# Combined effect of Beam-Beam and Impedance

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FCC Week 2022

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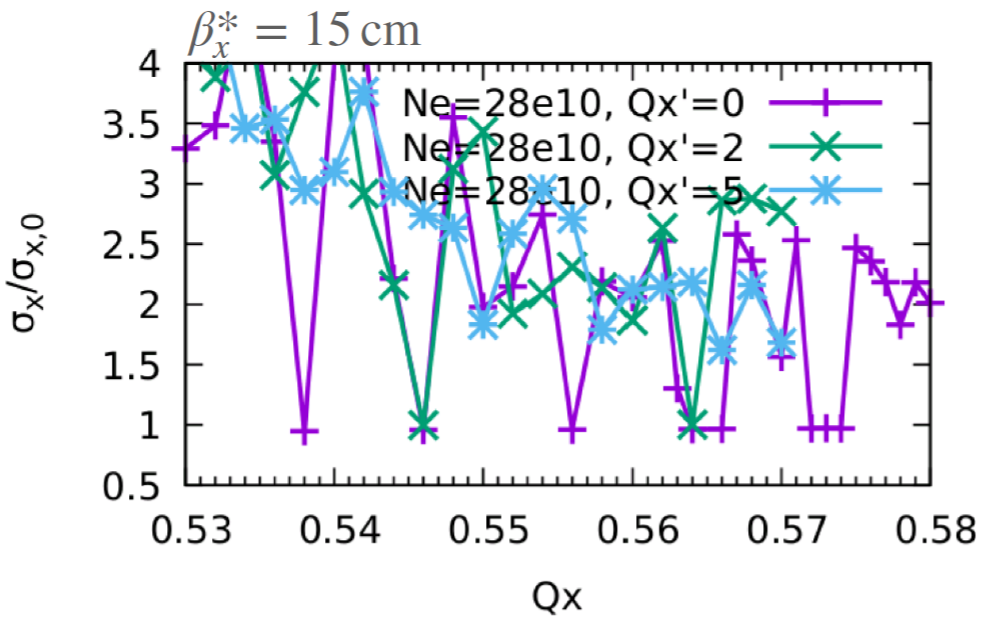
# Outline

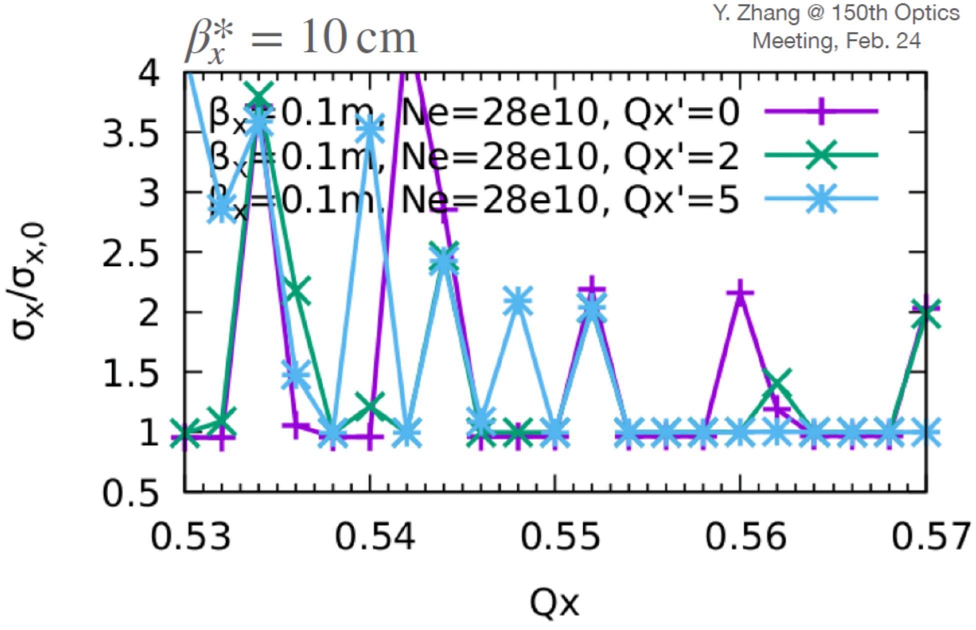
- Simulation with Longitudinal Impedance
- Introduction of Analysis work (BB+ZL)
- Simulation with both longitudinal and transverse impedance
  - FCCee
  - Other Machines (CEPC/SuperKEKB)
- Initial analysis work (BB+ZT)
- Summary

K. Oide, Mar. 17, 2022 @151st FCC-ee Optics Design Meeting
& 22nd FCCIS WP2.2 Meeting

# Parameters Update

- It has been pointed out by D. Shatilov, Y. Zhang, M. Zobov that the $\beta_x^* = 15$ cm optics in Nov. 2021 for Z does not have stable tune space against the coherent beam-beam instability including long. impedances.
  - The CDR has a $\beta_x^* = 10$ cm optics for Z.
- Then let us make a $\beta_x^* = 10$ cm optics and examine its performance.




Y. Zhang @ 150th Optics Meeting, Feb. 24

FUTURE CIRCULAR COLLIDER

# Parameters

$\beta_x^* = 10\,\text{cm}$ @Z

| Beam energy | [GeV] | 45.6 | 80 | 120 | 182.5 |
|---|---|---|---|---|---|
| Layout | | PA31-1.0 | | | |
| # of IPs | | 4 | | | |
| Circumference | [km] | 91.174117 | | 91.174107 | |
| Bending radius of arc dipole | [km] | 9.937 | | | |
| Energy loss / turn | [GeV] | 0.0391 | 0.370 | 1.869 | 10.0 |
| SR power / beam | [MW] | 50 | | | |
| Beam current | [mA] | 1280 | 135 | 26.7 | 5.00 |
| Bunches / beam | | 10000 | 880 | 248 | 40 |
| Bunch population | [$10^{11}$] | 2.43 | 2.91 | 2.04 | 2.37 |
| Horizontal emittance $\varepsilon_x$ | [nm] | 0.71 | 2.16 | 0.64 | 1.49 |
| Vertical emittance $\varepsilon_y$ | [pm] | 1.42 | 4.32 | 1.29 | 2.98 |
| Arc cell | | Long 90/90 | | 90/90 | |
| Momentum compaction $\alpha_p$ | [$10^{-6}$] | 28.5 | | 7.33 | |
| Arc sextupole families | | 75 | | 146 | |
| $\beta^*_{x/y}$ | [mm] | 100 / 0.8 | 200 / 1.0 | 300 / 1.0 | 1000 / 1.6 |
| Transverse tunes/IP $Q_{x/y}$ | | 53.563 / 53.600 | | 100.565 / 98.595 | |
| Energy spread (SR/BS) $\sigma_\delta$ | [%] | 0.038 / 0.132 | 0.069 / 0.154 | 0.103 / 0.185 | 0.157 / 0.219 |
| Bunch length (SR/BS) $\sigma_z$ | [mm] | 4.38 / 15.4 | 3.55 / 8.01 | 3.34 / 6.00 | 2.00 / 2.80 |
| RF voltage 400/800 MHz | [GV] | 0.120 / 0 | 1.0 / 0 | 2.08 / 0 | 4.0 / 7.25 |
| Harmonic number for 400 MHz | | 121648 | | | |
| RF freuqeuncy (400 MHz) | MHz | 399.994581 | | 399.994627 | |
| Synchrotron tune $Q_s$ | | 0.0370 | 0.0801 | 0.0328 | 0.0826 |
| Long. damping time | [turns] | 1168 | 217 | 64.5 | 18.5 |
| RF acceptance | [%] | 1.6 | 3.4 | 1.9 | 3.1 |
| Energy acceptance (DA) | [%] | ±1.3 | ±1.3 | ±1.7 | -2.8 +2.5 |
| Beam-beam $\xi_x/\xi_y$ [a] | | 0.0023 / 0.135 | 0.011 / 0.125 | 0.014 / 0.131 | 0.091 / 0.139 |
| Luminosity / IP | [$10^{34}$/cm$^2$s] | 182 | 19.4 | 7.26 | 1.24 |
| Lifetime (q + BS) | [sec] | – | | 1065 | 5090 |
| Lifetime (lum) | [sec] | 1129 | 1070 | 596 | 752 |

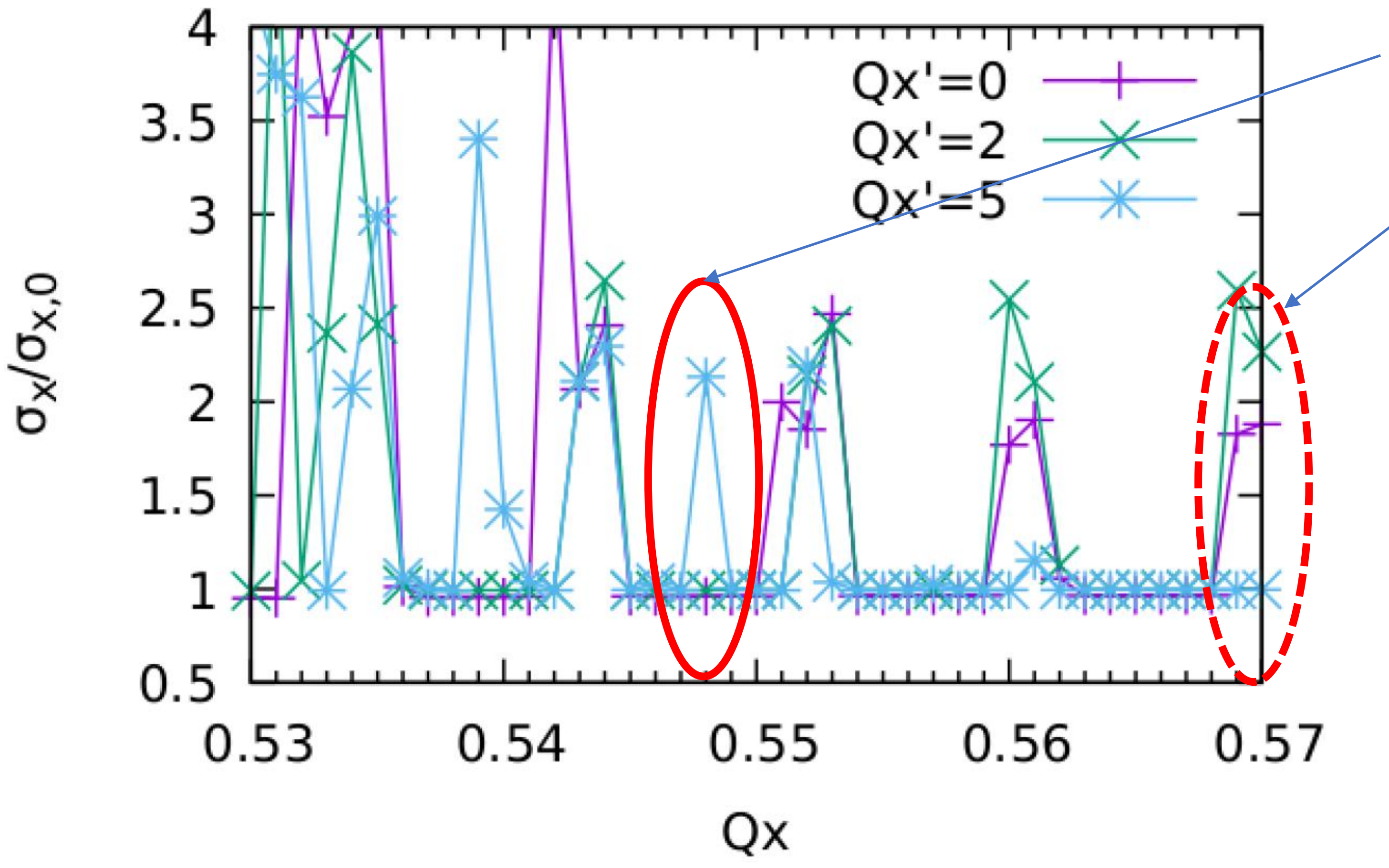
[a] incl. hourglass

$\beta_x^* = 15\,\text{cm}$ (Nov . 29)

| Beam energy | [GeV] | 45.6 | 80 | 120 | 182.5 |
|---|---|---|---|---|---|
| Layout | | PA31-1.0 | | | |
| # of IPs | | 4 | | | |
| Circumference | [km] | 91.174117 | | 91.174107 | |
| Bending radius of arc dipole | [km] | 9.937 | | | |
| Energy loss / turn | [GeV] | 0.0391 | 0.370 | 1.869 | 10.0 |
| SR power / beam | [MW] | 50 | | | |
| Beam current | [mA] | 1280 | 135 | 26.7 | 5.00 |
| Bunches / beam | | 9600 | 880 | 248 | 36 |
| Bunch population | [$10^{11}$] | 2.53 | 2.91 | 2.04 | 2.64 |
| Horizontal emittance $\varepsilon_x$ | [nm] | 0.71 | 2.16 | 0.64 | 1.49 |
| Vertical emittance $\varepsilon_y$ | [pm] | 1.42 | 4.32 | 1.29 | 2.98 |
| Arc cell | | Long 90/90 | | 90/90 | |
| Momentum compaction $\alpha_p$ | [$10^{-6}$] | 28.5 | | 7.33 | |
| Arc sextupole families | | 75 | | 146 | |
| $\beta^*_{x/y}$ | [mm] | 150 / 0.8 | 200 / 1.0 | 300 / 1.0 | 1000 / 1.6 |
| Transverse tunes/IP $Q_{x/y}$ | | 53.563 / 53.600 | | 100.565 / 98.595 | |
| Energy spread (SR/BS) $\sigma_\delta$ | [%] | 0.039 / 0.130 | 0.069 / 0.154 | 0.103 / 0.185 | 0.157 / 0.229 |
| Bunch length (SR/BS) $\sigma_z$ | [mm] | 4.37 / 14.5 | 3.55 / 8.01 | 3.34 / 6.00 | 2.02 / 2.95 |
| RF voltage 400/800 MHz | [GV] | 0.120 / 0 | 1.0 / 0 | 2.08 / 0 | 4.0 / 7.25 |
| Harmonic number for 400 MHz | | 121648 | | | |
| RF freuqeuncy (400 MHz) | MHz | 399.994581 | | 399.994627 | |
| Synchrotron tune $Q_s$ | | 0.0370 | 0.0801 | 0.0328 | 0.0826 |
| Long. damping time | [turns] | 1168 | 217 | 64.5 | 18.5 |
| RF acceptance | [%] | 1.6 | 3.4 | 1.9 | 3.1 |
| Energy acceptance (DA) | [%] | ±1.3 | ±1.3 | ±1.7 | -2.8 +2.5 |
| Beam-beam $\xi_x/\xi_y$ [a] | | 0.0040 / 0.152 | 0.011 / 0.125 | 0.014 / 0.131 | 0.096 / 0.151 |
| Luminosity / IP | [$10^{34}$/cm$^2$s] | 189 | 19.4 | 7.26 | 1.33 |
| Lifetime (q + BS) | [sec] | – | | 1065 | 2405 |
| Lifetime (lum) | [sec] | 1089 | 1070 | 596 | 701 |

[a] incl. hourglass

# Effect of Chromaticity on X-Z instability




- Non-zero tune chromaticity bring new resonance
- In the high order resonance region (0.5+n*nus), some resonance may be suppressed

Future work：

- Analysis work considering linear tune chromaticity
- Simulation work considering realistic chromaticity (from lattice model)

Courtesy of Chuntao Lin

# PWD with Longitudinal Impedance

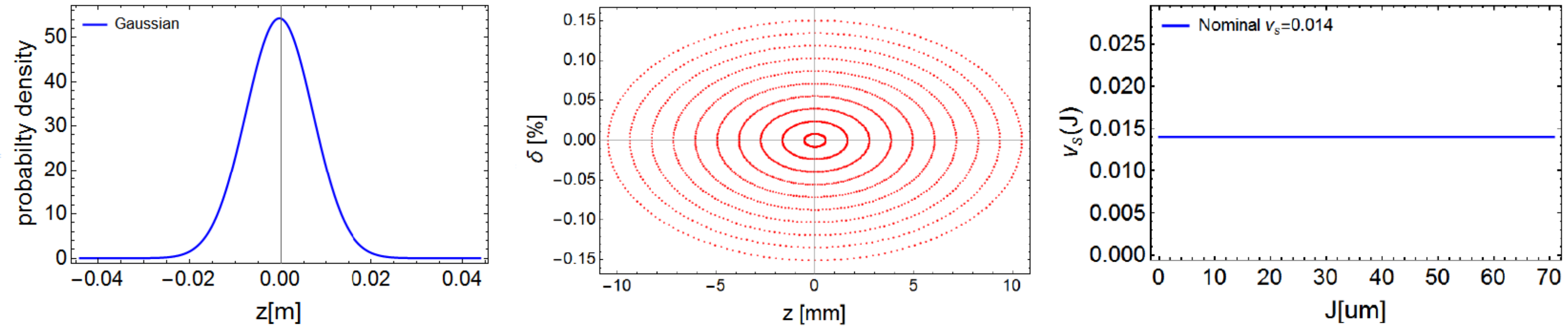
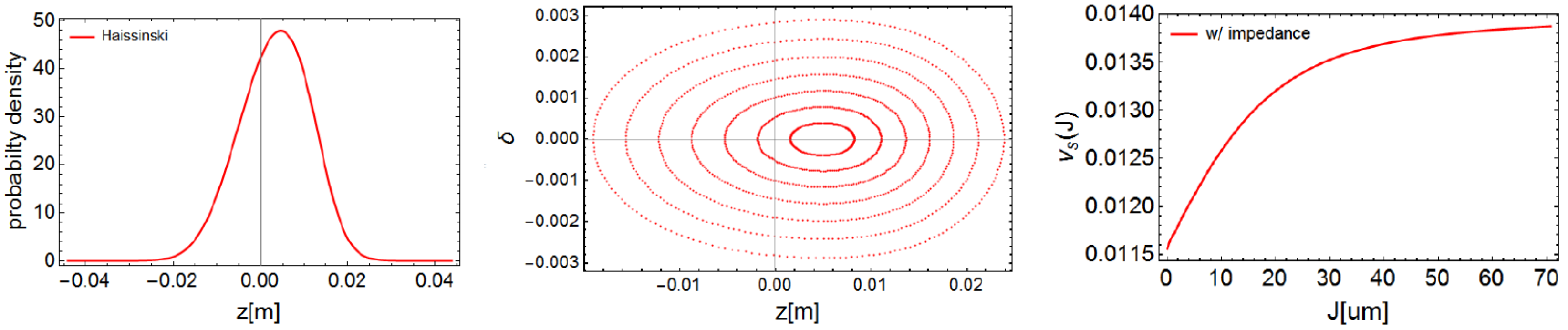



Figure: Longitudinal beam dynamics without longitudinal impedance



# Analysis **without** PWD

- Azimuthal and Radial Mode Expansion:

$$x(J,\phi;t) = \sum_{l,k} x_{kl}(t)\sqrt{\frac{k!}{(|l|+k)!}}\hat{J}^{|l|/2}L_k^{(|l|)}(\hat{J})e^{il\phi}$$

$$p_x(J,\phi;t) = \sum_{l,k} p_{kl}(t)\sqrt{\frac{k!}{(|l|+k)!}}\hat{J}^{|l|/2}L_k^{(|l|)}(\hat{J})e^{il\phi}$$

- Synchrotron-betatron motion of dipole moment vector $(x_{kl}, p_{kl})$ in Arc

$$M_0 = e^{-2\pi i l \nu_z}\delta_{kk'}\delta_{ll'}\begin{pmatrix}\cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x\end{pmatrix}.$$

For example,
$k_{\max} = 80,\ l_{\max} = 8$

- Momentum Kick due to localized Wake force(beam-beam)

$$M_W = \begin{pmatrix}1 & 0 \\ -2M_{klk'l'} & 1.\end{pmatrix} \qquad \Delta p_{kl}(t) = -2\sum_{k'l'} M_{klk'l'}x_{k'l'}(t).$$

$$M_{k\ell,k'\ell'} = \pm\frac{\beta_x}{2}i^{l-l'-1}\int_{-\infty}^{\infty} d\omega Z(\omega)g_{kl}(\omega)g_{k'l'}(\omega),$$

where $g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}}\left(\frac{\omega\sigma}{\sqrt{2}c}\right)^{|l|+2k} e^{-\omega^2\sigma^2/2c^2}.$

- Stability analysis of $M_0 M_W$

C. Lin, K. Ohmi and Y. Zhang, PRAB 25, 011001 (2022)

# Analysis **with** PWD

- Azimuthal Mode Expansion

$$x(J,\phi)=\sum_{l=-\infty}^{\infty} x_l(J)e^{il\phi},\quad p_x(J,\phi)=\sum_{l=-\infty}^{\infty} p_l(J)e^{il\phi}$$

- Synchro-betatron motion in Arc

$$\begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} = e^{-2\pi i l \nu_s(J)} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix}$$

- Action Discretization

we truncate $l$ at $\pm l_{max}$, and discretize $J$ at $J_1, J_2, \ldots, J_{n_J}$.

For example, $n_J = 40$ , $l_{\max} = 8$

$$\begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} = e^{-2\pi i l \nu_s(J_i)} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix}$$

- Momentum Kick due to localized Wake force(beam-beam)

$$M_W = \begin{pmatrix} 1 & 0 \\ \beta_x M_{lil'i'} & 1 \end{pmatrix}.$$

$$\Delta p_l(J_i) = \mp\frac{\beta_x}{2\pi}\sum_{l'}\sum_{i'}\Delta J_{i'} W_{ll'}(J_i, J_{i'})\,\psi(J_{i'})\,x_{l'}(J_{i'}) \equiv \beta_x M_{lil'i'} x_{l'}(J_{i'}).$$

- Stability analysis of $M_0 M_W$

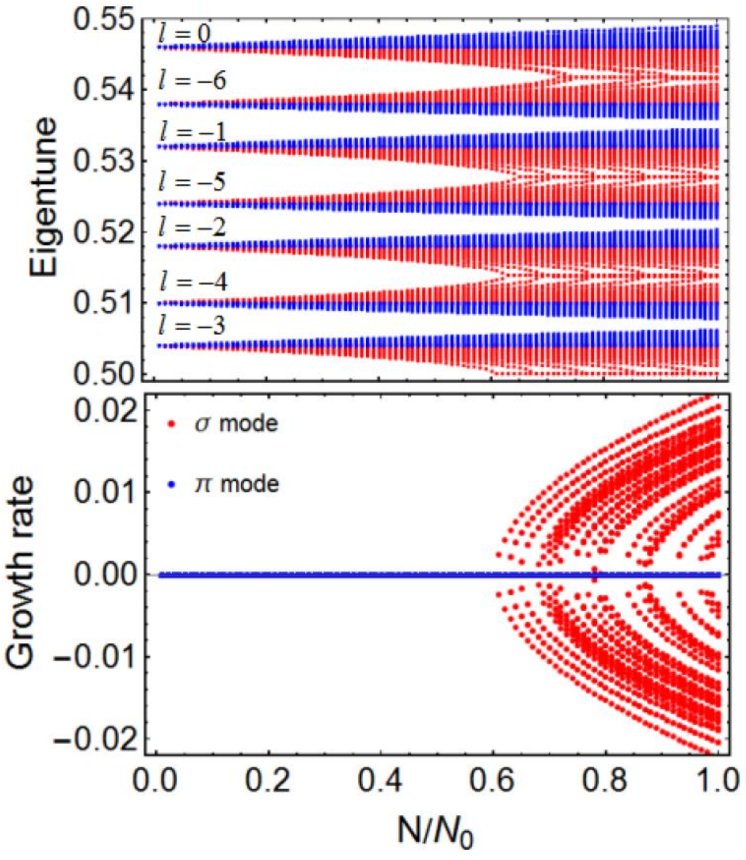
$$W_{ll'}(J_i, J_{i'}) = \sum_j\sum_{j'} e^{-il\phi_j + il'\phi_{j'}} W_x\left(z(J_i,\phi_j) - z(J_{i'},\phi_{j'})\right)\Delta\phi_j\Delta\phi_{j'}.$$

C. Lin, K. Ohmi and Y. Zhang, PRAB 25, 011001 (2022)

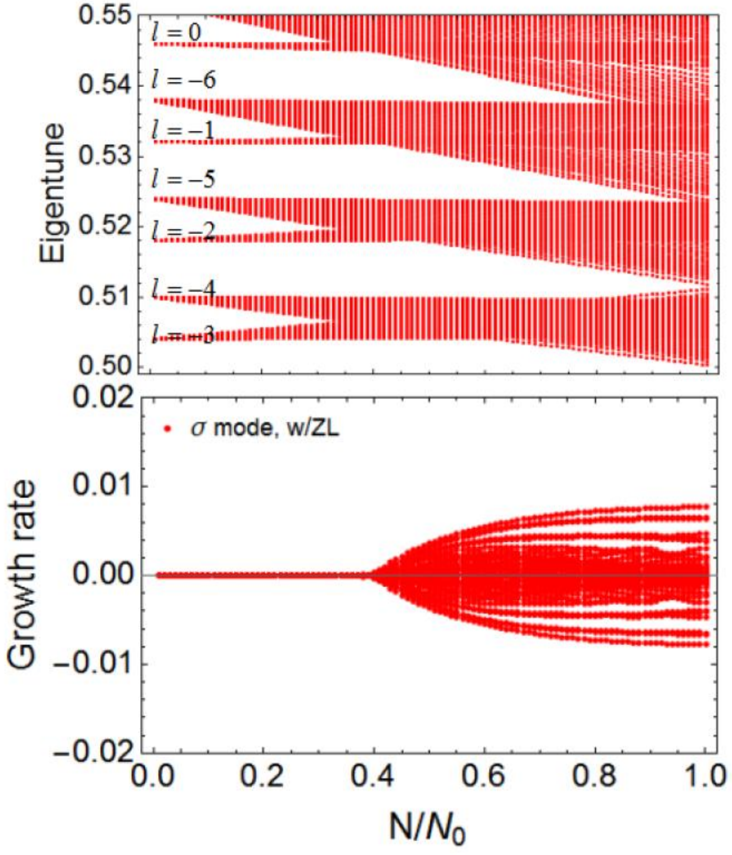
$\nu_x$=0.546, CEPC CDR

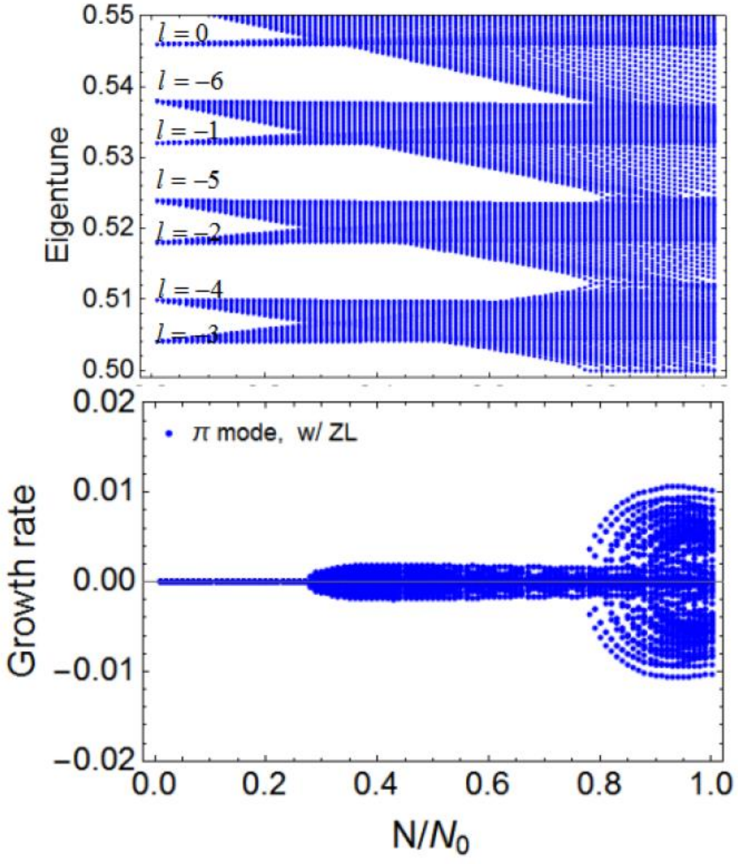
# Eigen-Mode Analysis w/o and w/ ZL

w/o ZL




w/ ZL

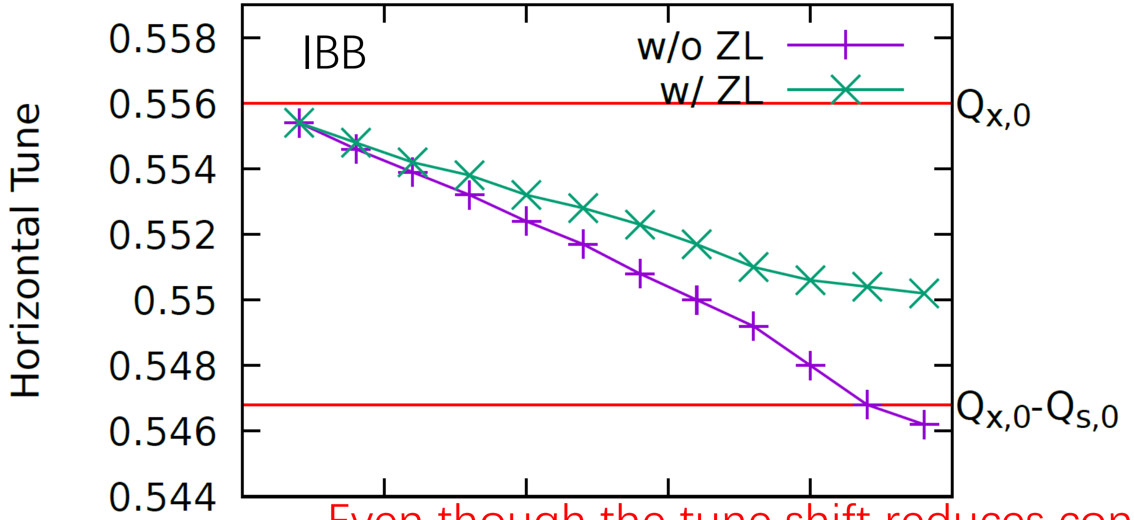


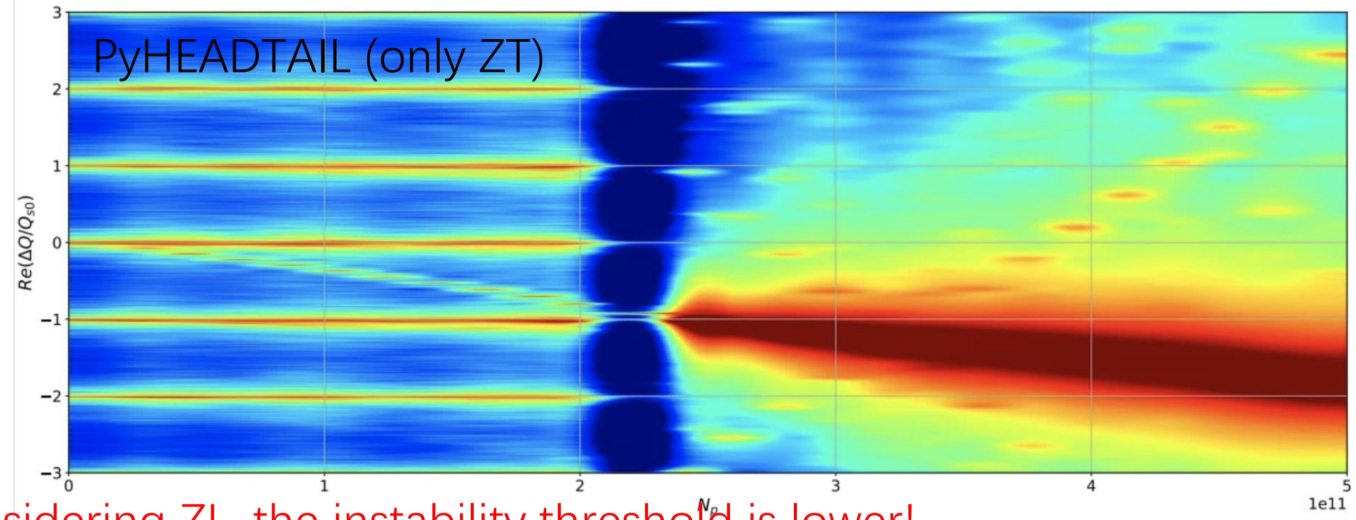
# Frequency Analysis of Single Beam with Transverse Impedance (ZT)

Courtesy of Mauro Migliorati

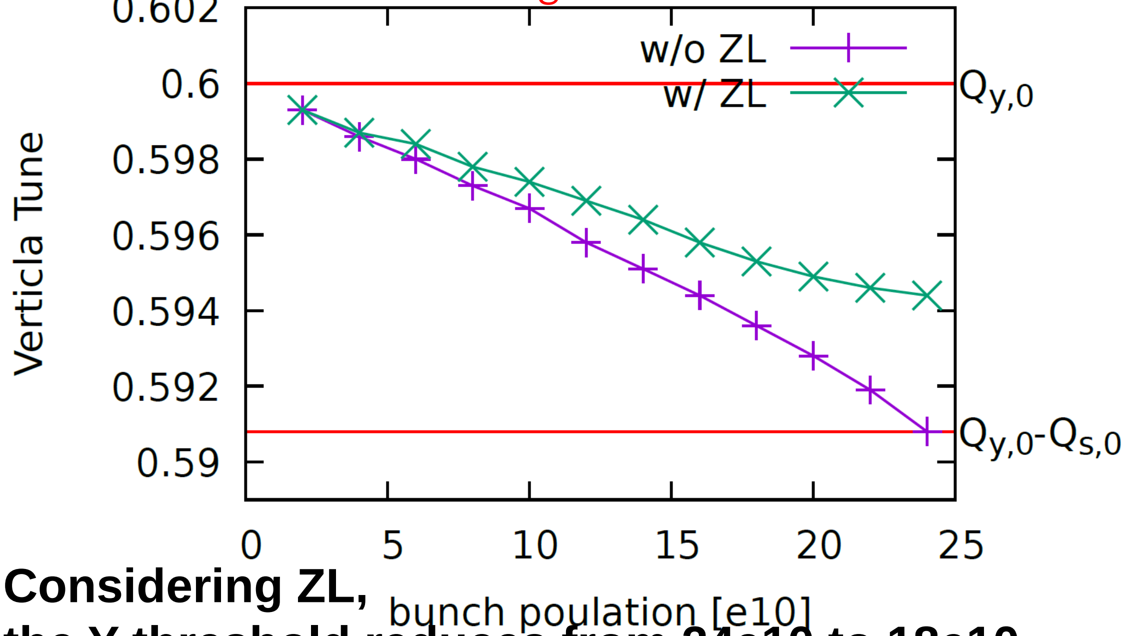
**Considering ZL,**
**the X threshold reduces from 22e10 to 18e10.**

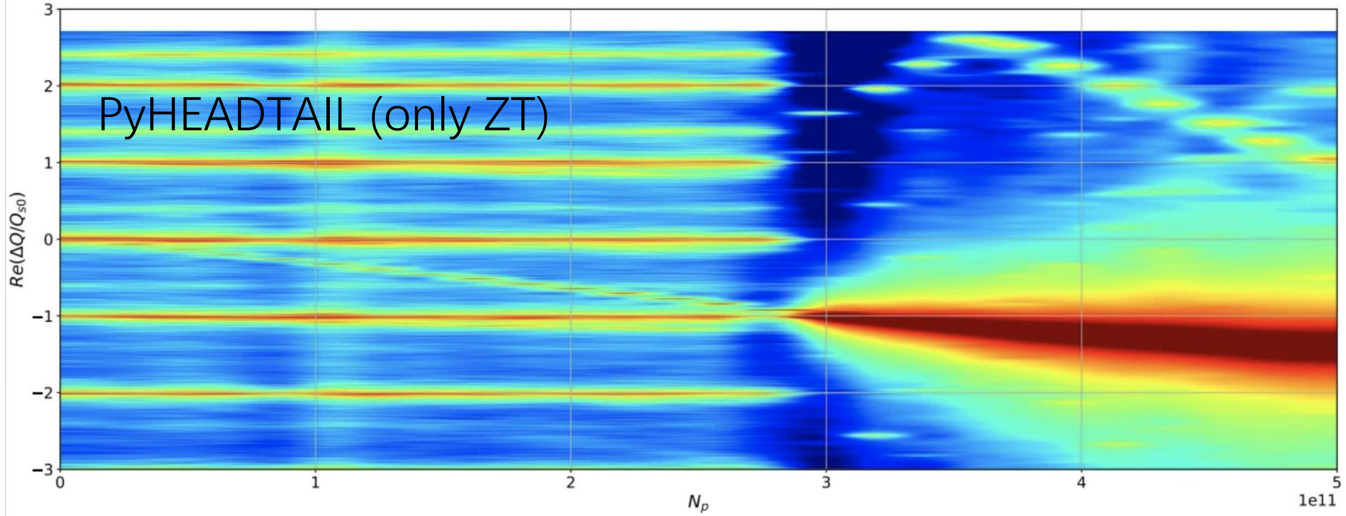



**Considering ZL,**
**the X threshold reduces from 22e10 to 18e10.**




Even though the tune shift reduces considering ZL, the instability threshold is lower!



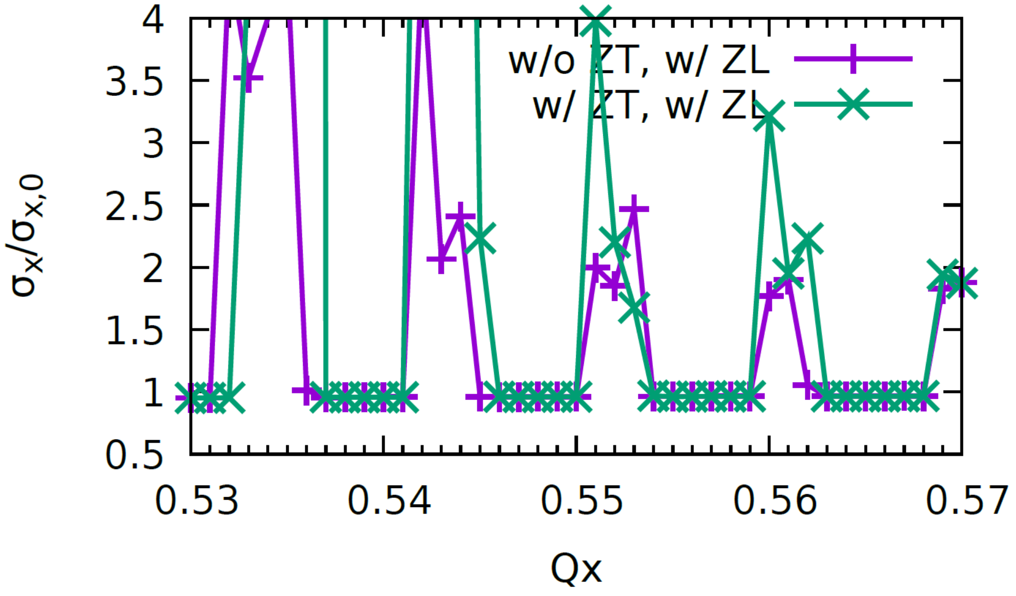



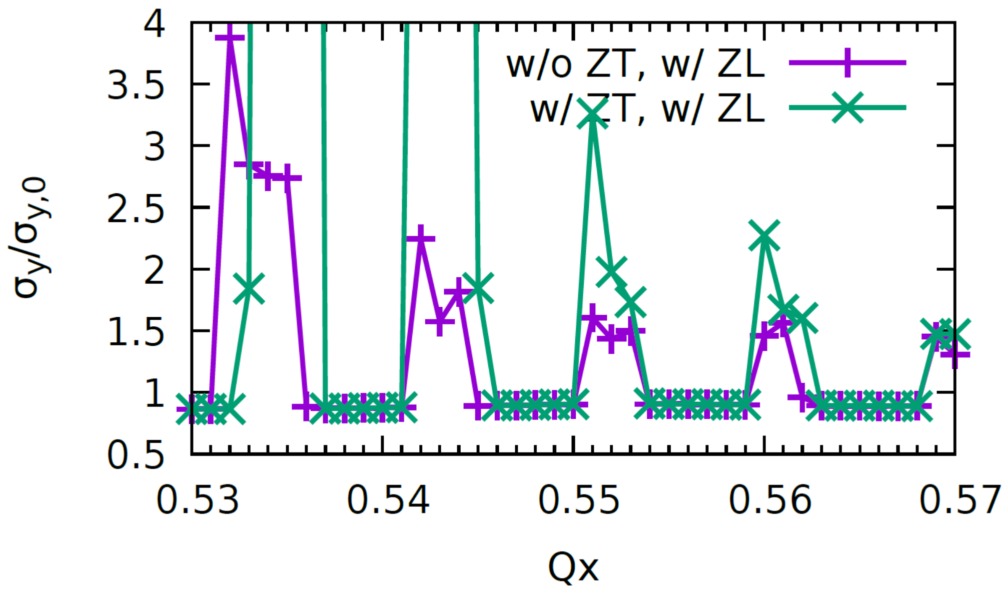

**Considering ZL,**
**the Y threshold reduces from 24e10 to 18e10.**

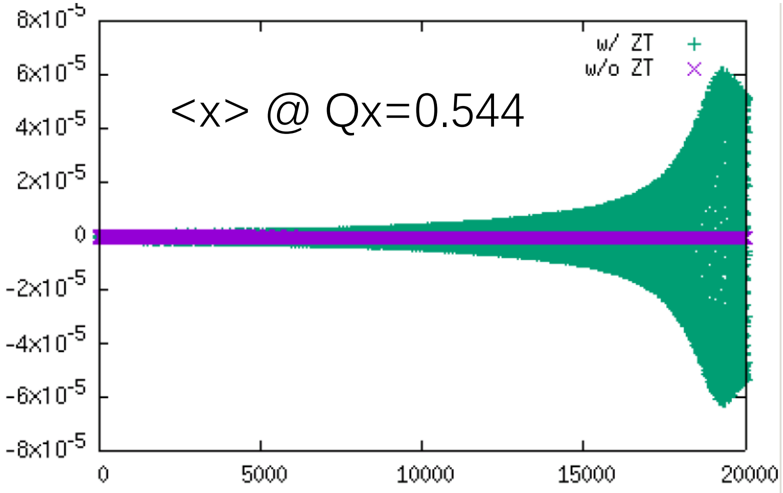
**Considering ZL,**
**the Y threshold reduces from 30e10 to 20e10.**

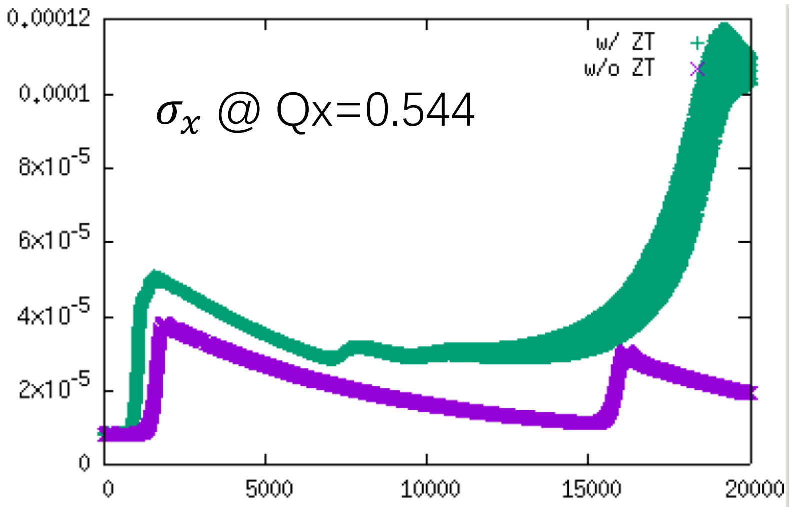
- no clear effect in stability region considering the present transverse impedance.
- **Combined X-Z instability and TMCI instability**

# Collision Stability considering ZT



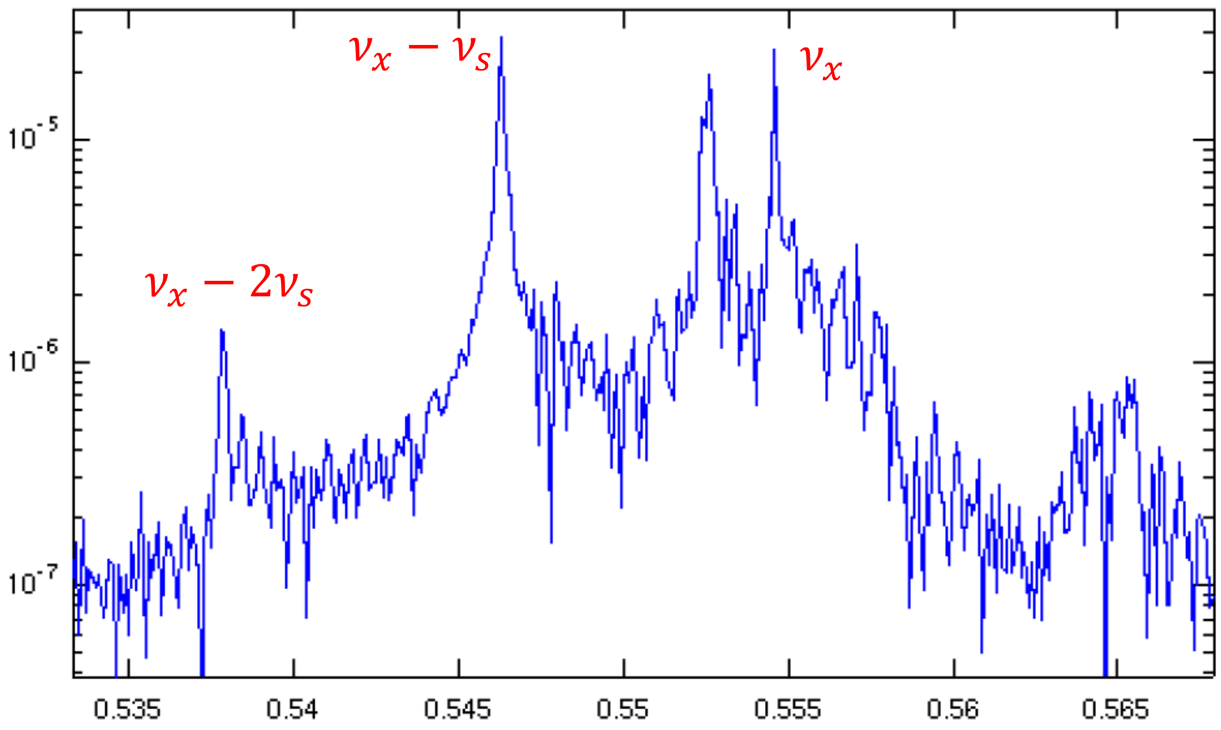



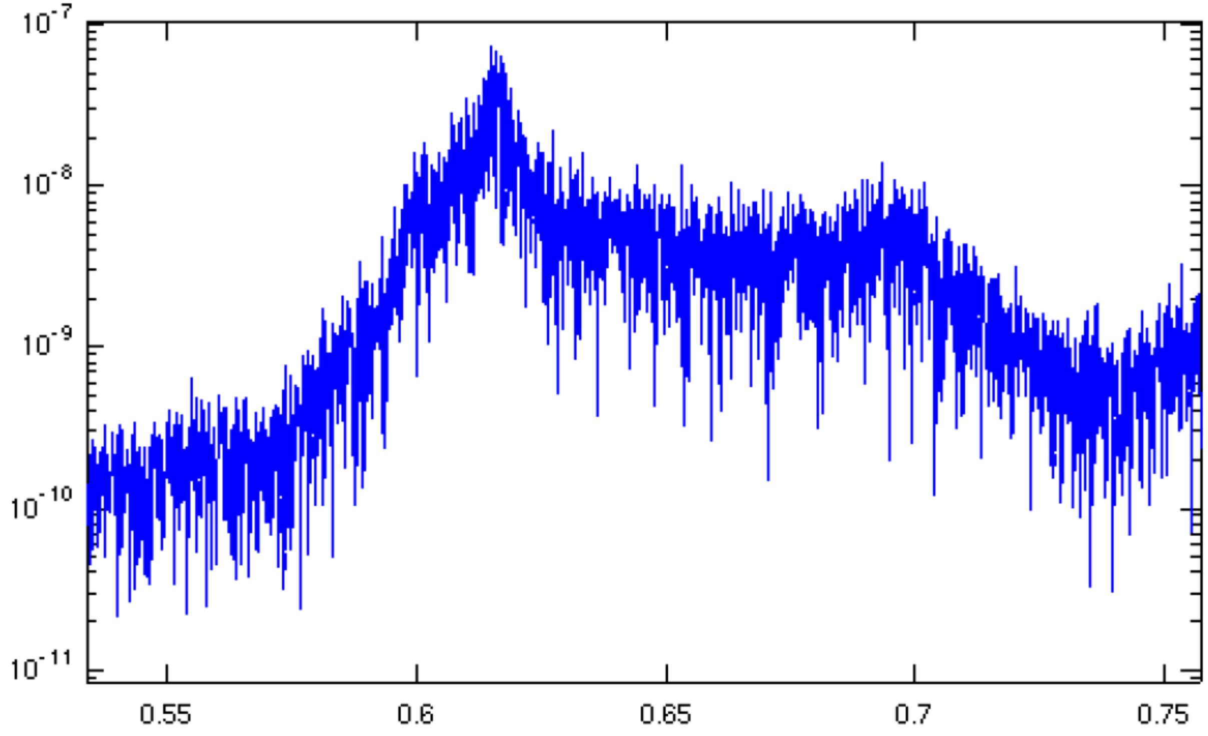



# Dipople Spectrum@Qx0=0.556/Qy0=0.60 (stable)

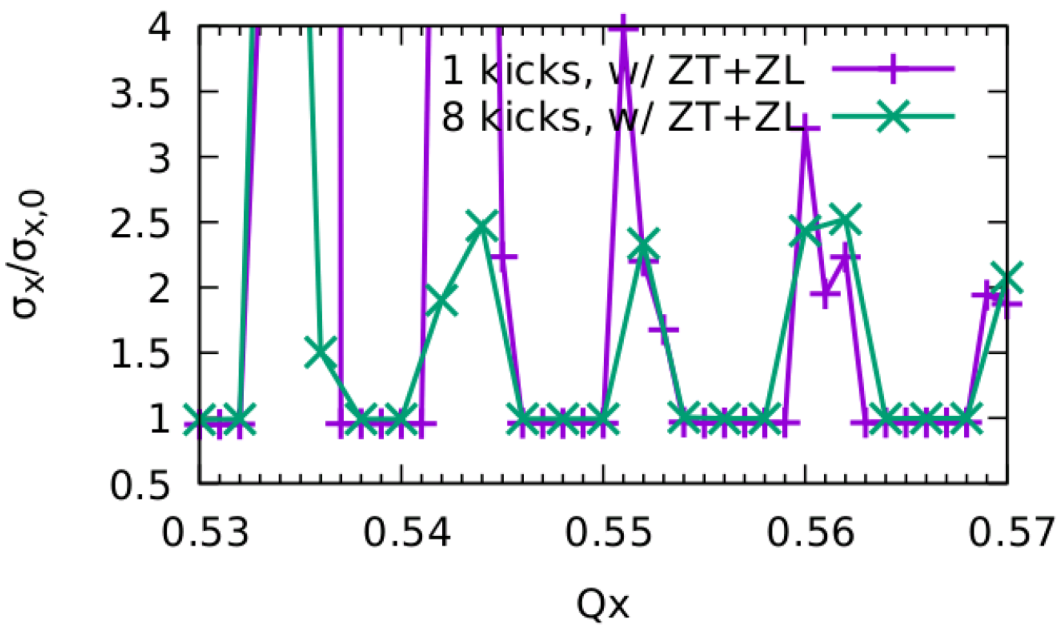
- There does not exist mode coupling in horizontal direction

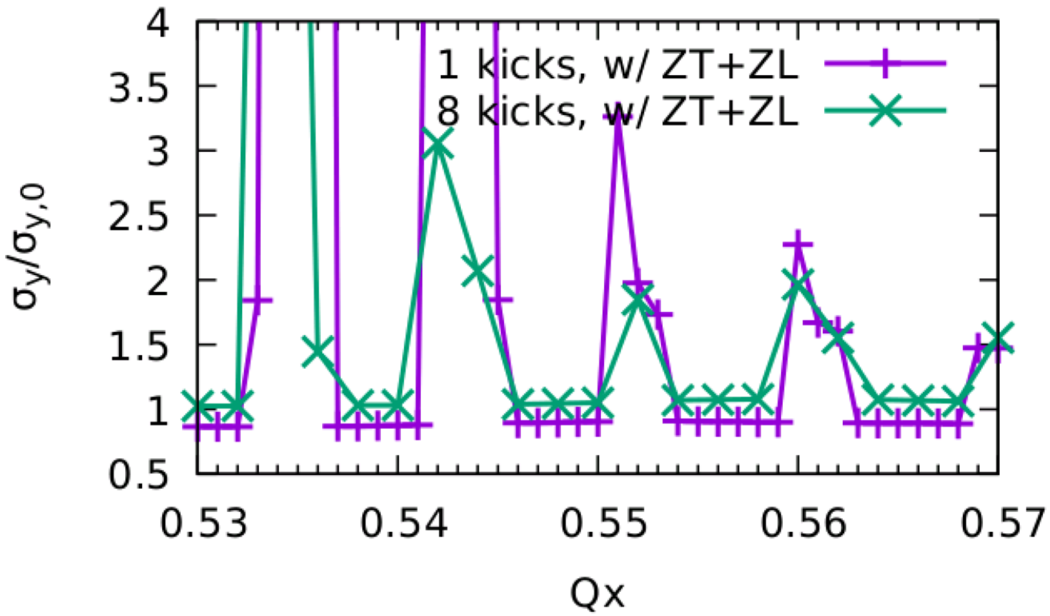


# Distribute ZT at 8 positions

- There does not exist any clear difference between 1 kick and 8 kicks of transverse impedance

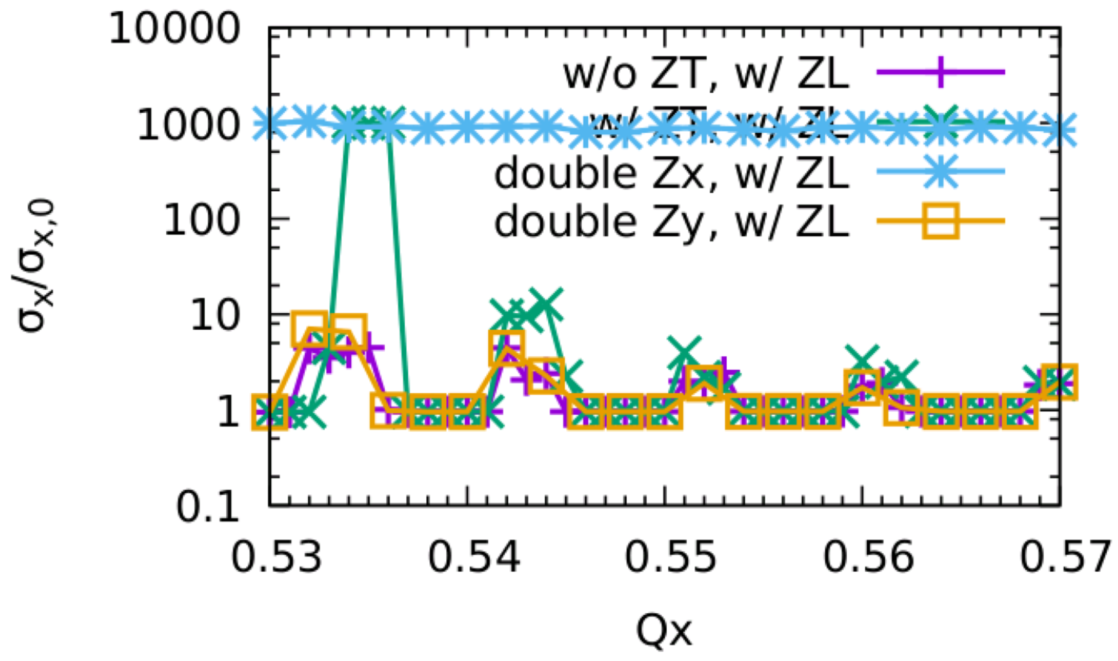


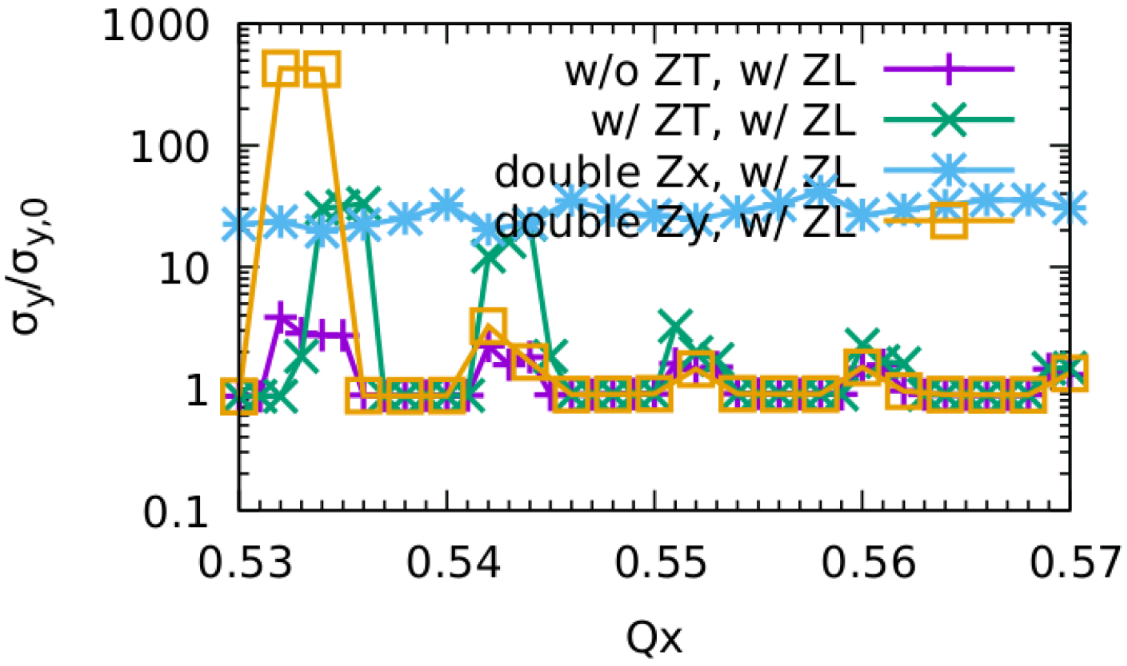




It is believed that the difference of equilibrium $\sigma_y$ does not come from the kick number, but come from the model of ring (local RF cavity vs smooth ring)

# If we double ZT,

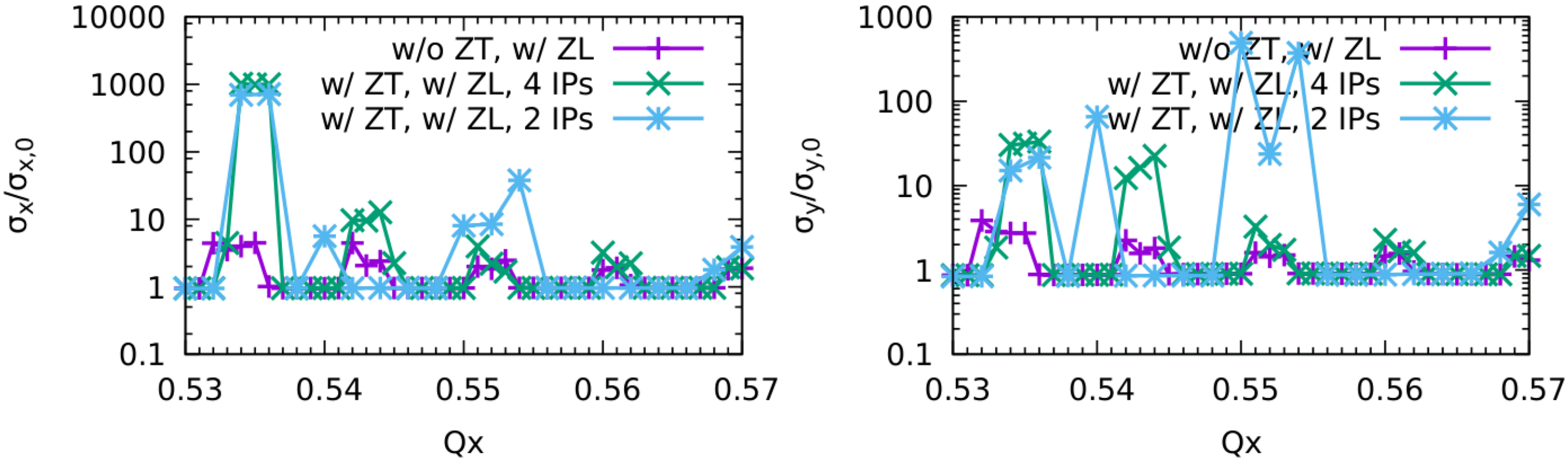
- There does not exist stable tune area
- It seems the TMCI instead of X-Z instability dominates



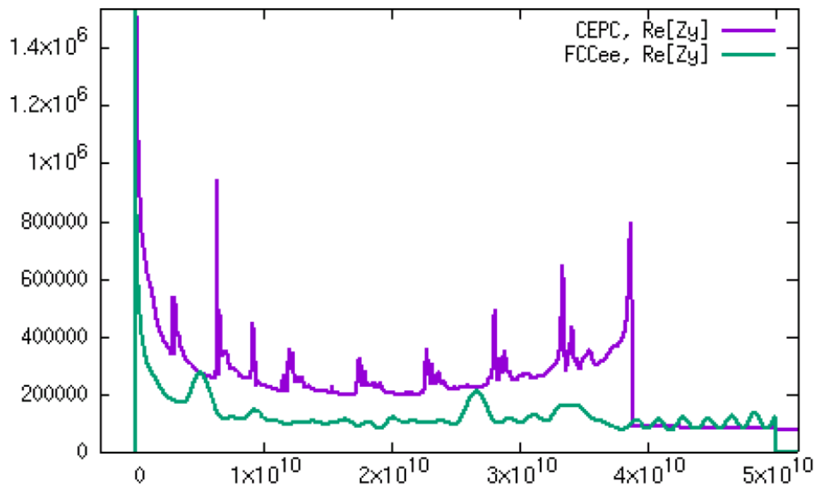



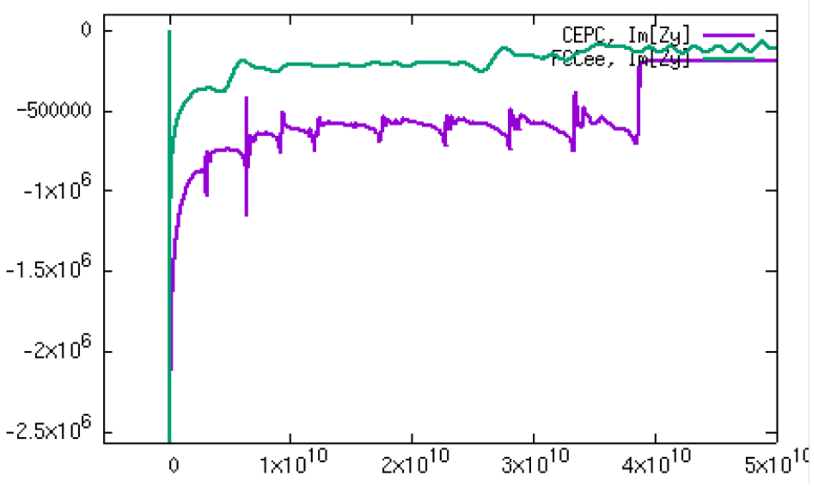
# If there exist 2 IPs instead of 4

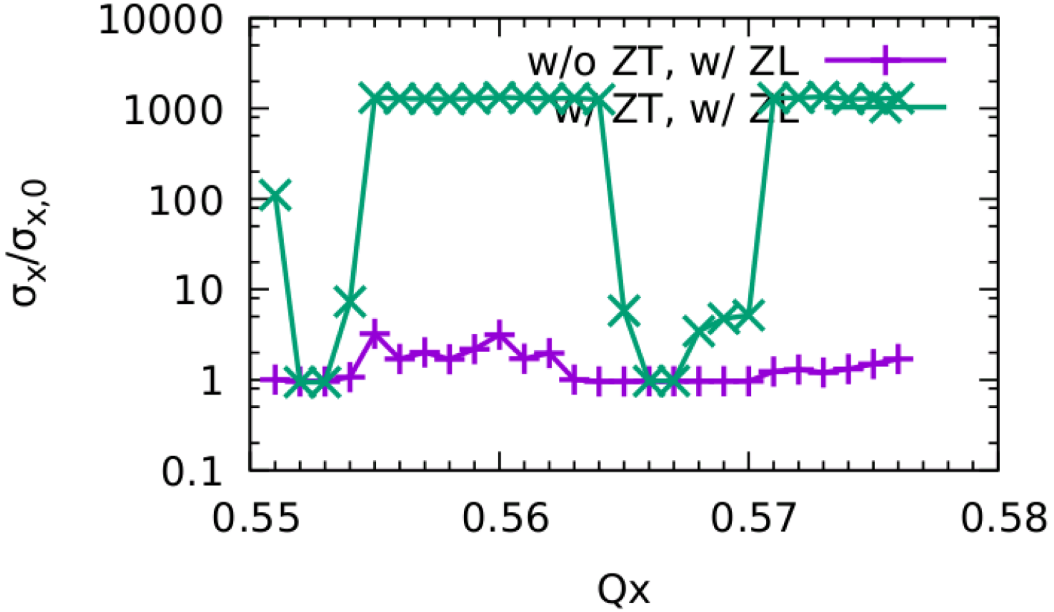
- There exist large enough stable tune area. The machine still work
- X: combined effect of X-Z instability and TMCI
- Y: TMCI like instability

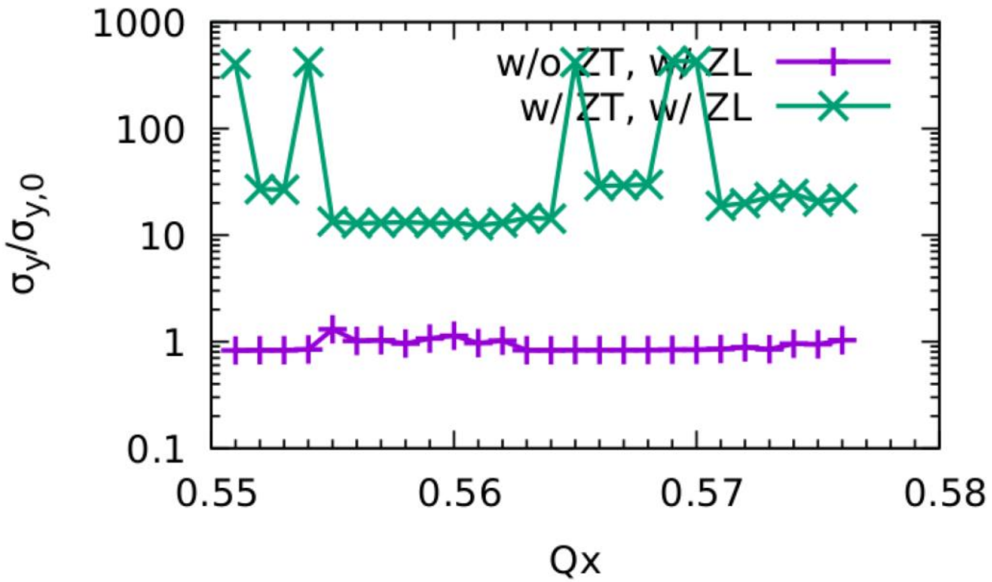


# What happens at CEPC considering ZT

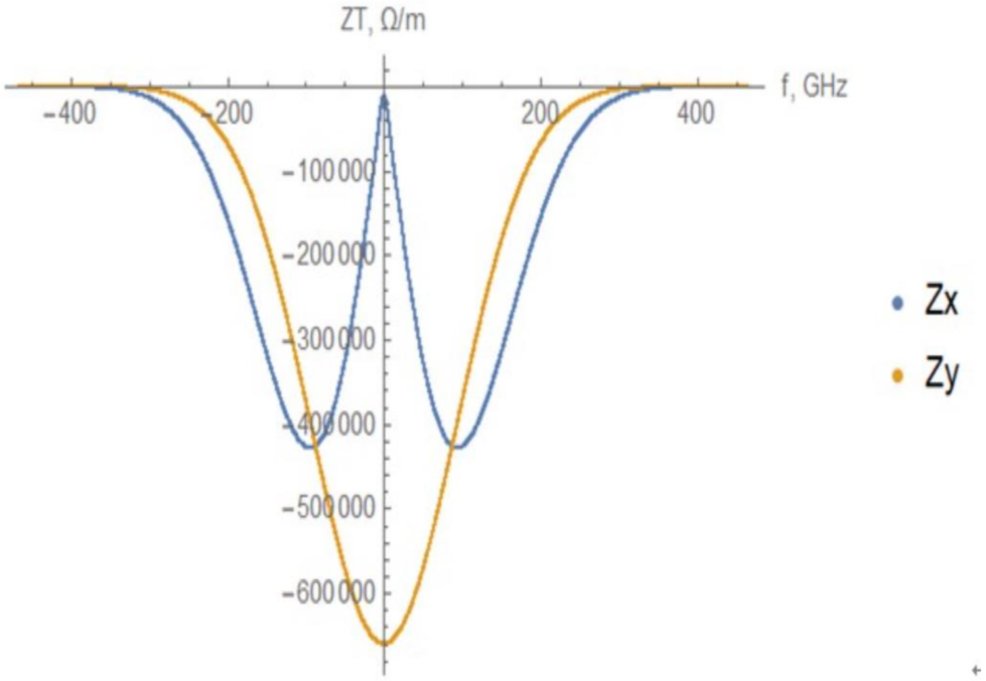


# Initial Analysis – Smooth Approximation of Beam-Beam

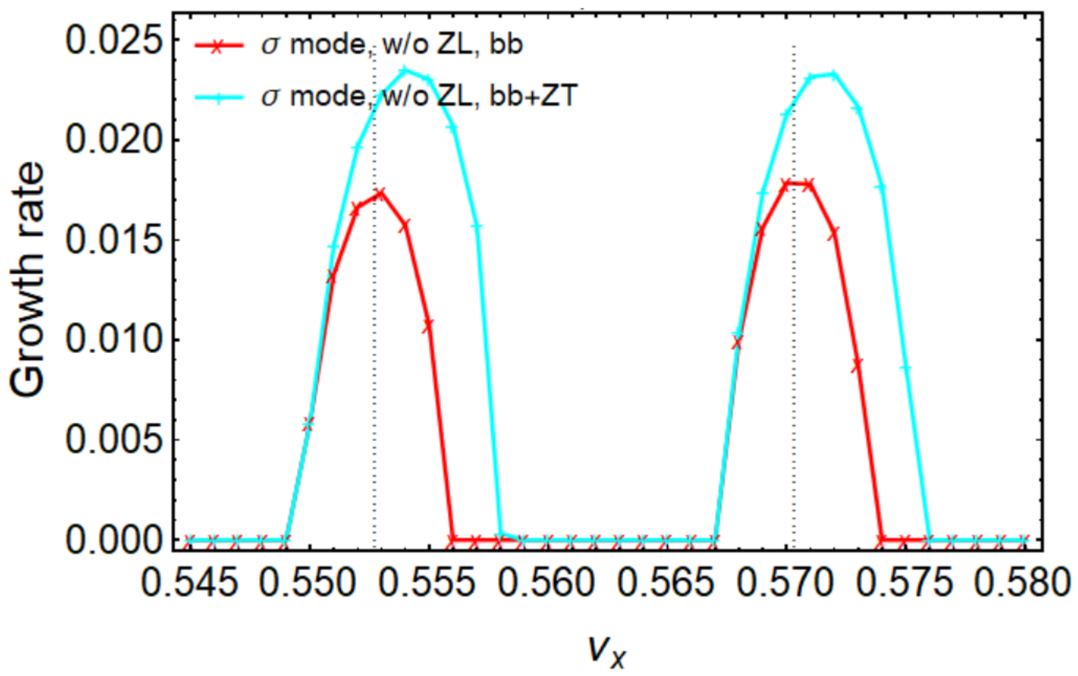


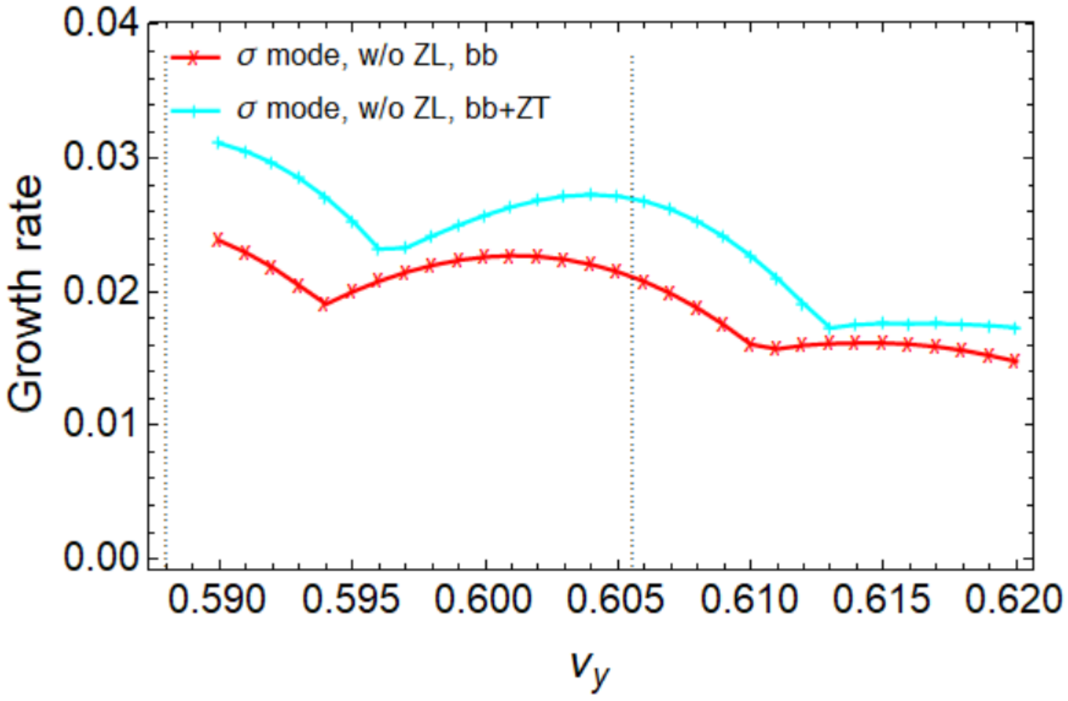

Cross-Wake of Beam-Beam

- Both horizontal and vertical Cross Wake force is obtained
- Beam-Beam Wake is included in the total ring impedance budget
- Conventional TMCI analysis is used:
  - Horizontal Threshold reduces from 22.6e10 to 21.9e10, when BB wake is considered
  - Vertical Threshold reduces from 22.6e10 to 15.6e10, when BB wake is considered

# Intitial Analysis - Local Beam-Beam Model

- In horizontal direction, considering ZX
  - the instability growth rate is faster,
  - unstable tune area increases
- In vertical direction,
  - pure beam-beam is unstable due to ignorance of strong nonlinearity ?
  - It is also found enhance of instability when considering ZY

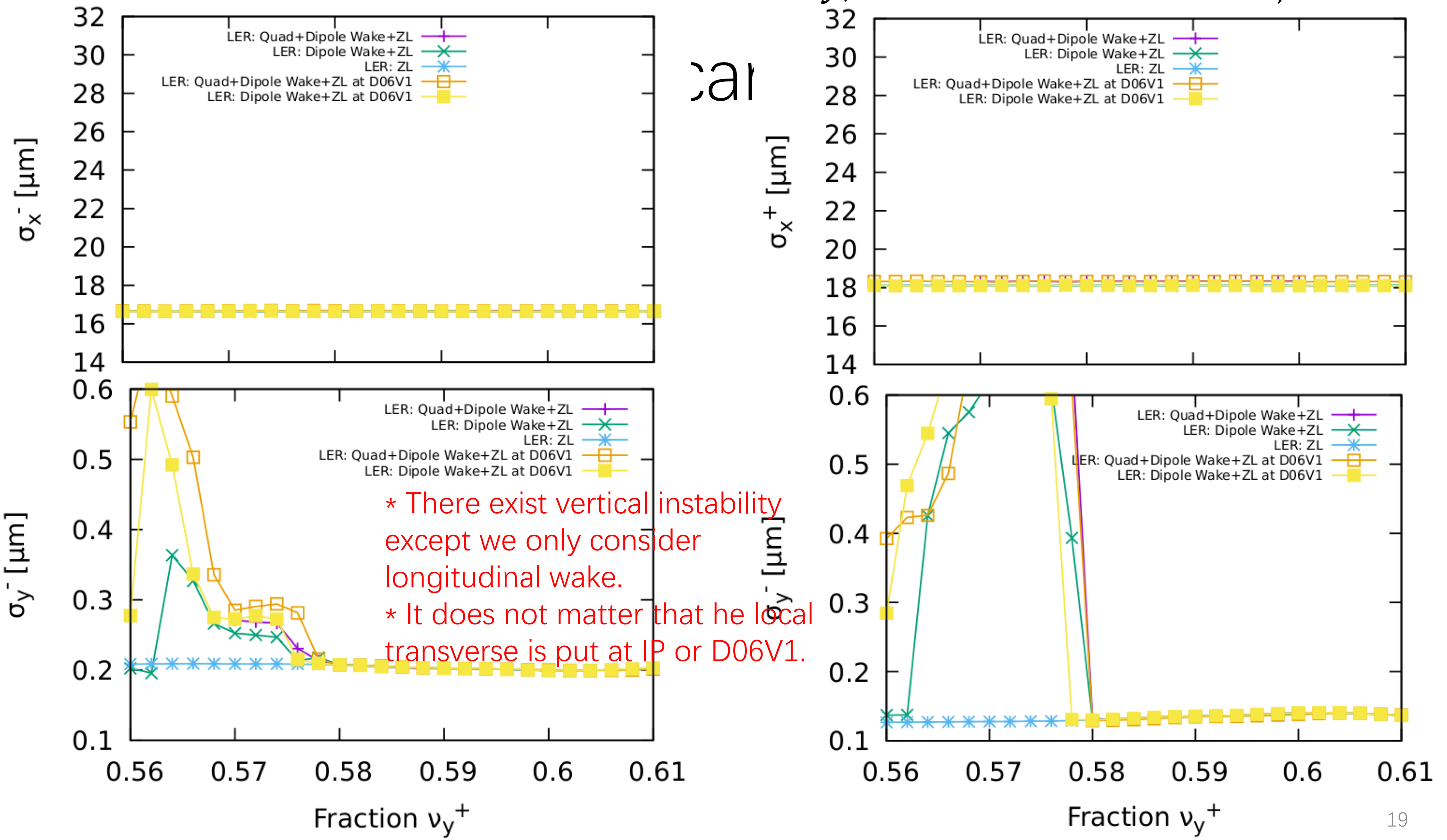


**SKEKB LER nuy scan**

# Collision versus $\nu_{y,0}^{+}$

$\nu_{x,0}^{+} = 0.525$

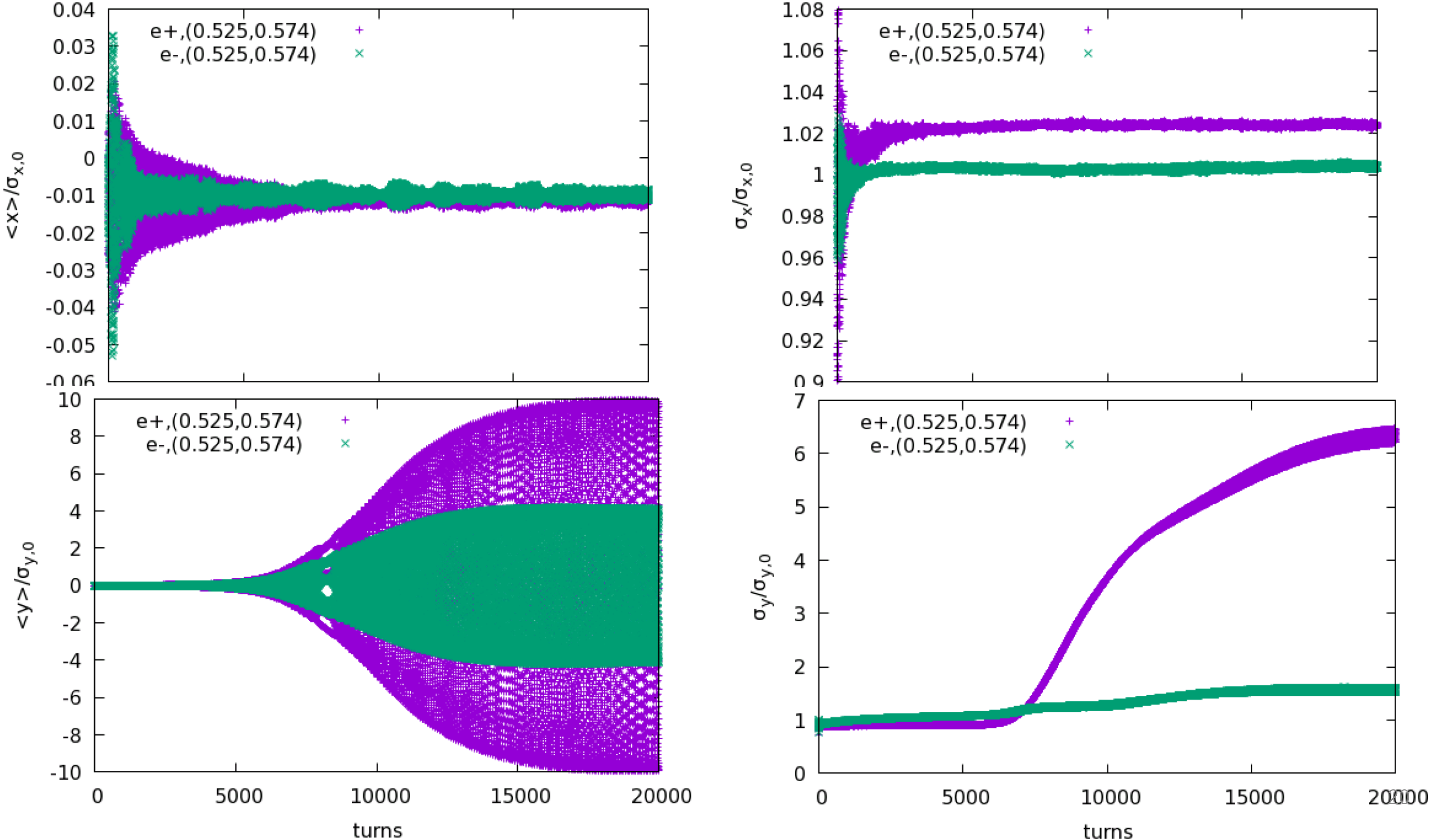



* There exist vertical instability except we only consider longitudinal wake.

* It does not matter that he local transverse is put at IP or D06V1.

# LER: dipole+quad wake + ZL, HER: ZL

SKEKB



# Summary

- The FCCee (Z) parameters are evolving, and the combined effect of beam-beam and impedance has been studied by simulation.
- The simulation results show that stable tune areas can be found for the present parameter set with 4 IPs even if both ZL and ZT are taken into account.
- However, if the impedance is doubled the stable tunes no longer exist.
- The numerical simulations also indicate that there exists a combined instability of X-Z and TMCI in the horizontal direction when the transverse impedance is considered.
- The analysis work is underway to study the crosstalk between the two transverse instabilities.