FCC week 2022, 01/06/2021 H. Burkhardt

Solenoid modeling for the FCC-ee IR

2T solenoid, strongest magnet in the lattice arc bends 0.016 T at Z-energies

average transverse component $0.015 \times 2T =$ **0.03 T** **2 ×** stronger

peak fringe fields depending on geometry details **~10 ×** stronger than main bends

Model : input **B(x, y, z)** shown here for ideal analytical solenoid and 3D fieldmap by M.K.

determine the trackjectory numerically (FieldStep)

reproduce on MAD-X level by orbit correctors $\propto x'', y''$ **+ thin solenoid slices** $\propto$ Bz

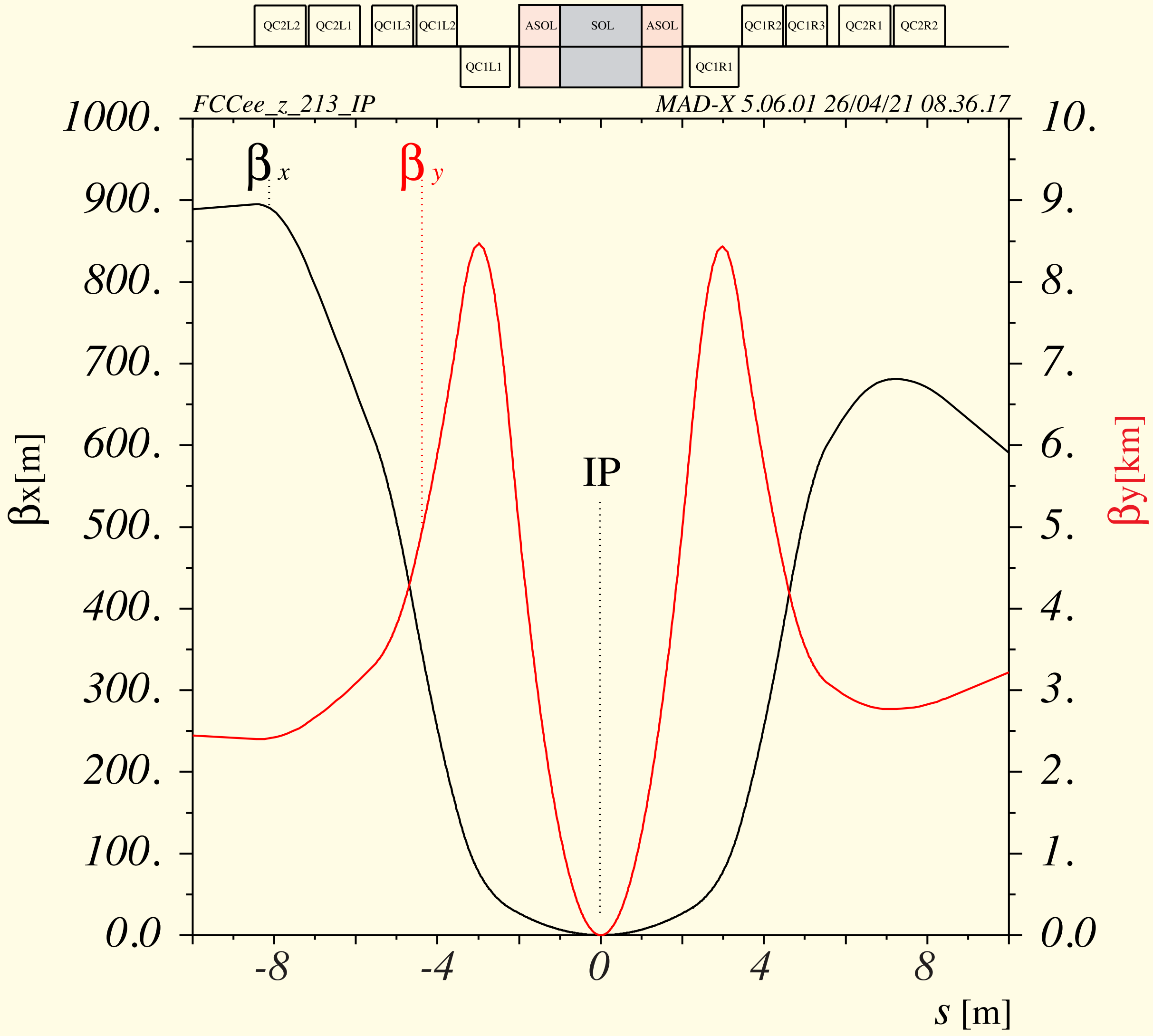
~ factorization ;

tilt generates bump modelled with corrector slices

+ coupling + focusing as in untilted solenoid

Acknowledgement : many fruitful discussions with CERN/FCC colleagues including
Rogelio Tomas, Tobias Persson, Leon van Riesen-Haupt, Riccardo de Maria, Laurent Deniau, Katsunobu Oide, Frank Zimmermann, Anton Bogomyagkov, Dmitry Shatilov, Kyrre Sjobaek, Manuela Boscolo, Barbara Dalena, Mike Koratzinos

FCC-IR



βy max = 8471m
βy min = 0.8 mm
107 variation**

IP region : strongly varying twiss parameters

here illustrated using MAD-X with many slices to get smooth curves

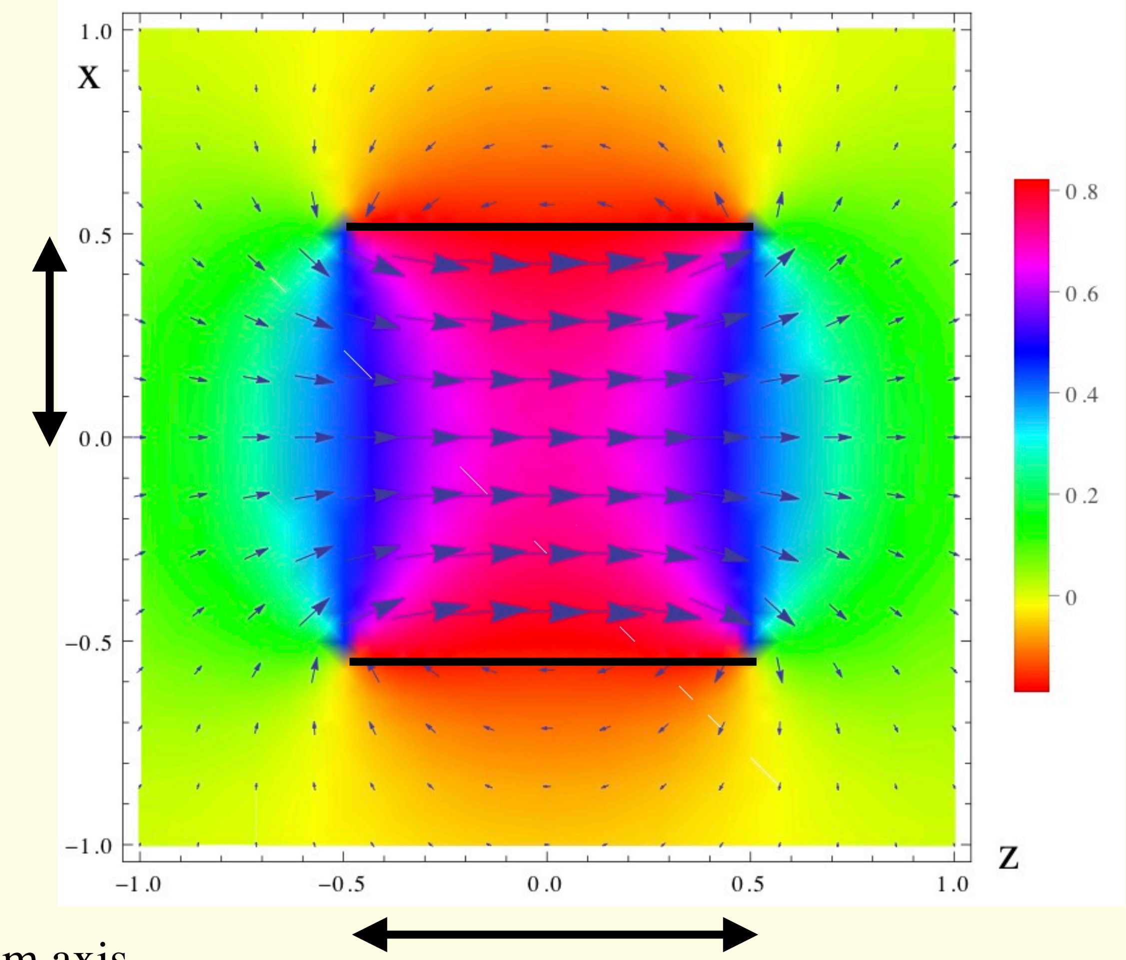
Analytical solenoid

Analytic expression of solenoid field

in terms of complete elliptic integrals

of 1st, 2nd, 3rd kind   K, E, Π

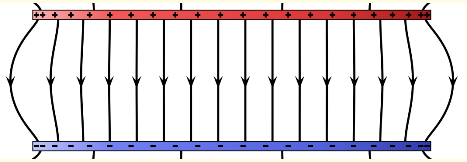
available in Mathematica, SymPi and C++17



R = 0.5

L = 1

in other magnets



flux more contained

Vector potential

cylindrical coordinates z, ρ - distance from axis

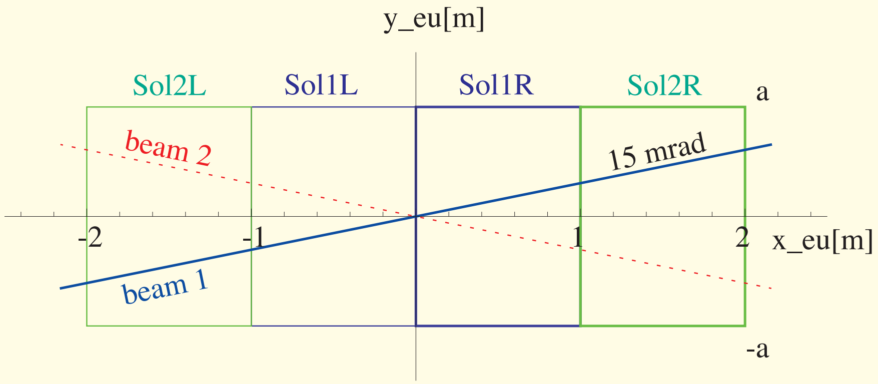
$$A_\phi(\rho,z) = \frac{\mu_0\, I}{2}\;\frac{z\;k}{2\,\pi\,L}\sqrt{\frac{R}{\rho}}\left[\frac{k^2+h^2-h^2k^2}{h^2k^2}K(k^2)-\frac{1}{k^2}E(k^2)+\frac{h^2-1}{h^2}\Pi(h^2,k^2)\right]_{z-L/2}^{z+L/2}$$

$$h^2 = \frac{4R\rho}{(R+\rho)^2} \qquad k^2(\rho,z) = \frac{4R\rho}{(R+\rho)^2+z}$$

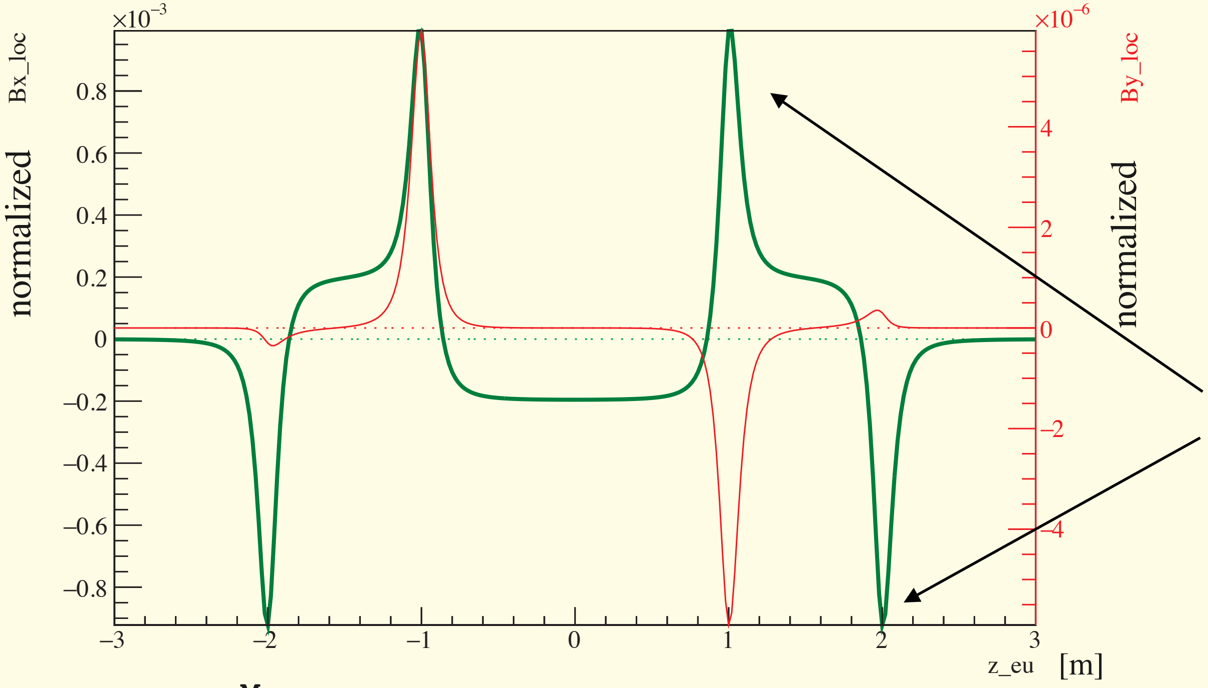
K.F. Müller, Berechnung der Induktivität von Spulen
01/05/1926, doi = 10.1007/BF01655986

IR transverse fields, horizontal 15 mrad angle

four 1m solenoid
anti-solenoid
pieces, 2T field



fields in
local system
main field Bx
vertical bump



peaks and SR
by fringe increase
when R (here 0.1 m)
is reduced

	Min	Max		
Bx/Brho	-0.9221e-3	0.99422e-3		kick y
By/Brho	-0.0059e-3	0.00591e-3	170× smaller	kick -x

Bρ = 152.1 Tm   max Bx = 0.15 T   nearly 10×   the 0.016 T of arc bends

at IP   ~ 0.015 × 2 T = 0.03 T   2×   arc bends

Solenoid in lattice programs

A. Dragt + students, G. Ripken et al.

Maps from **Hamiltonian in presence of fields** $-\sqrt{p_t^2/c^2 - m^2c^2 - (p_x - q\,A_x)^2 - (p_y - qA_y)^2} - qA_z$

symplectic for canonical coordinates derived from the Hamiltonian V. Arnold, H. Goldstein, Landau-Lifschitz

standard machine magnets (quadrupole) have mainly Az and small (ideally 0) Ax, Ay

the analytic **solenoid** has only nonzero Aϕ (Ax, Ay)

Transformation in transport (real space) coordinates with separate edges (Talman lectures)

$$R_{\mathrm{sol}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -K & 0 \\ 0 & 0 & 1 & 0 \\ K & 0 & 0 & 1 \end{pmatrix}}_{\text{edge1}} \underbrace{\begin{pmatrix} 1 & \frac{SC}{K} & 0 & \frac{S^2}{K} \\ 0 & C^2 - S^2 & 0 & 2CS \\ 0 & -\frac{S^2}{K} & 1 & \frac{SC}{K} \\ 0 & -2CS & 0 & C^2 - S^2 \end{pmatrix}}_{\text{solenoid body}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & K & 0 \\ 0 & 0 & 1 & 0 \\ -K & 0 & 0 & 1 \end{pmatrix}}_{\text{edge2}}$$

the product Rsol is symplectic, the individual matrices are not

Dragt ebook, in chapter 16.2 : *It is therefore highly desirable, in the case of a solenoid, to treat fringe-field effects with care (which must be done numerically) using realistic profiles b[n](z)*

Willeke, Ripken Methods of beam optics, AIP Proc. 184, 758 (1989), Matrix U, *Coupled case*, Eq. (3.51)

MAD-X Solenoid, thick, thin

```
label: SOLENOID, L=length, KS=ksval;                             ! thick version
label: SOLENOID, L=0, Lrad=length, KS=ksval, KSI:=ksval*Lrad;    ! thin version
```

more recently also possible to add permanent dtheta tilt

Note : currently in MAD-X no information about solenoid radius — extend of fringe fields

fringe field effect taken into account as seen from outside

Example, FCC-ee solenoid pieces, Bsol = 2 T beam pc = 45.6 GeV

```
Thick  L = 1 m ,    ksval := Bsol / beam->brho = 0.0131488
```

Slicing, MAD-X module MAKETHIN

```
Thin  solenoid, lrad:= length / nslices , ks:=ksval , ksi:= ksval * lrad;
```

Literature :

MAD8 Physics Guide

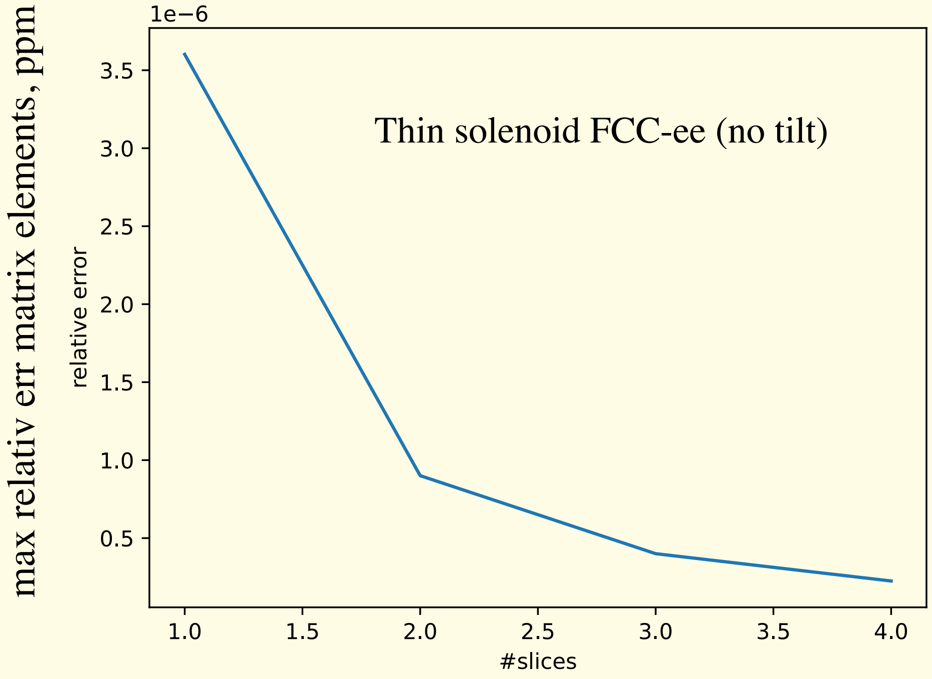
On The Implementation Of Experimental Solenoids In MAD-X And Their Effect On Coupling In The LHC, A. Koschik, H. Burkhardt, T. Risselada, F. Schmidt EPAC 2006

Upgrade of slicing and tracking in MAD-X, H. Burkhardt, L. Deniau, A. Latina, IPAC2014

MAD-X Thin solenoid

Thin solenoid
converges well to reproduce the thick solenoid

noted in our EPAC 2006 paper :
slicing a thick solenoid into several thin ones ...
converges to the thick lens solution …
does in general not give the correct edge focusing effect



The MAD-X thin solenoid transfer matrix can be written as product of a rotation about the s-axis and “a thin quadrupole” focusing in both x, y

$$\begin{bmatrix} \cos(\psi) & 0 & \sin(\psi) & 0 & 0 & 0 \\ 0 & \cos(\psi) & 0 & \sin(\psi) & 0 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) & 0 & 0 & 0 \\ 0 & -\sin(\psi) & 0 & \cos(\psi) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\psi = \mathrm{L\,KS}/2 = \mathrm{KSI}/2$ 6.5744 mrad FCC-ee Z

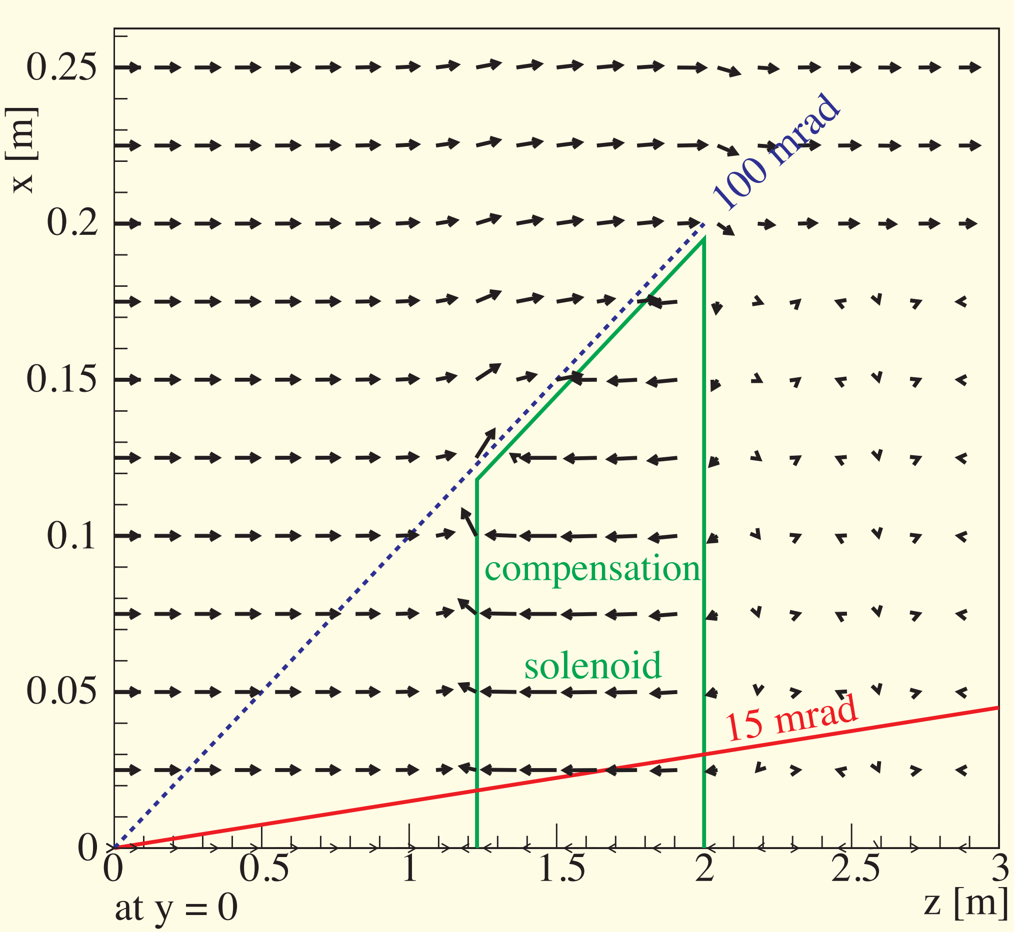
$\frac{1}{f} = \frac{\mathrm{KS}}{2} \; \frac{\mathrm{KSI}}{2}$ 4.32e-5 / m

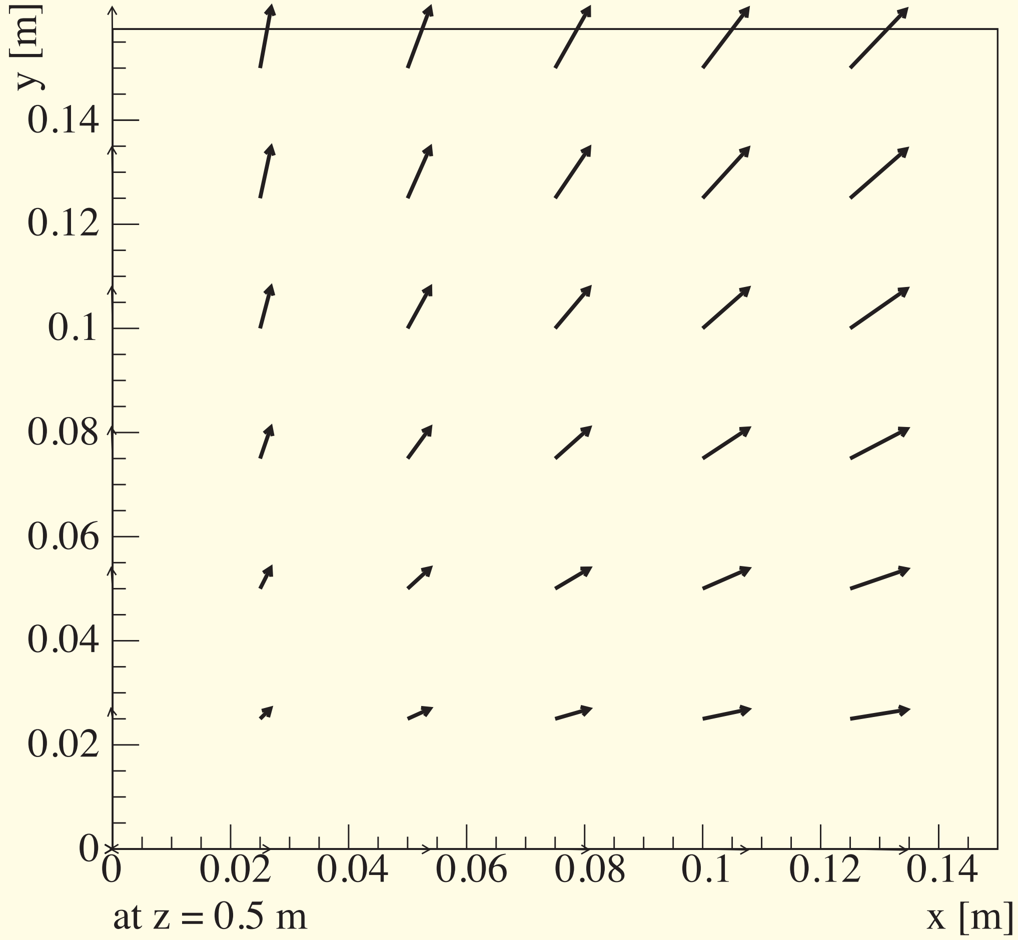
for the 1m long solenoid piece

Fieldmap 3D

3D_map_compantation_only_best_so_far.xls Mike Koratzinos 26/11/2021

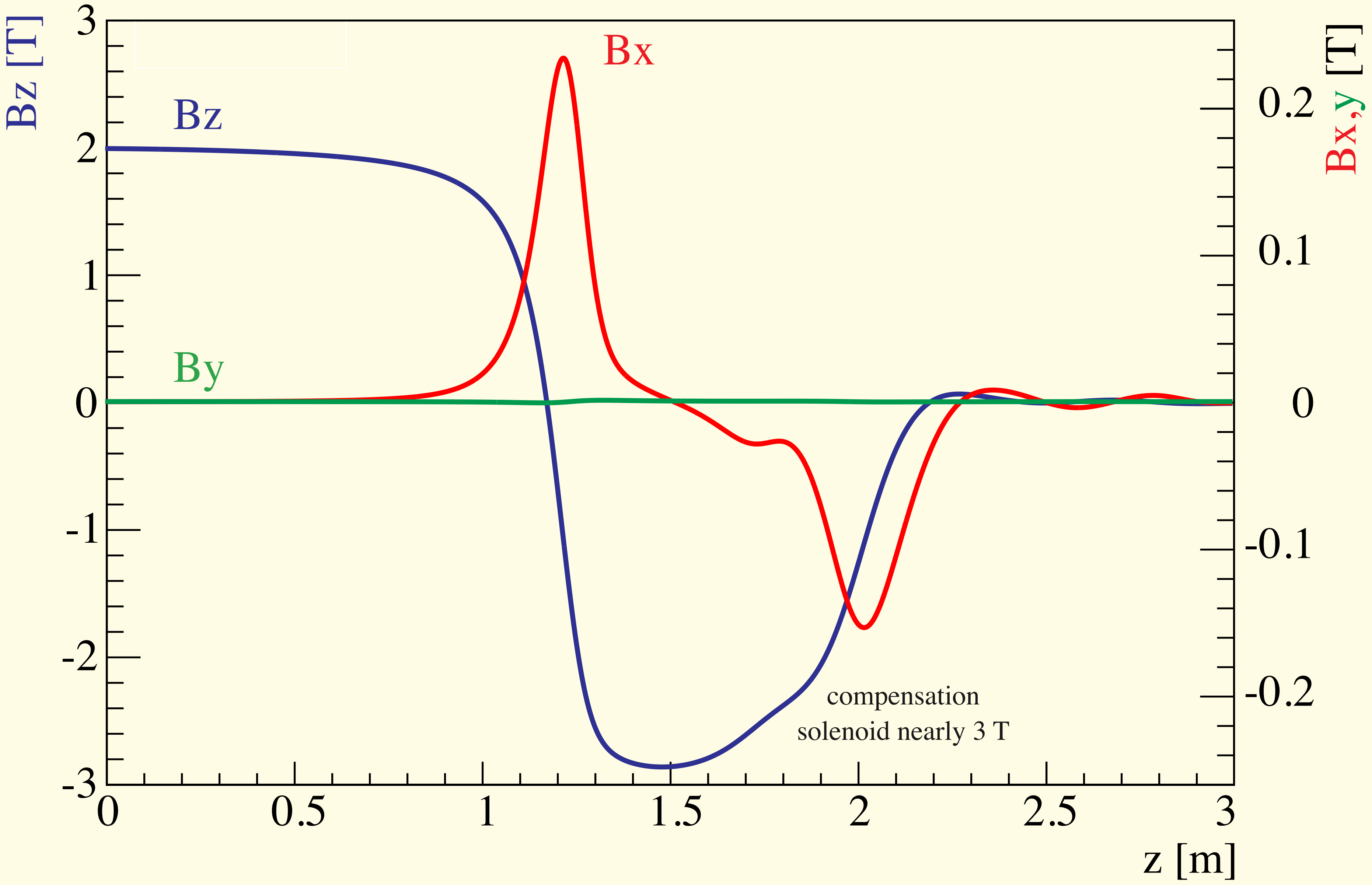
defined for 0 to 0.5 m in x,y and 0 to 3 m in z, in steps of 0.025 m





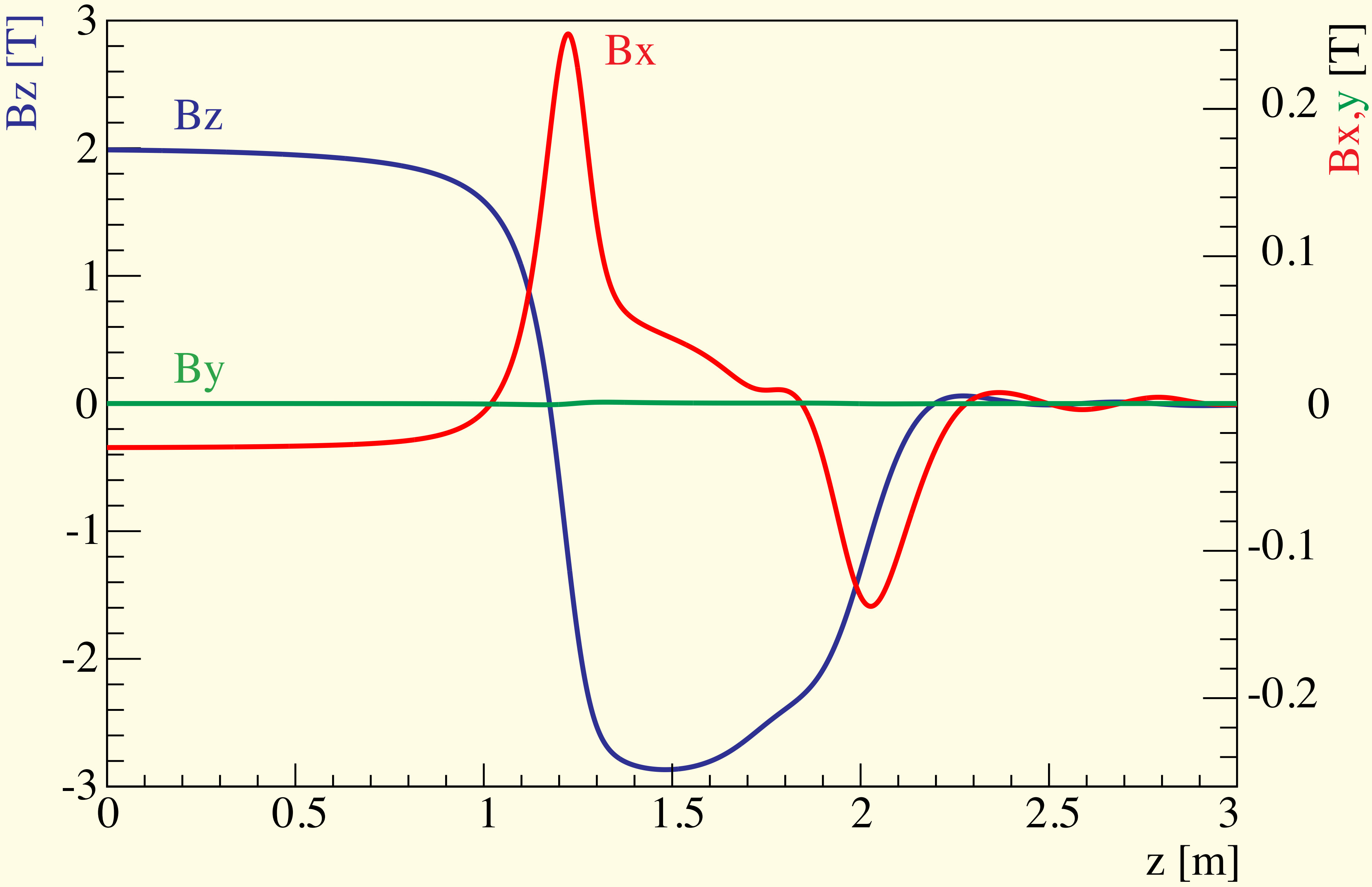
0 **side view** **3 m** **look in z-direction along the solenoid axis**

FieldMap, fields on 15 mrad line, detector (eu) coordinates



compensation
solenoid nearly 3 T

FieldMap, fields on 15 mrad line (cs)



Numerical solution, FieldStep tracking

$$\begin{aligned} \mathbf{x}'' &= \mathbf{x}' \times \mathbf{B_n} \\ \mathbf{x}' += & \, \Delta s \, \mathbf{x}'' \\ \mathbf{x} += & \, \Delta s \, \mathbf{x}' \end{aligned}$$

numerical solution of equations of motion:

input : B(x,y,z)

determine design passage by tracking

converges quickly, excellent agreement with GEANT4 and (my) FieldStep

very fast for small lattice sections like ± 5 m around IP

output :

track coordinates at every step

optionally track design particle and 6 particles with small offsets in all coordinates

(multi-treading, ~ same speed as single particle)

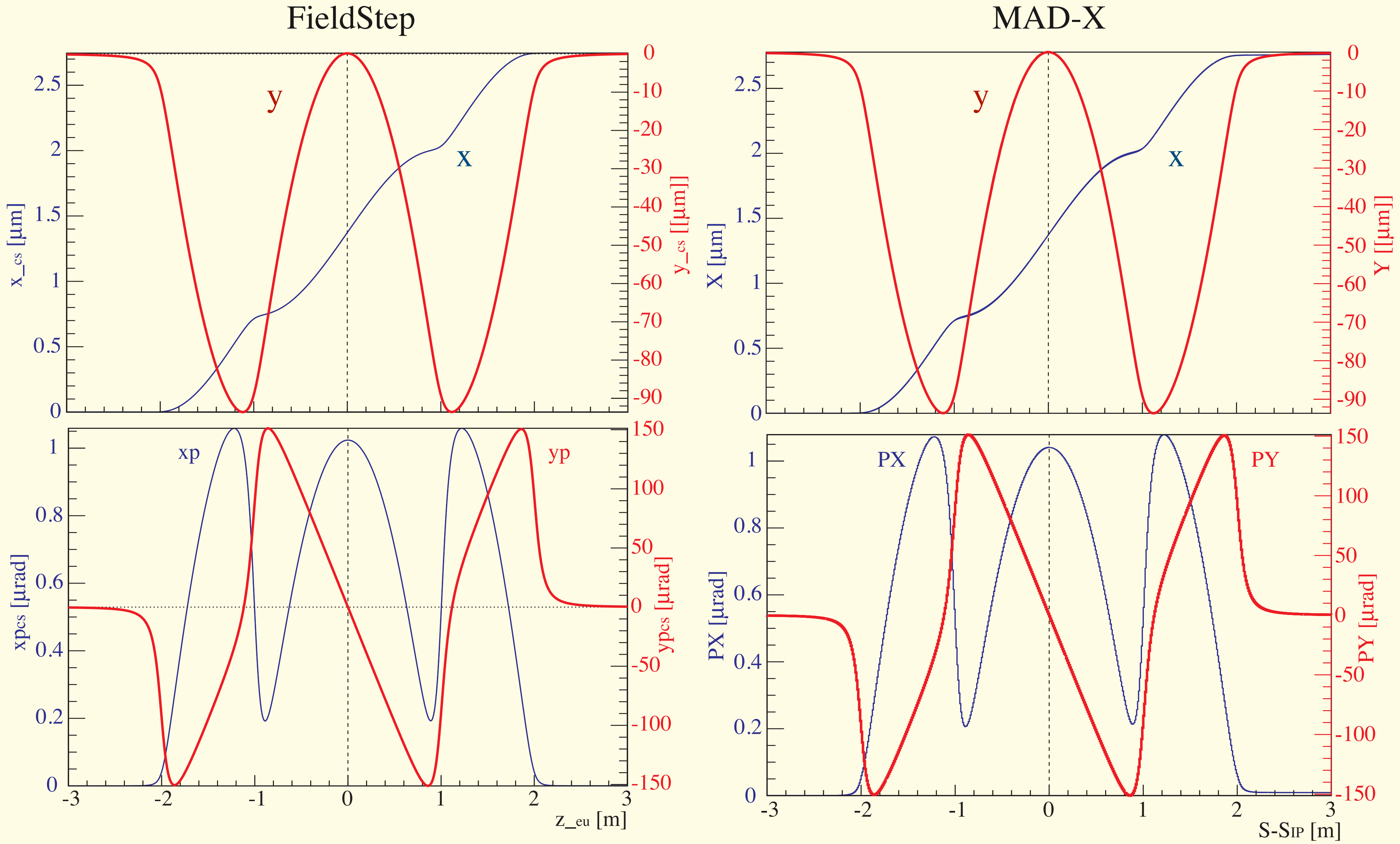
transfer matrices in real space coordinates from numerical Jacobian, symplectic when Ax = Ay = 0

outside : reproducing the Solenoid map (for small solenoid radius)

inside : transport (real space) positions and angles x, x'

use x'' as corrector strength to reproduce solenoid bump on the MAD-X level

Analytical Solenoid, position and direction

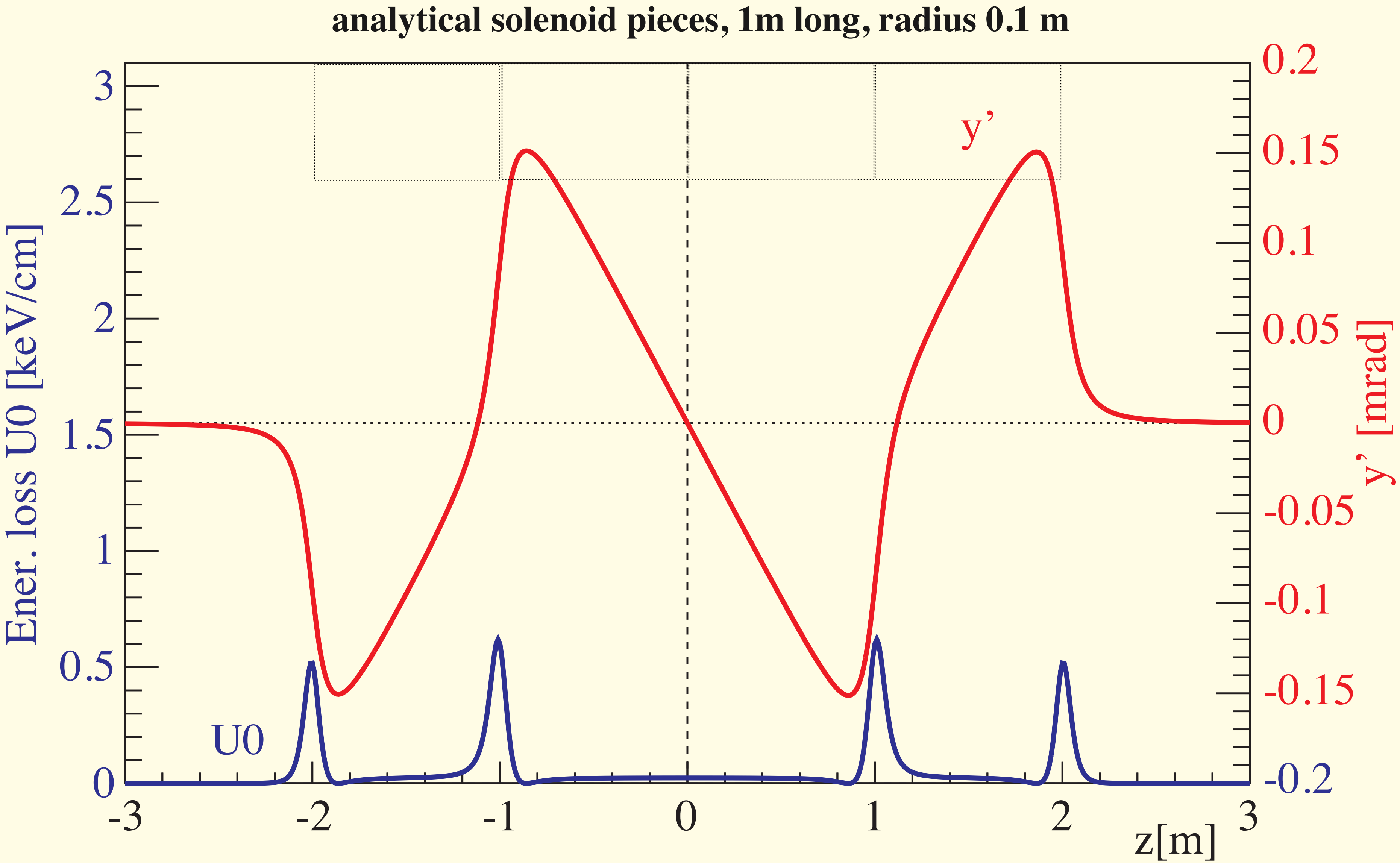


excellent agreement in bump shape

x : 0 to 2.7 μm   0 to 1.05 μrad

y : 0 to -94 μm   ± 151 μrad

Energy loss U0 and direction y' of radiation

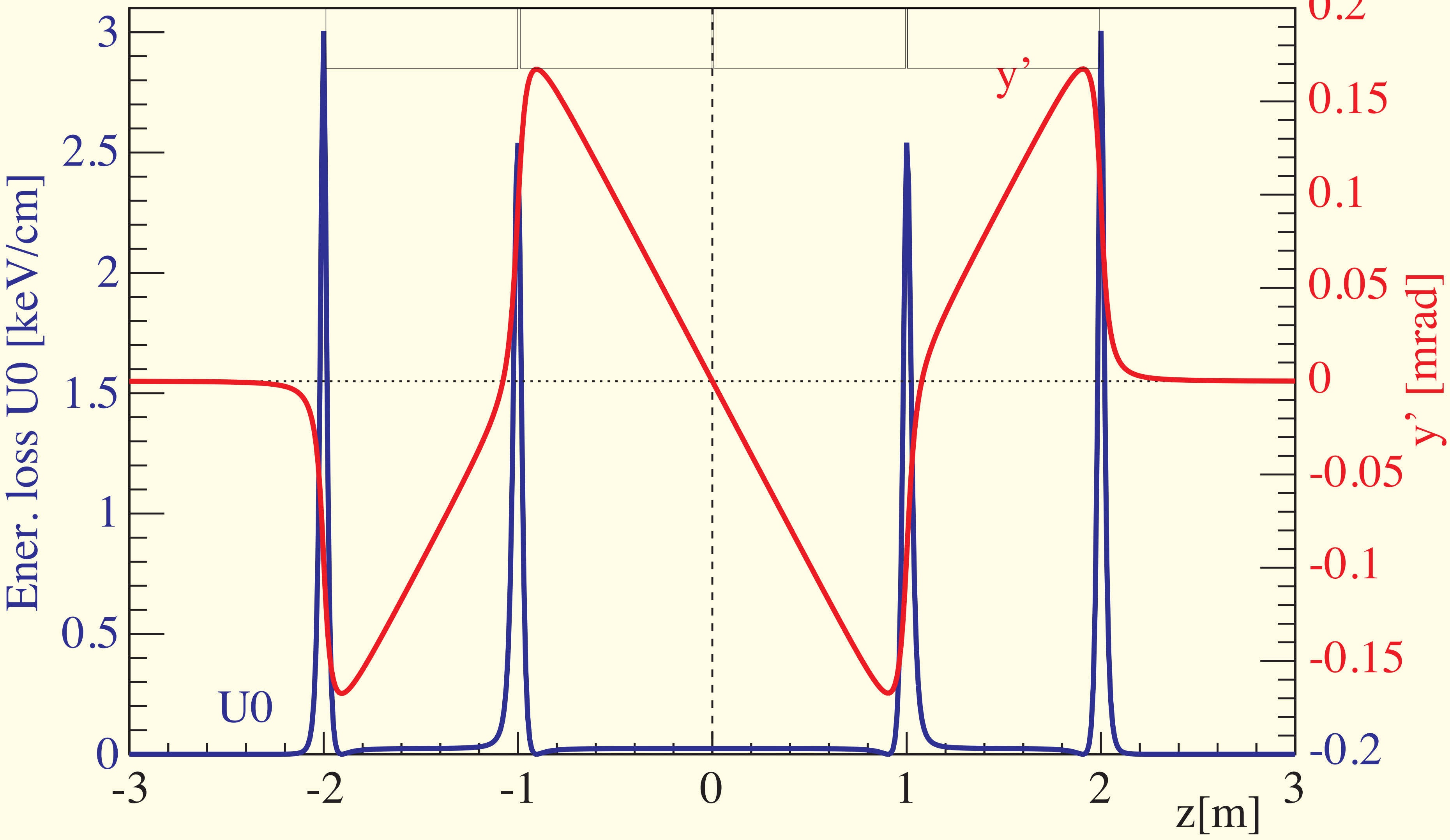


FCC-ee Z, 1.28 A beam current IR solenoid SR **41 kW / beam**

other FCC-ee energies 2T solenoid			
	80 GeV,	135 mA,	13.3 kW
	120 GeV,	26.7 mA,	5.9 kW
	182.5 GeV,	5 mA,	2.6 kW

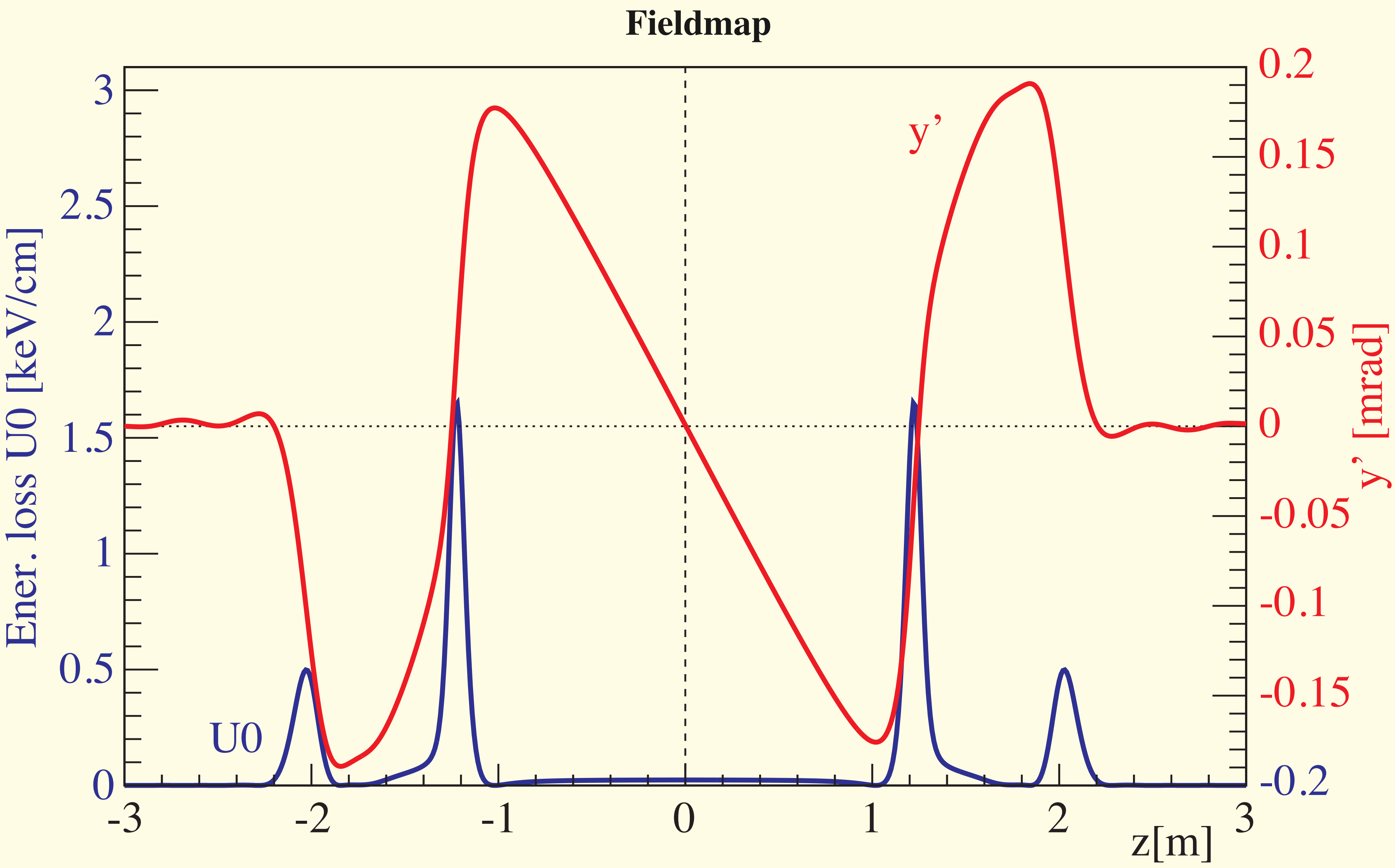
Energy loss U0 and direction y’ of radiation

analytical solenoid pieces, 1m lomg, radius 0.05 m



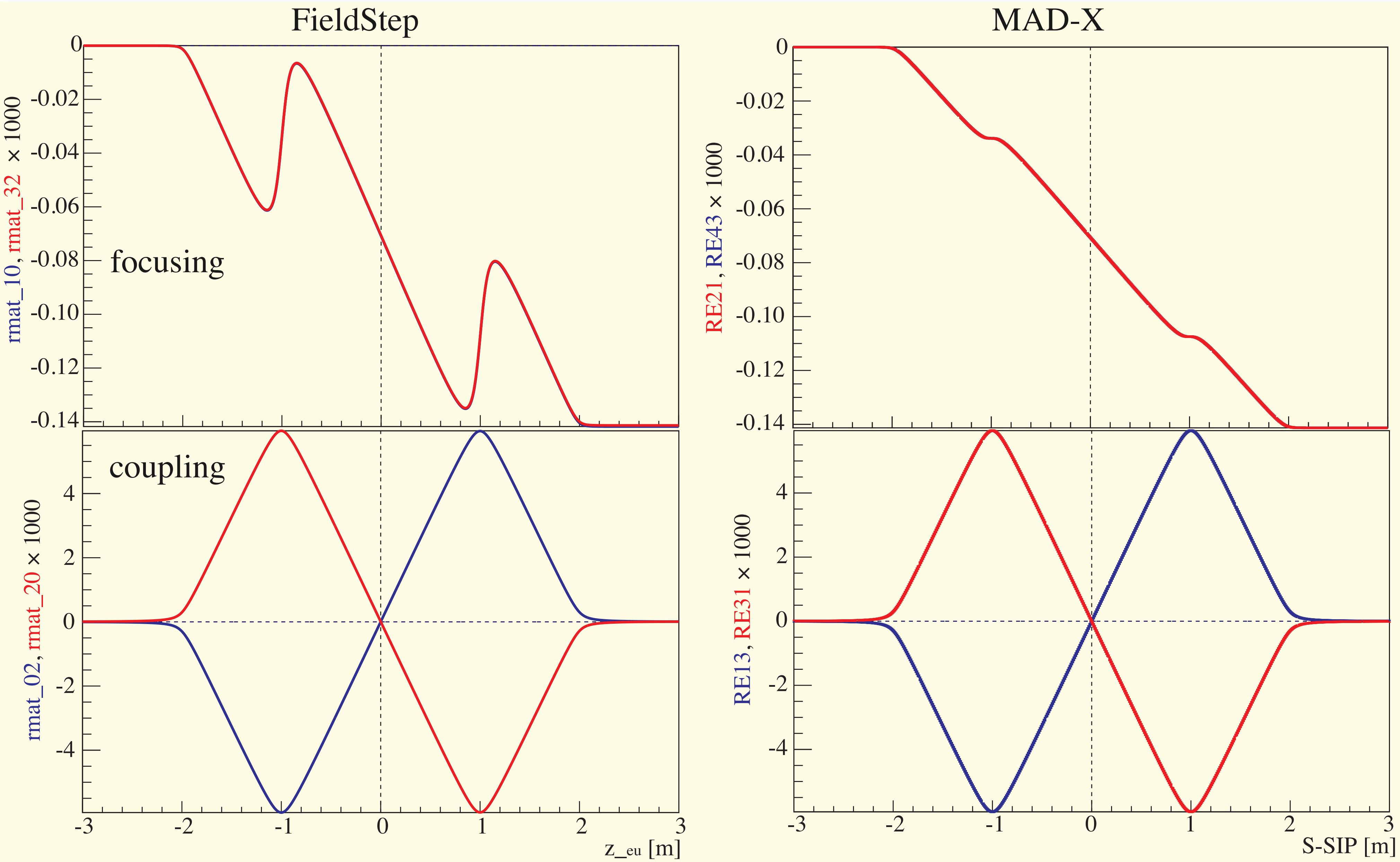
FCC-ee Z, 1.28 A beam current IR solenoid SR **81 kW / beam**

Energy loss U0 and direction y' of radiation



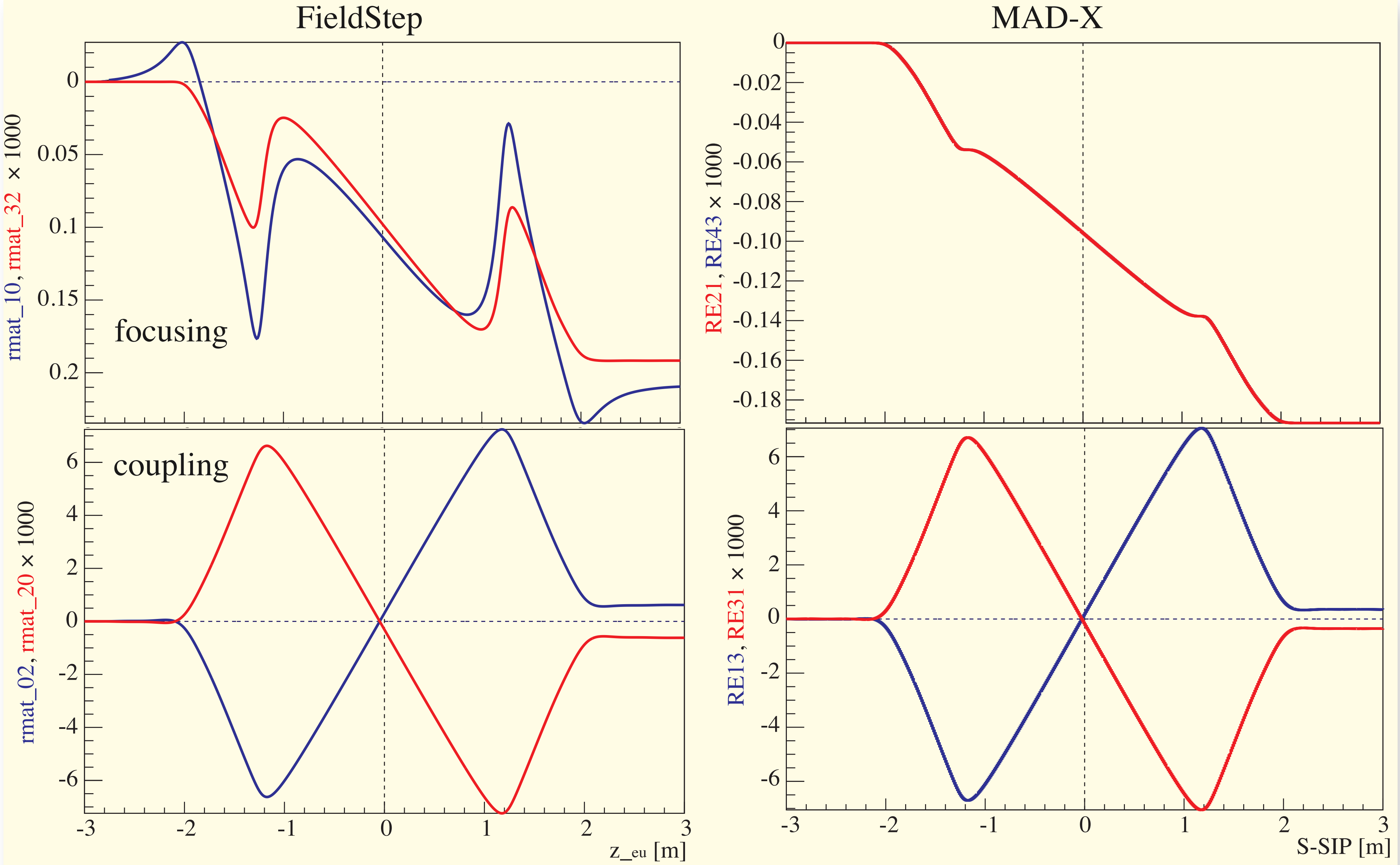
FCC-ee Z, 1.28 A beam current IR solenoid SR **77 kW / beam**

Analytical Solenoid, focusing + coupling



coupling well reproduced. overall (weak) focusing correct on average, ok as seen from outside

Fieldmap, focusing + coupling



3D fieldmap : imperfections visible, radial symmetry and closure not exact

Concluding remarks

Given B(x , y, z) (analytical formulas or fieldmap)

the passage through the interaction region can be well modelled on the MAD-X level

with orbit correctors (kicker elements, describing the bump generated by the tilted solenoid detailed fringe fields)

+ thin solenoid slices (coupling + overall focusing)

positions, directions, dispersion and energy loss agree very well with numerical fieldstepping

in and outside the solenoid

Backup

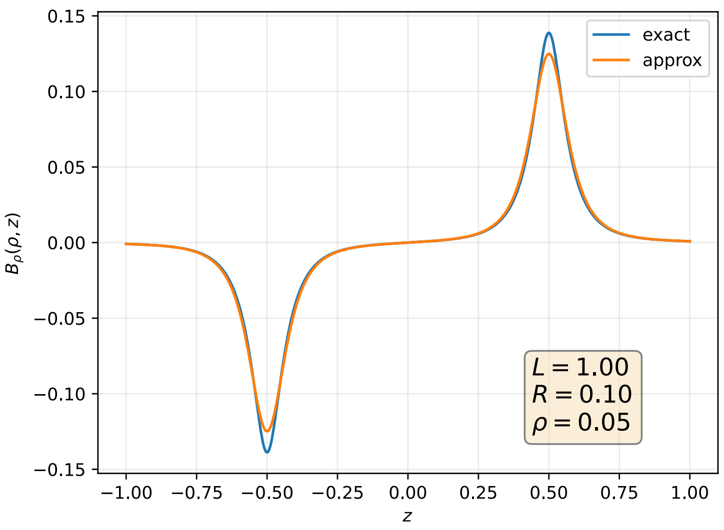
Sliced sequences generated, insert in IR with seqedit

```
hv_corr_install.madx
..
install,element=hv_corr_m00020,at =-0.02/cos(0.015),from=IP;
install,element=hv_corr_m00010,at =-0.01/cos(0.015),from=IP;
install,element=hv_corr_p00000,at =    0/cos(0.015),from=IP;
install,element=hv_corr_p00010,at = 0.01/cos(0.015),from=IP;
install,element=hv_corr_p00020,at = 0.02/cos(0.015),from=IP;
..
hv_corr_def.madx
hv_corr_m00020 :kicker,lrad:=0.01/cos(0.015), hkick:=    5.0977934605683e-10*h_on , vkick:=   -1.95543847556181e-06*v_on ;
hv_corr_m00010 :kicker,lrad:=0.01/cos(0.015), hkick:=   2.54894409450189e-10*h_on , vkick:=   -1.95547204643059e-06*v_on ;
hv_corr_p00000 :kicker,lrad:=0.01/cos(0.015), vkick:=  -1.95548319190777e-06*v_on ;
hv_corr_p00010 :kicker,lrad:=0.01/cos(0.015), hkick:=  -2.54894409450189e-10*h_on , vkick:=    -1.9554719453329e-06*v_on ;
hv_corr_p00020 :kicker,lrad:=0.01/cos(0.015), hkick:=   -5.0977934605683e-10*h_on , vkick:=   -1.95543827316283e-06*v_on ;
```

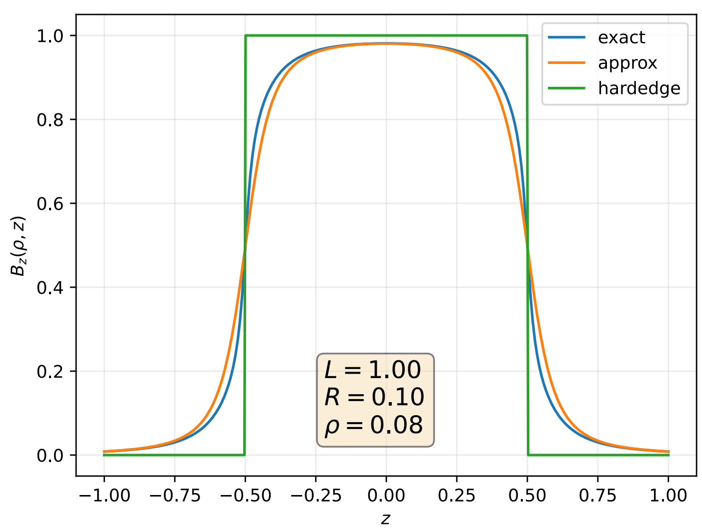
```
sol_install.madx
..
install,element=avesol_m00020 ,at =-0.02/cos(0.015),from=IP;
install,element=avesol_m00010 ,at =-0.01/cos(0.015),from=IP;
install,element=avesol_p00000 ,at =    0/cos(0.015),from=IP;
install,element=avesol_p00010 ,at = 0.01/cos(0.015),from=IP;
install,element=avesol_p00020 ,at = 0.02/cos(0.015),from=IP;
..
avesol_m00020:solenoid, lrad:=     0.0100011251054784,  ks:=     0.0130330719303976*ksi_on, ksi:=    0.000130345382884605*ksi_on;
avesol_m00010:solenoid, lrad:=     0.0100011251054784,  ks:=     0.0130331837298234*ksi_on, ksi:=     0.00013034650100465*ksi_on;
avesol_p00000:solenoid, lrad:=     0.0100011251054784,  ks:=     0.0130332209718128*ksi_on, ksi:=    0.000130346873466445*ksi_on;
avesol_p00010:solenoid, lrad:=     0.0100011251054784,  ks:=     0.0130331837305903*ksi_on, ksi:=    0.000130346501012319*ksi_on;
avesol_p00020:solenoid, lrad:=     0.0100011251054784,  ks:=     0.0130330719319335*ksi_on, ksi:=    0.000130345382899966*ksi_on;
..
select, flag=error, range=avesol_m00020; ealign,dx:=    1.3520966226094e-06*dx_on ,dy:=    9.12901013449553e-08*dy_on, dphi:=     3.9320328843862e-06*dphi_on; select, flag=error, clear;
select, flag=error, range=avesol_m00010; ealign,dx:=   1.36233178875816e-06*dx_on ,dy:=    1.20738713487934e-07*dy_on, dphi:=    1.97657591949412e-06*dphi_on; select, flag=error, clear;
select, flag=error, range=avesol_p00000; ealign,dx:=   1.37256953163144e-06*dx_on ,dy:=    1.30630423739617e-07*dy_on, dphi:=    2.10964898685578e-08*dphi_on; select, flag=error, clear;
select, flag=error, range=avesol_p00010; ealign,dx:=   1.38280730200639e-06*dx_on ,dy:=    1.20965120604878e-07*dy_on, dphi:=   -1.93438300117683e-06*dphi_on; select, flag=error, clear;
select, flag=error, range=avesol_p00020; ealign,dx:=   1.39304255065988e-06*dx_on ,dy:=    9.17429165901021e-08*dy_on, dphi:=   -3.88984015056126e-06*dphi_on; select, flag=error, clear;
..
```

Fields close to axis, fringe field kick

$$\frac{B_\rho(\zeta,\rho)}{\rho} = B_{\rho\_\mathrm{over}\_\rho}(\zeta) \approx -\frac{R^2}{4\,(R^2+\zeta^2)^{3/2}}$$



$$B_z(\zeta) \approx \frac{\zeta}{2\sqrt{R^2+\zeta^2}}\Bigg|_{z=\zeta-L/2}^{z=\zeta+L/2}$$



related by $$\frac{dB_z(\rho,\zeta)}{d\zeta} = \frac{R^2}{2\,(R^2+\zeta^2)^{\frac{3}{2}}} = -2\,B_{\rho\_\mathrm{over}\_\rho}$$

$$\begin{aligned} B_\rho &= & -&\frac{\partial A_\varphi}{\partial z} \\ B_\phi &= & &0 \\ B_z &= & &\frac{1}{\rho}\,\frac{\partial(\rho A_\varphi)}{\partial \rho} \end{aligned}$$

as expected for only non-zero Aϕ

$$\int_{-\infty}^{\infty} B_\rho(\zeta,\rho)\;dz = \mp\frac{2}{\rho}$$ fringe field kick close to axis

cartesian close to axis $$\mathbf{B}(x,y,z) = (x\,B_{\rho\_\mathrm{over}\_\rho}, y\,B_{\rho\_\mathrm{over}\_\rho}, B_z)_{z-L/2}^{z+L/2}$$

good for insight and solving equations of motion in real space coordinates

not needed for numerical evaluation, very fast to evaluate exact formulas or later to use measured field map

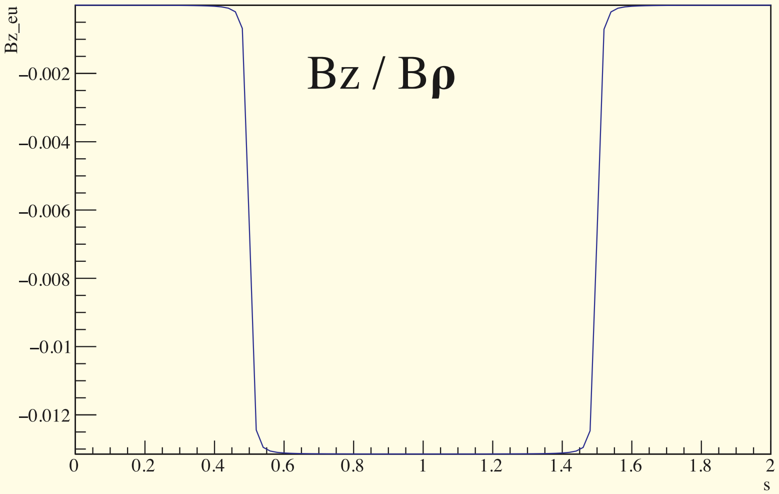
Numerical transfer matrix in real space coordinates

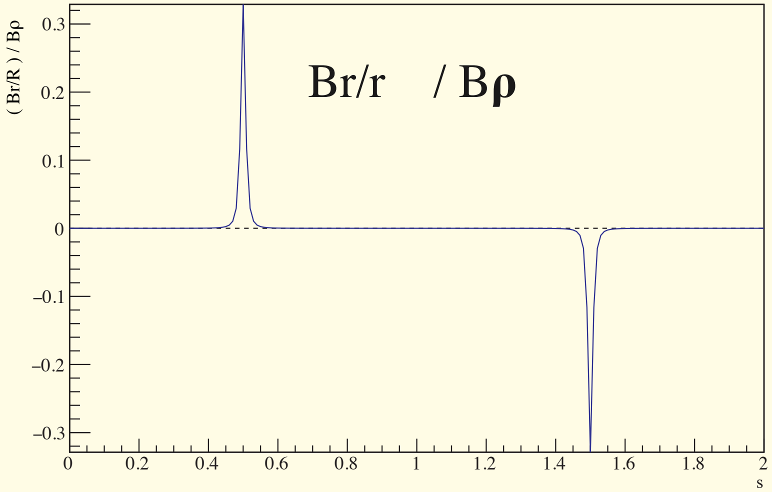
Example **no tilt single L = 1m solenoid, R = 0.1 m, 2 T**,
Jacobian from tracking with small offsets
done in 6D, numbers here just for the 4D part

```
s=0.5  solenoid entry
             1          -0.5   -3.2196e-05    1.5952e-05
   -1.5004e-07             1      -0.00327     0.0016026
    3.2196e-05   -1.5952e-05             1          -0.5
       0.00327    -0.0016026   -1.5004e-07             1

s=1   solenoid center
       0.99999    1.7235e-06    -0.0032864   -5.8978e-07
   -4.2867e-05             1    -0.0065724    -0.0032869
     0.0032864    5.8978e-07       0.99999    1.7235e-06
     0.0065724     0.0032869   -4.2867e-05             1

s=2   0.5 meter from solenoid    back symplectic
       0.99994       0.99998    -0.0065725    -0.0065751
   -4.2547e-05       0.99998     2.794e-07    -0.0065751
     0.0065725     0.0065751       0.99994       0.99998
    -2.794e-07     0.0065751   -4.2547e-05       0.99998
always det = 1
```



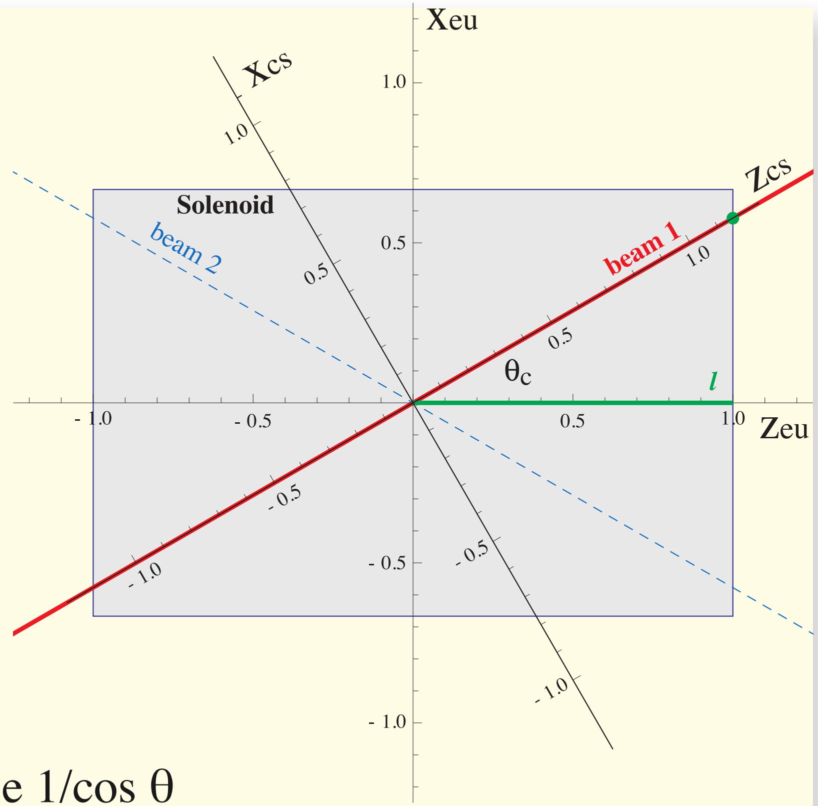


possible to read these into MAD-X and use with option, sympl=false; consequences with SR and emit would need reconsideration, maybe not such a good idea for an overall small effect ?

Of some interest for other applications where SR not essential like solenoid focusing at low energy ?

Geometry of tilted solenoid

Illustrated for $\theta_c = \pi/6 = 30^o$



$$\mathrm{Rot}_{y,3D}(\theta_c) = \begin{bmatrix} \cos(\theta c) & 0 & \sin(\theta c) \\ 0 & 1 & 0 \\ -\sin(\theta c) & 0 & \cos(\theta c) \end{bmatrix}$$

$$\mathbf{x}_{\mathrm{cs}} = \begin{bmatrix} x_{\mathrm{cs}} \\ y_{\mathrm{cs}} \\ z_{\mathrm{cs}} \end{bmatrix} \qquad \mathbf{x}_{\mathrm{eu}} = \mathrm{Rot}_{y,3D}(\theta_c)\ \mathbf{x}_{\mathrm{cs}} = \begin{bmatrix} x_{\mathrm{cs}} \cos(\theta c) + z_{\mathrm{cs}} \sin(\theta c) \\ y_{\mathrm{cs}} \\ -x_{\mathrm{cs}} \sin(\theta c) + z_{\mathrm{cs}} \cos(\theta c) \end{bmatrix}$$

eu : detector system, solenoid axis z_{eu}

cs : beam reference system, solenoid rotated, length increase 1/cos θ

FCC-ee : beam divergence ~ 40 µrad small compared to (half) crossing angle θc = 15 mrad

fields seen and main effects of tilted solenoid similar for all beam particles

possible to approximate rather well using MAD-X with

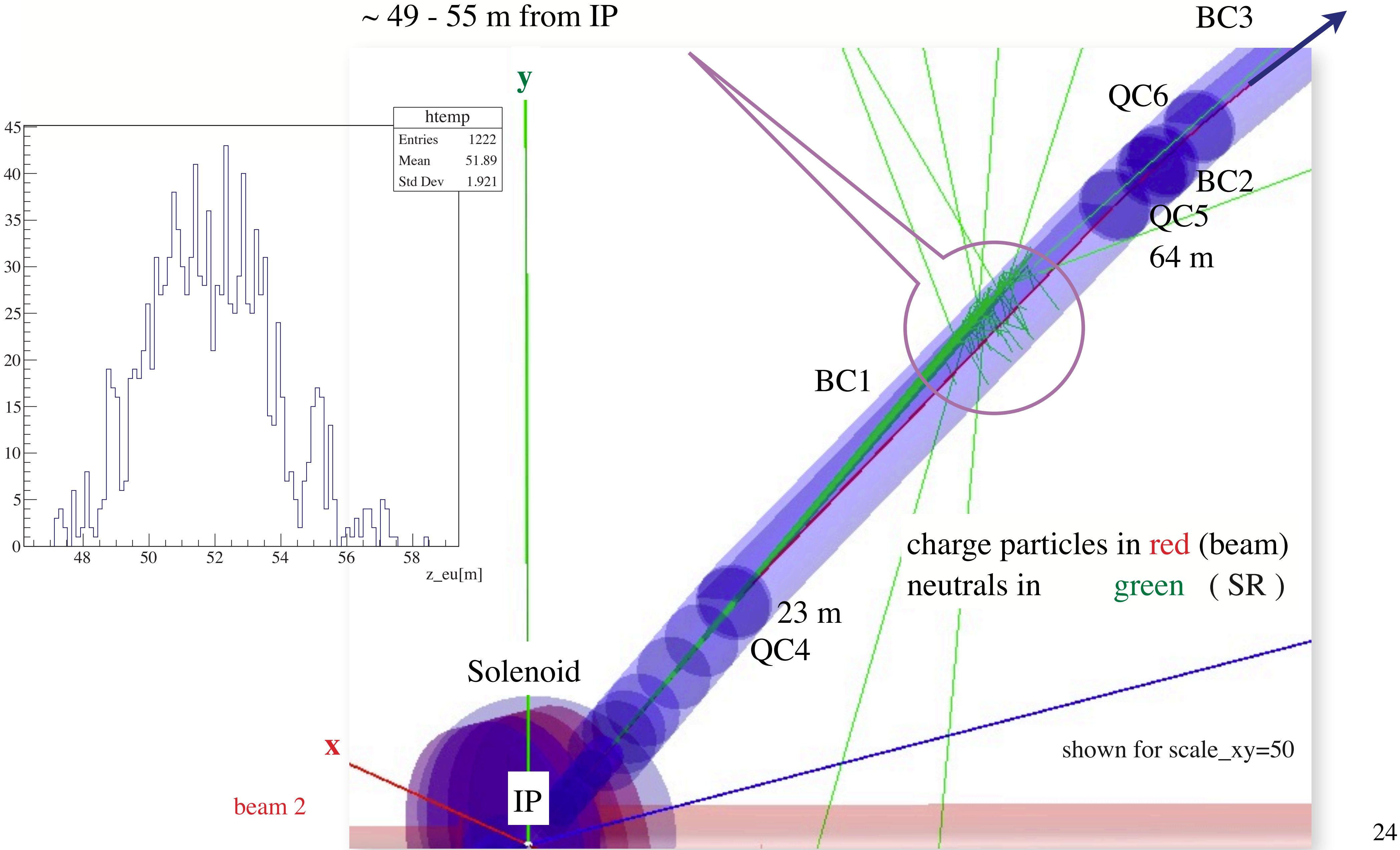
1) orbit correctors reproducing the design particle trajectory determined with tracking
2) slices of on axis-solenoid aligned to design particle track (work in progress)

Loss location of the solenoid generated photons

Using MDISim (GEANT4, geometry automatically generated from MAD-X, display with ROOT)

hitting beam pipe in first downstream bend BC1

~ 49 - 55 m from IP



charge particles in red (beam)
neutrals in green (SR)

shown for scale_xy=50