z-resonance: QED and EW

1. Z-resonance and $\gamma, Z', \ldots \longrightarrow$ Laurent series,

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n \; B^{(n)}, \quad s_0 = \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z.$$

2. We want to extract EW Z-vertex couplings and definitions like $\sin^2 \theta^f_{\text{eff}}$, but in reality, we deal with complicated process

$$e^+ e^- \to f^+ f^- \quad + \text{invisible} \; (n \; \gamma + \text{e}^+ \text{e}^- \text{pairs} + \cdots)$$

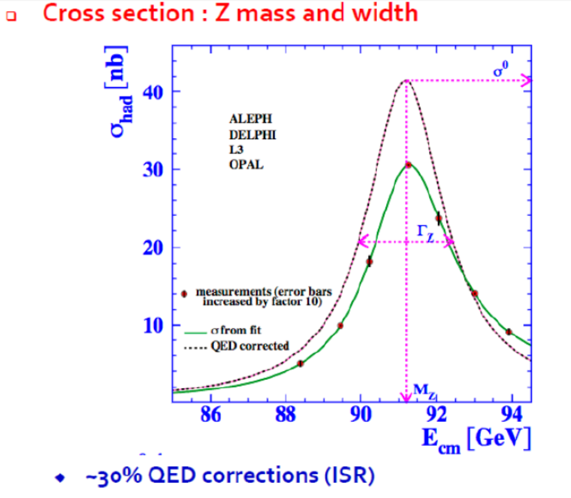
$$\sigma^{e^+e^- \to f^+f^- + \cdots}(s) = \int dx \; \overbrace{f(x)} \;\; \underbrace{\sigma^{e^+e^- \to f^+f^-}(s')} \; \delta(x - s'/s)$$

$\longrightarrow$ form factors, QED separation/deconvolution, non-factorizations, ...

To determine the structure function/flux function kernels and hard scattering ansatz for data preparation or for unfolding is one of the challenges of FCC-ee-Z physics.

QED unfolding

Altogether $17 \cdot 10^6$ Z-boson decays at LEP



How to unfold - rough scheme

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma),$$

S-matrix Ansatz in the complex energy plane

$$\mathcal{A}^{e^+e^-\to b\bar{b}} = \underbrace{\frac{R_Z}{s-s_Z} + \overbrace{\frac{R_\gamma}{s}}^{}}_{\gamma - Z\ interference}\!\!\!\!\overbrace{\phantom{\frac{R_\gamma}{s}} + S + (s-s_Z)S' + \ldots}^{Background},$$

$$s_Z = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z$$

- $R, S, S', \ldots$ are individually gauge-invariant and UV-finite - unitarity and analyticity of the S-matrix. IR-finite, when soft and collinear real photon emission is added. [Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994] [Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Awramik, Czakon, Freitas, 2006].

The term $R_\gamma(s)/s$ is part of the the background

- The poles of $\mathcal{A}$ have complex residua R_Z and R_γ.
- There is only ONE pole in mathematics, while in physics we observe two of them: photon exchange at $s = 0$, Z exchange at $s_0 = s_Z$. Mathematicaly, the appearance of the photon pole is result of summing of part of background around Z pole, $s_0 = s_Z$

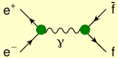
[T. Riemann, APPB 2015]

$$
\begin{aligned}
\frac{R_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s_0 - (s_0 - s)} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \; \frac{1}{s_0} \; \frac{1}{1 - \frac{s_0 - s}{s_0}} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \; \frac{1}{s_0} \; \left[1 + \frac{s_0 - s}{s_0} + \left(\frac{s_0 - s}{s_0} \right)^2 \cdots \right];
\end{aligned}
$$

QED unfolding, S-matrix approach, slide by A.Freitas, AWLC2020, pdf

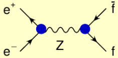
Consistent (gauge-invariant) theory setup:

Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

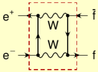


$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$



$$S = \left[\frac{1}{M_Z^2}g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\prime} + g_Z^{e\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$



$g_V^f(s)$: effective $Vf\bar{f}$ couplings

At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Current state of art: full one-loop for S, T

→ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed) — see, e.g., Bardin, Grünewald, Passarino '99

→ Sensitivity to some NP beyond EWPO

QED unfolding, ISR, slide by A.Freitas, Snowmass 2020, pdf

Z lineshape

Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

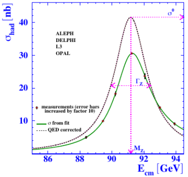
$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

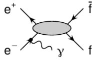
Universal ($m=n$) logs known to $n=6$,
also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools → talk by Jadach

LEP EWWG '05

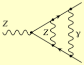




QED unfolding, FSR, slide by A.Freitas, Snowmass 2020, pdf

Factorization of massive and QED/QCD FSR:

$$\Gamma_f \approx \frac{N_c M_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma_Z'}\right]_{s=M_Z^2}$$



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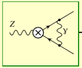
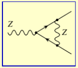
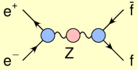


+ finite, with $\otimes =$


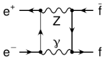
$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

g_V^f, g_A^f, Σ_Z': Electroweak corrections

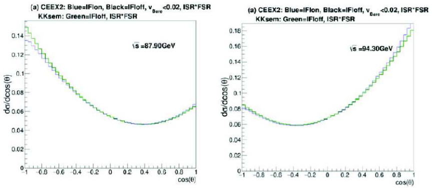


QED unfolding, IFI, slide by A.Freitas, Snowmass 2020, pdf

- Interference between ISR and FSR suppressed by Γ_Z/M_Z on Z resonance



- Still relevant for high precision an off-resonance



Jadach, Yost '18

- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost '18
Greco, Pancheri-Srivastava, Srivastava '75

EWPOs - refers to $|M|^2$; EWPPs - refers to M

Beyond Born level, one can write

$$\mathcal{M}_\gamma^{(0)}(e^-e^+ \to f^-f^+) = \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \, \gamma_\alpha \otimes \gamma^\alpha,$$

$$\mathcal{M}_Z^{(0)}(e^-e^+ \to f^-f^+) = 4ie^2 \frac{\chi_Z(s)}{s} \big[M_{vv}^{ef} \, \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha$$
$$- M_{va}^{ef} \, \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5 \big].$$

In the pole scheme, where $\bar{M}_Z$ is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2}\; 8\pi\alpha_{em}} K_Z(s) \simeq \frac{1}{1 + i\frac{\bar{\Gamma}_Z}{\bar{M}_Z}} \frac{s}{s - \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2} \, \Gamma_Z$$

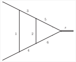
EWPOs - refers to $|M|^2$; EWPPs - refers to M

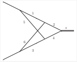
Definitions are related:

$$\bar{M}_Z \approx M_Z - \frac{1}{2}\,\frac{\Gamma_Z^2}{M_Z} \;\approx\; M_Z - 34\ \mathrm{MeV},$$
$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2}\,\frac{\Gamma_Z^3}{M_Z^2} \;\approx\; \Gamma_Z - 0.9\ \mathrm{MeV}.$$

- Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- However, at FCC-ee $\delta\Gamma_Z \sim 0.1$ MeV. Non-facotrization effects must be added properly beyond 1-loop.
- Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- At this precision it is important which parameters are taken as input parameters in schemes.

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) + a_b(s)\gamma_5] = \cdots + \overbrace{\underbrace{\text{[diagram 1]} + \text{[diagram 2]}}_{\text{planar,non−planar}}}^{\text{fermionic,bosonic}} + \cdots$$





Note approximate factorization of weak couplings

$$A_{FB} = \frac{\left[\int_0^1 dcos\theta - \int_{-1}^0 dcos\theta\right] \frac{d\sigma}{dcos\theta}}{\sigma} \sim \overbrace{\frac{2a_e v_e}{a_e^2+v_e^2}}^{A_e}\overbrace{\frac{2a_f v_f}{a_f^2+v_f^2}}^{A_f} + \text{corrections}$$

$$A_f = \frac{2\Re e\frac{v_f}{a_f}}{1+\left(\Re e\frac{v_f}{a_f}\right)^2} = \frac{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}}{1-4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}+8(Q_f\sin^2\theta_{\rm eff}^{\rm f})^2},$$

$$\sin^2\theta_{\rm eff}^{\rm f} = F\left(\Re e\frac{v_f}{a_f}\right)$$

EWPOs, Z pole

$$\sigma^0_{\rm had} = \sigma[e^+e^- \to \text{hadrons}]_{s=M_Z^2},$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$

$$R_\ell = \frac{\Gamma[Z \to \text{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau,$$

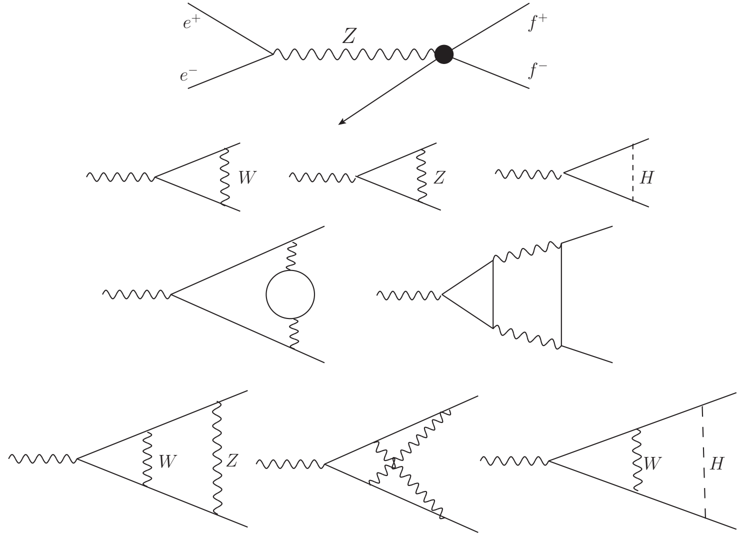
$$R_q = \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \text{hadrons}]}, \quad q = u, d, s, c, b.$$

The remaining EWPOs are cross section asymmetries, measured at the Z pole, e.g., forward-backward asymmetry

$$A^f_{\rm FB} = \frac{\sigma_f\left[\theta < \frac{\pi}{2}\right] - \sigma_f\left[\theta > \frac{\pi}{2}\right]}{\sigma_f\left[\theta < \frac{\pi}{2}\right] + \sigma_f\left[\theta > \frac{\pi}{2}\right]},$$

where θ is the scattering angle between the incoming e^- and the outgoing f.

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections



General remarks on usefulness of EWPOs

1. EWPOs encapsulate experimental data after extraction of well known and controllable QED and QCD effects, in a model-independent manner.
2. They provide a convenient bridge between real data and the predictions of the SM (or SM plus New Physics).
3. Contrary to raw experimental data (like differential crosssections), EWPOs are well suited for archiving and long term exploitation.
4. In particular archived EWPOscan be exploited over long periods of time for comparisons with steadily improving theoretical calculationsof the SM predictions, and for validations of the New Physics models beyond the SM.
5. They are also useful for comparison and combination of results from different experiments.

- In general, there are many different approaches. Which measured parameters to choose as an independent input parameters? E.g. recently Piccinini et al, Durham talk

  https://indico.cern.ch/event/801961/contributions/3361495/attachments/1823019/2982558/piccinini.pdf

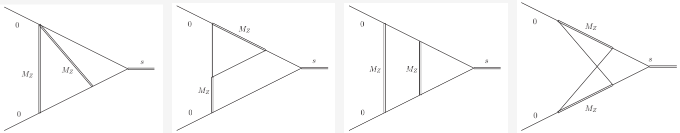
  are proposing to take for LHC $(\alpha/G_\mu, \sin^2\theta^f_{\text{eff}}, M_Z)$

  $\sin^2\theta^f_{\text{eff}}$ fixed at measured leptonic $\sin^2\theta^f_{\text{eff}}$ requiring v_l/a_l does not get radiative corrections. Procedure independent of QED corrections (both couplings get the same QED corrections and we have a ratio).

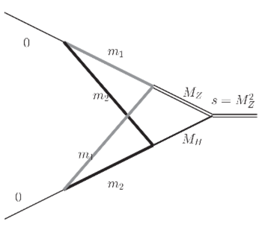
MB and SD methods are very much complementary!

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR);
  SD more useful for integrals with many internal masses

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.



2-loops $\longrightarrow$ 3-loops



$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

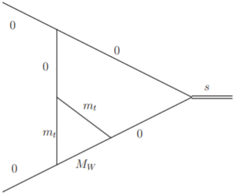
$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Towards 3-loop results (Report "1")

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \to^{(\mathbf{A})} 7 \to^{(\mathbf{B})}$ **5**	$211 \to^{(\mathbf{A})} 84 \to^{(\mathbf{B})}$ **50**
Number of diagrams	15	$2383 \to^{(\mathbf{A},\mathbf{B})}$ **1114**	$490387 \to^{(\mathbf{A},\mathbf{B})}$ **120187**
Fermionic loops	0	150	**17580**
Bosonic loops	15	**964**	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

soft7 ϵ^0:[MB - 3 dim] [SD - 5 dim], ϵ^{-1}:[MB - 2 dim] [SD - 4 dim], ϵ^{-2}:[MB - 1 dim] [SD - 3 dim]



MB	$0.060266486557699\boldsymbol{9}\,\epsilon^{-2}$	
SD - 90 Mio	$0.06026648655\boldsymbol{5}\,\epsilon^{-2}$	
MB	$(-0.031512489\boldsymbol{0}3$	$+0.18933275\boldsymbol{4}2i)\,\epsilon^{-1}$
SD - 90 Mio	$(-0.03151248\boldsymbol{1}6$	$+0.1893327\boldsymbol{6}96i)\,\epsilon^{-1}$
MB 1	$(-0.22823186751\boldsymbol{1}1$	$-0.088247945 6\boldsymbol{9}1i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.22823186751\boldsymbol{5}1$	$-0.0882479457\boldsymbol{3}9i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.228226\boldsymbol{5}3$	$-0.088245\boldsymbol{9}6i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.2281\boldsymbol{6}2$	$-0.0882\boldsymbol{0}9i) + \mathcal{O}(\epsilon)$

1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics: besides top Yukawa y_t and Higgs self-coupling λ

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions** $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

50% non-perturbative

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$
$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$
$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 0.9 \div 1.6 \times 10^{-4} \quad \text{(present : lost } 10^5 \text{ in precision!)}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad \mathtt{(FCC-ee/ILC\ requirement)}$$

LEP/SLD: $\sin^2\Theta_{\mathrm{eff}} = (1 - v_l/a_l)/4 = 0.23148 \pm 0.00017$

$\delta\Delta\alpha(M_Z) = 0.00020 \Rightarrow \delta\sin^2\Theta_{\mathrm{eff}} = 0.00007$ **;** $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

affects most precision tests and new physics searches!!!

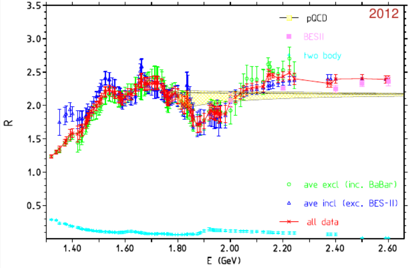
$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

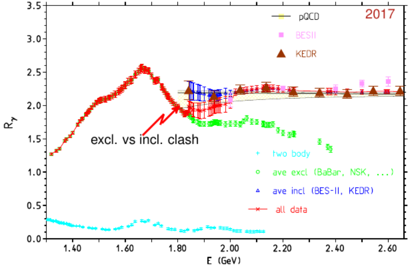
For pQCD contributions very crucial: precise QCD parameters α_s, m_c, m_b, $m_t \Rightarrow$ **Lattice-QCD**

SM precision parameters determination: $\alpha(M_Z^2)$

❐ Still an issue in HVP

❐ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty





● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Three approaches should be further explored for better error estimate

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	present	direct	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	Adler QCD 0.2%	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	via $A_{\mathrm{FB}}^{\mu\mu}$ off Z	3×10^{-5}

- **Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$**

$$A_{\mathrm{FB}}^{\mu\mu} = A_{\mathrm{FB},0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z}+\mathcal{G}}$$

where

$\gamma - Z$ interference term	$\mathcal{I} \propto \alpha(s)\,G_\mu$
Z alone	$\mathcal{Z} \propto G_\mu^2$
γ only	$\mathcal{G} \propto \alpha^2(s)$
v vector Z coupling	also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$
a axial Z coupling	sensitive to ρ-parameter (strong M_t dependence)

❑ **using v, a as measured at Z-peak**

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$

$\sigma_{\mu\mu}$:

1. the photon-exchange term, $\mathcal{G}$, proportional to $\alpha^2(s)$;
2. the Z-exchange term, $\mathcal{Z}$, proportional to G_F^2 (where G_F is the Fermi constant);
3. the Z-photon interference term, $\mathcal{I}$, proportional to $\alpha(s) \times G_F$

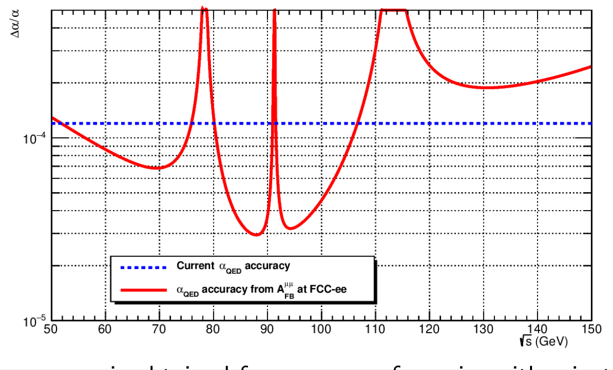
The muon forward-backward asymmetry, $A_{\mathrm{FB}}^{\mu\mu}$, is maximally dependent on the interference term

$$A_{\mathrm{FB}}^{\mu\mu} = A_{\mathrm{FB},0}^{\mu\mu} + \frac{3}{4}\frac{2}{2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}},$$

varies with $\alpha_{\mathrm{QED}}(s)$ as follows:

$$\Delta A_{\mathrm{FB}}^{\mu\mu} = \left(A_{\mathrm{FB}}^{\mu\mu} - A_{\mathrm{FB},0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$



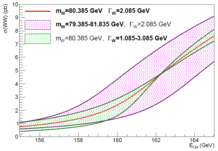
The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.

W-mass, slide by A.Freitas, Snowmass 2020, pdf

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

  $\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$

  b) Non-resonant contributions are important

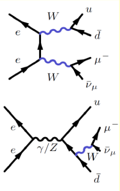


- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\text{th}}M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\text{th}}M_W \lesssim 0.6$ MeV



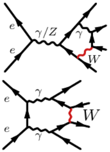
SM W-physics, FCC-ee-W, 11th FCC-ee workshop 2019, pdf

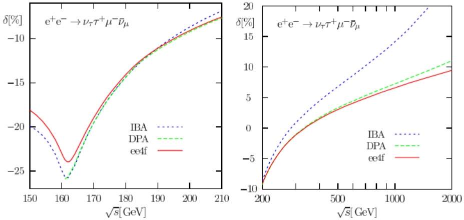
Full NLO calculation for $e^+e^- \to 4f$ (Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals

⇒ pioneering methods for six-point diagrams
now automated for LHC: `RECOLA`, `OpenLoops`, `MadLoops`

- complex mass scheme for W decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for $\sqrt{s} > 500$ GeV





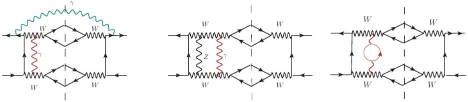
SM W-physics, FCC-ee-W

EFT expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$ (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)



- Numerical effect: $\Delta\sigma_{WW} \sim 5‰$; $[\delta M_W] \lesssim 3\,\text{MeV}$

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)			
$\sqrt{s}$ [GeV]	NLO_{EFT}	NLO_{ee4f} [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

Future improvements of theory predictions?

Implementation of state-of-the art calculations in public tools?

- NLO-EW $e^-e^+ \to 4f$ now possible with standard tools

  (RECOLA, OpenLoops, MadLoops + SHERPA, MadGraph, WHIZARD...)

  but not (yet) optimized for e^-e^+ (ISR, Beamstrahlung)

- Two-loop Coulomb-enhanced corrections for differential observables doable; (related: $t\bar{t}$ with Coulomb resummation in WHIZARD)

  (no guarantee of formal accuracy for general distributions)

Full NNLO in EFT for total cross section

- Soft $\log \beta$ terms can be adapted from QCD results
- NNLO $\log(m_e/M_W)$ terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections

  (from amplitudes for $e^+e^- \to W^+W^-$ at threshold: border of current capabilities)

  resulting uncertainty from cross-section calculation

  $$\Delta\sigma^{(2}_{\text{hard}} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)‰ \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

Full NNLO for $e^+e^- \to 4f$: completely new methods needed

SM W-physics, M. Skrzypek, FCC-ee-W: hybrid approach, 11th FCC-ee workshop 2019, pdf

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Conclusions and outlook

- KoralW+YFSWW3: LEP2 precision is 0.5%.
  Factor of 20 $\div$ 50 improvement is needed for FCCee
- Lesson from LEP2: be pragmatic, split into Double- and Single-Pole, pick only numerically dominant terms:
  - $\mathcal{O}(\alpha^1)$ for $e^- e^+ \to 4f$ must be implemented in MC with explicit split into Double Pole and Single Pole. Calculations exist
  - $\mathcal{O}(\alpha^2)_{DP}$ calculations for the Double-Pole production and decay parts are needed! Feasible?
  - $\mathcal{O}(\alpha^2)_{SP}$ and $\mathcal{O}(\alpha^3)$ seem to be negligible
- More detailed analysis at the threshold may be instrumental
  - EFT methods promising, but for now inclusive results only
  - Non-factorizable soft interferences can be exponentiated within YFS scheme. How much of the higher order corrs. would be reproduced this way?

The overall precision tag $\sim 2 \times 10^{-4}$ feasible (?)

YFSWW3$\oplus$KoralW with new exponentiation
look like a good starting point

QED unfolding, ISR, slide by A.Freitas, Snowmass 2020, pdf

- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV could be achievable

- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp}$	$[20\text{ MeV}]_{\rm exp}$	
$\oplus\,[50\text{ MeV}]_{\rm QCD}$	$\oplus\,[30\text{ MeV}]_{\rm QCD}$	(h.o. resummation)
$\oplus\,[10\text{ MeV}]_{\text{mass def.}}$	$\oplus\,[10\text{ MeV}]_{\text{mass def.}}$	
$\oplus\,[70\text{ MeV}]_{\alpha_s}$	$\oplus\,[15\text{ MeV}]_{\alpha_s}$	($\delta\alpha_s \lesssim 0.0002$)
> 100 MeV	$\lesssim 50$ MeV	

Conclusions

- Top pair threshold scan allows precise mass determination

$$\Delta m_t < 100\,\text{MeV}$$

- Theory-dominated error, $\sim 3\%$ QCD scale uncertainty
- Known corrections:
  - N^3LO QCD + Higgs
  - N^2LO electroweak + non-resonant
  - LL initial state radiation
- All corrections included in version 2 of `QQbar_threshold`
  `https://qqbarthreshold.hepforge.org/`

SM FCC-ee-t, Daniel Samitz, shower cuts dependence, 11th FCC-ee workshop 2019, pdf

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MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of $\mathbf{m_t^{MC}}$?

- picture of "top quark particle" does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
  $\rightarrow$ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\mathrm{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\mathrm{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...