Simulations of the Spin Polarization for the Future Circular Collider e+e− using Bmad

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Outline

1. Motivation
2. Introduction to the Spin Polarization Theory
3. Linear Spin Polarization Simulations in Bmad
4. Benchmark between Tao (Bmad) and SITF (SITROS)
5. Nonlinear Spin Tracking in Bmad

Motivation

- Center-of-mass collision energy calibration with high precision
- Precise beam energy calibration using resonant depolarization
- Spin simulations for the validation of the energy calibration method
- Bmad, a simulation tool that allows full lattice control and the spin simulations
- Sufficient polarization levels under various orbital conditions are required for the energy calibration

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Bmad Home Page, https://www.classe.cornell.edu/bmad/

Spin Precession

The spin precession under electromagnetic field can be described by the Thomas-BMT equation

$$\frac{\mathrm{d}\hat{S}}{\mathrm{d}s} = \left(\vec{\Omega}^{c.o}(s) + \vec{\omega}^{s.b}(\vec{u}; s)\right) \times \hat{S}$$

$$\vec{u} \equiv (x, x', y, y', z, \delta)$$

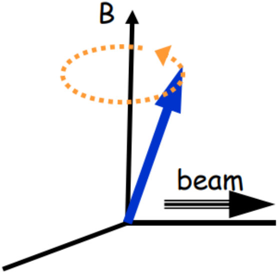

Figure from Bai, M. (2010, December). Polarized protons and siberian snakes.

Important Definitions about the Spin Quantities

- $\hat{n}_0(s)$
  - the periodic and stable spin direction on the closed orbit
  - the precession axis for spins on the closed orbit
- ν_0
  - closed orbit spin tune,
  - the number of spin processions around $\hat{n}_0$ per turn on the closed orbit
  - $\nu_0 = a\gamma$ in the perfectly aligned flat ring without solenoids
  - $\nu_0 \neq a\gamma$ in general
- $\hat{n}(\vec{u}; s)$
  - invariant spin *field*
  - the one-turn periodic unit vector that satisfies the T-BMT equation depending on $(\vec{u}; s)$
  - $\hat{n}(\vec{u}; s)=\hat{n}(\vec{u}; s + C)$

Polarization Build-Up

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- Sokolov-Ternov (ST) effect: spin-flip synchrotron radiation emission

$$P_{ST} = \frac{W_{\uparrow\downarrow} - W_{\downarrow\uparrow}}{W_{\uparrow\downarrow} + W_{\downarrow\uparrow}} \simeq 92.38\% \quad \text{and} \quad \tau_{ST}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_e |\rho|^3}$$

- Baier-Katkov-Strakhovenko (BKS) polarization level

$$\vec{P}_{BKS} = -\frac{8}{5\sqrt{3}} \hat{n}_0 \frac{\oint \mathrm{d}s \frac{\hat{n}_0(s) \cdot \hat{b}(s)}{|\rho(s)|^3}}{\oint \mathrm{d}s \frac{\left[1 - \frac{2}{9}(\hat{n}_0 \cdot \hat{s})^2\right]}{|\rho(s)|^3}}$$

$$\tau_{BKS}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_e} \frac{1}{C} \oint \mathrm{d}s \frac{\left[1 - \frac{2}{9}\left(\hat{n}_0 \cdot \hat{s}\right)^2\right]}{|\rho(s)|^3}$$

Polarization Build-Up with Radiative Depolarization

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- Radiative depolarization due to the spin diffusion
- ST effect + radiative depolarization → equilibrium polarization
- Derbenev–Kondratenko–Mane (DKM) formula when radiative depolarization is considered

$$P_{DK} = -\frac{8}{5\sqrt{3}} \times \frac{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \hat{b} \cdot \left(\hat{n} - \frac{\partial \hat{n}}{\partial \delta} \right) \right\rangle_s}{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \left(1 - \frac{2}{9} \left(\hat{n} \cdot \hat{s} \right)^2 + \frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2 \right) \right\rangle_s}$$

$$\tau_{DK}^{-1} = \tau_{BKS}^{-1} + \tau_{dep}^{-1}$$

$$\tau_{dep}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_e} \frac{1}{C} \oint \mathrm{d}s \left\langle \frac{\frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2}{|\rho(s)|^3} \right\rangle_s$$

- $\partial \hat{n} / \partial \delta$: the spin-orbit coupling function

Spin-Orbit Resonances

- The spin-orbit resonances

$$\nu_0 = m + m_x Q_x + m_y Q_y + m_z Q_z$$

  $|m_x| + |m_y| + |m_z| = 1$ first order spin-orbit resonances
- Away from resonance $\Rightarrow$ $\hat{n}(\vec{u}; s)$ almost aligned with $\hat{n}_0(s)$
- Near resonances $\Rightarrow$ $\hat{n}(\vec{u}; s)$ deviates from $\hat{n}_0(s)$ $\Rightarrow$ large $\partial\hat{n}/\partial\delta$ $\Rightarrow$ lower polarization

Spin Polarization Simulations in Bmad

- Sufficient polarization level should be available for the energy calibration using resonant depolarization
- Tao (Bmad)
  - the linear polarization calculation module in Bmad
  - check the influence of the 1st order spin-orbit resonances
- Long-Term Tracking
  - the nonlinear spin tracking module
  - check the influence of the higher order resonances, which may become significant at higher energies

Bmad Home Page, https://www.classe.cornell.edu/bmad/

Main Lattice Parameters

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Sequence 217 at Z energy is used in the simulations

Circumference (km)	97.756
Beam energy (GeV)	45.6
β_x^* (m)	0.15
β_y^* (mm)	0.8
ϵ_x (nm)	0.27
ϵ_y (pm)	1
Synchrotron tune Q_z	0.025
Horizontal tune Q_x	269.139
Vertical tune Q_y	269.219

Table: Main parameters at Z energy

FCC collaboration. (2019). FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2. European Physical Journal: Special Topics, 228(2), 261-623.

Effective Model

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- Use an effective model to simulate realistic orbital motions after lattice correction
- The errors are randomly distributed obeying the truncated Gaussian distributions (truncated at $2.5\,\sigma$)

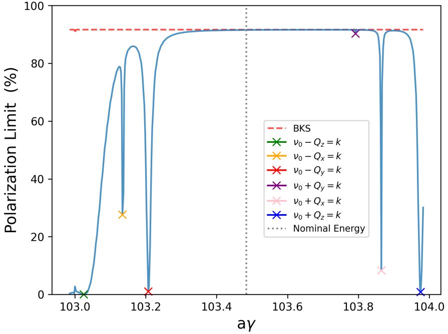
Type	$\sigma_{\Delta \mathrm{X}}$ (μm)	$\sigma_{\Delta \mathrm{Y}}$ (μm)	$\sigma_{\Delta \mathrm{S}}$ (μm)	$\sigma_{\Delta \mathrm{PSI}}$ (μrad)	$\sigma_{\Delta \mathrm{THETA}}$ (μrad)	$\sigma_{\Delta \mathrm{PHI}}$ (μrad)
Arc quadrupole	0.1	0.1	0.1	2	2	2
Arc sextupole	0.1	0.1	0.1	2	2	2
Dipoles	0.1	0.1	0.1	2	0	0
IR quadrupole	0.1	0.1	0.1	2	2	2
IR sextupole	0.1	0.1	0.1	2	2	2

Table: An effective model for the small error generation used in the spin-orbit simulations

Energy Scan in Tao

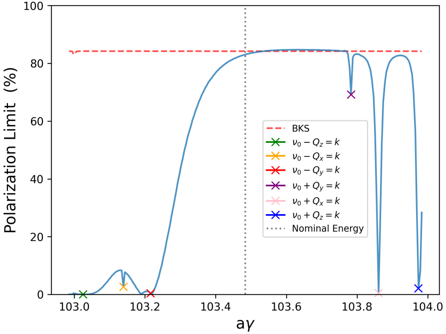
- Energy scans using two error seeds generated from the effective model
- Six first order spin-orbit resonances between two integer spin tunes

$(\Delta y)_{\text{rms}} = 43.7\,\mu\text{m}$



91.6% near nominal energy

$(\Delta y)_{\text{rms}} = 148\,\mu\text{m}$



84.6% near nominal energy

Robustness of the Error Generation Method

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- The effective model is an efficient way for the proceeding of the current spin polarization research
- 100 error seeds were generated to check the robustness of the effective model

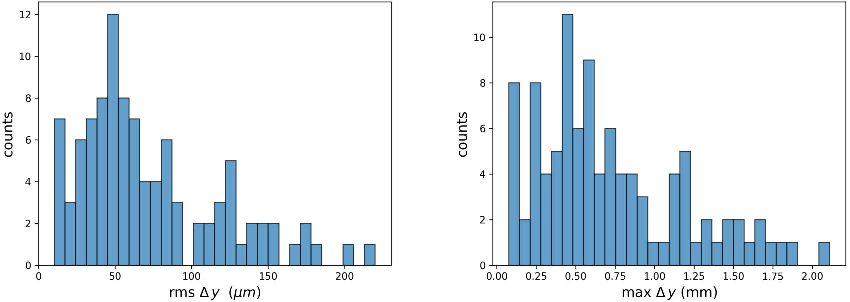

Figure: Distribution of the rms (left) and maximum (right) vertical orbits deviation of 100 produced errors

A more robust error generation method are needed in the future

Benchmark between Tao (Bmad) and SITF

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- SITF, the linear spin simulation module in SITROS
- Both SITF and Tao (Bmad) belong to SLIM family
- Underlying differences between two codes exist → check step by step

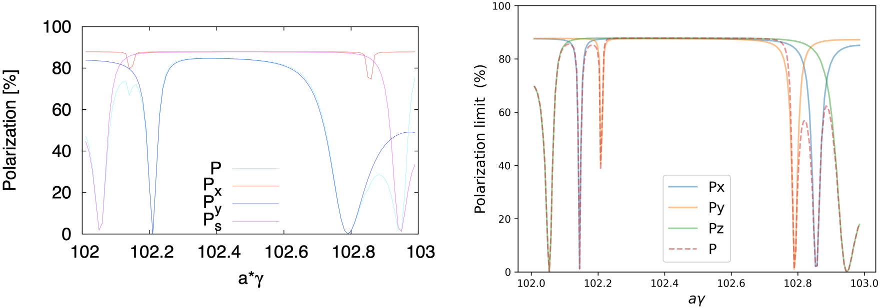

Figure: Energy scan using sequence version 213 seed 13 in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Parameter Comparisons using Clean Lattice

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- Clean lattice without misalignments at 45.6 GeV

	Q_x	Q_y	Q_z	x_{rms} [mm]	y_{rms} [mm]	β_x at IP.1 [m]	β_y at IP.1 [mm]
MADX	269.1354	269.2105	0.0247	0.027	0	0.1495	0.8
Tao	269.1354	269.2105	0.0247	0.027	0	0.1495	0.8
SITF	269.1354	269.2108	0.0247	0.027	0	0.1495	0.8

- Simple lattice with 10 nm x and y misalignments in one IR quadrupole (QC1L1.1)

	Q_x	Q_y	Q_z	x_{rms} [mm]	y_{rms} [mm]	β_x at IP.1 [m]	β_y at IP.1 [mm]
MADX	269.1354	269.2105	0.0247	0.027	0.004	0.1495	0.8
Tao	269.1354	269.2105	0.0247	0.027	0.004	0.1495	0.8
SITF	269.1354	269.2106	0.0247	0.027	0.004	0.1495	0.8

$\hat{n}_0$ Deviation Comparison

- $\hat{n}_0$, the central quantity for the spin polarization description
- Away from integer spin tune $\Rightarrow$ $\hat{n}_0$ almost aligned with the vertical
- Near integer spin tune $\Rightarrow$ $\hat{n}_0$ deviates from the vertical

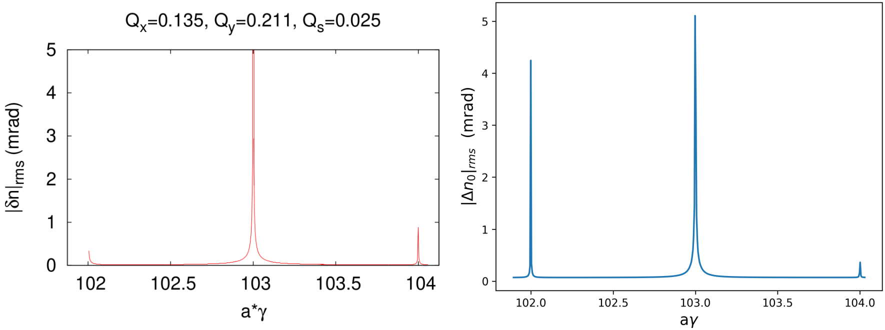


Figure: Variation of the rms $\hat{n}_0$ deviation from the vertical in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Benchmark between Tao, SITF and SLIM

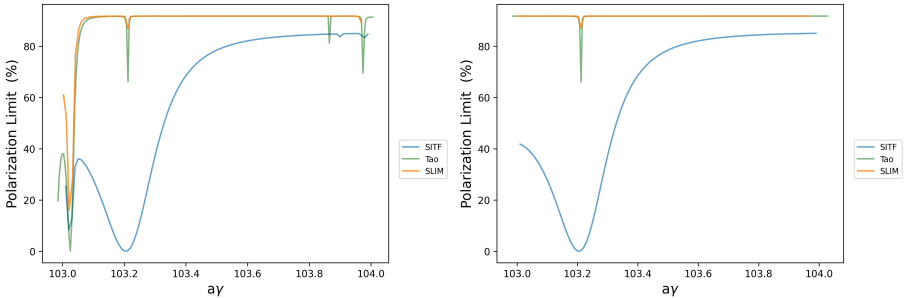


Figure: Energy scan of the equilibrium polarization (left) and the vertical mode polarization (right) by three codes

The difference may lie in the computation for the spin-orbit coupling function $\partial\hat{n}/\partial\delta$.

SITF and SLIM data are from Eliana Gianfelice-Wendt

Nonlinear Spin Tracking

- The higher order resonances may become prominent at high energies and affect the achievable polarization level
- Obtain τ_{dep} via Monte-Carlo spin tracking, while P_{BKS} and τ_{BKS} are computed at closed orbit

$$P(t) = P_{DK}\left[1 - e^{-t/\tau_{DK}}\right] + P_0 e^{-t/\tau_{DK}} \simeq P_0 e^{-t/\tau_{dep}}$$

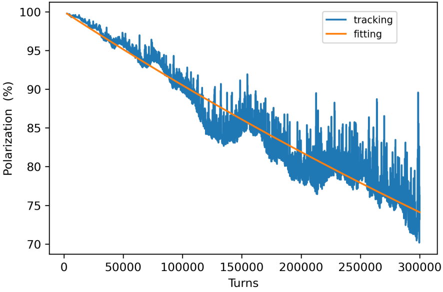
$$P_{eq} \simeq P_{BKS}\frac{\tau_{dep}}{\tau_{BKS} + \tau_{dep}}$$

Long-Term Tracking in Bmad

10 electrons, PTC

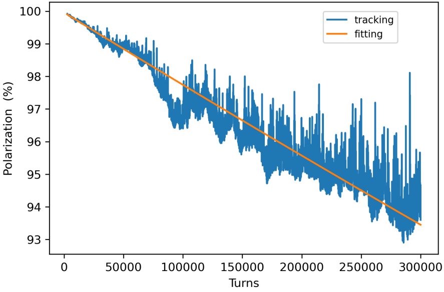
$\nu_0 = m + Q_y$

$P_{eq} = 0.03\%$



$\nu_0 = m + Q_y - Q_s$

$P_{eq} = 0.15\%$

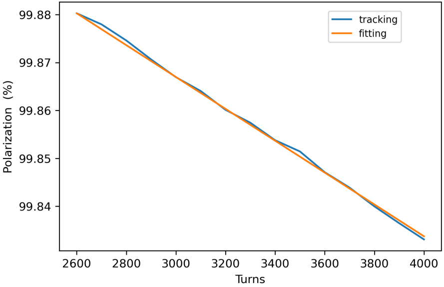


Large fluctuations, need more particles

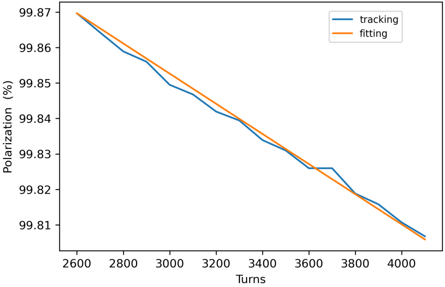
Long-Term Tracking in Bmad

500 electrons, PTC

$$\nu_0 = m + Q_y - Q_s$$
$$P_{eq} = 0.099\%$$



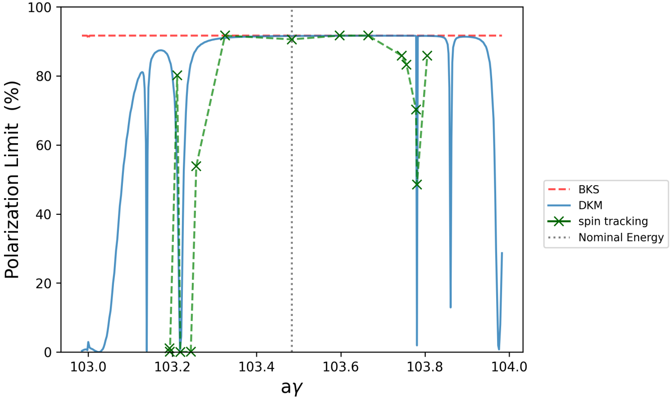
$$\nu_0 = m + Q_y + Q_s$$
$$P_{eq} = 0.077\%$$



Small fluctuations, but **time consuming**

Preliminary Results of Nonlinear Spin Tracking

100 particles, 10000 turns, PTC



Need over 1000 particles, over 10000 turns

Summary

- The exploration of the FCC-ee spin polarization simulations using Bmad shows promising results
- Linear polarization simulations offer a proof of concept, manifesting the influence of the 1st order resonances
- Benchmarks with SITROS in the linear spin calculation regime reveal underlying differences between codes
- First attempts at nonlinear spin trackings highlight the technical challenges associated with such simulations
- Further results will be presented in the EPOL meetings

Thank you!

Backup Slides

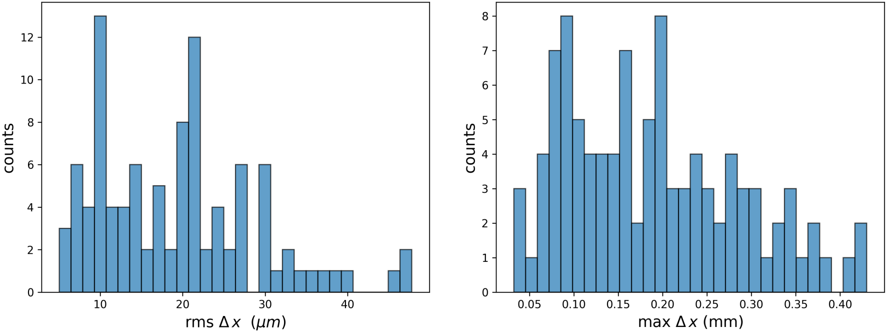


Figure: Distribution of the maximum horizontal (left) and vertical (right) orbits deviation of 100 produced errors

Backup Slides

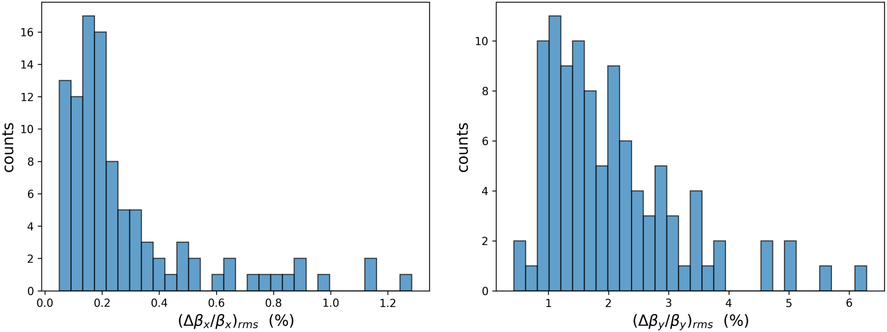


Figure: Distribution of the rms horizontal (left) and vertical (right) beta beating of 100 produced errors

A more robust error generation model is needed.

Backup Slides

EPFL

- Match the main parameters with the designed values
- Simplified matching: using the elements in RF section
- Optimized matching: adding BPMs, kickers and correctors

	Step order	"Data"	"Variables"
No err	1	x and z at IPs, Q_z	phi0, voltage
	2	β^*, Q_x, Q_y	correctors, RF Quad
	3	(recheck Data in step 1)	(phi0, voltage)
	4	save orbits at BPMs	
Add err	5	orbits at BPMs and IPs (higher weight)	kickers
	6	β^*, Q_x, Q_y	correctors, RF quad
	7	x and z at IPs, Q_z	phi0, voltage

Table: The optimized procedures for the parameter matching

Match the main parameters with the designed value

- Simplified matching: using the elements in RF section
- Optimized matching: adding BPMs, kickers and correctors

Attributes	Designed value	With RF Section	With Kickers, Correctors	Deviation (%)
β_x^* at IP.1/4 (m)	0.15	0.15	0.15	0
β_y^* at IP.1/4 (mm)	0.8	0.7977	0.79941	0.074
β_x^* at IP.2/3 (m)	0.15	0.15	0.15	0
β_y^* at IP.2/3 (mm)	0.8	0.79	0.79947	0.066
x at IP.1/4 (nm)	0	-180	10	N.A.
z at IP.1/4 (nm)	0	20	1.5	N.A.
x at IP.2/3 (nm)	0	-270	390	N.A.
z at IP.2/3 (nm)	0	-20	1.5	N.A.
Synchrotron tune Q_s	0.025	0.0247	0.025	0
Horizontal tune Q_x	269.139	269.139	269.139	0
Vertical tune Q_y	269.219	269.219003	269.219	0

Spin-Orbit Coupling Function Comparison

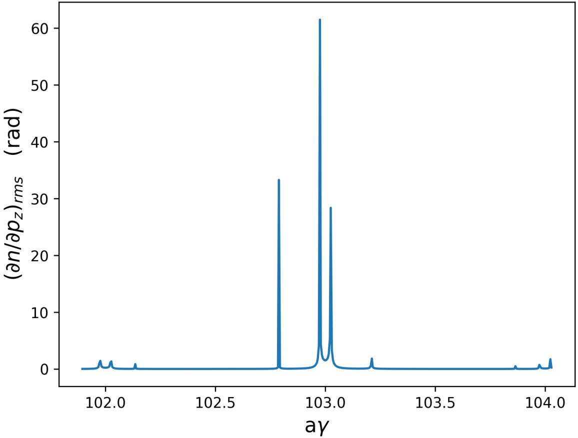


Figure: Variation of the rms spin-orbit coupling function $\partial\hat{n}/\partial\delta$ computed by Tao

Spin-Orbit Resonances

The ensemble average of the polarization

$$\left\langle \vec{P}_{DK} \right\rangle_{ens}(s) = P_{DK} \left\langle \hat{n} \right\rangle_{s}$$

Energy Scan Comparison with Simple Lattice

- Main difference comes from the vertical mode polarization

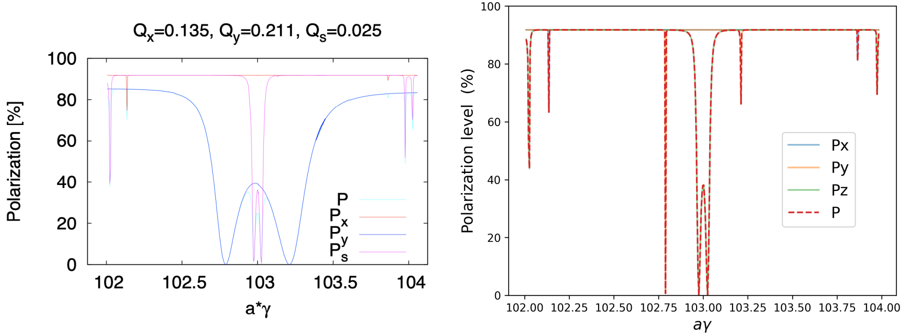


Figure: Energy scans using the simple lattice with one misalignment in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt